Monotonicity and non-monotonicity at PR-STV elections

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Presented at annual conference of EPOP, University of Lancaster, 13 September 2013

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Abstract
This paper explores an aspect of the STV electoral system (both PR-STV and the alternative vote), namely its property of being unable to guarantee the principle of monotonicity, which is that additional support cannot possibly damage a candidate. Put simply, this means that in an STV election it is possible to encounter the paradoxical situation where additional support can damage a candidate and can in fact result in his/her non-election, whereas without that additional support he or she would have been elected. STV shares this vulnerability with all electoral systems based on runoffs or eliminations, the best known of which is the two-round system used to elect presidents in many countries. As a result of this theoretical vulnerability, ie that STV cannot absolutely guarantee the principle of monotonicity, STV tends to be evaluated unfavourably by social choice theorists, whereas students of electoral systems tend to dismiss this as an arcane possibility that could arise in theory but probably never occurs in practice. So far, empirical evidence as to how frequently this arises has been lacking. This paper examines the empirical evidence from elections in both the Republic of Ireland and Northern Ireland from 1922 to 2011 in order to establish the frequency with which non-monotonicity actually occurs.

Monotonicity is the property that additional support cannot be damaging to a candidate.[1] It means that we cannot possibly arrive at a situation where we can reflect that a particular candidate was not elected but, had she won less support, she would have been elected – or that a candidate who was elected would not have won had she received more support. While the concept of non-monotonicity is familiar to students of social choice theory, it is often initially baffling to those who have never previously encountered it and who are mystified as to how it could possibly happen that additional support might have a negative impact on a candidate's fortunes. A common reaction is to assume that only a social choice function that has been deliberately designed to produce irrational outcomes could permit non-monotonicity to occur, and it can be surprising to discover that some widely employed means of filling seats, such as run-off methods and the single transferable vote (STV), do not guarantee monotonicity. In this paper, having reviewed some of the abstract literature on monotonicity, we seek to assess how often in practice it is violated in

1. Choice might, of course, refer to a policy option or a variety of other possibilities; for simplicity's sake, we assume in this article that our social choice function is being employed to choose among candidates.

elections held under PR-STV (proportional representation by the single transferable vote).

1. Non-monotonicity in action

Non-monotonicity is possible under electoral systems such as PR-STV, and run-off / elimination methods in general, because these operate by the elimination of the lowest-placed candidate(s), and thus the outcome of the election may depend on the order of elimination. A candidate may be able to increase the probability of her election by diverting some of her own support to a second candidate in order to ensure that a third candidate, whose voters rank her above the second candidate, is eliminated.

The simplest example is supplied by Doron and Kronick (1977: 309) and is illustrated in Figure 1.

Figure 1: simple example of non-monotonicity

As a demonstration that non-monotonicity is possible under STV, consider two cases, illustrated below, each involving 17 voters and 1 seat, so the quota is 9.

In case 1:

6 voters give their first preference to A, their second to B and their third to C.
6 voters give their first preference to B, their second to C and their third to A.
5 voters give their first preference to C, their second to A and their third to B.

C is eliminated, and A is elected as he receives all 5 votes transferred from C.

Case 2 is the same as case 1, except that two voters switch from B to A (so A has 8 first preferences and B has 4). Now B finishes bottom and is eliminated, and C will be elected as she receives all 4 votes transferred from B. As a result of A's acquisition of extra support, he loses the election.

Case 1	*First preferences*	*Second stage: Transfer of C's votes*	
A	6	+ 5	**11**
B	6	+ 0	6
C	5	– 5	
Total	17		17

Case 2	*First preferences*	*Second stage: Transfer of B's votes*	
A	8	+ 0	8
B	4	– 4	
C	5	+ 4	**9**
Total	17		17

In case 1, C, as the candidate with fewest votes, is eliminated, and all of his votes transfer to A, who as a result is elected. In case 2, the situation is the same except that two voters have switched from B to A. A is now indisputably more popular than in case 1 but fails to secure election. The fact that she has gained support from B means that B, rather than C as in case 1, is eliminated, and since B's supporters all have C as their second choice, C is now elected. Hence, it seems, in case 2 A has suffered from being 'too popular' – if she had not won additional support from B, she would have been elected.

Cases 1 and 2 might be, for example, parallel and simultaneous contests, in which case it seems anomalous that the candidate of party A in case 1 is elected while the more popular candidate in case 2 is not. They might, alternatively, be successive iterations of the same contest, in which case the injustice seems even greater. Candidate A, due to her performance during her term of office, has increased her support – and as a result, it seems, has lost her seat.

To give a less abstract example, consider the following example, this time held under the two-round system, which is the most common method by which presidents are elected (Blais et al 1997). Here, voters do not rank all candidates; rather, the second round is contested by the two candidates who won most votes in the first round. Figure 2 shows two scenarios among a group of 100 voters who vote entirely along Downsian lines, arranging the candidates in order of preference according to how the candidates are to their own position.

Figure 2: hypothetical case of non-monotonicity

Scenario 1	First round	Second round
Centre-left candidate	39	54
Centre-right candidate	30	
Far-right candidate	31	46

Scenario 2	First round	
Centre-left candidate	41	41
Centre-right candidate	30	59
Far-right candidate	29	

In scenario 1, the centre-left and far-right candidates, being the first- and second-placed candidates, proceed to the second round, while the centre-right candidate is eliminated. In the second round, supporters of the centre-right candidate either do not turn out to vote or, if they do vote, are equally likely to vote for each of the two

remaining candidates, so the centre-left candidate retains her lead over the far-right candidate. In scenario 2, the centre-left candidate has gained 2 votes at the expense of the far-right candidate. Now the far-right candidate is eliminated, and on the second round his supporters are much more likely to vote for the centre-right candidate than for the centre-left candidate, taking the centre-right candidate to victory. In scenario 2, it seems, the centre-left candidate has suffered from her gain in popularity; if she had not received the additional support, or if she had encouraged two of her supporters to vote tactically for the far-right candidate, she would have been elected. The French presidential election of 2002, when in the first round the centre-right candidate Jacques Chirac had 20 per cent of the votes, the far-right Jean-Marie Le Pen 17 per cent, and the centre-left Lionel Jospin 16 per cent, bears some resemblance to this case.

Essentially the same situation can be created under PR-STV, since the principle is exactly the same under STV in single-member constituencies, known as the alternative vote (AV), and under PR-STV, employed in multi-member constituencies. Violation of monotonicity is more than the simple observation that in one constituency a candidate with x per cent of the votes is elected while in another one a candidate with $x+y$ per cent is defeated. Clearly, the specific distribution of votes among candidates or parties, and the extent of fragmentation, makes this perfectly possible – for example, a candidate with 48 per cent is defeated when another candidate wins 52 per cent of the votes, while one with 38 per cent in another contest is elected because that figure represents a plurality of support. The particular characteristic of non-monotonicity is that a candidate has failed to be elected because she has gained additional support, and that we can construct an alternative scenario, otherwise identical, in which she loses support and as a result secures election.

Even if the concept of monotonicity is not exactly part of everyday discourse, the basic principle is clearly understood by many voters involved in small-electorate run-off contests, such as intra-party elections. Voters want their own candidate to make it through to the final stage and, moreover, they want him or her to face as unpopular a candidate as possible at that stage. In particular, if there is a Condorcet winner, supporters of other candidates know that unless this person is eliminated before the final run-off, their own candidate cannot win because, by definition, the Condorcet winner will prevail in a straight fight against any other candidate. Candidates other than the Condorcet winner need to ensure the elimination of the Condorcet winner (if there is one) at a pre-final stage if they are to have any hope of winning.

In the British Conservative Party leadership in the autumn of 2005, in which MPs selected two candidates (by successive eliminations of the bottom-placed candidate) to go to a ballot of the full party membership, it was rumoured that some supporters of the favourite, David Cameron, confident that Cameron had enough votes to be sure of making it to the final stage, were switching their votes to the most

right-wing candidate, Liam Fox, to make it even more certain that Cameron would win the run-off among the members. (On the second round, Cameron had 90 votes, David Davis had 57 and Liam Fox 51, so if seven of Cameron's 90 supporters had switched to Fox then Cameron would indeed have had his supposedly weakest rival as his opponent in the decisive stage.) Whether this actually happened is beside the point – the point is that MPs readily grasped the notion that their favourite candidate might benefit from receiving less support. Similarly, the possibility of non-monotonicity was among the reasons why Labour's Plant Report of the mid-1990s assessed PR-STV unfavourably.

2. How concerned should we be about the possibility of non-monotonicity?

Among some social choice theorists, the vulnerability of runoff methods to non-monotonicity suffices to earn these low approval ratings. Doron and Kronick (1977: 307) maintain that:

> A social choice function that did not fulfill this condition [monotonicity] – a function that permitted an increased vote for a candidate to cause a decline in that candidate's rank in the social ordering – would probably strike most of us as a rather absurd, even perverse, method of arriving at a social choice.

In the view of Brams and Fishburn (1984: 147, 151):

> Monotonicity may be the most basic of the conditions in requiring that more first-placed votes can never hurt a candidate, which to us seems a sine qua non of a democratic voting procedure … The fact that more first-place votes can hurt, rather than help, a candidate under STV violates what, in our opinion, is a fundamental democratic ethic.

Elsewhere, the same authors describe the possibility of non-monotonicity as one of the 'problems and paradoxes that plague preferential voting' (Fishburn and Brams 1983: 211).

For Nurmi (2004: 47):

> Of the numerous desiderata one can hope the voting system to satisfy monotonicity seems perhaps the most obvious. If the point of conducting an election is to go to the people, then it would seem self-evident that additional support should not diminish an alternative's likelihood of being adopted.

For Dummett (1997: 103) the inability to guarantee monotonicity is 'a grave defect' of both the alternative vote and PR-STV. Riker (1984: 106) declares that 'to my mind, the failure to ensure monotonicity … is the *worst* possible sin an electoral system can commit', while acknowledging that 'other people have other perspectives'. Doron and Kronick (1977: 310) conclude their article by conjuring up this nightmare scenario:

> Most voters would probably be alienated and outraged upon hearing the hypothetical (but theoretically possible) election night report: "Mr. O'Grady did not obtain a seat in today's election, but if 5,000 of his supporters had voted for him in second place instead of first place, he would have won!"

In contrast, students of electoral systems from a political science perspective, along with some social choice theorists, tend to be more phlegmatic about the possibility of non-monotonicity, acknowledging that it is a theoretical possibility but suggesting that we should not lose sleep over it. For one thing, they suggest, it is hardly a major consideration in the evaluation of electoral systems – there are more important things to worry about when assessing a method of converting votes into seats. Reference is frequently made to Arrow's impossibility theorem or to the work of Gibbard (1973) and

Satterthwaite (1975) which, in homely terms, demonstrate that even starting with a small set of clearly desirable criteria it can be shown that no electoral system satisfies these, and that no electoral system meeting a set of desirable criteria can guarantee that no voter ever has an incentive to vote strategically as opposed to sincerely.

Tideman, for example, maintains (2006: 195) that 'The lack of monotonicity in the alternative vote should probably not be considered a serious defect since many competing rules lack this property as well'. Bartholdi and Orlin (2003: 14, 19) conclude

> even if voters know that non-monotonicity is possible, its distortions, since hidden, might be perceived as "random" ... For STV elections we have established the formal difficulty of recognizing when strategic voting is possible and of recognizing instances of non-monotonicity. Most immediately, this suggests that STV might rise somewhat in our estimation, since two of its weaknesses—susceptibility to strategic voting and non-monotonicity—are perhaps less threatening than previously thought.

Austen-Smith and Banks (1991: 536) argue that as a normative critique 'the nonmonotonicity of electoral rules and systems has no bite' and that the supposed problem of non-monotonicity is in reality a 'non-issue'. Others maintain that, even if we can construct artificial scenarios such as those above in which the principle of monotonicity is violated, such violation will surely occur very rarely, if ever, in practice. Bowler and Grofman outline the sheer impossibility of voters having knowledge about the preference orderings of all other voters and developing not only strategies to maximise the impact of their own vote but also the necessary 'counterstrategies against all the other voters who would similarly be calculating what will happen if they altered their preference ordering over the parties'. In consequence, they argue, 'STV generally presents such difficult calculations to voters seeking to behave tactically that it seems to make little sense to do anything other than register a sincere preference for the party that they would most like to see win' (Bowler and Grofman 2000: 268). Still, as with Bartholdi and Orlin's suggestion that non-monotonicity can be seen as a random effect, voters might still be disturbed by the impact of non-monotonicity even if they cannot take advantage of it through strategic voting. The same would be true of some kind of random 'jittering' of the votes that took place within the black box of the vote-counting process, but we would hardly regard that as an acceptable feature of an electoral system.

The contending assessments of the inherent seriousness of the possibility of non-monotonicity show that no resolution to this can be found through argument in the abstract. This suggests that the frequency with which non-monotonic outcomes are likely to occur should enter the equation when we come to assess its significance. If, in real life, the probability of their occurring is remote, then a shrug of the shoulders would be an understandable response. It would make no more sense to abandon an otherwise meritorious electoral system

on the basis of its vulnerability to non-monotonicity than it would be to cease all outdoor activity because of the risk of being struck by an asteroid. If, in contrast, such outcomes occur with non-negligible frequency, then perhaps we should be more concerned. In the words of Nurmi (1996: 48): 'It is worth emphasising that the result showing that STV is non-monotonic tells us nothing about the empirical frequency of situations where an elected candidate would not be elected if he / she had some more support'.

Some previous attempts have been made to estimate the frequency with which non-monotonicity is likely to occur. Allard (1995: 49) calculates the figure at 0.00028, meaning that if the UK were divided into 138 4-seat constituencies, with elections every four years, 'we would expect in the whole country less than one incidence every century of monotonicity failure under STV'. Dummett (1997: 103) describes this as a 'surely ludicrous underestimate' and, while acknowledging that 'only a mass of empirical data could settle the question', suggests that 'a rough calculation suggests that, with three candidates under AV, 2% would be a conservative estimate'. Lepelley et al, on the basis of a more sophisticated estimation technique, concur, estimating that in a large electorate with three candidates and single-peaked preferences, monotonicity will be violated in 1.74 per cent of cases (Lepelley et al 1996: 143).

The former Chief Electoral Officer for Northern Ireland, where PR-STV has been used to elect local councils and a regional assembly since the early 1970s, inclines more towards Allard's view, stating that

> the various records examined do not reveal any hint whatsoever of the operation in practice of the paradox professors Dummett and Riker have assigned to STV – namely that a vote intended in a candidate's favour has actually told against that candidate ... The experience of the use of STV in Northern Ireland over the past 22 years, involving a range of election types and sizes, reveals no evidence to support *in practice* the lack of monotonicity (Bradley 1995: 47).

However, before we take this as the definitive word on the subject, we need to bear in mind that non-monotonicity does not, so to speak, leap out of an election result waving a flag and shouting for attention. In order to be sure that non-monotonic outcomes have not occurred, we have to be clear about what we are looking for.

3. What are we looking for when we look for cases of non-monotonicity?

The results reported later in this paper derive from examination of actual election results, scrutinised in an attempt to identify cases where there is reason to believe that a candidate might have been elected if some of his/her support can gone to some other candidate.

It would help to have some outline of the general case. This involves 3 candidates, whom we will call FR (front runner, the leading candidate), A and B. B's supporters prefer A to FR, while A's supporters are, broadly, collectively indifferent as between FR and B, or might in fact prefer FR to B. The preferences of FR's supporters are irrelevant, as FR is in no danger of being eliminated from the count. Thus, in a head-to-head contest, A would beat FR, while FR would beat B. Whether A would beat B, or vice versa, is irrelevant, again because FR will not be eliminated from the count, so A is not necessarily a Condorcet winner.

Then:

if B is first eliminated, A will be elected rather than FR;
if A is first eliminated, FR will be elected rather than B.

Scenario 1	Scenario 2
FR	FR
A	B
B	A

Scenario 1 is dangerous for FR, because despite his lead over the other candidates, the known leaning of B's supporters towards A means that upon B's elimination, A might receive sufficient additional support to overtake FR. Scenario 2 is safe for FR, because when A is eliminated her transfers will not disturb FR's lead over B. Hence, if FR knew that Scenario 1 was set to occur, with A only slightly ahead of B and FR well ahead of them both, it would be in FR's interest to channel some of his support to B, thus bringing about scenario 2.

Thus we are looking for situations where:

FR > A > B, and
(FR – A) > (A – B).

With these conditions, the number of votes that FR 'loses' to B will suffice to bring B above A (thus resulting in A's elimination and FR's election) but will not be so many that B also overtakes FR – which by definition would mean that FR now becomes the trailing candidate and is eliminated. Note that FR must not merely 'lose' votes but must lose them specifically to B.

There are of course any number of such patterns involving three candidates. The relevant ones are cases where there is some reason to believe that

B's next preferences would go predominantly to A, while
A's next preferences would either go predominantly to FR or do nothing to alter the existing lead of FR over B.

If we knew that all candidates could be placed on the sole dimension of competition, and all voters voted for candidates on the basis of their position on this dimension, then we would be looking for a situation where A is located between FR and B, and is no nearer to B than to FR. Thus A's elimination will either favour FR *vis-à-vis* B or at least will leave FR's existing lead over B undisturbed.

So, in general, we are looking for cases where the centre party (A) is second, the party to which it is ideologically closer (FR) is leading, and the party from which it is further (B) is trailing. FR and B are located on opposite sides of A.

B——————A————FR

FR will be elected if A is eliminated but A will be elected if B is eliminated.

The key situations to look for as examples of non-monotonicity in operation, then, are ones where the eliminated candidate (B) is not far behind the candidate above her (A), where each candidate has sufficient votes to make a difference to the overall outcome, where A is ultimately elected partly as a result of transfers received from B, and where there is reason to believe that A's later preferences would not have benefited B compared with a candidate (FR) who at that stage was ahead of both A and B but who was ultimately not elected.

Cases where FR is elected due to the elimination of A are also examples, in that FR can be seen to have benefited from not having taken some additional support from B – if he had done so, that would have cost him the election.

In other words, when A and B are close, we can see the danger of non-monotonicity regardless of the actual outcome, because either way we can say that FR would have been elected had he lost some support to B, or would not have been elected had he won some support from B.

4. Data

The data come from Dáil elections and by-elections held under PR-STV and the alternative vote between 1922–2011, and from elections and (where held under the alternative vote) by-elections to the Northern Ireland parliament (known colloquially as 'Stormont') 1921–65, the Northern Ireland Assembly 1973–75, 1982, and 1998–2011, and the Northern Ireland Constitutional Convention 1975. From 1929 to 1965 inclusive, only one constituency at Stormont, the 4-seat Queen's University constituency, returned members by PR-STV, the others employing single-member plurality.

In all, there are 1,064 cases (constituencies) of Dáil elections held under PR-STV, 127 Dáil by-elections held under the alternative vote, and 135 Northern Ireland cases held under PR-STV, making a total of 1,326.

The Irish party system is not entirely one-dimensional and, in particular, the left–right dimension has always been relatively weak by European standards, both in structuring the party system and in underpinning voting behaviour (Marsh 2010: 177–9). In broad terms, the traditional largest two parties, Fianna Fáil and Fine Gael, could be classified as centre or centre-right parties, as could the now-departed Progressive Democrats, with Labour and a number of smaller parties on the left.

In terms of Irish politics, north and south, some possibilities matching the scenarios outlined above (ie in which B's next preferences would go predominantly to A, while A's next preferences would either go predominantly to FR or do nothing to alter the existing lead of FR over B) are:

B	A	FR
Democratic Left	Labour	Fine Gael
Workers' Party	Labour	Fine Gael
Sinn Féin	Labour	Fine Gael
Clann na Poblachta	Labour	Fine Gael
any party	Ind	any party (usually)
Fine Gael	PDs	Fianna Fáil (1997 and 2002)
nationalist party	Alliance	unionist party
unionist party	Alliance	nationalist party

There is also the possibility of there being an impact on the specific candidates who are elected even if not on the distribution between parties, for example if the relevant dimension is geography rather than ideology.

In order to be certain whether a case of non-monotonicity has arisen, we would like to be able to examine all votes to determine what would have transpired in the counter-factual situation. This is not possible. Votes are not archived, still less made available for

public scrutiny. Consequently we have to make probabilistic judgements based on our knowledge of voters' preferences as revealed by transfer patterns. In a situation of imperfect information we can, and indeed have no option but to, make inferences based on national patterns or on what happened in comparable cases in adjacent constituencies, for example, but we are making best estimates rather than being able to adjudge with certainty. These estimates are based on our information about observed transfer patterns: when a candidate is eliminated from the count, or has surplus votes distributed, we can observe the pattern of next preferences among the supporters of each party (for details of the way votes are counted under PR-STV see Sinnott 2010: 117–24). For instance, regarding the example from the Cork North-Central constituency in 2011 discussed in the next section, we know from nationwide transfer patterns that when votes were transferred from one Fine Gael candidate in a situation where another Fine Gael candidate was available to receive transfers, 68 per cent of these votes passed to another Fine Gael candidate (Gallagher 2011: 163). Hence, we can be entirely confident that if Murphy of FG had been eliminated in the counterfactual scenario, the transfer of his votes would have substantially increased his running mate Burton's lead over Gilroy of Labour. The same logic is applied to the assessment of the outcomes in the other counter-factuals.

5. Non-monotonicity in action: some examples

Before discussing the aggregate results, it would help to present a few examples of non-monotonicity in action.

First, in Cork West in 1957 the front runner was a Fine Gael candidate, who was trailed by an Independent and by a Fianna Fáil candidate. The Independent, as is usually the case in a group of three candidates, the other two of whom are from mutually antagonistic parties, was clearly a Condorcet winner. Hence, if Collins faced the Independent Wycherley on the final count he was bound to lose, whereas if Wycherley were eliminated Collins was certain to retain his lead over the FF candidate Finn. (In this particular case there is every reason to believe that the elimination of Wycherley would in fact have led to Collins's extending his lead over Finn, as Wycherley inclined towards the second Inter-Party government of 1954–57.) In the event, Finn was eliminated, and his transfers, many of them passing through the surpluses of other elected candidates, took Wycherley into the last seat, finishing 52 votes ahead of Collins. However, if Collins had siphoned or channelled a number of votes (at least 542)[2] to Finn, then Collins would have been elected upon Wycherley's elimination. To express this another way, if Scenario 2 had been about to occur, and Collins then undertook intensive campaign activity which resulted in his gaining 542 votes from Finn, then we can say that this additional support cost him the election.

Cork West, 1957

Scenario 1 – actual result

FR	Collins	FG	5,869
A	Wycherley	Ind	4,666
B	Finn	FF	4,125

Elimination of Finn leads to election of Wycherley, Collins not elected.

Scenario 2 – hypothetical result, if Collins had lost or 'given' 542 votes to Finn:

FR	Collins	FG	5,327
B	Finn	FF	4,667
A	Wycherley	Ind	4,666

Now, Wycherley is eliminated, Collins retains or increases lead over Finn and is elected.

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2. It is not possible to state precisely the maximum number of votes that Collins could safely have channelled to Finn. Had he channelled 871 votes, this would have kept him ahead of Finn (4,998 to 4,996) and hence he would have been elected on the assumption that Wycherley's transfers did not favour Finn over Collins. Given that Wycherley's transfers would probably have in fact favoured Collins over Finn, it may be that even if Collins had lost or channelled 1,202 votes to Finn, leaving him just 1 vote ahead of Wycherley, he would still have received sufficient transfers from Wycherley's elimination to overtake Finn and win a seat.

In the second example, Cork North-Central in 2011, the principle is the same. Murphy was a Condorcet winner, being from the same party as Burton and from the same part of the constituency as Gilroy. If Gilroy was eliminated then Murphy was certain to beat the front-runner Burton (as happened in practice), whereas if Burton could have siphoned 64 votes to Gilroy then this drop in support would have secured his election by leading to the elimination of Murphy.

Cork NC, 2011

Scenario 1 – actual result

FR	Burton	FG	7,816
A	Murphy	FG	7,356
B	Gilroy	Lab	7,293

Elimination of Gilroy leads to election of Murphy, Burton not elected.

Scenario 2 – hypothetical result, if Burton had lost or 'given' 64 votes to Gilroy:

FR	Burton	FG	7,752
B	Gilroy	Lab	7,357
A	Murphy	FG	7,356

Now, Murphy is eliminated, Burton extends lead over Gilroy and is elected.

Similarly, in Armagh in 1982 if News had been eliminated then his transfers would have left his running mate O'Hanlon well ahead of French, whereas it turned out that French's supporters had a strong preference for News rather than O'Hanlon, so French's elimination deprived O'Hanlon of election. Again, if O'Hanlon did something during the campaign that caused 287 of French's erstwhile supporters to desert him in favour of O'Hanlon, then this apparently successful piece of campaigning by O'Hanlon actually cost him the election.

Armagh, 1982

Scenario 1 – actual result

FR	O'Hanlon	SDLP	4,801
A	News	SDLP	3,781
B	French	WP	3,495

Elimination of French leads to election of News, O'Hanlon not elected.

Scenario 2 – hypothetical result, if O'Hanlon had lost or 'given' 287 votes to French:

FR	O'Hanlon	SDLP	4,514
B	French	WP	3,782
A	News	SDLP	3,781

Now, News is eliminated, O'Hanlon extends lead over French and is elected.

6. Frequency of occurrence of non-monotonicity

In all, 17 cases of non-monotonicity have been identified south of the border (four of these being intra-party cases), and three cases in the north (one of these being intra-party). The Appendix gives a full list of the cases identified. Table 1 shows that the total number amounts to around 1.5 per cent of all cases, a figure close to the estimate of Lepelley et al cited above.

Table 1: Occurrences of non-monotonicity in Ireland 1922–2011

Elections	*Total cases where could have arisen*	*Inter-party non-monotonicity*	*Intra-party non-monotonicity*	*%*
Rep Ireland	1,191	13	4	1.4
N Ireland	135	2	1	2.2
Ireland total	1,326	15	5	1.5

7. Conclusion

The record of elections held under PR-STV and AV in Ireland between 1922 and 2011 has been examined, and it has been found that non-monotonicity has arisen in around 1.5 per cent of all cases. How concerned should we be by this? If a failure to guarantee monotonicity in all circumstances is a cardinal shortcoming in an electoral system, as several writers quoted above maintain, then even 20 cases out of a possible 1,326 is highly disturbing and this evidence disproves the contention that non-monotonicity is something that could happen in theory but in practice never does. On the other hand, if guaranteeing monotonicity is regarded as merely a desirable, as opposed to an essential feature of an electoral system, then the fact that it does not arise in 98.5 per cent of cases would lead to our taking a relaxed view of the phenomenon.

Appendix: identified cases of non-monotonicity in Ireland 1922–2011

Election	*Constituency*	*Front runner*	*A (elected)*	*B (eliminated)*
1927 Jun	Limerick	Nolan CG	Clancy Lab	Bourke FF
1947 (B)	Tipperary	Hayes FF	Kinane CPob	Ryan FG
1948	Louth	Walsh FF	Connolly Lab	Roe FG
1948	Wicklow	Sweetman FG	Cogan Ind	Byrne FF
1951	Dublin SW	O'Daly FF	MacBride CPob	O'Higgins FG
1951	Wicklow	Ledwidge FF	Cogan Ind	Deering FG
1957	Cork W	Collins FG	Wycherley Ind	Finn FF
1957	*Sligo–Leitrim	Roddy FG	Rogers FG	McDonagh FF
1987	Dublin NE	Haughey FF	McCartan WP	Holman PD
1992	Dublin NE	Cosgrave FG	Broughan Lab	McCartan DL
1992	*Laois–Offaly	Enright FG	Flanagan FG	Moloney FF
1997	Limerick E	Jackman FG	Kemmy Lab	Ryan DL
2002	*Cork E	Bradford FG	Stanton FG	Mulvihill Lab
2002	Longford–Roscommon	Kelly FF	Sexton PD	Belton FG
2007	Donegal NE	Mac Lochlainn SF	McDaid FF	Keaveney FF
2007	Dun Laoghaire	Boyd Barrett PBPA	Cuffe Grn	Regan FG
2011	*Cork NC	Burton FG	Murphy FG	Gilroy Lab
Northern Ireland				
1982	*Armagh	O'Hanlon SDLP	News SDLP	French WP
1998	Antrim E	McKee DUP	Hutchinson UKUP	Steele UUP
2003	Strangford	Boyle SDLP	McCarthy All	Little UUP

* denotes intra-party case.

Note: The 1992 Laois–Offaly case was particularly complicated, with a fourth candidate being involved; had Enright (FG) been able to channel some support to Moloney (FF), then this fourth candidate (Connolly FF) would have been eliminated and Enright would almost certainly have overtaken Flanagan and won a seat.

Much the same applies to the 2002 Cork E case, with Bradford (FG) having sufficient spare votes to have secured the elimination of Sherlock (Lab), which would have led to the election of Mulvihill and Bradford, rather than Sherlock and Stanton as actually happened.

Likewise, in Donegal NE in 2007, a fourth candidate was involved. A small shift of votes from Mac Lochlainn (SF) to Keaveney (FF) would have led to the elimination of Blaney (FF), and it is likely that Mac Lochlainn and McDaid would have been elected rather than, as actually happened, Blaney and McDaid.

In Antrim E in 1998, too, a fourth candidate (O'Connor SDLP) was involved.

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