FLORENCE NIGHTINGALE DAY 17TH DECEMBER 2015

QUIZ SOLUTIONS

Note: almost all problems are taken from Martin Gardner, Mathematical Circus, Penguin Books, 1985.

Q1.

A dish needs 15' cooking. You only have two hour glasses (filled with sand), one of 7' and another of 11'. How can you cook the dish for 15' exactly?

Solution. Several possibilities. The fastest: start both hour glasses at the same time, when the egg is dropped into boiling water. When sand finish dropping in the 7' hour glass turn it over. When the 11' one finishes, turn over once again the 7' hour glass. When this latter stops, 15' will have elapsed.

Q2.

Draw six line segments of equal length to form eight equilateral triangles.

Solution. One solution is drawn in Figure 1.

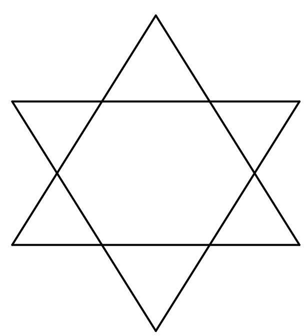

FIGURE 1. One solution, many other possibilities!

Q3.

A man travels $5,000$ miles in a car with one spare tyre. He rotated the tyres on his car so that at the end of the trip, each tyre had traveled the same number of miles. For how many miles was each tyre used?

Solution. Each tyre is used for $\frac{4}{5}$-th of the journey, hence $4,000$ miles.

Q4.

A magic square is an array of numbers such that the sum of each row and each column add up to the same total. Below is a 7×7 magic square containing the integers $1, 2, \ldots, 49$.

Solution. Note the cyclic pattern!

26	17	8	6	46	37	35
18	9	7	47	38	29	27
10	1	48	39	30	28	19
2	49	40	31	22	20	11
43	41	32	23	21	12	3
42	33	24	15	13	4	44
34	25	16	14	5	45	36

Q5.

You have ten stacks of coins of 20p. Exactly one entire stack is counterfeit, but you do not know which one. You know the exact weight of a single coin of 20p, and you also know that a single counterfeit coin weighs 1g more than a genuine 20p coin. You may weigh the coins on a pointer scale. What is the minimum number of weighings necessary to determine which stack is counterfeit?

Solution.

A single weighing suffices. Take one coin from the first stack, two from the second, and so forth to the entire 10 coins tenth stack (so that's 55 coins in total). The excess weight in grams of the lot tells you how many counterfeit coins you have in it, and hence from which stacks these come. (E.g. if that weighs 3 more grams that 55 genuine coins would weigh, then the third stack of coins is counterfeit.)

Q6.

Two or more spots are placed anywhere on the circumference of a circle. Every pair is joined by a straight line, dividing the disk into a certain number of regions. Figure 2 shows the maximum number of regions into which the circle can be divided given six spots.

The formula giving the maximum number of regions with n spots is

$$n + \binom{n}{4} + \binom{n-1}{2} = \frac{n^4 - 6n^3 + 23n^2 - 18n + 24}{24}$$

giving indeed 31 with $n = 6$.

Q7.

A knight's tour is a path taken by a knight on a chessboard: two cells in one direction and one cell perpendicular. The rules for such "tour" is that the knight can visit a cell at

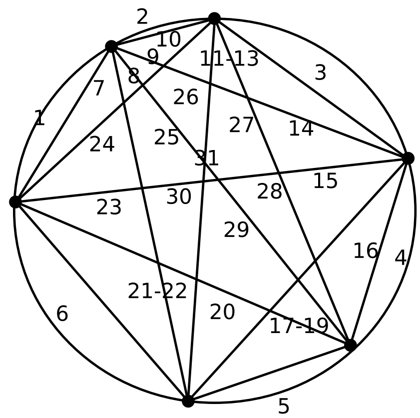

FIGURE 2. 6 Spots on a circle

most once, and cannot cross his path. On a 6×6 chessboard, the maximum length of a knight's tour is drawn in Figure 3.

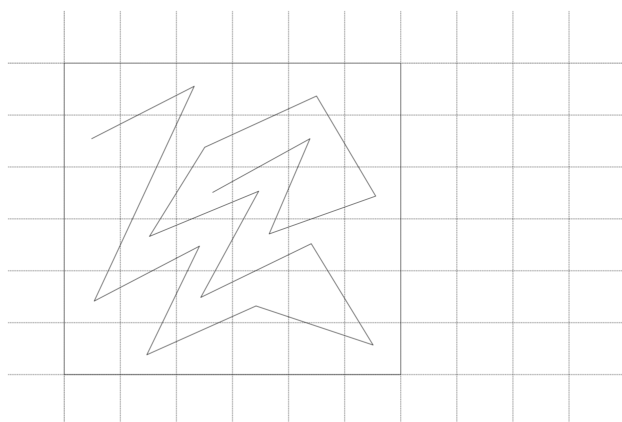

FIGURE 3. Knight's move: 17 individual moves is the maximum.

Q8.

A cyclic number is a number X formed by n digits, such that for each integer $k = 1, \ldots, n$, the multiple kX of X contains the same digits listed in the same cyclic order (that is $abc \sim bca \sim cab$). Trivially, 1 is a cyclic number, since $1 \cdot 1 = 1$.

Solution. The following six digits number is a cyclic number

$$1\ 4\ 2\ 8\ 5\ 7$$

Indeed, we calculate the first six successive multiples

$$142857 \quad 285714 \quad 428571 \quad 571428 \quad 714285 \quad 857142$$

Q9.

Planar maps are coloured according to the rule that no two regions sharing (part of) a common edge can have same colour.

Solution. The six edges in the map of Figure 4 have same length, and the minimum number of colours required is 4.

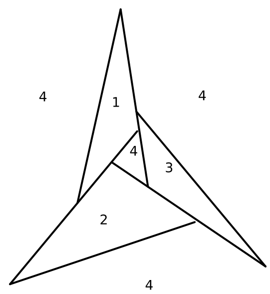

FIGURE 4. A map requiring 4 colours at least.

Q10.

Ali, Ben and Cathy lunch together every day. If Ali orders a coffee, then so does Ben. Either Ben or Cathy always orders a coffee, but never both during the same lunch. Either Ali or Cathy or both always order a coffee, and if Cathy orders a coffee, then so does Ali.

You decide to join them for a lunch on Tuesday. Who amongst Ali, Ben and Cathy will order a coffee then?

Solution. We represent all combinations in a Venn diagram, where: A=Alice, B=Ben, C=Cathy, and a "-" in front of A,B,C is a negation (i.e. no coffee). This is Figure 5. Then we eliminate the impossibles given by the constraints:

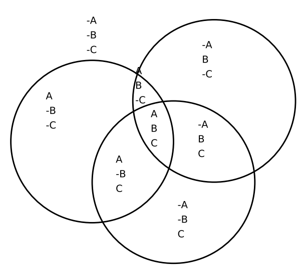

FIGURE 5. All combinations for coffees.

(1) If Ali orders a coffee, then so does Ben: rules out A,-B,C and A,-B,-C.
(2) Either Ben or Cathy always orders a coffee, but never both during the same lunch: rules out the remaining A,B,C and -A,B,C and -A,-B,-C.
(3) Either Ali or Cathy or both always order a coffee: rules out the remaining -A,B,-C.
(4) If Cathy orders a coffee, then so does Ali: rules out the remaining -A,-B,C.

Thus, by elimination, the only option is A,B,-C order coffee at every lunch. In other words, when you join them, Ali and Ben will order a coffee, but not Cathy.

Q11.

There is a unique number of 10 digits which has the property that the first digit gives the number of 0 in it, the second digit that of 1, and so forth, up to the tenth digit which is the number of 9 forming it. Find this number.

Solution. This is mainly guesswork, unless... a few obvious facts: this number cannot have any 8 nor 9, and there are many 0. So, with some more trials, we get the answer:

$$6,210,001,000$$