

# Geometric Rigidity Workshop

Lancaster University

Abstracts

June 2016

## **Bob Connelly (Cornell) - Universally rigid complete bipartite graphs**

A graph is universally rigid if it is rigid not just in the dimension it sits in, but in all higher dimensions as well. It turns out that there is a very simple criterion to detect universal in the case when the graph is complete bipartite. For the generic case, the graph is universally rigid if and only if the partitions cannot be separated by a quadric. For the non-generic case, it is more complicated, but still depends on stress matrices and quadrics. This is joint work with S. Gortler.

## **Elad Hahn (Tel Aviv) - A Universal Induction Step Algorithm for Obtaining Assur Graphs**

An algorithm is proposed used for obtaining Assur Graphs. The algorithm uses a single induction step and is based on the property of strong-connectivity of Assur Graphs. The algorithm is named "universal" since it can be applied to both Body-Bar and Bar-Joint graphs, creating a complete set of Assur Graphs, which are the building blocks of all engineering systems. A general idea of the soundness of the algorithm will be provided.

## **Oleg Karpenkov (Liverpool) - Some open questions on geometry of tensegrities.**

In this talk we briefly discuss several open questions regarding the stratification of the configuration space of all tensegrities.

## **Wayne Lam (TU-Berlin) - Holomorphic quadratic differentials on planar meshes**

Every discrete holomorphic quadratic differential yields a discrete minimal surface. In this talk, we discuss holomorphic quadratic differentials from the viewpoint of discrete complex analysis. We focus on their relations to discrete harmonic functions and infinitesimal conformal deformations.

## **Stephen Power (Lancaster) - Crystal flex complexity**

The RUM spectrum, RUM multiplicity and RUM dimension give some measures of the infinitesimal flex "complexity" of a 3-periodic bar-joint framework. We describe a more simple minded crystal flex complexity index in  $\mathbb{N} \cup \infty$ , arising from an infinite dimensional vector space perspective, and point out some connections and conjectured connections with the RUM spectrum and a more general spectrum, the "geometric flex spectrum". (Work in progress with Derek Kitson and Ghada Badri.)

### **Bernd Schulze (Lancaster) - Rigidity preserving transformations**

It is well known that infinitesimal rigidity can be transferred between a bar-joint framework in Euclidean d-space and a bar-joint framework on the d-sphere which has no joints on the equator. By allowing joints to lie on the equator we extend this correspondence and show that infinitesimal rigidity can be transferred between a bar-joint framework in d-space with a subset of points lying on a hyperplane and a point-hyperplane framework in d-space. In particular, this allows us to extend a result of Jackson and Jordan characterising the rigidity of bar-joint frameworks in the plane with three collinear points to an arbitrary number of collinear points. We also briefly discuss further connections to parallel drawings of d-scenes and to the rigidity of frameworks with various types of slider constraints. This is joint work with Yaser Eftekhari, Bill Jackson, Anthony Nixon, Shin-ichi Tanigawa and Walter Whiteley.

### **Brigitte Servatius (WPI) - From combinatorial zeolites to geometric realizations**

Combinatorial zeolites are line graphs of three regular graphs. For planar vertex transitive 3-regular graphs there is always a unit distance realization of the line graph and we conjecture that this is also the case if we relax the condition of vertex transitivity to finitely many vertex orbits.

### **Meera Sitharam (Florida) - Constructions and generalizations of d-flattenable graphs**

We give a forbidden minor type characterization of graphs  $G$  and nonedges  $f$  such that for every assignment of lengths to the edges of  $G$ , the set of realizable lengths for  $f$  is a single interval. The characterization is general and works for any realization dimension  $d$ .

Parameterizing realization spaces of linkages using nonedge lengths (Cayley parameter) has proven useful in multiple applications from CAD to molecular and materials modeling. This result generalizes two previous results of the speaker and coauthors: a similar result for  $d=2$ ; and the equivalence of two properties:  $d$ -flattenable of the graph and existence of a

convex Cayley realization space (set of realizable edge lengths) of the graph in  $d$ -dimensions, for lengths defined by any norm.

**Shin-Ichi Tanigawa (CWI) - Singularity degree of graphs**

As observed by Connelly and Gortler (2014), the singularity degree of the Euclidean distance matrix completion problem indicates the complexity of characterizing the universal rigidity. In this talk I will show several relations between the singularity degree and graph properties.

**Louis Theran (St Andrews) -  $K_{7,4}$  is GGR in 3d**

I'll show how to construct super stable realisations of  $K_{7,4}$ . The argument is an exercise with points and quadrics in  $P^3$ . This is joint work with Bob Connelly and Shlomo Gortler.