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Further Empirical Evidence on the Consumption-Real Exchange Rate Anomaly

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Abstract

This paper adopts a nonlinear framework to model the deviations of the real exchange rate from its fundamental value implied by International Real Business Cycle models with complete asset markets. By focusing on the post Bretton Woods era, we find that in several cases there is a long run relationship between real exchange rates and consumption series in line with international risk sharing. Further, linearity tests indicate that the majority of the deviation processes exhibit significant smooth transition nonlinearity. Exponential Smooth Transition Autoregressive models appear parsimoniously to capture the nonlinear adjustment. These findings provide an explanation for the empirical regularities noted in the literature on the relation between the real exchange rate and consumption, such as the Backus and Smith (1993) puzzle. Finally, Generalized Impulse Response functions show that shock absorption is significantly faster than suggested in the Purchasing Power Parity puzzle.

Keywords: Real Exchange Rates, Consumption, Nonlinearity
JEL Classification: F41, C22, C52

1. Introduction

In the early 1990s the lack of evidence supporting Purchasing Power Parity (PPP) led researchers to focus on the identification of potential pitfalls concerning the empirical approaches employed till then as well as to provide theoretical justifications for the observed behaviour of real exchange rates.

Three of the most important avenues of research that emerged have focused on: (i) the effect of the sample size, (ii) the presence of nonlinearities in the adjustment mechanism, and (iii) the fact that real variables may affect the equilibrium real exchange rate. As far as the latter factor is concerned, International Real Business Cycle (IRBC) models, with complete or incomplete asset markets, establish a relationship between the equilibrium real exchange rate and consumption series on the basis of international risk sharing (e.g., Backus and Smith, 1993; Kollmann, 1995; Chari et al., 2002). However, the findings of a number of studies cast doubts on the empirical validity of this implication (see, e.g., Benigno and Thoenissen, 2008). The main objective of the present paper is to reassess the implied relationship between the real exchange rate and consumption by extending the sample used by previous studies and by allowing the presence of nonlinearity in the adjustment mechanism. The rest of the introductory section outlines recent advances in the literature that motivate our approach.

As noted by Frankel (1986), the tests typically employed during the 1980s to investigate whether real exchange rates are stationary may have low power when applied to small spans of data during the recent floating rate period. Following Frankel a number of researchers supported this view by using long span of data (e.g., Lothian and Taylor, 1996) and panel unit root tests (Frankel and Rose, 1996). Even though these studies provided evidence that real exchange rates mean revert in the long-run, the implied half life of deviations from PPP ranged from three to five years. The fact that real shocks cannot account for such a high degree of persistence gave rise to Rogoff's (1996) PPP puzzle.

Perhaps the most important explanation of the Rogoff puzzle is provided by theoretical models which demonstrate how transactions costs or the sunk costs of international arbitrage induce nonlinear but mean reverting adjustment of the real exchange rate (see, e.g., Dumas, 1992; Sercu et al., 1995; O'Connell and Wei, 2002). Whilst globally mean reverting, these nonlinear processes have the property of exhibiting near unit root behaviour for small deviations from PPP, since small deviations are left uncorrected if they are not large enough to cover transactions costs or the sunk costs of international arbitrage. On the other hand, large deviations are much less persistent. Hence, the low power of stationarity tests and the excess volatility of the real exchange rate may be attributed to the presence of nonlinearities in the data. In his seminal paper Dumas (1992) summarised this position as follows

> "*Linear equations are unlikely clearly to identify a process such as the one for* lnp[1] ... *in which long-run behaviour is very different from short-term behaviour, since reversion manifests itself only when deviations from parity has become wide enough.*"
>
> Dumas (1992, p. 171)

The set of parametric models that can capture the nonlinearity postulated includes the Threshold Autoregressive (TAR) model of Tong (1983) and the Smooth Transition Autoregressive (STAR) model of Granger and Teräsvirta (1993) and Teräsvirta (1994). There are two common forms of the STAR model. The one is the Exponential STAR (ESTAR) model in which transitions between a continuum of regimes are assumed to occur smoothly and symmetrically. The appealing feature of the ESTAR model is that the speed of mean reversion is increasing with the size of the deviation from the equilibrium, which implies that the corresponding half life of a shock depends on its size. The smooth adjustment process is suggested in the analysis of Dumas (1992) and demonstrated by Berka (2002). Furthermore, Teräsvirta (1994) argues that if an aggregated process is observed, regime changes may be smooth rather than discrete as long as heterogeneous agents do not act simultaneously even if they individually make dichotomous decisions, which favours the use of the ESTAR model over TAR model.

Michael et al. (1997), Taylor et al. (2001) and Kilian and Taylor (2003) among others show that ESTAR models can parsimoniously fit a number of real exchange rates. Nonlinear impulse response functions derived from the estimated models suggest that large shocks mean revert much faster than the ones previously reported for linear models, for which the speed of mean reversion is independent of the size of the shock. These findings therefore seem to go some way towards solving Rogoff's PPP puzzle. However, deviations from PPP still dissipate very slowly.

Although the early studies of PPP assumed a constant equilibrium rate, it is well recognised that even in relatively short spans of data real effects on the equilibrium exchange rate may be important. A variety

[1] where lnp denotes the real exchange rate.

of theoretical models, such as Balassa (1964) and Samuelson (1964), Lucas (1982) and Stein et al. (1995), demonstrate how real factors drive real exchange rates' movements and imply a non-constant equilibrium. Neglecting the influence of such factors may result in an omitted variable bias, which could account for the slow mean reversion reported in the empirical literature. The significance of real factors has been documented in panel data analysis (see, e.g., Canzoneri et al., 1996; Chinn and Johnston, 1996), as well as, in studies adopting a country by country nonlinear framework for long span of data (Lothian and Taylor, 2008; Paya and Peel, 2006).

International Real Business Cycle (IRBC) models predict a close relation between movements in the real exchange rate and relative consumption levels. (e.g., Backus and Smith, 1993; Kollmann, 1995). However, the evidence in favour of a link between real exchange rate and relative consumption is scarce. Backus and Smith (1993) are the first to document the lack of a systematic pattern governing the movements of real exchange rates and relative consumption by comparing the means, standard deviations and autocorrelations of the first differences of the two series. Kollmann (1995) employs the methods proposed by Park (1992) and Phillips and Ouliaris (1990) to investigate whether consumption and real exchange rates are cointegrated. By using quarterly data for the recent floating period he concludes that the complete markets model cannot match the observed consumption and real exchange rate growth rates. This result also holds using panel data (Koedijk et al., 1996). More recently, Sercu and Uppal (2000) examine a different set of countries than the set used by Kollmann (1995) for the post-Bretton Woods era and find that there is a long-run relation between consumption and real exchange rates, on the basis of the Johansen (1991) test. However, the authors do not specify if the cointegration equation is consistent with the implications of IRBC models. Finally, Head et al. (2004) employ the generalized method of moments and reject the hypothesis that there is a link between real exchange rates and relative consumption levels.

As noted by Obstfeld and Rogoff (2000) the fact that real exchange rates and consumption appear to be disconnected should be of no surprise given the high volatility of real exchange rates under floating together with the low volatility of consumption. The discrepancy between theory and empirical evidence is known as the "Backus and Smith puzzle" or the "consumption real exchange rate anomaly".

We argue that the empirical failure of IRBC models in previous studies may be due to the linear framework adopted in conjunction with the relatively short span of data available for the post-Bretton Woods era. Our line of reasoning is that factors such as the cost of arbitrage, the presence of heterogeneous agents (noise traders and rational speculators) in the market, and the fact that the equilibrium rate cannot be observed directly by the arbitrageurs may lead to persistent and inherently nonlinear deviations from economic fundamentals (e.g., Frankel and Froot, 1990; Kilian and Taylor, 2001; De Grauwe and Grimaldi, 2006). Moreover, we show that the sample correlation between the real exchange rate and relative consumption levels may be small or negative even though there is a well defined long-run structural relationship between the variables. Essentially the structural relationship is a nonlinear dynamic one so that the sample contemporaneous correlation may be misleading.[2]

When deviations from the equilibrium are small, arbitrageurs, who may be uncertain about the exact value of the equilibrium exchange rate, may be dominated by noise traders who can drive the exchange rate in the opposite direction. Hence, small misalignments of the exchange rate will be left uncorrected. However, when deviations from equilibrium become large a consensus is developed that the currency is overvalued or undervalued which, eventually, will result in driving the exchange rate towards its fundamental value. In this setting deviations from the equilibrium exhibit a high degree of persistence and smooth threshold

[2]We are aware that real business cycle models that include incomplete asset markets , non-traded goods or other market frictions can explain the contemporaneous correlation (see, e.g., Chari et al., 2002; Kehoe and Perri, 2002; Benigno and Thoenissen, 2008; Selaive and Tuesta, 2006).

dynamics. The hypothesis of a nonlinear adjustment to the equilibrium is also motivated by the empirical regularities noted by Backus and Smith (1993) and Obstfeld and Rogoff (2000), and with the difficulty of finding cointegration when linear models are used to analyse short spans of data.

The present study re-examines the validity of IRBC models during the recent floating period. By expanding the span of data used by previous studies we attempt to mitigate the low power of linear cointegration tests and to approximate the long-run relationship using the Johansen (1991) method. Subsequently, we apply the linearity test of Escribano and Jordá (1999) to the deviations from the IRBC equilibrium. We also consider two recent modifications of the linearity test which account for conditional heteroskedasticity. Our findings support the presence of smooth transition nonlinearity, which provides an explanation for the failure of cointegration tests based on relatively short span of data (e.g., Pippenger and Goering, 1993). It appears that STAR models produce parsimonious fits to the deviation series. The results of the Generalized Impulse Response Function (GIRF) suggest a fast adjustment process with half-lives between one to three years.

The rest of the paper is structured as follows. Section 2 provides a brief discussion of IRBC models based on complete asset markets and ESTAR models. The next section describes the data, the empirical methodology and the experimental results. The final section concludes.

2. The Equilibrium Real Exchange Rate in IRBC models

International Real Business Cycle models comprise an extension of the closed economy Real Business Cycle models to an international setting where transactions take place both in goods as well as in financial markets (e.g., King et al., 1988). In this setting, as long as financial markets are complete, risk sharing takes place across countries with the real exchange rate being proportional to the ratio of marginal utilities of consumption (see, e.g., Chari et al., 2002; Apte et al., 2004). It follows that IRBC models with complete markets predict that higher real consumption abroad lowers the real value of foreign currency.

To analyse this statement more formally we follow Kollmann (1995) and assume a world with K countries indexed by $k = 1, \dots, K$, each represented by an infinitely lived agent. Furthermore, it is assumed that the goods consumed differ across countries, which implies a non-constant real exchange rate. Each country's preferences are given by

$$U_k = E_s \left[\sum_{t=s}^{\infty} \beta_k^{t-s} u_{k,t}(C_{k,t}) \right], \qquad k = 1, \dots, K, \tag{1}$$

where, E is the expectations operator, $\beta_k \in (0, 1)$ is country k's subjective discount factor, $u_{k,t}(\cdot)$ is country k's instantaneous utility function in period t, and $C_{k,t}$ denotes consumption of country k. In equilibrium, the risk sharing condition for any country pair (i, j) and for all periods and states is

$$Q_t = \Lambda_{i,j} \frac{\beta_i^t m_{i,t}}{\beta_j^t m_{j,t}}, \tag{2}$$

where Q_t is the real exchange rate[3] in period t, $\Lambda_{i,j}$ is a constant, and $m_{k,t}$ is the marginal utility of consumption for country $k = i, j$. The above relation should hold even if there are frictions in goods and labour

[3]The real exchange rate is defined as

$$Q_t = S_t \frac{P_{j,t}}{P_{i,t}}$$

where S_t denotes the nominal exchange rate, units of currency i per unit of currency j, and $P_{k,t}$ denotes consumer prices for country k.

markets, such as sticky prices and sticky wages, because their effect is already reflected in consumption choices.

Taking logs and assuming that the utility function is iso-elastic with exponent $1 - n_k$, where n_k denotes the coefficient of relative risk aversion of country $k = i, j$, Equation (2) yields the model tested by Kollmann (1995) and Backus and Smith (1993)[4]

$$q_t = \lambda_{i,j} + \ln\left(\frac{\beta_j}{\beta_i}\right) t + n_i c_{i,t} - n_j c_{j,t} + z_t, \tag{3}$$

where $q_t, \lambda_{i,j}, c_{i,t}$ and $c_{j,t}$ denote the logarithms of $Q_t, \Lambda_{i,j}, C_{i,t}$ and $C_{j,t}$, respectively, and z_t denotes the deviation from the equilibrium implied by the model. Given that the coefficient of risk aversion takes positive values, a country undergoing a real depreciation should experience relative consumption growth, with a rate depending on the elasticity of intertemporal substitution in consumption.

2.1. Nonlinear Adjustment to Equilibrium

Recently, a number of authors have provided evidence in favour of smooth transition dynamics in the deviations of nominal exchange rates from macroeconomic fundamentals such as those suggested by the monetary model and the PPP (see, e.g., Taylor and Peel, 2000; Taylor et al., 2001; Paya and Peel, 2006). A model that seems to parsimoniously capture the nonlinear mean reversion postulated is the ESTAR. An ESTAR model for the process $\{z_t\}$ may be written

$$z_t - \mu = \sum_{p=1}^{\bar{p}} \phi_p (z_{t-p} - \mu) \exp\left(-\gamma \left(z_{t-1} - \mu\right)^2\right) + \epsilon_t, \tag{4}$$

where $\gamma \in (0, \infty)$ is the smoothness parameter, which determines the transition speed of function $G(z_{t-1}; \gamma, \mu) = \exp\left(-\gamma \left(z_{t-1} - \mu\right)^2\right)$ towards the inner or outer regime. The error term, ϵ_t, is assumed to follow a white noise process with mean 0 and variance σ_ϵ, and μ is a constant. Equation (4) is a popular reformulation of the ESTAR model proposed by Granger and Teräsvirta (1993). The exponential transition function, $G(\cdot)$, is particularly applicable because it implies symmetric adjustment for positive and negative deviations from the equilibrium. Further, the speed of adjustment is increasing with the smoothness parameter γ and the absolute value of the past deviation from the equilibrium. A particularly interesting case is when $\sum_{p=1}^{\bar{p}} \phi_p = 1$. In this case, at the equilibrium $G(\cdot) = 1$ and z_t will behave as a unit root process, while for larger deviations $G(\cdot) \in [0, 1)$ and z_t will mean revert. Hence, although z_t is a globally mean reverting nonlinear process, it may exhibit a high degree of persistence, which provides an explanation for the low power of stationarity test. Kilian and Taylor (2003) propose a different ESTAR parameterisation. They argue that it is more intuitive to allow the effect of the deviations from the equilibrium on the nonlinear dynamics to be cumulative. To this end, the authors suggest modifying Equation (4) to

$$z_t - \mu = \sum_{p=1}^{\bar{p}} \phi_p (z_{t-p} - \mu) \exp\left(-\gamma \sum_{d=1}^{\bar{d}} \left(z_{t-d} - \mu\right)^2\right) + \epsilon_t, \tag{5}$$

where $\bar{d}$ is a positive integer. Suppose that $\bar{d}$ differs from unity and that the smoothness parameter, γ, is significant. Then cumulative deviations are a more informative indicator of whether the market is moving towards the equilibrium value rather than a single past deviation of the process.

[4]Backus and Smith (1993) derive a restricted model with identical risk aversion coefficients, as well as, subjective discount factors across countries. Whilst, a more general model than the one of Kollmann (1995) is provided by Apte et al. (2004).

3. Data, Empirical Methodology and Experimental Results

We use quarterly data for private consumption, nominal exchange rates and consumer price indices obtained from the International Financial Statistics database for Canada, Germany, France, Japan, Sweden, the United Kingdom and the United States. The sample period is from 1973:I to 2004:IV, except for Germany and France, for which the sample period ends at 1998:IV. We set the U.S. dollar as the reference currency for the empirical analysis.

In order to investigate whether the consumption real exchange rate anomaly is present in the examined data set, we initially utilise the correlation coefficients between real exchange rates and relative consumption. These correlations vary between -0.575 and 0.101,[5] indicating that the "Backus and Smith puzzle" remains for the extended sample period. However, the correlation statistic may be an inappropriate measure for testing the validity of IRBC models due to the presence of time trends in the equilibrium equation, different risk aversion parameters and nonlinear dynamics.

3.1. Cointegration Analysis

IRBC models clearly predict that there should be a long-run relationship between real exchange rates and consumption, or equivalently if the variables are integrated of order one, I(1), they should form a cointegrating system. By the Granger Representation Theorem (Engle and Granger, 1987) the above set of variables must posses a Vector Error Correction Model (VECM) representation in which the error term, z_t, in Equation (3) comprises the deviations from the equilibrium. Let $\boldsymbol{y}_t = [q_t, c_{i,t}, c_{n,t}]$ denote the 3×1 vector of the system's variables, the VECM is written

$$\Delta \boldsymbol{y}_t = \sum_{i=1}^{p} \boldsymbol{\Gamma}_i \Delta \boldsymbol{y}_{t-i} + \boldsymbol{\Pi} \boldsymbol{y}_{t-1} + \boldsymbol{u}_t, \tag{6}$$

where Δ is the difference operator. The rank of matrix $\boldsymbol{\Pi}$ determines the number of cointegrating relationships. If matrix $\boldsymbol{\Pi}$ is of full rank, $r = 3$, the VECM reduces to a vector autoregression (VAR) and $\boldsymbol{y}_t$ is a stationary process. If $\boldsymbol{\Pi}$ is the null matrix, $r = 0$, then the system's variables are not cointegrated and the underlying process is not stationary. Finally, if $\boldsymbol{\Pi}$ is neither of full rank nor the null matrix, $0 < r < 3$, then there are r cointegrating relationships and $\boldsymbol{\Pi}$ can be decomposed

$$\boldsymbol{\Pi} = \boldsymbol{\alpha} \boldsymbol{n}' \tag{7}$$

where $\boldsymbol{n}$ are the r cointegrating vectors determining the long-run equilibrium, and $\boldsymbol{\alpha}$ denotes the matrix of the adjustment coefficients.

It is well recognised that depending on the properties of the series under examination cointegration techniques may have low power when applied to short spans of data. Further, due to serious small sample bias the coefficients obtained in cointegration analysis can vary widely across country pairs making economic interpretation very difficult (Froot and Rogoff, 1995). We examine this scenario by extending the data set used by previous studies and applying the Johansen (1991) methodology.

Table 1 reports the trace and λ-max (maximum eigenvalue) statistics for cointegration, and the long-run coefficients for consumption. Overall, the cointegration results support the existence of a long-run relationship among real exchange rates and consumptions. On the basis of both the trace and λ-max statistics

[5] These values are similar to the ones reported in the literature (see, e.g., Chari et al., 2002).

the null of no cointegration can be rejected for all countries but Japan at the 10 % significance level.[6] Both tests indicate that there is a single cointegrating relationship between the system's variables. As the last two columns of Table 1 report, the long-run coefficients, n_{US} and n_j, are correctly signed for Canada, France, Sweden and the United Kingdom suggesting that higher (lower) real consumption abroad lowers (increases) the real value of foreign currency (see Equation (3)). The implied relative risk aversion parameters are sometimes higher than the upper limit of ten suggested as reasonable by Rajnish et al. (1985). However, recent work by Barro (2005) suggests that higher values may be realistic. It is noted that the correlation coefficients for these countries with the exception of France are negative. Therefore, the difference in the relative risk aversion coefficients and/or the presence of a time trend (due to different discount rates between countries) may result in negative contemporaneous correlations between the real exchange rate and relative consumption.

TABLE 1

This is illustrated in Table 2 which reports correlation coefficients between the real exchange rate and consumptions for three different cases. The first case (ρ) corresponds to the Backus and Smith (1993) model where the relative risk aversion coefficients are assumed to be identical and a time trend is not included in the equilibrium value. In the second case (ρ_{ra}), the assumption of identical risk aversion coefficients is relaxed by using the estimates from Table 1. Finally, we also consider the effect of different risk aversion coefficients and a time trend (ρ_{tra}) for the cases that the latter is significant in the cointegration analysis.

TABLE 2

As far as Germany is concerned, although there is evidence of cointegration the results are not in line with IRBC models since the coefficient of relative risk aversion is negative. In summary, these findings support mean reversion towards the time-varying equilibrium specified by IRBC models.

3.2. Linearity Testing

A complementary reason for the empirical regularities reported in the IRBC literature, such as the estimated values of correlation coefficients (Backus and Smith, 1993), the difference in volatility (Obstfeld and Rogoff, 2000), and the difficulty of finding cointegration (Kollmann, 1995), may be that the deviations process is governed by nonlinear dynamics. Next, we investigate whether the deviations series exhibit significant STAR nonlinearity of the type suggested by Kilian and Taylor (2003). Escribano and Jordá (1999) developed a linearity test that provides useful insights concerning the presence of STAR nonlinearity, and the specification of the transition variable.

In deriving an LM test for the null of linearity against STAR nonlinearity we adopt the typical STAR model (Teräsvirta, 1994; Escribano and Jordá, 1999; van Dijk et al., 2002)

$$z_t = \boldsymbol{\phi}'\boldsymbol{x}_t + \boldsymbol{\theta}'\boldsymbol{x}_t F(s_t, \gamma, c) + u_t, \tag{8}$$

[6] Japan is excluded from the remaining analysis since no cointegration was found.

where $\boldsymbol{x}_t = (1, z_{t-1}, \ldots, z_{t-p})'$, $\boldsymbol{\phi} = (\phi_0, \ldots, \phi_p)$, $\boldsymbol{\theta} = (\theta_0, \ldots, \theta_p)$, s_t is the transition variable, γ is the transition parameter and c is a constant. The transition function $F(\cdot)$ for the ESTAR model is defined by

$$F(s_t, \gamma, c) = \left[1 - \exp\left(-\gamma(s_t - c)^2\right)\right]. \tag{9}$$

In the case of a Logistic STAR (LSTAR) model

$$F(s_t, \gamma, c) = \left[1 + \exp\left(-\gamma(s_t - c)\right)\right]^{-1}. \tag{10}$$

Testing linearity in this framework is not straightforward due to the presence of unidentified-nuisance parameters (Davies, 1977). Luukkonen et al. (1988) overcome the identification problem by replacing $F(\cdot)$ with a Taylor series approximation. The resulting equation permits the use of LM tests which asymptotically posses the χ^2 distribution. Escribano and Jordá (1999) extended the work of Luukkonen et al. (1988) and Teräsvirta (1994) and proposed a new specification strategy to choose between ESTAR and LSTAR models based on the following equation[7]

$$z_t = \boldsymbol{\delta}_0'\boldsymbol{x}_t + \boldsymbol{\delta}_1'\boldsymbol{x}_t s_t + \boldsymbol{\delta}_2'\boldsymbol{x}_t s_t^2 + \boldsymbol{\delta}_3'\boldsymbol{x}_t s_t^3 + \boldsymbol{\delta}_4'\boldsymbol{x}_t s_t^4 + u_t. \tag{11}$$

- Estimate Equation (11) and obtain the p-value, p_1, for the null hypothesis of linearity, $H_0^1 : \boldsymbol{\delta}_1' = \boldsymbol{\delta}_2' = \boldsymbol{\delta}_3' = \boldsymbol{\delta}_4' = 0$.
- If linearity is rejected,
  - test the null $H_0^L : \boldsymbol{\delta}_2' = \boldsymbol{\delta}_4' = 0$ with an F-test and obtain the corresponding p-value, p_E.
  - test the null $H_0^L : \boldsymbol{\delta}_1' = \boldsymbol{\delta}_3' = 0$ with an F-test and obtain the corresponding p-value, p_L.
- If $p_E < p_L$ select ESTAR, otherwise select LSTAR.

The implementation of the above procedure requires the specification of the lag length p and the transition variable s_t. We follow previous studies and set $p = 2$ for all countries but Sweden and the United Kingdom, for which we set $p = 4$ so as to deal with residual autocorrelation. Kilian and Taylor (2003) argue that although most studies employ a single past deviation as the transition variable, it is more intuitive to allow the effects of persistent deviations to be cumulative. To this end, we consider $s_t = (\sum_{d=1}^{\bar{d}} z_{t-d}^2)^{1/2}$, where $\bar{d}$ denotes the lag with the minimum p-value for the null of linearity, H_0^1, and we allow a maximum of 8 lags.

An important issue when testing the presence of STAR nonlinearities is the presence of conditional heteroskedasticity in the model's residuals. For example, Lundbergh and Teräsvirta (1998) examine the linearity test of Teräsvirta (1994) and conclude that conditional heteroskedasticity may result in severe size distortions and that the robust version of Granger and Teräsvirta (1993) appears to have very low power.[8] Pavlidis et al. (2009) show that the Escribano and Jordá (1999) test exhibits similar problems as the test of Teräsvirta (1994) and investigate the performance of possible alternatives to improve its properties (size and size-adjusted power). Their findings suggest that the use of the Heteroskedasticity Consistent Covariance Matrix Estimator (HCCME) of MacKinnon and White (1985) improves upon the size, but results in very low size-adjusted power. On the other hand, the Fixed Design Wild Bootstrap appears to lead to a marked improvement both in terms of size and size-adjusted power.

[7] This new procedure appears to be consistent and to generate much higher correct selection frequencies (see Paya and Peel, 2005).

[8] See Lundbergh and Teräsvirta (1998) for the specification, estimation and evaluation of models with nonlinear behaviour in the mean (STAR) and in the conditional variance (STGARCH), the STAR-STGARCH model.

TABLE 3

The results of the Escribano and Jordá procedure using the Least Squares Covariance Matrix (LS), the Heteroskedasticity Consistent Covariance Matrix of MacKinnon and White (1985) (HC), and the Fixed Design Wild Bootstrap (WB) are presented in Table 3. Overall, linearity is rejected for the majority of cases. For Canada, France, and the United Kingdom the results are qualitatively similar between the three versions of the LM test. The null hypothesis, H_0^1, is rejected at least at the 10% significance level and the same transition variable is selected for each country by all tests, $\bar{d} = 8, 7$ and 2. The latter finding supports the use of the model proposed by Kilian and Taylor instead of the ESTAR model usually adopted in the literature with $\bar{d} = 1$. In the case of Sweden, only the original version of the Escribano and Jordá procedure rejects the null of linearity at the 5% significance level, which implies that $\bar{d} = 1$. Germany is the only country for which the deviations from the equilibrium do not appear to follow a STAR process. Although the results suggest the selection of an LSTAR model rather than an ESTAR for all countries but Sweden the difference in the associated p-values is marginal. Given that there is no prior reason for an asymmetric adjustment, the remaining analysis focuses on ESTAR models.

3.3. Estimation of the ESTAR models and the Wild Bootstrap

We examine the performance of the two ESTAR models (4) and (5) (discussed in Section 2.1) in capturing the nonlinear dynamics of the deviations series, z_t. While the former model is used for all countries, the model proposed by Kilian and Taylor is only employed for Canada, France and the United Kingdom. This is due to the linearity test results, which indicate that the effect of the deviations are not cumulative, i.e. $\bar{d} < 2$, for Germany and Sweden. Furthermore, we cannot reject the restriction that z_t follows a unit root process at the equilibrium, $H_0 : \sum \phi_p = 1$. Table 3 shows the estimates of the restricted ESTAR models, the standard error of the regression, the corresponding t-statistic, the Ljung-Box Q-statistic for serial correlation in the residuals and the LM test statistic (ARCH) for conditional heteroskedasticity up to lags 1 and 4. The Q-statistic does not indicate the presence of serial correlation in the regression residuals. However, there is some evidence of conditional heteroskedasticity for Sweden and the United Kingdom.

In order to test the significance of the smoothness parameter, γ, in the presence of conditional heteroskedasticity or non-normality we employ the Fixed Design Wild Bootstrap (see, e.g., Wu, 1986; Mammen, 1993; Davidson and Flachaire, 2001). The asymptotic validity of the Fixed Design Wild Bootstrap for stationary autoregressions with known finite lag order when the error term exhibits conditional heteroskedasticity of unknown form is established in Gonçalves and Kilian (2004). Their results cover as special cases the N-GARCH, t-GARCH and asymmetric GARCH models, as well as, stochastic volatility models. The procedure we follow is to impose the null H_0: $\gamma = 0$ and simulate 1,000 series for z_t, denoted by z_t^b according to

$$z_t^b = \hat{\mu} + \sum_{p=1}^{\bar{p}} \hat{\phi}_p (z_{t-p} - \hat{\mu}) + \epsilon_t^b. \tag{12}$$

The residuals ϵ_t^b are constructed by multiplying the residuals obtained by the ESTAR model, $\hat{\epsilon}_t$, by a random variable, η_t, that follows the Rademacher distribution

$$\eta_t = \begin{cases} -1 & \text{with probability } p = 0.5 \,, \\ 1 & \text{with probability } (1-p). \end{cases}$$

The η_t are mutually independent drawings from a distribution independent of the original data. The distribution has the properties that $E(\eta_t) = 0$, $E(\eta_t^2) = 1$, $E(\eta_t^3) = 0$, and $E(\eta_t^4) = 1$. A consequence of these properties is that any heteroskedasticity or symmetric non-normality in the estimated residuals ($\hat{\epsilon}_t$) is preserved in the newly created residuals.[9]

This procedure provides an empirical distribution for $\hat{\gamma}$ and the associated standard errors. The idea in $1,000$ replications is to determine the appropriate t-values so we do not reject the null of $\hat{\gamma} = 0$. These critical values can then be used to determine whether the estimates of $\hat{\gamma}$ reject the null or not (see also Paya and Peel, 2006). The Wild Bootstrap p-values under the null H_0: $\gamma = 0$, are also reported in Table 4.

TABLE 4

The Wild Bootstrap p-values imply that the estimated transition parameters are in each case significant for all conventional levels, which supports the nonlinear nature of the deviation processes. Therefore, the difficulty of detecting cointegration in short samples may be attributed to large and persistent deviations generated by the ESTAR adjustment mechanism. Further, the high short term volatility of the real exchange rates compared to the volatility of the consumption series is, also, in accordance with the implications of the ESTAR model. We conduct a Monte Carlo experiment in the following section, which illustrates the above points.

3.4. Generating the Puzzle

We are interested in examining the behaviour of the correlation coefficients between the real exchange rate and relative consumption and the properties of linear cointegration tests when the true DGP is nonlinear.

To this end, we calibrate nonlinear models by using parameter values similar to the estimated ones. For simplicity we assume that the two consumption series follow a driftless random walk

$$
\begin{aligned}
c_{i,t} &= c_{i,t-1} + u_{i,t}, \qquad u_{i,t} \sim N(0, 0.02), \\
c_{j,t} &= c_{j,t-1} + u_{j,t}, \qquad u_{j,t} \sim N(0, 0.02).
\end{aligned}
$$

The DGP for the real exchange rate is given by

$$
\begin{aligned}
q_t \quad = \quad & -0.07t + 6c_{i,t} - 9c_{j,t} + (1.2(q_{t-1} + 0.07(t-1) - 6c_{i,t-1} + 9c_{j,t-1}) - \\
& -0.1(q_{t-2} + 0.07(t-2) - 6c_{i,t-2} + 9c_{j,t-2}) + 0.2(q_{t-3} + 0.07(t-3) - \\
& -6c_{i,t-3} + 9c_{j,t-3}) + 0.1(q_{t-4} + 0.07(t-4) - 6c_{i,t-4} + 9c_{j,t-4})) \cdot \\
& \cdot \exp\left(-0.3(q_{t-1} + 0.07(t-1) - 6c_{i,t-1} + 9 * c_{j,t-1})^2\right) + \epsilon_t,
\end{aligned}
$$

where $\epsilon_t \sim N(0, 0.15)$. We set the sample size equal to 128 observations and generate 1,000 series for each variable. In turn, we obtain the correlation coefficients between the "fake" real exchange rate series and the "fake" relative consumption. The percentage of negative correlation coefficients is 46.4, implying that the likelihood of observing a small or negative correlation is large.

Further, we examine the power of the Johansen (1991) test to detect cointegration between the "fake" q_t, $c_{i,t}$ and $c_{j,t}$. The null hypothesis of no cointegration can be rejected in 43.1 percent of the cases when the nominal significance level is 10 percent. However, if we change the sample size to 70 observations, which is about the sample size used by Kollmann (1995), the power deteriorates to only 15.7 percent, indicating the importance of the sample length.

[9] The Wild Bootstrap matches the moments of the observed error distribution around the estimated regression function at each design point ($\widehat{z}^b$). Liu (1988) and Mammen (1993) show that the asymptotic distribution of the Wild Bootstrap statistics are the same as the statistics they try to mimic.

3.5. Generalized Impulse Response Functions

In this context, it is also of importance to investigate whether the estimated nonlinear models, as well as, the inclusion of the equilibrium determinants can explain the PPP puzzle regarding the slow rate at which shocks appear to damp out. Impulse response analysis addresses this issue by focusing on the effect of a shock on the behaviour of the deviation process. However, a number of studies have shown that impulse response analysis is considerably more complex for nonlinear models when compared to linear models (see Gallant et al., 1993; Koop et al., 1996; Potter, 2000; van Dijk et al., 2007). In particular, impulse responses produced by nonlinear models depend on (i) the history set, that is the initial conditions, (ii) the size and sign of the current shock, and (iii) the shocks that occur in future periods. Koop et al. (1996) propose a measure, the Generalized Impulse Response Function (GIRF), which deals with the complications entailed in impulse response analysis for nonlinear models. The GIRF is defined as the average difference between two realizations of the stochastic process, z_{t+h}, which start with identical histories up to time $t-1$, but only the first realization is hit by a shock of magnitude δ_t at period t.

$$\text{GIRF}(h, \delta_t, \omega_{t-1}) = E\left[z_{t+h}|\epsilon_t = \delta_t, \omega_{t-1}\right] - E\left[z_{t+h}|\omega_{t-1}\right], \tag{13}$$

where $h = 1, 2 \ldots$ denotes horizon, $\epsilon_t = \delta_t$ is an arbitrary shock occurring at time t, and ω_{t-1} is the history set of z_t. Given that the GIRF$(h, \delta, \omega_{t-1})$ is a function of δ_t and ω_{t-1}, which are realizations of random variables, the GIRF$(h, \delta, \omega_{t-1})$ itself is a realization of a random variable. It follows that various conditional versions of the GIRF can be defined. For example, we can condition on the shock and treat the variables generating the history as random. Alternatively, we can consider a specific history and treat the GIRF as a random variable in terms of the shock. In general, we can condition on a subset of shocks and a subset of histories, depending on the specific application. In this work, we choose to condition upon "all past histories" so as to examine the time profile of the effects of shocks of different magnitudes on the future patterns of the series variable.

Due to the fact that analytic expressions for the conditional expectations involved in (13) are usually not available for $h > 1$, we use bootstrap integration methods (see Koop et al., 1996, for a detailed description) to overcome the issue of future shocks intrinsically incorporated in the model.[10] In particular, for each available history 200 repetitions are implemented to average out future shocks, where future shocks are drawn with replacement from the models residuals, and then the results across all histories are averaged. The maximum impulse response horizon is set to 48 quarters and we consider shocks of magnitude $\delta_t = \psi\hat{\sigma}_\epsilon$, where $\hat{\sigma}_\epsilon$ is the residual standard deviation and $\psi = 1,\ 3,\ 5$.

In order to measure the rate at which the final effect of an impulse, δ_t, is attained we compute the π-life or π-absorption time (see van Dijk et al., 2007)

$$N(\pi, \delta_t, \omega_{t-1}) = \sum_{m=0}^{\infty}\left(1 - \prod_{h=m}^{\infty} I(\pi, h, \delta_t, \omega_{t-1})\right), \tag{14}$$

where $0 \leq \pi \leq 1$ and $I(\pi, h, \delta_t, \omega_{t-1})$ is the indicator function which takes the value of 1 if at least a fraction $1-\pi$ of the difference between the initial and ultimate effects of δ_t has been absorbed after h periods and 0 otherwise.[11] The π-life corresponds to the minimum horizon beyond which the difference between the

[10] An analytical expression of the "impulse response function" for the deterministic skeleton of a restricted ESTAR model is provided by Venetis et al. (2007).

[11] The indicator function is defined as

$$I(\pi, h, \delta_t, \omega_{t-1}) = I\left[|\text{GIRF}(h, \delta_t, \omega_{t-1}) - \text{GIRF}^{\infty}(\delta_t, \omega_{t-1})| \leq \pi|\delta_t - \text{GIRF}^{\infty}(\delta_t, \omega_{t-1})|\right]$$

impulse responses at all longer horizons and the ultimate response is less than or equal to the fraction π of the difference between the initial impact and the ultimate response. Note that the above definition of π-life differs from the definition usually adopted in the literature, which is the shortest horizon at which at least a fraction $1-\pi$ of the initial effect, δ_t, has been absorbed. This is an appealing feature since monotonicity is not granted. That is, $I(\pi, h, \delta_t, \omega_{t-1}) = 1$ does not necessarily imply $I(\pi, h+j, \delta_t, \omega_{t-1}) = 1$, $\forall j > 0$.

TABLE 5

Table 5 displays the π-lives of shocks for the estimated ESTAR models of the deviation series (see Table 4). The results reported further illustrate the nonlinear nature of the real exchange rate with time-varying equilibrium models, with the absorption time decreasing with the size of the shock. Moreover, the reduction in the time needed to absorb fraction $(1-\pi)$ of different size shocks depends on the proportion $(1-\pi)$. In other words, if the shock increases from $1 \times \hat{\sigma}_\epsilon$ to $5 \times \hat{\sigma}_\epsilon$ the reduction in the time needed to absorb 25% of both shocks is not generally the same as the reduction in time needed to absorb 50% of the shocks. The half-lives corresponding to the smallest shocks range between 7 and 14 quarters, while for the largest shocks the half-lives range between 3 and 9 quarters. The absorption time also depends on the specific ESTAR formulation. For Canada and France the absorption time is much smaller when the Kilian and Taylor ESTAR model is adopted, but not for the United Kingdom. However, the results are qualitatively similar. Given that consensus estimates of linear models suggest a half-life between 3 and 5 years (see Rogoff, 1996), these results, in accordance with the results of other studies adopting a nonlinear framework, seem to go some way towards solving the PPP puzzle.

4. Conclusion

The present study adopts an IRBC framework, where the equilibrium real exchange rate is determined by consumption series. By focusing on the recent float, we find evidence in favour of a long-run relationship in line with the risk sharing condition implied by IRBC models with complete markets for most of the countries under examination. The results of linearity tests indicate that the deviations from the equilibrium, as estimated by the Johansen (1991) method, exhibit STAR nonlinearity. We fit ESTAR models and employ the Fixed Design Wild Bootstrap so as to draw inferences in the presence of conditional heteroskedasticity. The estimated models appear to parsimoniously fit the deviation processes. The nonlinear nature of the series provides an explanation for the empirical regularities noted in literature and the results of studies using shorter spans of data. Finally, we address the PPP puzzle regarding the slow absorption rate of shocks by employing GIRFs. Our findings suggest that shocks to the deviations from the IRBC equilibrium have short half-lives.

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Table 1: Cointegration Results

			Trace			λ-max				
Country j	p	trend	$r <= 2$	$r <= 1$	$r = 0$	$r <= 2$	$r <= 1$	$r = 0$	n_{US}	n_j
Canada	7		0.24	8.64	40.87***	0.24	8.40	32.22***	12.85	16.01
									(2.51)	(2.97)
France	12	$\surd$	3.77	16.52	41.69*	3.77	12.75	25.17*	7.75	1.01
									(1.15)	(0.98)
Germany	7	$\surd$	4.53	16.04	42.28*	4.53	11.51	26.24**	4.26	-1.42
									(0.67)	(0.23)
Japan	7	$\surd$	4.92	16.91	34.79	4.92	11.99	17.88	—	—
									(—)	(—)
Sweden	8	$\surd$	7.28	18.24	63.74***	7.28	10.96	45.50***	16.57	9.43
									(2.49)	(1.68)
United Kingdom	3		0.01	11.09	41.75***	0.01	11.08	30.66***	5.05	4.59
									(1.21)	(1.17)

NOTE: The system variables are $(q_t, c_{t,US}, c_{t,j})$, where q_t is the real exchange rate and $c_{t,k}$ is the real consumption at time t for country $k = i, j$. The cointegrating vector has been normalised with respect to the real exchange rate. Figures in parentheses denote standard errors. The lag length of the Vector Autoregression (VAR) was determined on the basis of the Schwartz Information Criterion (SIC), allowing for a maximum length of 8 lags. However, in the case that the LM test indicated that the VEC residuals exhibited serial correlation up to order 8 the lag length was increased. ***, **, * denote significance at the 1%, 5%, and 10% significance level, respectively.

Table 2: The Consumption Real Exchange Rate Anomaly

Correlation	Canada	France	Sweden	United Kingdom
ρ	-0.575	0.101	-0.553	-0.274
ρ_{ra}	0.393	0.045	-0.556	0.031
ρ_{tra}	—	0.255	0.149	—

NOTE: ρ denotes the correlation coefficient between real exchange rate and relative consumption, while ρ_{ra} is the correlation coefficient adjusted for the different levels of relative risk aversion and ρ_{tra} is the coefficient adjusted for both the different levels of relative risk aversion and a time trend.

Table 3: Linearity Testing

Country j		p	$\bar{d}$	p_1	p_E	p_L
Canada						
	OLS	2	8	0.002	0.008	0.003
	HC	2	8	0.023	0.165	0.023
	WB	2	8	0.008	0.028	0.010
France						
	OLS	2	7	0.066	0.029	0.017
	HC	2	7	0.051	0.030	0.017
	WB	2	7	0.080	0.052	0.030
Germany						
	OLS	2	3	0.298	0.133	0.118
	HC	2	2	0.235	0.122	0.109
	WB	2	3	0.364	0.226	0.194
Sweden						
	OLS	4	1	0.033	0.015	0.036
	HC	4	6	0.308	0.692	0.753
	WB	4	5	0.226	0.142	0.194
United Kingdom						
	OLS	4	2	0.006	0.014	0.012
	HC	4	2	0.001	0.040	0.030
	WB	4	2	0.036	0.044	0.040

NOTE: The length of the autocorrelation is denoted by p, while $\bar{d}$ shows the number of lags included in the transition variable for which the p-value for the null of linearity, p_1, is the lowest. p_E and p_L are the p-values for the null hypotheses of LSTAR and ESTAR nonlinearity, respectively.

Table 4: Estimated ESTAR Models

Canada

(a) Typical ESTAR Model Parameterisation

$$\hat{z}_t = -\underset{[1.913]}{0.287} + (\underset{[11.105]}{1.002}(z_{t-1} + \underset{[1.913]}{0.287}) + (1 - \underset{[11.105]}{1.002})(z_{t-2} + \underset{[1.913]}{0.287}))\cdot$$
$$\cdot \exp(-\underset{\substack{[3.217]\\(0.010)}}{0.069}(z_{t-1} + \underset{[1.913]}{0.287})^2)$$

$s = 0.146$, $Q_1 = 0.784\ (0.376)$, $Q_4 = 3.803\ (0.433)$,
$\text{ARCH}_1 = 0.005\ (0.946)$, $\text{ARCH}_4 = 0.579\ (0.678)$

(b) Kilian and Taylor Parameterisation

$$\hat{z}_t = -\underset{[1.783]}{0.145} + (\underset{[9.467]}{0.893}(z_{t-1} + \underset{[1.783]}{0.145}) + (1 - \underset{[9.467]}{0.893})(z_{t-2} + \underset{[1.783]}{0.145}))\cdot$$
$$\cdot \exp(-\underset{\substack{[2.966]\\(0.000)}}{0.032}\mathbf{\Sigma}_{d=1}^{8}(z_{t-d} + \underset{[1.783]}{0.145})^2)$$

$s = 0.137$, $Q_1 = 0.011\ (0.917)$, $Q_4 = 2.739\ (0.602)$,
$\text{ARCH}_1 = 1.829\ (0.179)$, $\text{ARCH}_4 = 0.684\ (0.605)$

France

(a) Typical ESTAR Model Parameterisation

$$\hat{z}_t = \underset{[0.900]}{0.040} + (\underset{[14.423]}{1.382}(z_{t-1} - \underset{[0.900]}{0.040}) + (1 - \underset{[14.423]}{1.382})(z_{t-2} - \underset{[0.900]}{0.040}))\cdot$$
$$\cdot \exp(-\underset{\substack{[3.153]\\(0.000)}}{0.643}(z_{t-1} - \underset{[0.900]}{0.040})^2)$$

$s = 0.074$, $Q_1 = 0.242\ (0.623)$, $Q_4 = 3.378\ (0.497)$,
$\text{ARCH}_1 = 0.087\ (0.769)$, $\text{ARCH}_4 = 0.294\ (0.881)$

(b) Kilian and Taylor Parameterisation

$$\hat{z}_t = \underset{[1.403]}{0.066} - (\underset{[12.945]}{1.310}(z_{t-1} - \underset{[1.403]}{0.066}) + (1 - \underset{[12.945]}{1.310})(z_{t-2} - \underset{[1.403]}{0.066}))\cdot$$
$$\cdot \exp(-\underset{\substack{[2.551]\\(0.003)}}{0.121}\mathbf{\Sigma}_{d=1}^{7}(z_{t-d} - \underset{[1.403]}{0.066})^2)$$

$s = 0.077$, $Q_1 = 0.231\ (0.630)$, $Q_4 = 2.140\ (0.710)$,
$\text{ARCH}_1 = 0.271\ (0.604)$, $\text{ARCH}_4 = 0.602\ (0.662)$

Germany

(a) Typical ESTAR Model Parameterisation

$$\hat{z}_t = -\underset{[0.774]}{0.028} + (\underset{[11.797]}{1.190}(z_{t-1} + \underset{[0.774]}{0.028}) + (1 - \underset{[11.797]}{1.190})(z_{t-2} + \underset{[0.774]}{0.028}))\cdot$$
$$\cdot \exp(-\underset{\substack{[2.508]\\(0.000)}}{2.340}(z_{t-1} + \underset{[0.774]}{0.028})^2)$$

$s = 0.066$, $Q_1 = 0.293\ (0.588)$, $Q_4 = 2.138\ (0.710)$,
$\mathrm{ARCH}_1 = 1.582\ (0.211)$, $\mathrm{ARCH}_4 = 0.751\ (0.560)$

Sweden

(a) Typical ESTAR Model Parameterisation

$$\hat{z}_t = \underset{[0.152]}{0.010} + (\underset{[13.342]}{1.180}(z_{t-1} - \underset{[0.152]}{0.010}) - \underset{[0.709]}{0.093}(z_{t-2} - \underset{[0.152]}{0.010}) + \underset{[1.536]}{0.200}\cdot$$
$$\cdot(z_{t-3} - \underset{[0.152]}{0.010}) + (1 - \underset{[13.342]}{1.180} + \underset{[0.152]}{0.093} - \underset{[1.536]}{0.200})(z_{t-4} - \underset{[0.152]}{0.010}))\cdot$$
$$\cdot \exp(-\underset{\substack{[3.770]\\(0.000)}}{0.305}(z_{t-1} - \underset{[0.152]}{0.010})^2)$$

$s = 0.159$, $Q_1 = 0.303\ (0.582)$, $Q_4 = 0.540\ (0.969)$,
$\mathrm{ARCH}_1 = 4.046\ (0.047)$, $\mathrm{ARCH}_4 = 1.136\ (0.343)$

United Kingdom

(a) Typical ESTAR Model Parameterisation

$$\hat{z}_t = -\underset{[1.151]}{0.051} + (\underset{[12.547]}{1.134}(z_{t-1} + \underset{[1.151]}{0.051}) + \underset{[0.259]}{0.034}(z_{t-2} + \underset{[1.151]}{0.051}) + \underset{[0.590]}{0.078}\cdot$$
$$\cdot(z_{t-3} + \underset{[1.151]}{0.051}) + (1 - \underset{[12.547]}{1.134} - \underset{[0.259]}{0.034} - \underset{[0.590]}{0.078})(z_{t-4} + \underset{[1.151]}{0.051}))\cdot$$
$$\cdot \exp(-\underset{\substack{[3.276]\\(0.000)}}{0.652}(z_{t-1} + \underset{[1.151]}{0.051})^2)$$

$s = 0.075$, $Q_1 = 0.045\ (0.832)$, $Q_4 = 2.379\ (0.666)$,
$\mathrm{ARCH}_1 = 5.212\ (0.024)$, $\mathrm{ARCH}_4 = 2.163\ (0.078)$

(b) Kilian and Taylor Parameterisation

$$\hat{z}_t = -\underset{[1.198]}{0.046} + (\underset{[12.233]}{1.093}(z_{t-1} + \underset{[1.198]}{0.046}) + \underset{[0.617]}{0.082}(z_{t-2} + \underset{[1.198]}{0.046}) + \underset{[0.725]}{0.095}\cdot$$
$$\cdot(z_{t-3} + \underset{[1.198]}{0.046}) + (1 - \underset{[12.233]}{1.093} - \underset{[0.617]}{0.082} - \underset{[0.725]}{0.095})(z_{t-4} + \underset{[1.198]}{0.046}))\cdot$$
$$\cdot \exp(-\underset{\substack{[3.684]\\(0.000)}}{0.387}\boldsymbol{\Sigma}_{d=1}^{2}(z_{t-d} + \underset{[1.198]}{0.046})^2)$$

$s = 0.074$, $Q_1 = 0.008\ (0.930)$, $Q_4 = 1.645\ (0.897)$,
$\mathrm{ARCH}_1 = 4.936\ (0.028)$, $\mathrm{ARCH}_4 = 1.801\ (0.133)$

NOTE: Figures in square brackets denote the ratio of the absolute value of the estimated coefficient to the estimated standard error of the coefficient estimate. The Wild Bootstrap $p-$value for the γ coefficient is reported in parentheses below the coefficient estimate. s is the standard error of the regression. Q_1 and Q_4 denote the Ljung-Box Q-statistic for serial correlation up to order 1 and 4, respectively. ARCH_1 and ARCH_4 denote the LM test statistic for conditional heteroskedasticity up to order 1 and 4, respectively.

Table 5: Estimated π-lives of Shocks

		Shock Absorption		
Country	δ_t	0.25	0.50	0.80
	$1 \times \hat{\sigma}_\epsilon$	6 (4)	14 (10)	34 (22)
Canada	$3 \times \hat{\sigma}_\epsilon$	5 (4)	11 (8)	30 (18)
	$5 \times \hat{\sigma}_\epsilon$	3 (3)	9 (6)	25 (13)
	$1 \times \hat{\sigma}_\epsilon$	7 (7)	10 (9)	17 (15)
France	$3 \times \hat{\sigma}_\epsilon$	7 (6)	10 (9)	17 (15)
	$5 \times \hat{\sigma}_\epsilon$	5 (5)	8 (8)	15 (13)
	$1 \times \hat{\sigma}_\epsilon$	4 (-)	7 (-)	13 (-)
Germany	$3 \times \hat{\sigma}_\epsilon$	3 (-)	5 (-)	12 (-)
	$5 \times \hat{\sigma}_\epsilon$	2 (-)	3 (-)	9 (-)
	$1 \times \hat{\sigma}_\epsilon$	7 (-)	9 (-)	12 (-)
Sweden	$3 \times \hat{\sigma}_\epsilon$	6 (-)	8 (-)	11 (-)
	$5 \times \hat{\sigma}_\epsilon$	4 (-)	6 (-)	9 (-)
	$1 \times \hat{\sigma}_\epsilon$	7 (9)	9 (11)	11 (14)
United Kingdom	$3 \times \hat{\sigma}_\epsilon$	5 (7)	7 (9)	10 (13)
	$5 \times \hat{\sigma}_\epsilon$	4 (5)	5 (7)	8 (11)

NOTE: The table reports the absorption time for the typical ESTAR parameterisation and the Kilian and Taylor parameterisation. Figures in parentheses correspond to the latter. In the cases of Germany and Sweden only the typical ESTAR parameterisation is employed.