MODELLING THE SEASONALITY OF EXTREME WAVES IN THE GULF OF MEXICO

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Context

- Ocean structures must be safe.
- Estimation of extreme environments is important.
- Gap to fill between regulatory requirements, engineering practice and latest statistical approaches.
- Regulatory requirements ad-hoc (if not inconsistent) w.r.t. accommodation of covariate effects and estimation of (e.g.) directional and seasonal design values.
- Statistics literature provides framework for consistent and rational estimation.

Procedure in a nut shell: I

- Hindcast data for multiple locations in neighbourhood. Extract *storm peaks* (to eliminate temporal dependence) over threshold u.
- Assume extremal characteristics of all locations marginally identical, although dependent. Goal is to estimate distribution of n-year return value q_n for any single location.
- Estimation using NHPP: storm arrival rate μ, GP shape γ and scale σ.
- Accommodate covariate effects: μ, γ, σ and u vary with covariates (e.g. direction, season, time). u estimated before hand as high (local) quantile (sensitivity to threshold choice).
- Maximise likelihood, penalised by parameter roughness w.r.t. covariates. Diagnostics for model fit. Cross-validation for optimal roughness. Block bootstrapping for parameter uncertainty pointwise.

Procedure in a nut shell: II

- Simulate to estimate properties of q_n.
- Estimate q_n also for partitions w.r.t. covariates. Estimate and accommodate storm dissipation effects.
- Present findings in engineering terms.

Recent work

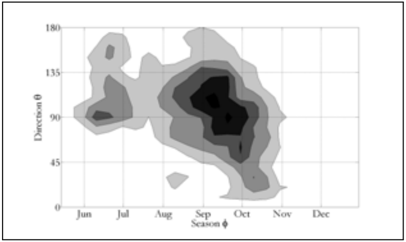
Large body of statistical and engineering literature on extremes. Our recent contributions include:

- Effect of combining locations on estimation uncertainty (Jonathan and Ewans 2007b).
- Illustrations of extent of covariate effects on extreme quantile estimates (Jonathan et al. 2008).
- Modelling directional extremes in the Gulf of Mexico and Northern North Sea (Jonathan and Ewans 2007a, Ewans and Jonathan 2008).

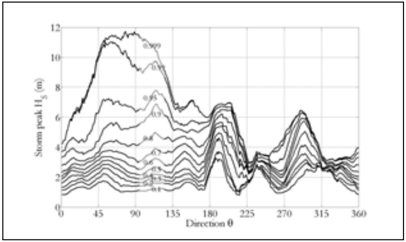
Procedure

- Significant wave height H_S values from GOMOS Study (Oceanweather, 2005), for September 1900 to September 2005 inclusive, at 30-minute intervals.
- For two typical locations (henceforth "A" and "B"), selected 78 grid points on 13 x 6 rectangular lattice with spacing with 0.125 (14 km).
- For each storm period for each grid point, isolated storm peak significant wave height, H_S^{sp}, corresponding wave direction, θ, and storm peak season ϕ. 315 storms.
- Define season ϕ on the interval $[0, 360)$ corresponding to one year and refer to a value of ϕ as a *seasonal degree*, approximately equal to day of the year.

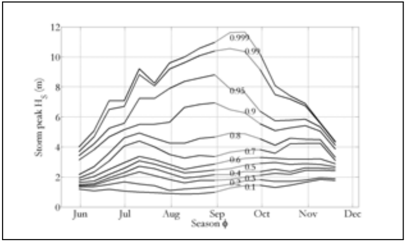
Exploratory analysis at A



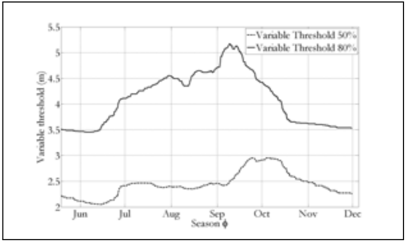
Density with direction, season



Quantiles by direction

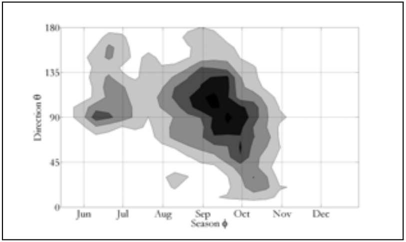


Quantiles by season

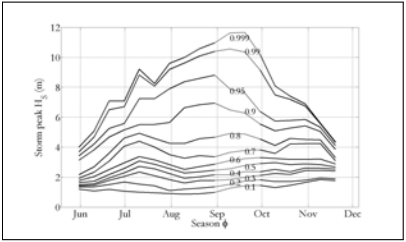


Threshold by season

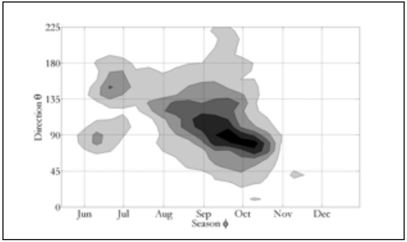
Comparing exploratory analysis at A and B



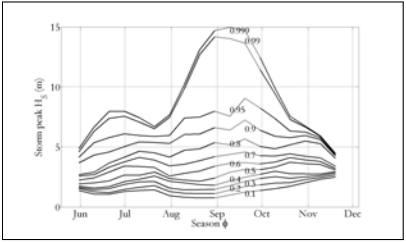
Density with direction, season at A



Quantiles by season at A



Density with direction, season at B



Quantiles by season at B

Generalised Pareto Modelling: I

Given $\{X_i\}_{i=1}^{n}$, $\{\phi_i\}_{i=1}^{n}$, distribution of storm peaks above variable threshold $u(\phi)$ assumed GP with cdf $F_{X_i|\phi_i,u}$:

$$
\begin{aligned}
F_{X_i|\phi_i,u}(x) &= P\left(X_i \leq x|\phi_i, u(\phi_i)\right) \\
&= 1 - \left(1 + \tfrac{\gamma(\phi_i)}{\sigma(\phi_i)}\left(x - u(\phi_i)\right)\right)_{+}^{-\frac{1}{\gamma(\phi_i)}}
\end{aligned}
$$

γ and σ vary smoothly with season, assumed to follow Fourier form $\sum_{k=0}^{p}\sum_{b=1}^{2} A_{abk}\, t_b(k\phi)$.

Generalised Pareto Modelling: II

Penalised negative log likelihood is l^*:

$$l^* = \sum_{i=1}^{n} l_i + \lambda \left(R_\gamma + \frac{1}{w} R_\sigma \right)$$

Unpenalised negative log likelihood is:

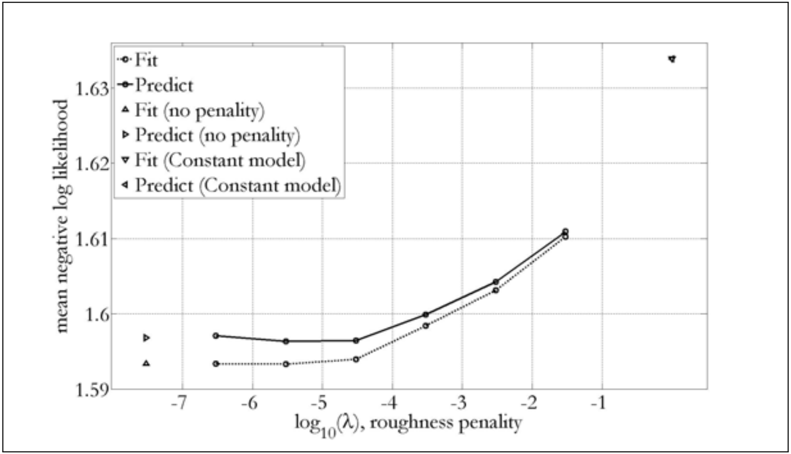
$$l_i = \log \sigma(\phi_i) + \left(\frac{1}{\gamma(\phi_i)} + 1 \right) \log \left(1 + \frac{\gamma(\phi_i)}{\sigma(\phi_i)} (X_i - u(\phi_i)) \right)_+$$

Roughness of γ is given by:

$$R_\gamma = \int_0^{2\pi} \left(\frac{\partial^2 \gamma}{\partial \phi^2} \right)^2 d\phi = \sum_{k=1}^{p} \pi k^4 \left(\sum_{b=1}^{2} A_{1bk}^2 \right)$$

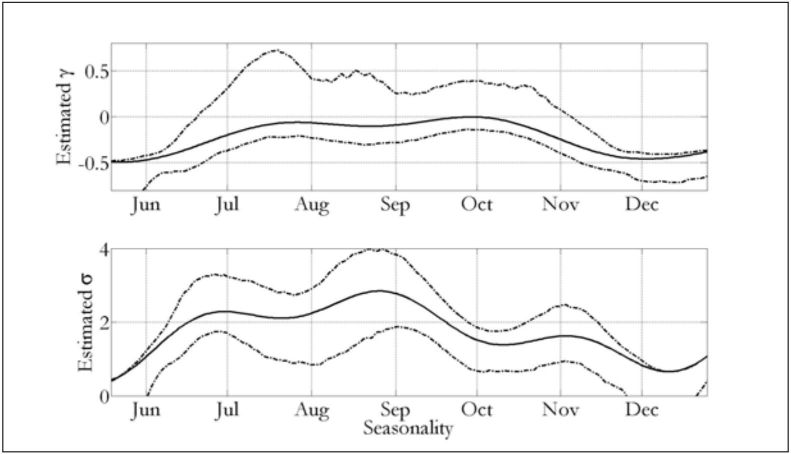
Analogous expression for roughness of σ

Cross-validation for roughness



Value of λ, 50% threshold at A

Forms of γ and σ



50% threshold at A, with block bootstrap 95% confidence interval

Poisson Modelling: I

Non-homogeneous Poisson process model. The negative log-likelihood written:

$$l(\mu,\gamma,\sigma) = l_N(\mu) + l_W(\gamma,\sigma)$$

where l_N is the (negative) log-density of the total number of exceedances (with rate argument μ), and l_W is the (negative)log-conditional-density of exceedances given a known total number N). Inferences on μ made separately from those on γ and σ.
The Poisson process log-likelihood, for arrivals at times $\{t_i\}_{i=1}^{n}$ in period P_0 is:

$$l_N(\mu) = -(\sum_{i=1}^{n} \log \mu(t_i) - \int_{P_0} \mu(t)dt)$$

Poisson Modelling: II

Or approximately (Chavez-Demoulin and Davison 2005):

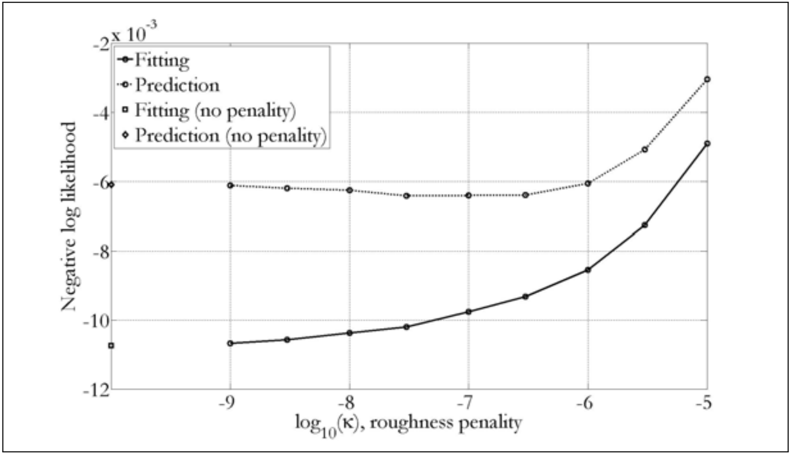
$$\hat{l}_N(\mu) = -(\sum_{j=1}^{m} c_j \log \mu(j\delta) - \delta \sum_{j=1}^{m} \mu(j\delta))$$

where $\{c_j\}_{j=1}^{m}$ is the number of occurrences in each of the m sub-intervals. W
We estimate storm occurrence rate adopting a Fourier form for Poisson intensity μ, penalising its roughness R_μ:

$$\hat{l}_N^*(\mu) = \hat{l}_N(\mu) + \kappa R_\mu$$

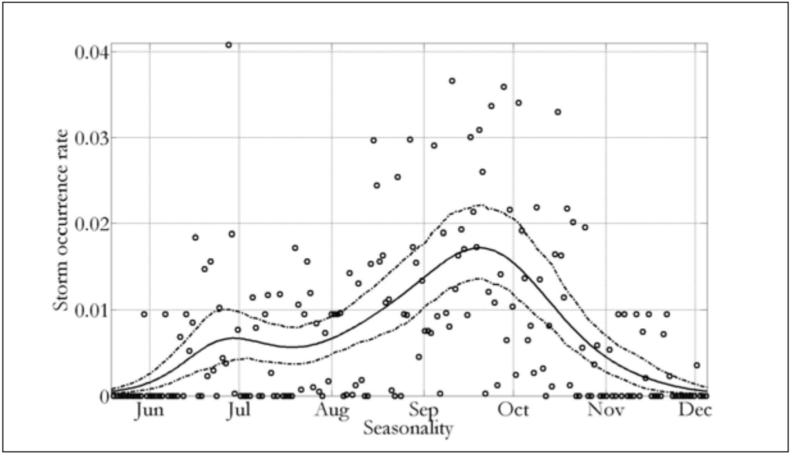
R_μ has form analogous to that of R_γ or R_σ.

Cross-validation for roughness



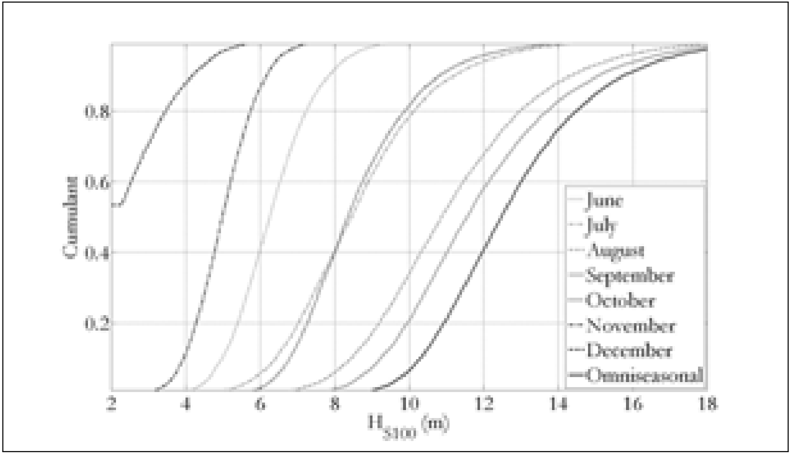
Value of κ, 50% threshold at A

Form of μ



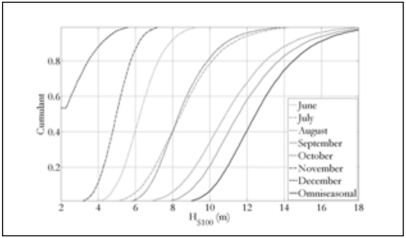
50% threshold at A, with block bootstrap 95% confidence interval

100-year storm peak cdf

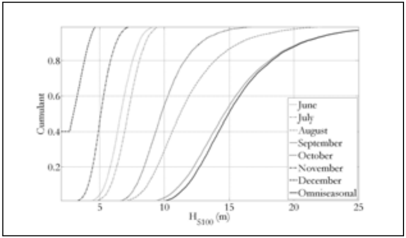


50% threshold at A

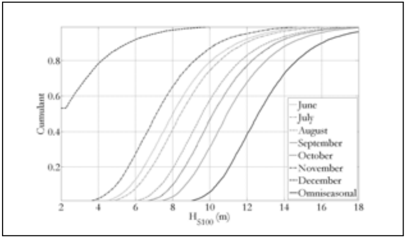
Comparing seasonal and constant models



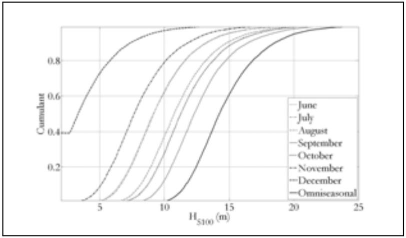
Seasonal, 50% threshold at A



Seasonal, 50% threshold at B

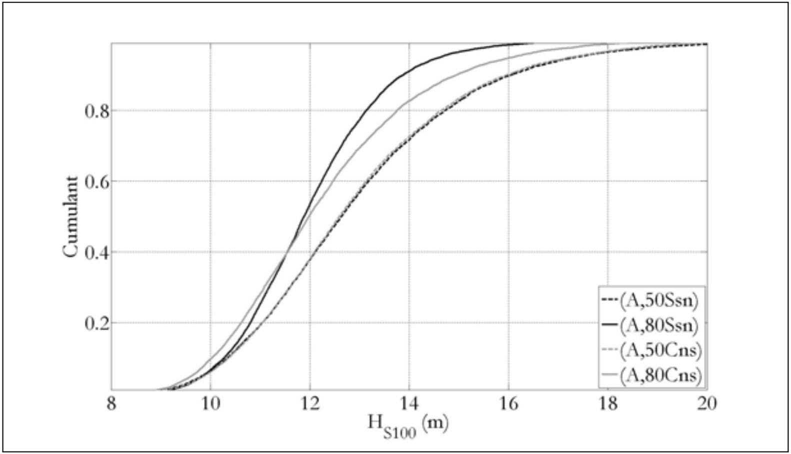


Constant, 50% threshold at A



Constant, 50% threshold at B

Comparing models and threshold choices at A



Design values for different strategies at A

Seasonal model								
	Variable threshold set at 50%				Variable threshold set at 80%			
	Omniseasonal		Equal monthly NEP		Omniseasonal		Equal monthly NEP	
Month	Hs (m)	NEP	Hs (m)	NEP	Hs (m)	NEP	Hs (m)	NEP
January	12.5	1.00	3.0	0.93	11.9	1.00	5.5	0.94
February	12.5	1.00	3.8	0.93	11.9	1.00	3.9	0.94
March	12.5	1.00	3.9	0.93	11.9	1.00	4.2	0.94
April	12.5	1.00	3.3	0.93	11.9	1.00	5.8	0.94
May	12.5	1.00	4.5	0.93	11.9	1.00	5.0	0.94
June	12.5	1.00	8.2	0.93	11.9	1.00	7.8	0.94
July	12.5	0.96	11.9	0.93	11.9	0.92	12.2	0.94
August	12.5	0.74	15.2	0.93	11.9	0.75	13.1	0.94
September	12.5	0.65	15.9	0.93	11.9	0.69	14.1	0.94
October	12.5	0.97	11.4	0.93	11.9	0.96	11.1	0.94
November	12.5	1.00	6.4	0.93	11.9	1.00	6.5	0.94
December	12.5	1.00	4.6	0.93	11.9	1.00	4.3	0.94
Constant model								
January	12.6	1.00	3.2	0.94	12.0	1.00	5.0	0.94
February	12.6	1.00	3.9	0.94	12.0	1.00	4.0	0.94
March	12.6	1.00	4.1	0.94	12.0	1.00	4.3	0.94
April	12.6	1.00	3.7	0.94	12.0	1.00	5.7	0.94
May	12.6	1.00	6.5	0.94	12.0	0.99	8.1	0.94
June	12.6	0.94	12.5	0.94	12.0	0.97	10.9	0.94
July	12.6	0.93	12.9	0.94	12.0	0.91	12.5	0.94
August	12.6	0.87	14.1	0.94	12.0	0.84	13.5	0.94
September	12.6	0.75	15.3	0.94	12.0	0.73	14.6	0.94
October	12.6	0.83	14.5	0.94	12.0	0.88	13.2	0.94
November	12.6	0.96	11.4	0.94	12.0	0.98	10.2	0.94
December	12.6	1.00	6.6	0.94	12.0	1.00	7.3	0.94

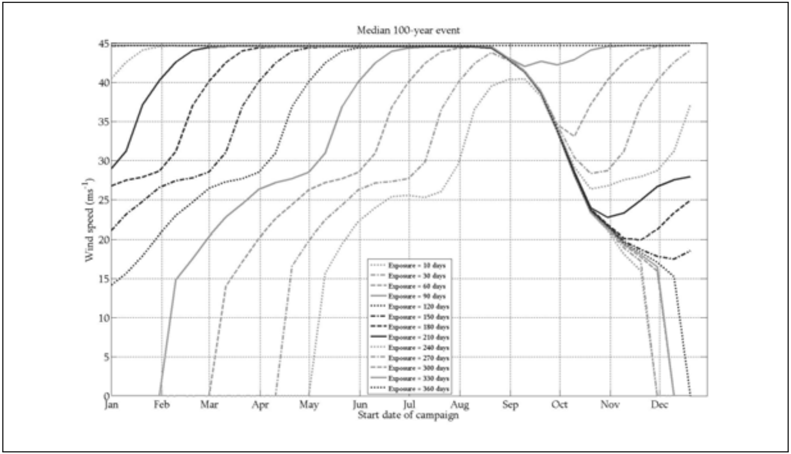
Main findings

- Rational, consistent approach to ocean design.
- Sensitivity to more arbitrary choices (e.g. threshold).
- Accommodation of covariate effects.
- Allowing threshold to vary w.r.t. covariates captures a considerable amount of the covariate effect.
- Combining locations is less advantageous for seasonal covariate than for direction
    - Storm occurrences across locations are more localised seasonally than directionally

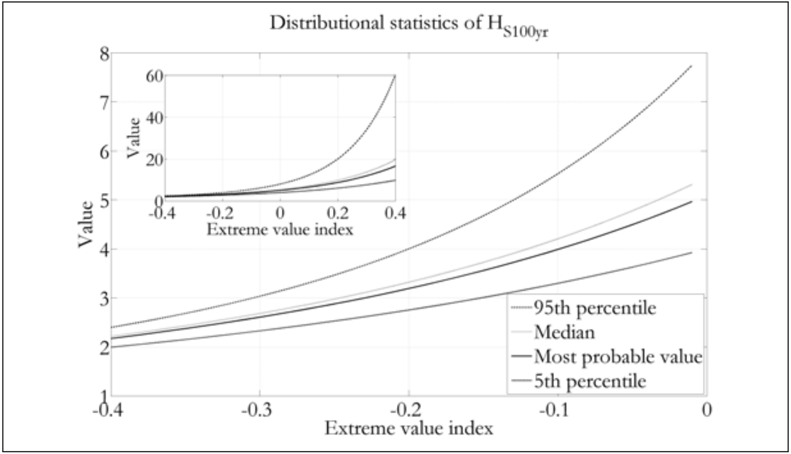
Future work

- Jointly model spatial and temporal dependency. Extreme quantiles for region rather than single location.
- Jointly model multiple variables (wind, waves, current, e.g. Heffernan and Tawn 2004), compare inferences with *response-based* approaches.
- Model multiple covariate effects (e.g. more general non-parametric smoothers).
- Improved modelling of dissipation effects
- Estimates for extreme quantiles incorporating uncertainties from model and threshold specification
- Influence design practice. Regulators (e.g. API) currently reviewing methods for seasonal and directional design.
- Bridge industry and academia, communicate.

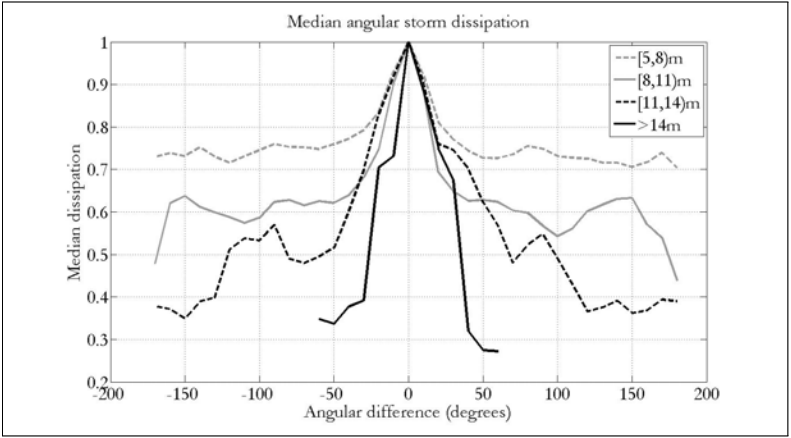
Communicating results intuitively



Interpreting parameters



Improved modelling of storm dissipation



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Thanks for listening.

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