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# Electoral competition with primaries and quality asymmetries


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**Abstract**

In two-dimensional two-party electoral competition under plurality rule, there are typically no equilibria, even when one of the dimensions refers to valence. The good news is that the introduction of either closed or open primaries acts as a stabilizing force since equilibria exist quite generally, serves as an arena for policy debates since all candidates propose differentiated platforms, and guarantees that each party's nominee is of higher quality than its primary opponent. Moreover, primaries tend to benefit the party whose median voter is closer to the overall median. The bad news is that the winner of the general election need not be the candidate with the highest overall quality since too competitive primaries can prove harmful. Given the differences between open and closed primaries, we show that the choice of primary type is particularly important and may determine the winner of the general election.



Keywords: Downsian model; primaries; valence

JEL Classification: C62, C72, D72



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# 1 Introduction

Party primaries have become an increasingly common method of nominating candidates for a general election. In the US, since just after WWII, primaries are by and large conducted in the same manner as a general election and run by the same electoral authorities. In Europe and Latin America, primary elections are a more recent phenomenon, and primaries are generally run by the parties themselves. Several questions of interest naturally arise. For example, how does the introduction of primaries influence candidates' policy proposals? How does the introduction of primaries affect the outcome of the general election? If we allow candidates to vary in policy positions on one dimension and also for there to be a second non-policy dimension of evaluation on which they may vary as well (what is called a valence or quality dimension and is commonly assessed by all voters), how does the differentiation of candidates in the party primary on these two dimensions affect the party's performance in the general election? Also, because there are varieties of primary type, with the most important distinction being between open primaries and closed primaries, with voting in the latter type restricted to party affiliates, how does the choice of primary type matter? Should each party wish to have the same electoral rules for primary nomination as its opposition?

In order to answer such questions we consider plurality two-party competition in a two-stage election (primary and general), where each party has two primary candidates. We first consider the situation where both parties have closed primaries in the framework of a standard Downsian model of electoral competition.[1] We allow all four candidates to differ both in policy platform and in terms of a commonly valued and commonly known non-policy characteristic, which we may label a valence or quality dimension. By introducing this quality heterogeneity among candidates, voters not only vote on the basis of policy proposals, but also on the basis of which candidate is considered to be "better" (for example, in terms of charisma, corruption allegations and experience). Regarding candidates' behavior, we posit that all candidates aim at maximizing their general election vote share.[2] This setup is very general and imposes no restrictions on parties' structure. Parties are treated as exogenous at the time of the primary but differ in the ideology of their median voter.

The introduction of primaries in this intuitive setup provides results in sharp contrast to the standard

[1]Standard assumptions of a Downsian model are office-motivated candidates, full commitment, and sincere voting among others. See Grofman (2004); Osborne (1993) for a list of assumptions that are generally perceived to best justify the description of a model as a Downsian one.

[2]We stress that this objective is perfectly in line with the more natural lexicographic objective of maximizing the probability of winning in the general election, and, when indifferent, maximizing the probability of winning in the primary. Actually, the vote share objective is a *refinement* of the lexicographic objective described above: an equilibrium of the game with the vote share objective is always an equilibrium with the lexicographic objective described above, while the opposite need not hold.

Downsian model with two candidates of unequal valence competing in the general election. While without primaries a pure strategy Nash equilibrium does not exist (Aragonès and Palfrey, 2002),[3] we show that with closed primaries *a unique equilibrium in pure strategies always exists*, and this is true for any distribution of voters' preferences. That is, while the standard Downsian model of electoral competition with valence asymmetries predicts that stability can not be reached, the introduction of primaries provides a clear stabilizing effect. This *stabilizing effect of primaries* on electoral competition is the first substantial result of our analysis.[4]

In addition to identifying the stabilizing effect of primaries, our setup permits the full characterization of this unique equilibrium, with several interesting properties arising. In equilibrium, each party's low valence candidate proposes a platform coinciding with the ideal policy of her party's median. Each party's high valence candidate is relatively more moderate than the low valence primary candidate and, targeting at the best electoral outcome in the general election, locates the closest possible to the society's median. The valence asymmetry between the two primary candidates ultimately determines how differentiated the two platforms will be. The larger the advantage of one candidate is, the more she is capable of moving towards moderate policies and hence becoming more appealing in the general election while guaranteeing a primary victory. As far as valence is concerned, since high valence candidates always win their primary, our result is in line with recent empirical evidence showing that primaries tend to be effective at selecting high quality types (Hirano and Snyder, 2014). In terms of policy proposals, primaries serve as the arena for meaningful intra-party policy debates since primary candidates propose differentiated platforms. Nevertheless, the intensity of such debates is crucial in determining the winner of the general election and primaries can prove harmful to the party with the highest valence candidate. Our results show that the low valence general election candidate may win if she faced a weak party primary opponent, while the highest valence candidate could not propose a moderate enough platform because of tight primaries inside the losing party. Hence, our divergent equilibrium result shows that primaries may end up being harmful to a party if they create too much competition during the nomination process. This *within-party competition effect* of primaries on electoral outcomes, is the second substantial result of our analysis and relates to the negative aspect of the divisive effect of primaries (Key, 1953; Agranov, 2016).

Third, our analysis suggests that primaries have a *matching effect*: candidates nominated by leftist

[3]To be precise, Aragonès and Palfrey (2002) consider that the two heterogeneous candidates are win-motivated and hold imperfect information regarding voters' policy preferences. As it is argued in Aragonès and Xefteris (2016), in these models, win-motivation with imperfect information about voters' preferences is technically *equivalent* to vote-share maximization and perfect information about voters' preferences.

[4]The search for stabilizing forces in multidimensional competition models has attracted previous attention and several proposals. Among others, Lin et al. (1999) consider probabilistic voting, Krasa and Polborn (2012); Dziubiński and Roy (2011); Aragonès and Xefteris (2014) allow for differentiated candidates and Bräuninger (2007) for costly voting.

(rightist) parties win more often when the society's median is leftist (rightist). This is a result of primaries making candidates more responsive to the policy preferences of their primary electorate rather than the general one. If for example the society becomes more leftist, the leftist party will win more often than before, since the high valence primary candidate of the rightist party cannot react to the median's shift. If the rightist high valence candidate were to propose leftist policies that would please the new median, this would potentially make her lose the primary. Notice that this otherwise intuitive feature of our equilibrium is, surprisingly, absent from most electoral competition models without primaries: models with office-motivated candidates usually generate equilibria in which candidates converge (either in deterministic or in probabilistic terms) and models with policy-motivated candidates usually predict that candidates will locate equidistantly from the society's median -and will hence tie- independently of whether the society's median is leftist or rightist. Overall, the matching effect may further strengthen the stability of the party system as it allows parties to form a more durable ideological framework. However, the asymmetry of party competition, with the party whose median voter is closer to the overall median being advantaged, allows us to recognize an important stylized fact about much political competition, namely that there may be (extended) periods during which one party is dominant (Merrill et al., 2008).

To the best of our knowledge, this is the first paper to point at these three effects of primaries simultaneously. The first and the third effect are obviously positive ones: primaries stabilize the electoral process and generate consistency between the party of the elected candidate and voters' policy preferences, and these in turn promote a sense of trust in the political system. The second effect, has, arguably, negative implications: a high valence candidate that faces hard within-party competition might end up losing the general election to a mediocre candidate that won in her party's primaries against a low quality opponent.

We propose several modifications to our analysis and find that our results are robust in a number of directions. First, we focus on open primaries. Once all candidates make their policy proposals, active citizens who are willing to participate in the procedure vote in the primary of the party in which their top-ranked candidate participates. This implies that by proposing moderate platforms, candidates not only increase their general election vote share, but also increase the amount of active voters participating in their party's primary. Hence, both parties' size as well as the ideal policy of the parties' medians are now endogenously determined. Interestingly, and despite the endogenous party formation described, the stabilizing effect of primaries prevails. That is, for a large class of voters' distributions we still obtain a unique equilibrium in pure strategies such that: a) in some instances, the highest valence candidate does not emerge as the winner of the general election (*within-party competition effect of primaries*) and b) the winner of the leftist (rightist) primary is more likely to be the general election's winner when the society's median is leftist (rightist) (*matching effect of primaries*). Further comparing open and closed primaries, it turns out that the "negative" within-party competition effect is stronger (weaker) in closed than open primaries, when the highest valence candidate is a candidate of the more (less) extreme party. That is, the highest valence candidate is the winner under open primaries more often than under closed primaries, only when this candidate belongs to the party with the most extreme political views. In other words, parties that are most benefited from the transition from a closed primaries system to an open one are those with relatively more extremist party members.

Finally, we investigate situations where a primary is held only in one of the two parties (either closed or open). This is of interest since often incumbents who run for reelection without going through a nomination process face a challenger who emerged from a primary. In our model, we assume that the position of the incumbent is fixed and that primary candidates in the opposition strategically choose their platforms (typically the incumbent has less flexibility than the challenger in credibly promising something different to the implemented policies). We find that when the incumbent implements socially detrimental (appealing) policies, the highest valence challenger is elected less (more) often under closed primaries than under open primaries. The reason why bad incumbents are less threatened by challengers that emerge from closed primaries than from open primaries, is that closed primaries hold candidates close to their party's median, who might be quite far from the society's one. Open primaries pose no such restriction and allow candidates to expand their primary electorate by moving towards the center. In other words, open primaries give incentives to high valence candidates of initially less moderate parties to move towards the center and, thus, to: a) win more often and, perhaps more importantly, b) make their parties more moderate by moving towards the centre and hence attracting new moderate voters for their primaries. These results point to an interesting effect of the organization of the party in opposition on the incumbent's decisions when the latter cares about reelection: incumbents have stronger incentives to implement moderate policies when the challenger's party holds open primaries than when it holds closed primaries.

Our paper complements the existing literature on primaries with valence asymmetries by adding several insightful new results.[5] Our work closely relates to Adams and Merrill (2008) since, to the best of our knowledge, this is the only other setup where all four primary candidates may differ in valence. Nevertheless, while Adams and Merrill (2008) focus on a probabilistic voting model we focus

[5] Research on primaries without valence issues was presented among others in Owen and Grofman (2006); Meirowitz (2005); Coleman (1971); Aranson and Ordeshook (1972). For papers interested in the non commitment of primary winners and flip flopping between primary and general elections see Hummel (2010); Agranov (2016). For the effect of sequential primaries on electoral competition see Callander (2007); Deltas et al. (2015). For work on different ways of candidates' nomination including primaries see among others Crutzen et al. (2010); Jackson et al. (2007); Hortala-Vallve and Mueller (2015); Kselman (2015); Amorós et al. (2016).

on a deterministic one.[6] As well known, probabilistic and deterministic voting models with valence asymmetries deliver very diverse predictions on candidates' equilibrium behavior and as we show this is precisely true in the context of primaries.[7] The presence of a random element eventually leads to primary candidates proposing identical platforms, while the point of convergence might differ across parties (Adams and Merrill, 2008). On the contrary, we show that primary candidates run on different platforms giving back to primaries the element of an internal battlefield.

Hummel (2013) also employs a non probabilistic valence model, but, unlike us, he considers that: a) the higher (lower) valence candidate of the leftist party has precisely the same valence with the higher (lower) valence of the rightist party and b) voters may strategically decide not to support their top-ranked candidates. Similar to Hummel (2013) high valence candidates propose more moderate policies than low valence ones. Nevertheless, by allowing all four candidates to differ in valence, we provide new results on the effect of primaries on the winner of the general election and demonstrate why and when the highest valence candidate may not win the general election.

Takayama (2014) uses similar assumptions to those of Hummel (2013) and models primaries only in the challenger's party with overall three candidates of different valence. She shows that as the incumbent's valence increases, the qualifying challenger becomes more moderate. Our results under the presence of an incumbent are different and depend on the primary type. We show that if the party runs closed primaries, the policy proposed by the high valence challenger is not affected by the incumbent's characteristics. On the contrary, when the party organizes an open primary, the challenger becomes more moderate as the incumbent's valence decreases.

In Kartik and McAfee (2007), as in Hummel (2013), there are two levels of valence but different to the aforementioned papers high valence types are committed to an exogenous platform. In Serra (2011); Snyder and Ting (2011) both primaries and the general election function as a valence revelation mechanism and their focus is more on the adoption or not of primaries rather than on primary candidates' platforms proposals. In Andreottola (2016) only primaries serve as a valence revelation mechanism and in contrast to us his results show that the high valence primary candidate proposes more extreme platforms than the low valence candidate. For models of primary elections with endogenous valence see Serra (2010); Casas (2013).

[6] In a probabilistic voting model one votes for a certain candidate with a probability that is increasing in the utility that one derives from the election of this candidate. That is, one need not vote for the candidate that she likes best, while voting for the top-ranked candidate is one's most probable action. In a deterministic voting model one always votes for the candidate one likes best.

[7] For example, in standard two party competition models, while probabilistic voting models do not rule out convergent equilibria when valence asymmetries are not very large (see, for example, Schofield 2007), this never occurs in deterministic voting models (see, for example, Aragonès and Palfrey 2002).

The remainder is structured as follows: In section 2 we present the model, in Section 3 we present our results for closed, open and one party primaries. In Section 4 we conclude.

## 2 The Model

The policy space is the $[0,1]$ interval. We have a unit mass of general-election voters whose ideal policies are distributed according to an absolutely continuous, strictly increasing and twice differentiable distribution function $\Phi : [0,1] \rightarrow [0,1]$ with a unique median, $m \in (0,1)$, defined by $\Phi(m) = \frac{1}{2}$. Two positive measure subsets of these voters form the two exogenously given parties and participate in a closed primary election where no other voters can participate. Let the median of the leftist party be the primary voter with ideal policy $l \in [0,1]$ and the median of the rightist party be the primary voter with ideal policy $r \in [0,1]$ with $l < m < r$.[8] Candidates $A$ and $B$ compete in the primary of the leftist party and candidates $C$ and $D$ compete in the primary of the rightist party. Each candidate $J \in \{A, B, C, D\}$ is characterized by a valence parameter $v_J \geq 0$ and strategically chooses and commits to an electoral platform $x_J \in S_J$, where $S_J = [0, m]$ if $J \in \{A, B\}$ and $S_J = (m, 1]$ if $J \in \{C, D\}$.[9] Let $v_A < v_B$, $v_D < v_C$ and $v_B > v_C$. This assumption is without loss of generality: it just provides equilibrium locations in order with candidates' "names" and places the highest valence candidate $B$ in the leftist party. We also assume that $m + 2v_B < r$, that is, valence differences are small enough such that the high valence candidate can not unambiguously win primaries and the general election by locating near the society's median.

The game has three stages. In stage one, all four candidates choose and announce their policy platforms simultaneously. In stage two, closed primary elections take place in each of the two parties. In stage three, the general election takes place and each voter votes for one of the two primaries' winners. All ties, either in primaries or in the general election are broken with equiprobable draws.

The utility of a voter with ideal policy $i \in [0,1]$ when candidate $J \in \{A, B, C, D\}$ is elected in office (or else, wins in the general election) is given by

$$u_i(x_J, v_J) = -|i - x_J| + v_J$$

in line with literature on electoral competition among candidates of unequal valence (see for example, Groseclose 2001; Aragonès and Palfrey 2002). Voters are sincere both in primaries and in the general

[8] Notice that we impose very little structure on the precise kind of closed primaries that each party holds. While for example one party may run a (primary) election where only "core" party members are eligible, another party may run a primary open to all party members.

[9] This assumption is only made to ensure a tractable structure of our proofs and is without loss of generality. That is, both the existence and the uniqueness of the characterized equilibrium are robust to allowing each candidate to locate anywhere in $[0,1]$.

election and vote for the candidate that offers them the highest utility.[10] In case of indifference we assume that a voter supports any of the two candidates with equal probability.

Since voters' behavior in stages two and three is essentially parametric, one may define the expected vote share of candidate $J$ in the general election as

$$P_J(x_J, x_{-J} : v, \Phi, l, r)$$

where $x_{-J}$ is the vector of platforms of the other candidates and $v = (v_A, v_B, v_C, v_D)$. Candidates are Downsian, that is, they maximize expected vote shares in the general election.[11] A Nash equilibrium in this setup is a vector $\hat{x} = (\hat{x}_A, \hat{x}_B, \hat{x}_C, \hat{x}_D)$ such that for every $J \in \{A, B, C, D\}$ it is true that $P_J(\hat{x}_J, \hat{x}_{-J} : v, \Phi, l, r) \geq P_J(\acute{x}_J, \hat{x}_{-J} : v, \Phi, l, r)$ for any $\acute{x}_J \in S_J$.

# 3 Results

Before presenting our main results, let us define two concepts of crucial relevance. In equilibrium each primary is won by the high valence candidate ($B$ wins the leftist primary and $C$ wins the rightist primary). We refer to the valence difference in the general election as the "toughness" faced by $B$ in the general election (defined as $T_G = -(v_B - v_C)$). Similarly, we refer to the valence difference in each party as the "toughness" candidates $B$ and $C$ face in their primary elections respectively (defined as $T_L = -(v_B - v_A)$ and $T_R = -(v_C - v_D)$ respectively). Notice that given our restrictions on candidates' valence characteristics, "toughness" takes generically negative values, approaching zero when both candidates are of almost equal valence.

**Proposition 1** *There exists a unique Nash equilibrium, $\hat{x}$, and it is such that $\hat{x}_A = l$, $\hat{x}_B = l + v_B - v_A = l - T_L$, $\hat{x}_C = r - v_C + v_D = r + T_R$ and $\hat{x}_D = r$. In equilibrium, candidates $B$ and $C$ win their parties' primaries and: a) candidate $B$ wins the general election if $l + r - 2m + T_R > T_G + T_L$, b) candidate $C$ wins the general election if $l + r - 2m + T_R < T_G + T_L$ and c) each of $B$ and $C$ wins with equal probability if $l + r - 2m + T_R = T_G + T_L$.*

The existence of a unique Nash equilibrium points at the *stabilizing* effect of primaries on electoral competition. While in the absence of primaries, and when the two candidates differ in valence, no

[10] Our equilibrium results would be robust to discrete primary electorates and a small fraction of strategic voters. If the share of strategic voters were large, our conjecture is that all candidates would have strong incentives to move towards the median of the primary election (similar to Woon 2016; Serra 2015). For full elaboration of such dynamics in a non-primary setting see Feddersen et al. 1990.

[11] Maximization of the general election vote share presents itself as the natural Downsian objective in this setup as it is compatible with (it is actually a refinement of) the following ordered pair of objectives: a) a candidate prefers all outcomes of the game in which she is the winner of the general election to any other outcome and b) among all outcomes in which a candidate does not win the general election, this candidate prefers the outcomes in which she wins her party's primaries.

equilibrium in pure strategies exists, this is no longer true when these two candidates have been selected through a primary race. Valence asymmetries in primaries create interesting electoral dynamics and in equilibrium lead to the divergence of proposed platforms in the primary race (in line with Hummel 2013 and in contrast to Adams and Merrill 2008). The low valence candidate locates at the party's median, while the high valence candidate is more moderate and locates closer to the society's median. How far towards the society's median the high valence candidate is able to move depends on the "toughness" of the primary. The higher the valence asymmetry inside a party, the more the winning candidate can converge towards the society's median, thus improving her future performance in the general election. On the contrary, if both candidates are of almost equal valence, then the high valence candidate is not able to differentiate much and this may have a negative impact in her performance in the general election. Our model predicts that in equilibrium the general election candidates are somewhere between their parties' and the general election median. This platform order tends to be accepted in the literature (Burden, 2001) and as we show this may be attributed to valence asymmetries and does not require the presence of strategic voting.

In the characterized equilibrium, all more extreme party members than the party's median are indifferent between the two candidates and therefore vote for any of the two primary candidates with equal probability. All more moderate party members than the party's median vote for the high valence candidate. Hence, the high valence candidate wins the primary with the support of three quarters of party members. Regarding the winner of the general election, any of the candidates may win depending on parties' and the society's medians as well as the valence asymmetries determining the "toughness" of both primaries and the general election. According to our results, the high valence leftist candidate $B$ wins the general election when $l + r - 2m + T_R > T_G + T_L$. The right hand side is capturing the aggregate "toughness" the leftist candidate $B$ is facing in terms of valence in the primary and in the general election. When the aggregate "toughness" is below (above) a given threshold (i.e., $l + r - 2m + T_R$) then the leftist candidate wins (loses) the general election. This threshold illustrates that the leftist candidate is favoured by a moderate leftist party (large $l$), an extreme rightist party (large $r$), a "leftist" overall electorate (small $m$) pointing at the *matching effect of primaries*, and tough primaries in the rightist party (large $T_R$). On the contrary, the leftist candidate is harmed by tough primary and general elections (large $T_L$ and large $T_G$). Tough primaries do not permit the qualifying candidate to move away from the party median and propose a moderate platform harming her general election performance (*within-party competition effect*). Tough general elections also harm the candidate's performance where voters directly compare the valence characteristics of the two general election candidates.

In order to further illustrate the effect of valence characteristics on $B$'s electoral performance consider the symmetric case in terms of ideologies (i.e., $m = 0.5$ and $l = 1 - r$). The condition for the victory of the highest valence candidate becomes:

$$T_R > T_G + T_L$$

that is, $B$ wins the general election when the aggregate "toughness" she faces is lower than the "toughness" in the rightist primary. In other words, $B$ may not win the general election when the primaries of the leftist party are significantly "tougher" than the primaries of the rightist party. As our model shows "toughness" of primaries is crucial for the determination of the general election's winner since it draws candidates near parties' medians and far from society's median. Hence, our model illustrates the presence of a detrimental *within-party competition effect* that may potentially harm the party's qualifying candidate.

### 3.1 Open parties

Having obtained a very general result for closed primaries a natural question is what occurs when parties hold open primaries. Open primaries are an increasing in popularity phenomenon with several parties electing their candidates using this method.[12] We therefore extend our setup allowing voters to decide in which primary to participate once all four candidates announce their platforms.[13]

For the analysis of open primaries some further assumptions are necessary. Let us consider that a subset of all voters are "active" and are participating in the primaries. Let active voters have ideal policies distributed according to any continuous log-concave distribution $F$ with a unique median $m^a$.[14] These "active" voters participate in the primary of the party where the candidate that gives them the highest utility is competing and vote for that candidate. Hence, parties are now endogenously formed and depend on candidates' policy proposals and valence characteristics. Finally, let each candidate $J \in \{A, B, C, D\}$ strategically choose and commit to an electoral platform $x_J \in S_J$, where $S_J = [0, m^a]$ if $J \in \{A, B\}$ and $S_J = (m^a, 1]$ if $J \in \{C, D\}$. Again we assume that valence differences are not very large.[15]

[12] In the US around one third of the states hold an open primary. In Europe the socialist parties of France, Greece and Italy run open primaries for their leaders as it is the case for the conservatives in the UK for some parliamentary candidates. The European Green Party ran a paneuropean open primary for 2014 EU election. In Latin America open primaries take place in Argentina. For mixed empirical evidence on the effect of primaries on political competition see Kaufmann et al. (2003); Gerber and Morton (1998); Kanthak and Morton (2001).

[13] Our open primaries model has a similar spirit with the literature modeling party platforms as a function of their members' preferences, with membership being in turn also endogenously determined by party platforms (e.g., Gomberg et al. 2015, 2004; Baron 1993).

[14] We consider that a continuous distribution function $F$ is log-concave if $\frac{\partial^2 \ln F(x)}{\partial x^2} < 0$ and $\frac{\partial^2 \ln[1-F(x)]}{\partial x^2} < 0$ for every $x \in (0, 1)$. That is, the notion of log-concavity that we employ implies that $F$ is strictly increasing and twice differentiable in its support too. Log-concavity of the distribution of voters' ideal policies is a general assumption and it is very widely used in political economics literature. See Bagnoli and Bergstrom (2005) for further properties of log-concave distributions.

[15] Given that party medians are now endogenously defined, the sufficient condition provided for the closed primaries

**Proposition 2** *Let $(l^*, r^*)$ be the unique values such that $2F(l^*) = F(\frac{l^*+v_B-v_A+r^*-v_C+v_D}{2} + \frac{v_B-v_C}{2})$ and $2[1-F(r^*)] = 1-F(\frac{l^*+v_B-v_A+r^*-v_C+v_D}{2} + \frac{v_B-v_C}{2})$. There exists a unique Nash equilibrium, $\hat{x}$, and it is such that $\hat{x}_A = l^*$, $\hat{x}_B = l^* + v_B - v_A = l^* - T_L$, $\hat{x}_C = r^* - v_C + v_D = r^* + T_R$ and $\hat{x}_D = r^*$. In equilibrium, candidates B and C win their parties' primaries and: a) candidate B wins the general election if $l^* + r^* - 2m + T_R > T_G + T_L$, b) candidate C wins the general election if $l^* + r^* - 2m + T_R < T_G + T_L$ and c) each of B and C wins with equal probability if $l^* + r^* - 2m + T_R = T_G + T_L$.*

As in closed primaries, an equilibrium exists and it is unique, guaranteeing the *stabilizing* effect of open primaries, when the distribution of ideal policies of active voters is nowhere "too convex" (log-concavity of $F$). Unlike, though, the case of closed primaries in which an equilibrium always exists, given the fixed party structure, when primaries are open, it might be the case that an equilibrium does not exist when $F$ is too convex in certain regions (that is, when the density of ideal policies of the active voters increases significantly when moving from $x$ to $x + \varepsilon$ for some $x \in (0,1)$). In specific, when $F$ is too convex about $\frac{l^*+v_B-v_A+r^*-v_C+v_D}{2} + \frac{v_B-v_C}{2}$ (if, for example, it is log-convex about this point) then an equilibrium does not exist.[16] Candidate $B$ can still win the primary by moving marginally to the right of $l^* + v_B - v_A$, increasing at the same time her general election vote share. This is so because when $F$ is too convex about $\frac{l^*+v_B-v_A+r^*-v_C+v_D}{2} + \frac{v_B-v_C}{2}$, a slight transition of $B$ from $l^* + v_B - v_A$ to $l^* + v_B - v_A + \varepsilon$, brings in the leftist primary many new supporters of $B$. Hence, $B$ still wins in the primary and improves her performance in the general election. All these suggest, that indeed the stabilizing effect of primaries holds even when primaries are open for a very general class of preference profiles, but, as expected, it is weaker compared to the closed primaries case. We note though that the fact that open primaries might stabilize electoral competition for such a general class of preference profiles, is more surprising, at least to us, than the fact that they are less prone to lead to stability compared to closed primaries. At first sight, one would expect that the dynamics that lead to the existence of an equilibrium when primary electorates are fixed, would disappear once we considered that parties are endogenous. As we have proved above this is far from being the case.

The equilibrium structure is similar to the one in closed primaries with primary losers locating on parties' medians and primary winners diverging from the party median towards the median of the society. What is different compared to closed primaries is that now party medians (i.e., $l^*$ and $r^*$) are endogenously determined and those depend on all four values of valence characteristics as well as the distribution of

case, $m + 2v_B < r$, is not applicable here. Instead we assume that $0 < m^a + 2v_B - 2v_C + v_A - v_D < 1$. Notice that $\lim_{v_B \to 0}(m^a + 2v_B - 2v_C + v_A - v_D) = m^a$. Hence, for any admissible $F$ the condition is guaranteed to hold for sufficiently modest valence differences.

[16] The notion of log-convexity that we employ is symmetric to the one of log-concavity. Note that the existence of a pair $(l^*, r^*)$ such that $2F(l^*) = F(\frac{l^*+v_B-v_A+r^*-v_C+v_D}{2} + \frac{v_B-v_C}{2})$ and $2[1-F(r^*)] = 1-F(\frac{l^*+v_B-v_A+r^*-v_C+v_D}{2} + \frac{v_B-v_C}{2})$ does not depend on the concavity of $F$; only its uniqueness does.

active voters (see Example 1). As it turns out, candidates $B$ and $C$ propose the two most moderate platforms and the indifferent voter between the two determines not only their vote shares in the general election but also the party sizes. Since active voters decide to participate in the primary where they can identify the candidate that gives them the highest utility, all active voters on the left (right) of the indifferent voter participate in the primary of the leftist (rightist) party. Once each party's primary electorate is endogenously determined, all more extreme voters than the party median are indifferent between the two primary candidates and randomize their vote. All more moderates than the party median support the high valence candidate. Similar to closed primaries, the high valence candidate obtains support of three quarters of the endogenously formed party and qualifies for the general election.

The condition such that the highest valence candidate wins the general election provides parallel insights as under closed primaries. Candidate $B$ wins the general election as long as the aggregate "toughness" she faces is lower than a given threshold (i.e., $l^* + r^* - 2m + T_R > T_G + T_L$). The difference compared to closed primaries is that the threshold value is endogenously determined since it depends on $l^*$ and $r^*$, which are defined by $F$ and $v$. Similar to closed primaries, the high valence candidate loses the general election if the primaries in the leftist party are much tougher in terms of valence than the ones of the rightist party. In order to be nominated, candidate $B$ has to move close to the endogenously determined party's median and far from the society's median that may result in a too extreme position to win the general election. Hence, as in closed primaries, open primaries can be harmful to high valence candidates through the *within-party competition effect.* Moreover, the *matching effect of primaries* where the leftist candidate benefits from a leftist electorate is still present since candidate $B$ wins more often as the society becomes more leftist (i.e., small $m$).

The following example further illustrates how platforms depend on the characteristics of all four candidates and not only on those of the primary candidates as was occurring in closed primaries. Moreover, it stresses that the identified *matching effect* is not a mere consequence of having different primary and general election electorates (that is, of $F \neq \Phi$ or, simpler, of $m \neq m^a$): it is still present when the preferences of the two sets of voters are correlated (that is, even if $F = \Phi$).

**Example 1** *Let both general election and active voters be identically and linearlydistributed on* $[0, 1]$.[17] *That is, let* $\Phi(x) = F(x) = \frac{2-a+ax}{2}x$, *where* $a \in [-2, 2]$ *(*$a = -2$ *corresponds to the triangular distribution with a pick at zero,* $a = 0$ *corresponds to the uniform distribution and* $a = 2$ *corresponds to the triangular distribution with a pick at one).*

1. *When both sets of voters are uniformly distributed on* $[0, 1]$, *that is when* $a = 0$, *a unique equilibrium*

[17]By "linearly distributed" we mean that the density function is linear (increasing, decreasing or flat).

*exists and it is given by,*

$$\hat{x}_A = \frac{1}{4} - \frac{1}{2}v_A + v_B - v_C + \frac{1}{2}v_D = \frac{1}{4} - \frac{1}{2}(T_L + T_G - T_R) = l^*,$$

$$\hat{x}_B = \frac{1}{4} - \frac{3}{2}v_A + 2v_B - v_C + \frac{1}{2}v_D = \frac{1}{4} - \frac{1}{2}(T_L + T_G - T_R) - T_L = l^* - T_L,$$

$$\hat{x}_C = \frac{3}{4} - \frac{1}{2}v_A + v_B - 2v_C + \frac{3}{2}v_D = \frac{3}{4} - \frac{1}{2}(T_L + T_G - T_R) + T_R = r^* + T_R,$$

$$\hat{x}_D = \frac{3}{4} - \frac{1}{2}v_A + v_B - v_C + \frac{1}{2}v_D = \frac{3}{4} - \frac{1}{2}(T_L + T_G - T_R) = r^*.$$

*As this example shows, all equilibrium platforms depend on all valence characteristics. The losing leftist candidate (i.e., candidate $A$) proposes a platform that coincides with the median of the endogenously formed party (i.e., $l^*$). Overall, high valence of the winning leftist and the losing rightist candidates acts as a centripetal force. On the contrary, high valence of the losing leftist and of the winning rightist acts as a centrifugal force. The winning leftist candidate (i.e., candidate $B$) proposes a more moderate platform than its primary opponent. With a similar intuition as our general result, the "toughness" of the primary determines by how much $B$ moderates. The above hold in a symmetric fashion for the rightist party.*

2. *When most voters are leftist ($a < 0$ or $m < \frac{1}{2}$), then $\lim_{v_B \to 0}(\frac{l^* + v_B - v_A + r^* - v_C + v_D}{2} + \frac{v_B - v_C}{2}) \in (m, \frac{1}{2})$ and when most voters are rightist ($a > 0$ or $m > \frac{1}{2}$), then $\lim_{v_B \to 0}(\frac{l^* + v_B - v_A + r^* - v_C + v_D}{2} + \frac{v_B - v_C}{2}) \in (\frac{1}{2}, m)$. That is, when the majority of general election voters are leftist and the valence differences are sufficiently modest, the candidate of the leftist party wins and vice versa.*

### 3.2 One party primaries

In reality not both parties need to hold a primary before the general election. A typical situation of interest for the absence of primaries in one party is when an incumbent runs for reelection. Let the incumbent be candidate $C$ whose ideal policy $x_C$ is known and fixed.[18] The incumbent is also described by a given level of valence that can be either higher or lower than any of the other candidates. The leftist candidates $A$ and $B$ may run in an open or a closed primary as before (without loss of generality we still assume that $v_B \geq v_A$ but we do not require that $v_B > v_C$). In this setup, a Nash equilibrium is a vector $\hat{x} = (\hat{x}_A, \hat{x}_B)$ such that none of the two leftist primary candidates has incentives to deviate.

[18]Formally, let $x_C > m > l$, and $m < x_C - |v_B - v_C|$ for the case of closed primaries, and $F[(m + v_B + x_C - v_C)/2] - F[m - v_B + v_A] < F[m - v_B + v_A]$ for the case of open primaries.

**Proposition 3** *If the leftist party runs a closed primary there exists a unique Nash equilibrium, $\hat{x}$, and it is such that $\hat{x}_A = l$, $\hat{x}_B = l + v_B - v_A = l - T_L$. In equilibrium, candidate $B$ wins the leftist primaries and: a) candidate $B$ wins the general election if $l + x_C - 2m > T_G + T_L$, b) candidate $C$ wins the general election if $l + x_C - 2m < T_G + T_L$ and c) each of $B$ and $C$ wins with probability $\frac{1}{2}$ if $l + x_C - 2m = T_G + T_L$.*

When the incumbent is challenged by a party organizing a closed primary, both primary candidates follow the same strategies as when both parties hold closed primaries. The low valence candidate between the two leftist candidates proposes a policy identical to the party's median (i.e., $\hat{x}_A = l$). The high valence candidate proposes a more moderate policy becoming as appealing as possible in the general election while guaranteeing the victory in the primary. How much the high valence can moderate depends again on the "toughness" of the primary race and not at all on the incumbent's characteristics (i.e., $\hat{x}_B = l - T_L$). Of course the incumbent's characteristics play a crucial role in determining the winner of the general election. Whether the leftist challenger succeeds in replacing the incumbent depends on the ideology of the leftist party, the ideal policy of the incumbent, as well as the "toughness" of the primary and of the general election in terms of valence. The aggregate "toughness" threshold condition such that the challenger $B$ wins (i.e., $l + x_C - 2m > T_G + T_L$) implies that the incumbent is of course harmed by her own extreme policies. The challenger is benefiting from a moderate leftist party, a leftist median voter, and non-competitive primaries. Clearly, the "toughness" of the general election in terms of the valence difference between the two candidates and who of the two candidates has a higher valence is also crucial.

Notice that in closed primaries candidates aim at winning nomination focus only on their party's median. This explains why the challenger's proposed platform is not affected by the incumbent's platform. As we describe in the following proposition this is no longer true when the party in opposition holds an open primary. Let as before $F$ denote the distribution of active voters. The natural way of extending sincere voting in this one-party primary scenario is to let active voters who like the most either candidates $A$ or $B$ participate in the primary of the leftist party supporting their favorite candidate, while active voters that like the most the incumbent do not participate in the primary.

**Proposition 4** *Let $l^*$ be the unique value such that $2F(l^*) = F(\frac{l^* + v_B - v_A + x_C}{2} + \frac{v_B - v_C}{2})$ for any log-concave distribution $F$. If the leftist party runs an open primary, there exists a unique Nash equilibrium, $\hat{x}$, and it is such that $\hat{x}_A = l^*, \hat{x}_B = l^* + v_B - v_A$. In equilibrium, candidate $B$ wins the leftist primaries and: a) candidate $B$ wins the general election if $l^* + x_C - 2m > T_G + T_L$, b) candidate $C$ wins the general election if $l^* + x_C - 2m < T_G + T_L$ and c) each of $B$ and $C$ wins with probability $\frac{1}{2}$ if $l^* + x_C - 2m = T_G + T_L$.*

Now $l^*$ is the unique median of the endogenously formed leftist party. Since $l^*$ is increasing in $x_C$ and decreasing in $v_C$, both leftist primary candidates propose relatively more moderate platforms when the incumbent is an extremist or of low valence. This movement towards moderate policies increases the leftist party's vote share in the general election. The aggregate "toughness" threshold condition such that the winning leftist candidate $B$ also wins the general election is intuitively similar to when party $B$ runs a closed primary.

The following example illustrates equilibrium proposals for the leftist candidates when primary voters are uniformly distributed.

**Example 2** *Let "active" voters be uniformly distributed across the policy space. If the leftist party runs an open primary,* $\hat{x}_A = \frac{1}{3}(x_C - v_A + 2v_B - v_C) = \frac{1}{3}(x_C - T_L - T_G)$, $\hat{x}_B = \frac{1}{3}(x_C - 4v_A + 5v_B - v_C) = \frac{1}{3}(x_C - 4T_L - T_G)$.

In contrast to closed primaries, all platforms depend on the quality characteristics of all candidates including those of the incumbent as well as the implemented policy. If we also assume that the society is uniformly distributed then the location of the indifferent voter and hence $B$'s vote share under an open primary is given by:

$$\frac{2}{3}(2v_B - v_A + x_C - v_C)$$

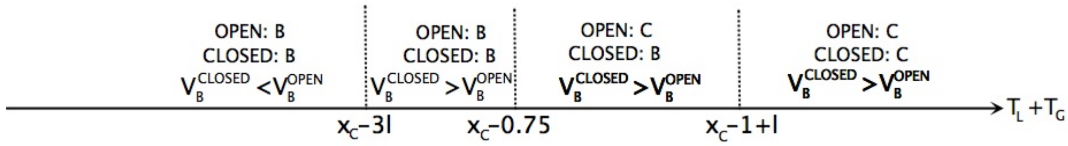
From Proposition 3, we know the proposed platforms in case of closed primaries (i.e., $\hat{x}_A = l$, $\hat{x}_B = l + v_B - v_A$). When the society is uniformly distributed the location of the indifferent voter and hence $B$'s vote share under a closed primary is given by:

$$\frac{l + v_B - v_A + x_C + v_B - v_C}{2}$$

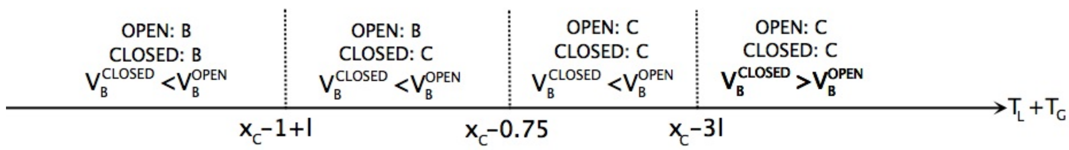
Given that the location of the indifferent voter varies across the two primary types, the selection of one system over the other clearly affects the electoral outcome and possibly the winner of the election. Comparing the location of the indifferent voter, the vote share of the challenger's party is larger under a closed primary rather than under an open primary if and only if $T_L + T_G > x_C - 3l$.

Figure 1 summarizes the comparisons across the two primary systems in terms of the challenger's vote share and the winner of the general election. Given the incumbent's characteristics $(x_C, v_C)$ and the ideology of the median voter in the leftist party $(l)$, the aggregate "toughness" the qualifying candidate $B$ is facing is crucial.[19] If the aggregate "toughness" $(T_L + T_G)$ is low the challenger wins regardless of

[19] Remember that $T_L = -(v_B - v_A)$ denotes the "toughness" in the leftist primary (with $v_A < v_B$). Large values of $T_L$ denote a very competitive primary that does not allow the primary winning candidate $B$ moderate enough and become




(a) A "moderate" party ($l > 0.25$).




(b) An "extreme" party ($l < 0.25$).

Figure 1: Winner of the general election and the challenger's vote share ($V_B$) for open and closed primaries in the challenger's party.

whether the party is running a closed or an open primary. On the contrary, if the aggregate "toughness" is high the incumbent remains in office regardless of whether the challenger won an open or a closed primary. Nevertheless, there exists an intermediate level of aggregate "toughness" where the type of primary actually determines the winner of the general election. Hence, the choice of one kind of primary over the other is of crucial importance.

Consider for example that the leftist party is "extreme" ($l = 0.1$) and that the ideal policy of the incumbent is moderately rightist ($x_C = 0.7$). Let both candidates of the general election have the same valence ($T_G = 0$) and candidate $B$ have higher valence than $A$ by 0.125 (i.e., $T_L = -0.125$). Then we are at the lower part of the graph (since $l < 0.25$) and at the interesting interval where $B$ wins the general election by organizing an open primary while it loses if it runs a closed primary (i.e., $-0.125 = T_L + T_G > x_C - 1 + l = -0.2$ and $-0.05 = x_c - 0.75 < T_L + T_G = -0.125$). This is because if $B$ wins in an open primary, the equilibrium platform is at 0.225 and hence $B$ obtains a 46.25% vote share in the general election. On the contrary, if the leftist party organizes an open primary $B$'s platform is at 0.4 and hence in the general election she obtains 55% of the total votes.

### 3.3 When the two parties use different types of primaries

When both parties hold primaries, we have so far focused on situations where both parties hold the same primary type. Given that we have previously established how open and closed primaries differ when a challenger emerging from primaries faces an incumbent, a natural followup question is what occurs when one party holds a closed primary and the other holds an open primary. Our results permit us to discuss

attractive in the general electorate. The "toughness" of the general election $T_G = -(v_B - v_C)$ also has a similar effect with larger values of $T_G$ being detrimental for the leftist candidate. Note here that while $T_L < 0$ is still true, for the general election we have that $T_G < 0$ if $v_C < v_B$ and $T_G > 0$ if $v_C > v_B$.

such situation. Let $S^A$ denote the set of all active voters participating in primaries, with a subset of them $S^C$ being eligible to participate in the closed primary. In the sincere setup we have been focusing, three natural cases emerge regarding how party members $S^C$ behave with respect to the open primary. Party members eligible to vote in the closed primary ($S^C$) still vote in a sincere manner and: a) participate only in the closed primary, b) participate in both primaries, or c) participate only in the one primary where they identify their preferred candidate.

If voters eligible to vote in the closed primary vote only in the latter, the situation is identical to the one we have described under the presence of an incumbent. In the closed primary, the low valence candidate will be locating at the party's median and the high valence candidate will be running on a more moderate platform. In the open primary, the two candidates will be focusing on the distribution of the remaining active voters ($S^A \setminus S^C$) and propose platforms as if they were facing an incumbent (i.e., the high valence candidate winning the closed primary). Hence, all intuition is as previously presented.

If voters eligible to vote in the closed primary participate in both primaries, candidates in the closed primary will be behaving exactly as before, focusing only on the party's median and the "toughness" of the closed primary. Candidates in the open primary will be running in an open primary as if they were facing an incumbent but now they will be focusing on the median of the whole set of voters ($S^A$). Hence, while the closed primary voters ($S^C$) participating only in the closed primary affect the open primary exclusively through the location of their winning candidate, voting in both primaries also affects the open primary through the distribution of voters participating in the open primary.

Finally, if party members eligible to vote in the closed primary vote only in the primary with the candidate they prefer, the situation is similar to both parties running an open primary. The difference is that while candidates running in open primaries will be focusing on all active voters ($S^A$), candidates running in the closed primary will be focusing only on a subset of them ($S^C$).

# 4 Conclusion

Neo-Downsian modeling has generated a huge literature, with the initial simplifying assumptions of Downs' classic model of two party plurality competition over a single policy dimension enriched with more realistic assumptions, including multidimensionality, party primaries (varying from open to closed), and valence as a basis for voter choice, as well as extensions to multiparty competition under electoral rules other than first past the post. Here we have contributed to that tradition by seeking to develop a model of primary competition for plurality two-party elections that matches various stylized facts about the real world, including perhaps most notably a prediction of party differentiation in the general election, and allowing for a party whose support base is closer to the position of the overall median voter to be advantaged, rather than assuming that electoral competition leads to tweedledum-tweedledee politics with each party having an equal probability of victory. We have also allowed for differences across primary type and for different results when there is or is not an incumbent. Moreover, from a theoretical perspective, the equilibrium results we have nicely complement the more common non-equilibrium results for multidimensional two-party competition.

## 5 Appendix

**Proof of Proposition 1.** This is a four-player, asymmetric and discontinuous game and hence not only there is no standardized way to characterize a unique equilibrium but even existence of an equilibrium is not trivially guaranteed. We will establish our result first by characterizing properties of an equilibrium and identifying a unique strategy profile that satisfies all of them (Step 1 Uniqueness of an equilibrium) and finally we will argue that this strategy profile is indeed a Nash equilibrium (Step 2 Existence of an equilibrium).

**Step 1** In this part of the proof we will establish a number of properties that a Nash equilibrium, $\hat{x}$, should satisfy.

I) In a Nash equilibrium, $\hat{x}$, $B$ should win the primaries of the leftist party with certainty and get a strictly positive vote share in the general election. The latter is obvious: given that $B$ is the candidate with the highest valence it trivially follows that if she runs in the general election she will be voted by a positive fraction of voters independently of the policy platforms that she and her opponent chose. As far as the first part of the claim is concerned we observe the following: since a) candidates maximize their general election vote shares, b) $B$ can always win with certainty the primaries of the leftist party by choosing, for example, any $x_B \in [l - v_B + v_A, l + v_B - v_A]$ and c) candidate $B$ always gets a strictly positive vote share if she runs in the general election against either $C$ or $D$ it follows that in equilibrium it must be the case that $B$ advances to the general election with strictly positive probability. So there are two cases: either $B$ advances to the general election with certainty or with probability $\frac{1}{2}$. Candidate $B$ advances to the general election with probability $\frac{1}{2}$ if and only if she receives exactly the same share of votes in the primaries of the leftist party as $A$. This may happen if and only if $|x_A - x_B| > v_B - v_A$ and either $\frac{x_A+v_A+x_B-v_B}{2} = l$ or $\frac{x_A-v_A+x_B+v_B}{2} = l$. In both cases $B$ can deviate marginally towards the location of candidate $A$ and secure a sure win in the primaries and practically double her expected general election vote share. Hence, it cannot be in equilibrium that $B$ advances to the general election with any probability smaller than 1.

II) In a Nash equilibrium, $\hat{x}$, it should be the case that $C$ wins the primaries of the rightist party with certainty and gets a strictly positive vote share in the general election. Since $C$ can always win with certainty the primaries of the rightist party and secure a strictly positive vote share in the general election by choosing, for example, $x_C = r - v_C + v_D$ it is obvious that in equilibrium it must be the case that $C$ advances to the general election with strictly positive probability and once in the general election against $B$ it receives a strictly positive vote share. So there are two cases: either $C$ advances to the general election with certainty or with probability $\frac{1}{2}$. Candidate $C$ advances to the general election with probability $\frac{1}{2}$ if and only if she receives exactly the same share of votes in the primaries of the rightist party as $D$. This may happen if and only if $|x_C - x_D| > v_C - v_D$ and either $\frac{x_C+v_C+x_D-v_D}{2} = r$ or $\frac{x_C-v_C+x_D+v_D}{2} = r$. In both cases $C$ can deviate marginally towards the location of candidate $D$ and secure a sure win in the primaries and practically double her expected general election vote share. Hence, it cannot be in equilibrium that $C$ advances to the general election with any probability smaller than 1.

III) In a Nash equilibrium, $\hat{x}$, it should be the case that $\hat{x}_A = \hat{x}_B - v_B + v_A$. Consider first that $\hat{x}_A < \hat{x}_B - v_B + v_A$. Since by (I) it is the case that $B$ wins with certainty the leftist party's primaries, it should be the case that $\frac{\hat{x}_A+v_A+\hat{x}_B-v_B}{2} < l$. By (II) we know that $C$ advances to general with certainty and that she receives there a strictly positive vote share. Hence, there exists $\varepsilon > 0$ such that if $B$ deviates to $\hat{x}_B + \varepsilon$ it will still be the case that $\frac{\hat{x}_A+v_A+\hat{x}_B+\varepsilon-v_B}{2} < l$ ($B$ wins the leftist party's primaries with certainty) and moreover $B$ will secure a strictly positive increase in her general election vote share. So in equilibrium it cannot be the case that $\hat{x}_A < \hat{x}_B - v_B + v_A$. Now consider that $\hat{x}_A > \hat{x}_B - v_B + v_A$. If $\hat{x}_A > \hat{x}_B + v_B - v_A$ then, given that $B$ wins with certainty the leftist party's primaries, it should be the case that $\frac{\hat{x}_A-v_A+\hat{x}_B+v_B}{2} > l$. Again, there exists $\varepsilon > 0$ such that if $B$ deviates to $\hat{x}_B + \varepsilon$ it will still be the case that $\frac{\hat{x}_A-v_A+\hat{x}_B+\varepsilon+v_B}{2} > l$ ($B$ wins the leftist party's primaries with certainty) and moreover $B$ will secure a strictly positive increase in her general election vote share. So in equilibrium it cannot be the case that $\hat{x}_A > \hat{x}_B - v_B + v_A$.

IV) In a Nash equilibrium, $\hat{x}$, it should be the case that $\hat{x}_A = l$. Consider first that $\hat{x}_A < l$. Then by (III) it follows that $\hat{x}_B = \hat{x}_A + v_B - v_A < l + v_B - v_A$. This suggests that there exists $\varepsilon > 0$ such that if $B$ deviates to $\hat{x}_B + \varepsilon$ it will be the case that $\frac{\hat{x}_A+v_A+\hat{x}_B+\varepsilon-v_B}{2} < l$ ($B$ wins the leftist party's primaries with certainty) and moreover $B$ will secure a strictly positive increase in her general election vote share. Hence, it cannot be in equilibrium that $\hat{x}_A < l$. Now consider that $\hat{x}_{A.} > l$. By (III) it follows that $\hat{x}_B = \hat{x}_A + v_B - v_A > l + v_B - v_A$. This suggests that there exists $\varepsilon > 0$ such that if $A$ deviates to $\hat{x}_A - \varepsilon$ it will be the case that $\frac{\hat{x}_A-\varepsilon+v_A+\hat{x}_B-v_B}{2} > l$ ($A$ wins the leftist party's primaries with certainty) and moreover $A$ will secure a strictly positive vote share in the general election. Hence, it cannot be in equilibrium that $\hat{x}_A > l$ either.

V) In a Nash equilibrium, $\hat{x}$, it should be the case that $\hat{x}_B = l + v_B - v_A$. This is a trivial implication of (III) and (IV).

VI) In a Nash equilibrium, $\hat{x}$, it should be the case that $\hat{x}_C = r - v_C + v_D$ and $\hat{x}_D = r$. Given (IV) and (V), that is, given that the candidates of the leftist party in equilibrium locate far from $m$ one can use similar arguments (actually simpler) to the ones employed in (III) and (IV) in order to establish that $\hat{x}_C = r - v_C + v_D$ and $\hat{x}_D = r$.

Hence, there exists a unique strategy profile that is a candidate for an equilibrium in our game;

$\hat{x} = (l, l + v_B - v_A, r - v_C + v_D, r)$.

**Step 2** In this part of the proof we have to show that no candidate has incentives to deviate from $\hat{x} = (l, l+v_B-v_A, r-v_C+v_D, r)$. First of all let us note that in this strategy profile $P_A(\hat{x}_A, \hat{x}_{-A} : v, \Phi, l, r) = P_D(\hat{x}_D, \hat{x}_{-D} : v, \Phi, l, r) = 0$ and $P_B(\hat{x}_B, \hat{x}_{-B} : v, \Phi, l, r) = 1 - P_C(\hat{x}_C, \hat{x}_{-C} : v, \Phi, l, r) = \Phi(\frac{\hat{x}_B+v_B+\hat{x}_C-v_C}{2})$.

Candidate $A$ will have incentives to deviate if there exists $\acute{x}_A \in [0, m]$ such that $P_A(\acute{x}_A, \hat{x}_{-A} : v, \Phi, l, r) > 0$. But if $A$ deviates to $\acute{x}_A < l$ then $\frac{\acute{x}_A+v_A+\hat{x}_B-v_B}{2} < l$ ($B$ wins the leftist party's primaries with certainty) and hence $P_A(\acute{x}_A, \hat{x}_{-A} : v, \Phi, l, r) = 0$. If $A$ deviates to $\acute{x}_A > l$ then at least all voters with ideal policies in will vote for candidate $B$ in the primaries of the leftist party. Notice that $l \in [0, \hat{x}_B]$ and hence $B$ wins the leftist party's primaries with certainty and $P_A(\acute{x}_A, \hat{x}_{-A} : v, \Phi, l, r) = 0$. Therefore, candidate $A$ has no incentives to deviate away from $\hat{x}_A = l$. Similar arguments rule out incentives for deviation away from $\hat{x}_D = r$ for candidate $D$ too.

If candidate $B$ deviates to $\acute{x}_B < l+v_B-v_A$ then, even if she wins in the leftist party's primaries, she gets a vote share of $\Phi(\frac{\acute{x}_B+v_B+\hat{x}_C-v_C}{2})$ that is strictly smaller than $\Phi(\frac{\hat{x}_B+v_B+\hat{x}_C-v_C}{2})$. That is her payoff is strictly smaller than $P_B(\hat{x}_B, \hat{x}_{-B} : v, \Phi, l, r)$. If candidate $B$ deviates to $\acute{x}_B > l + v_B - v_A$ then $\frac{\hat{x}_A+v_A+\acute{x}_B-v_B}{2} > l$ and hence $A$ wins the leftist party's primaries with certainty. That is, $P_B(\acute{x}_B, \hat{x}_{-B} : v, \Phi, l, r) = 0$. Therefore, candidate $B$ has no incentives to deviate from $\hat{x}_B = l + v_B - v_A$. Similar arguments rule out incentives for deviation from $\hat{x}_C = r - v_C + v_D$ for candidate $C$.

Finally, in this unique equilibrium $B$ is the winner if $\Phi(\frac{\hat{x}_B+v_B+\hat{x}_C-v_C}{2}) > \frac{1}{2} \iff l + 2v_B - v_A + r - 2v_C + v_D > 2m$, $C$ is the winner if $\Phi(\frac{\hat{x}_B+v_B+\hat{x}_C-v_C}{2}) < \frac{1}{2} \iff l+2v_B-v_A+r-2v_C+v_D < 2m$ and each of these candidates wins with probability $\frac{1}{2}$ if $\Phi(\frac{\hat{x}_B+v_B+\hat{x}_C-v_C}{2}) = \frac{1}{2} \iff l + 2v_B - v_A + r - 2v_C + v_D = 2m$.

■

**Proof of Proposition 2.** To prove existence of an equilibrium for any log-concave distribution, $F$, we first argue that for every log-concave, $F$, there exists a unique pair $(l^*, r^*) \in (0, 1)^2$ for which $2F(l^*) = F(\frac{l^*+v_B-v_A+r^*-v_C+v_D}{2} + \frac{v_B-v_C}{2})$ and $2[1 - F(r^*)] = 1 - F(\frac{l^*+v_B-v_A+r^*-v_C+v_D}{2} + \frac{v_B-v_C}{2})$; and then that no candidate has incentives to deviate from $\hat{x}_A = l^*$, $\hat{x}_B = l^* + v_B - v_A$, $\hat{x}_C = r^* - v_C + v_D$ and $\hat{x}_D = r^*$.

As far as existence of a unique pair $(l^*, r^*) \in (0, 1)^2$ with the described properties is concerned, notice that for every $q \in (0, 1)$ there exists a unique $(\bar{l}, \bar{r}) \in (0, 1)^2$ such that $2F(\bar{l}) = F(q)$ and $2[1 - F(\bar{r})] = 1 - F(q)$. When $q = 0$ we have $(\bar{l}, \bar{r}) = (0, m^a) \implies \frac{\bar{l}+v_B-v_A+\bar{r}-v_C+v_D}{2} + \frac{v_B-v_C}{2} = \frac{m^a+v_B-v_A-v_C+v_D}{2} + \frac{v_B-v_C}{2} > 0$; when $q = 1$ we have $(\bar{l}, \bar{r}) = (m^a, 1) \implies \frac{m^a+1+v_B-v_A-v_C+v_D}{2} + \frac{v_B-v_C}{2} < 1$; and both $\frac{\partial \bar{l}}{\partial q} \in (0, 1)$ and $\frac{\partial \bar{r}}{\partial q} \in (0, 1)$ - these derivatives are trivially positive and they are smaller than one due to log-concavity of $F$ (see, Le Breton and Weber 2003, 2005). That is, there should exist a unique $q \in (0, 1)$ for which $\frac{l^*+v_B-v_A+r^*-v_C+v_D}{2} + \frac{v_B-v_C}{2} = q$, which is equivalent to having that there exists a unique pair $(l^*, r^*) \in (0,1)^2$ for which $2F(l^*) = F(\frac{l^*+v_B-v_A+r^*-v_C+v_D}{2} + \frac{v_B-v_C}{2})$ and $2[1-F(r^*)] = 1-F(\frac{l^*+v_B-v_A+r^*-v_C+v_D}{2} + \frac{v_B-v_C}{2})$.

Finally, we argue that any unilateral deviation from the posited profile is unprofitable. Given that this profile is such that $A$ and $D$ locate exactly at the median of their party and $B$ and $C$ as close to the society's median voter as possible (that is, as long as they win the primaries with certainty), nobody has an incentive to deviate to a more extreme policy. But deviations towards more moderate policies cannot be trivially ruled out. Assume that candidate $B$ deviates towards the centre. Then, indeed she looses primary votes from the left but she gains primary votes from the right. For this deviation to be unprofitable we need that the gain from the right is smaller when compared to the loss from the left. Consider that $\hat{x}_B \in (l^*+v_B-v_A, m)$. In such a case the primary vote share of $B$ is $V(\hat{x}_B) = \frac{F(\frac{\hat{x}_B+v_B+\hat{x}_C-v_C}{2})-F(\frac{\hat{x}_B-v_B+\hat{x}_A+v_A}{2})}{F(\frac{\hat{x}_B+v_B+\hat{x}_C-v_C}{2})}$. We notice that: a) $\lim_{\hat{x}_B \to (l^*+v_B-v_A)^+} V(\hat{x}_B) = \frac{F(\frac{l^*+v_B-v_A+r^*-v_C+v_D}{2}+\frac{v_B-v_C}{2})-F(l^*)}{F(\frac{l^*+v_B-v_A+r^*-v_C+v_D}{2}+\frac{v_B-v_C}{2})} = \frac{1}{2}$; and b) $\frac{\partial V(\hat{x}_B)}{\partial \hat{x}_B} < 0$ for every $\hat{x}_B \in (l^*+v_B-v_A, m^a)$ due to log-concavity of $F$. In other words, if $B$ deviates to any $\hat{x}_B \in (l^*+v_B-v_A, m^a)$, she loses from $A$ in the primary and gets a payoff equal to zero. Similarly, one can show that $A$ has no profitable deviations to the left of the posited strategy. The arguments that prove that $C$ and $D$ have no incentives to change policy platforms are symmetric.

As in the proof of Proposition 1 in this unique equilibrium $B$ is the winner if $\Phi(\frac{\hat{x}_B+v_B+\hat{x}_C-v_C}{2}) > \frac{1}{2} \iff l^*+2v_B-v_A+r^*-2v_C+v_D > 2m$, $C$ is the winner if $\Phi(\frac{\hat{x}_B+v_B+\hat{x}_C-v_C}{2}) < \frac{1}{2} \iff l^*+2v_B-v_A+r^*-2v_C+v_D < 2m$ and each of these candidates wins with probability $\frac{1}{2}$ if $\Phi(\frac{\hat{x}_B+v_B+\hat{x}_C-v_C}{2}) = \frac{1}{2} \iff l^*+2v_B-v_A+r^*-2v_C+v_D = 2m$. ■

**Proof of Proposition 3.** For the same arguments as in Step 1 of the proof of Proposition 1: a) candidate $B$ must win the primary of the leftist party with certainty and get a strictly positive vote share in the general election b) in a Nash equilibrium, $\hat{x}$, it should be the case that $\hat{x}_A = \hat{x}_B - v_B + v_A$ and $\hat{x}_A = l$. Also the arguments of Step 2 of Proposition apply so that neither $A$ nor $B$ have incentives to deviate from $\hat{x} = l, l+v_B-v_A$. Finally, we observe that in this unique equilibrium $B$ is the winner if $\Phi(\frac{\hat{x}_B+v_B+x_C-v_C}{2}) > \frac{1}{2} \iff l+2v_B-v_A+x_C-v_C > 2m$, $C$ is the winner if $\Phi(\frac{\hat{x}_B+v_B+x_C-v_C}{2}) < \frac{1}{2} \iff l+2v_B-v_A+x_C-v_C < 2m$ and each of these candidates wins with probability $\frac{1}{2}$ if $\Phi(\frac{\hat{x}_B+v_B+x_C-v_C}{2}) = \frac{1}{2} \iff l+2v_B-v_A+x_C-v_C = 2m$. ■

**Proof of Proposition 4.** With similar arguments as in the proof of Proposition 2 there exists a unique value $l^*$ such that $2F(l^*) = F(\frac{l^*+v_B-v_A+x_C}{2} + \frac{v_B-v_C}{2})$ for any log-concave distribution $F$. For the same reasoning as in Proposition 2 we can then argue that neither $A$ nor $B$ have incentives to deviate from $\hat{x}_A = l^*, \hat{x}_B = l^*+v_B-v_A$. ■

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