“Optimizing Slot Allocation Decisions”

Konstantinos G. Zografos
Lancaster University Management School
Centre for Transport and Logistics (CENTRAL)
Department of Management Science

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Joint work with:

Dr Yu Jiang (Oxford University – Formerly with Lancaster University/CENTRAL)
Dr Konstantinos Androutsopoulos (Athens University of Economics and Business)
Dr Michael Madas (University of Macedonia)
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Objectives

1. Propose new airport slot-scheduling models that consider:
   - schedule efficiency
   - schedule fairness

2. Provide insights from the application of the proposed models to a medium size coordinated airport
Slot Modelling Concepts and Objectives (1/5)
The Concept of Slot Displacement
Slot Scheduling concepts and objectives (2/5)

Efficiency Related Slot Scheduling Objectives

1. Minimize total displacement
2. Minimize total weighted displacement
3. Minimize maximum displacement
4. Minimize violated slot assignments
Slot Modelling Concepts and Objectives (3/5)

Notation

- $T$: Set of coordination time intervals
- $D$: Set of calendar days
- $M$: Set of requested series of movements
- $P$: Set of movement pairs
- $C$: Set of airport capacity constraints
- $T_c$: Set of consecutive coordination time intervals
- $x_m^t \in \{0, 1\}$: It takes value 1 if movement $m$ ($a$ or $d$) is allocated to interval $t$
- $f_m^t$: Cost of allocating movement $m$ to coordination time interval $t$, calculated as $|t - t_m|$, where $t_m$ is the time interval originally requested for $m$
- $t_{ad}$: Minimum turnaround time corresponding to movement pair $a,d$
- $u_{ct}$: Capacity of constraint $c$ for day $D$ and coordination time interval $t$
- $a_m^d \in \{0, 1\}$: It takes value 1 if movement $m$ operates on day $D$
- $b_{mc} \in \{0, 1\}$: It takes value 1 if movement $m$ is an arrival(departure) and the constraint $c$ applies to arrivals(departures) only or total movements

It is postulated that the proportion of displacement received by an airline should be proportional to the requested number of slots.

$$\rho_a = \frac{\sum_{m \in M_a} \sum_{t \in T} |t - t_m| x_m^t}{\sum_{m \in M} \sum_{t \in T} |t - t_m| x_m^t}$$

- $\rho_a = 1.0$ airplane a is fairly treated
- $\rho_a < 1.0$ airplane a is a favoured airline
- $\rho_a > 1.0$ airplane a is a disfavoured airline
Minimize maximum deviation from the absolute fairness value (MMA):

\[
\min F_{\text{MMA}} = \max_{a \in A} \left| \rho_a - 1 \right|.
\]

Minimize maximum deviation from average fairness (MMR)

\[
\min F_{\text{MMR}} = \max_{a \in A} \left| \rho_a - \frac{\sum_{a' \in A} \rho_{a'}}{|A|} \right|.
\]
Indicative Formulations (1/5)

Single Airport Slot Allocation Model

\[
\begin{align*}
\text{minimise} & \sum_{m \in M} \sum_{t \in T} f_m^t x_m^t \\
\text{subject to} & \\
\sum_{t \in T} x_m^t &= 1, m \in M \quad \text{(2)} \\
\sum_{m \in M} \sum_{t \in T_c^s} a_m^d b_{mc} x_m^t &\leq u_c^{ds}, c \in C, d \in D, s \in T_c \quad \text{(3)}
\end{align*}
\]

Total absolute difference between the requested and allocated time

Every movement must be allocated to one time interval

Total movement consumption cannot exceed capacity

Indicative Formulations (2/5)

Single Airport Slot Allocation Model

Mathematical Formulation

\[
\sum_{t \in [0,k)} x_t^d + \sum_{t \in [k-t_{ad},n)} x_t^a \leq 1, \{a,d\} \in P, k \in [t_{ad},n)
\]

\[
x_t^d \in \{0,1\}, m \in M, t \in T
\]

Turnaround Time Constraints

- If the departure slot of a paired movement has been allocated at \( k \), it is not possible to schedule the corresponding arrival after interval \( k - t_{ad} \). It enforces the obvious restriction that an arrival must be separated from a departure by at least a specified number of coordination time intervals (\( t_{ad} \)), representing the minimum turnaround time.

### Indicative Formulations (3/5)

## Efficiency Metrics

<table>
<thead>
<tr>
<th>Total displacement</th>
<th>Weighted displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \sum_{m \in M} \sum_{t \in T} \left</td>
<td>t - t_m \right</td>
</tr>
</tbody>
</table>

- **\( T \)**: the set of coordination time intervals
- **\( M \)**: the set of movements
- **\( t_m \)**: requested time interval for movement \( m \)
- **\( x_m^t \)**: 
  - \( = 1 \) if movement \( m \) is allocated to interval \( t \)
  - \( = 0 \) otherwise
- **\( f_m \)**: aircraft seat capacity for movement \( m \)
- **\( d_m \)**: flight distance for movement \( m \)
Indicative Formulations (4/5)

Two bi-objective models

Model 1
unweighted displacement

\[
\min_{x' \in x'_m} Z(x) = \left( \sum_{m \in M_a} \sum_{t \in T} |t-t_m| x'_m, \max \left| \rho_a - \frac{\sum_{a' \in A} \rho_{a'}}{|A|} \right| \right)^T
\]

s.t.

\[
\sum_{m \in M} \sum_{t \in T_c} x'_m \leq u_c^d, \forall c \in C, d \in D
\]

\[
\sum_{t \in (0,k)} x'_a + \sum_{t \in (k-t_{ad},n)} x'_d \leq 1, \forall (a,d) \in P, k \in [t_{ad}, n)
\]

\[
\sum_{t \in T} x'_m = 1, \forall m \in M
\]

\[
x'_m \in \{0,1\}
\]

Model 2
weighted displacement

\[
\min_{x' \in x'_m} Z(x) = \left( \sum_{m \in M_a} \sum_{t \in T} |t-t_m| x'_m f_m d_m, \max \left| \rho_a - \frac{\sum_{a' \in A} \rho_{a'}}{|A|} \right| \right)^T
\]

s.t.

\[
\sum_{m \in M} \sum_{t \in T_c} x'_m \leq u_c^d, \forall c \in C, d \in D
\]

\[
\sum_{t \in (0,k)} x'_a + \sum_{t \in (k-t_{ad},n)} x'_d \leq 1, \forall (a,d) \in P, k \in [t_{ad}, n)
\]

\[
\sum_{t \in T} x'_m = 1, \forall m \in M
\]

\[
x'_m \in \{0,1\}
\]
Indicative Formulations (5/5)

Efficiency Metrics

Model 1 ($\mathcal{E}$)

\[
\min_{x=[x_m]} Z(x) = \sum_{m \in M} \sum_{t \in T} |t-t_m| x_m^t
\]

(9)

\[
\sum_{m \in M} \sum_{t \in T} x_m^t \leq u_c^d, \forall c \in C, d \in D
\]

(3)

\[
\sum_{a \in A} x_a^t + \sum_{k \in [t_{ad},n]} x_k^t \leq 1, \forall (a,d) \in P, k \in [t_{ad},n]
\]

(4)

\[
\sum_{t \in T} x_m^t = 1, \forall m \in M
\]

(5)

\[
x_m^t \in \{0,1\}
\]

(6)

\[
\frac{\sum_{m \in M} \sum_{t \in T} |t-t_m| x_m^t}{|M_a|} - \frac{1}{|A|} \sum_{a \in A} \frac{\sum_{m \in M} \sum_{t \in T} |t-t_m| x_m^t}{|M_a|} \leq \varepsilon \cdot \frac{\sum_{m \in M} \sum_{t \in T} |t-t_m| x_m^t}{|M_a|}, \forall a
\]

(7)

Model 2 ($\mathcal{E}$)

\[
\min_{x=[x_m]} Z(x) = \sum_{m \in M} \sum_{t \in T} |t-t_m| x_m^t f_m d_m
\]

(10)

\[
\sum_{m \in M} \sum_{t \in T} x_m^t \leq u_c^d, \forall c \in C, d \in D
\]

(3)

\[
\sum_{a \in A} x_a^t + \sum_{k \in [t_{ad},n]} x_k^t \leq 1, \forall (a,d) \in P, k \in [t_{ad},n]
\]

(4)

\[
\sum_{t \in T} x_m^t = 1, \forall m \in M
\]

(5)

\[
x_m^t \in \{0,1\}
\]

(6)

\[
\frac{\sum_{m \in M} \sum_{t \in T} |t-t_m| x_m^t}{|M_a|} - \frac{1}{|A|} \sum_{a \in A} \frac{\sum_{m \in M} \sum_{t \in T} |t-t_m| x_m^t}{|M_a|} \leq \varepsilon \cdot \frac{\sum_{m \in M} \sum_{t \in T} |t-t_m| x_m^t}{|M_a|}, \forall a
\]

(7)
Results from model b/d/f/h

- bi-objective model which minimizes displacement and fairness objectives and solved hierarchically
Results (2/10)

- Historical

- New entrant

- Others

Displacement objective (5-min intervals) vs. Fairness objective
## Results (3/10)

<table>
<thead>
<tr>
<th></th>
<th>Current allocation results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Displacement (5-min intervals)</td>
</tr>
<tr>
<td>Historical</td>
<td>159</td>
</tr>
<tr>
<td>New entrant</td>
<td>370</td>
</tr>
<tr>
<td>Other</td>
<td>899</td>
</tr>
<tr>
<td>Total</td>
<td>1428</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Model b/d/f/h</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Displacement objective</td>
</tr>
<tr>
<td></td>
<td>(5-min intervals)</td>
</tr>
<tr>
<td>Historical</td>
<td>136</td>
</tr>
<tr>
<td>New entrant</td>
<td>176</td>
</tr>
<tr>
<td>Others</td>
<td>2141</td>
</tr>
<tr>
<td>Total</td>
<td>2453</td>
</tr>
</tbody>
</table>

*This solution was obtained when $\varepsilon = 0.7$*

All $\varepsilon$-constraints are binding.
Results (4/10)

Actual slot allocation

Maximal proportion of displacement is reduced

Proportion of requests
- Proportion of displacement

Model b/d/f/h

Proportion of requests
- Proportion of displacement

Remark
Fairness metric
\[ \rho_a = \frac{\sum_{m \in M_{\text{tot}}} \sum_{t \in T} |t - t_m| |x_m' - x_m'|}{\sum_{m \in M_{\text{tot}}} |M_m|} \]

Historical

*This solution was obtained when \( \varepsilon = 0.7 \)
Results (5/10)

2 New entrant

Actual slot allocation

Model b/d/f/h

Less variance of proportion of displacement

*This solution was obtained when $\varepsilon = 0.7$
Results (6/10)

3. Others

Actual slot allocation

Model b/d/f/h

Displacement is more proportional to requests

*This solution was obtained when $\varepsilon = 0.7$
Results from model b/d/f/nh

bi-objective model which minimizes displacement and fairness objectives and solved non-hierarchically
Results (8/10)

Small sacrifice in efficiency leads to substantial fairness gains.

Small fairness gains require substantial sacrifice in efficiency.
Results (9/10)

Actual slot allocation

Model b/d/f/nh

Displacement is more proportional to requests
## Results (10/10)

### Current allocation results

<table>
<thead>
<tr>
<th>Category</th>
<th>Displacement (5-min intervals)</th>
<th>Computed fairness objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Historical</td>
<td>159</td>
<td>6.01</td>
</tr>
<tr>
<td>New entrant</td>
<td>370</td>
<td>3.67</td>
</tr>
<tr>
<td>Other</td>
<td>899</td>
<td>5.17</td>
</tr>
<tr>
<td>Total</td>
<td>1428</td>
<td>22.83</td>
</tr>
</tbody>
</table>

### Model b/d/f/nh

<table>
<thead>
<tr>
<th>Category</th>
<th>Displacement objective (5-min intervals)</th>
<th>Fairness objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Historical</td>
<td>531</td>
<td>1.16</td>
</tr>
<tr>
<td>New entrant</td>
<td>32</td>
<td>0.70</td>
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<tr>
<td>Others</td>
<td>309</td>
<td>2.83</td>
</tr>
<tr>
<td>Total</td>
<td>889</td>
<td>0.90</td>
</tr>
</tbody>
</table>

*This solution was obtained when $\varepsilon = 0.9$*
Modelling Insights (1/3)
Minimize total displacement & fairness

1. Trade off between fairness and total schedule displacement

2. Simultaneous consideration of slots improves fairness

3. Simultaneous consideration of slots improves total efficiency
Modelling Insights (2/3)
mini-max & total displacement

1. Trade-off between total and maximum total displacement

2. Maximum schedule displacement represents the worst case ‘Guaranteed service level’ it can be offered’

3. Slots are scheduled within a time range that will not exceed this worst-case ‘service level’ (maximum displacement)

4. Small sacrifice in total schedule displacement may lead to substantial improvements in maximum schedule displacement
1. Trade-off between total displacement and number of violated slot assignments.

2. For all values of tolerance limit significant gains in violated slot assignments can be achieved without sacrificing much on total displacement.

3. If airlines are ready to accept a reasonable level of maximum displacement (15-30 min) substantial benefits can be achieved in terms of both violated slot assignments and total displacement.
Concluding Remarks (1/2)

- Slot scheduling is a multi-objective, multi-stakeholder problem.
- There are multiple efficiency and fairness objectives.
- For the same profile of slot requests and the same level of airport declared capacity, a wide array of non-inferior schedules can be produced depending on the preference structure of stakeholders.
- In order to achieve a commonly acceptable solution it is necessary to explore efficiently a wide range of alternative solutions.
Concluding Remarks (2/2)

- The model results should be effectively communicated to all stakeholders at both aggregate (system-wide) and disaggregate (individual airline) level.

- The proposed model can provide the basis of a Decision Support System (DSS) for analysing alternative scheduling options of airport slots.
Acknowledgements

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Thank you for your attention