LECTURES 15-16: INPUT DEMAND

ANSWERS AND SOLUTIONS

True/False Questions

True_ Consider two firms, X and Y, that have the same cost function, $C(q)$, for a given set of input prices. The production function of firm X allows for greater substitutability between inputs, compared to the production function of firm Y. Then, if the price of any input changes, the cost function of firm X will be lower than that of firm Y.

False_ When a profit maximizing firm with a fixed proportions production function experiences a decline in the price of one of its inputs it will still produce the same amount of output.

True_ If a firm produces output using a fixed proportions production function, then an increase in the price of the output will increase the demand for all inputs proportionately.

False_ A firm with substantial input substitutability has a cost function that is more responsive to input price changes, relative to a firm with lower input substitutability.

False_ Suppose that when w_K doubles, the profit maximizing demand for capital goes down by 40%. For this firm, if instead it were w_L that doubled, the profit maximizing demand for capital would have gone down by less than 40%.

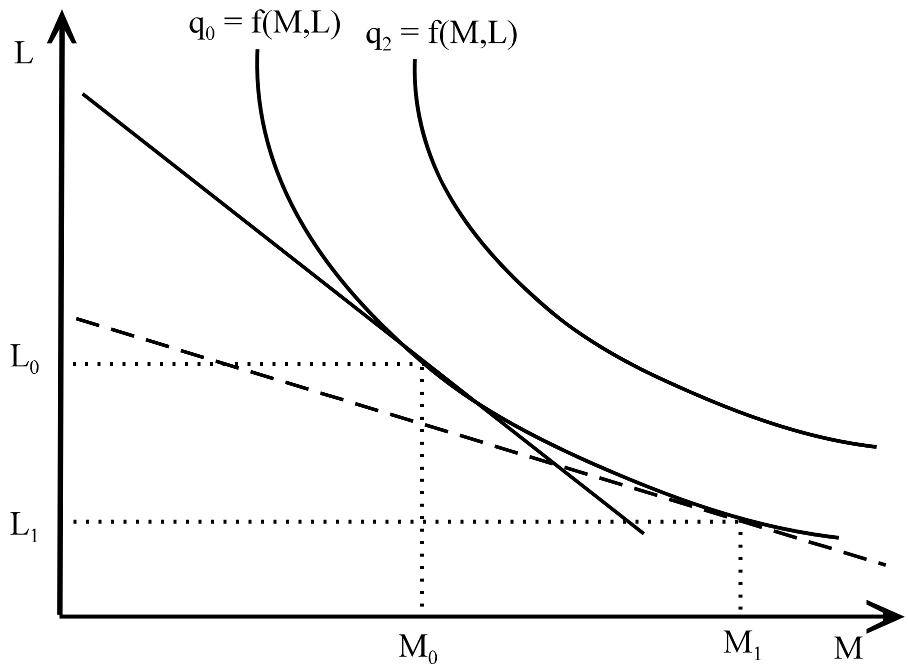
True_ An increase in the price of an input will lead to a larger reduction in the demand of that input for a perfectly competitive firm than for an otherwise identical firm facing a downward sloping demand curve.

True_ A decrease in the price of an input will lead to a larger increase in the demand of that input for a perfectly competitive firm than for an otherwise identical firm facing a downward sloping demand curve.

False_ A firm that has perfect substitutability between two inputs is not affected by the increase in the price of one of this inputs because it can always switch to using the other.

Short Questions

1. A firm is currently producing output q_0 using L_0 units of labor and M_0 units of other materials. The isoquant that corresponds to output level q_0 and the firm's optimal input choices are given in the figure below.



A. In the above graph, draw with a solid line the isocost that corresponds to the lowest cost of producing output q_0. [Hint: think of what must be the least cost isocost if L_0 and M_0 are the optimal input choices.]

Notice that the isocost is tangent to the $q=q_0$ isoquant at the point $\{M_0, L_0\}$.

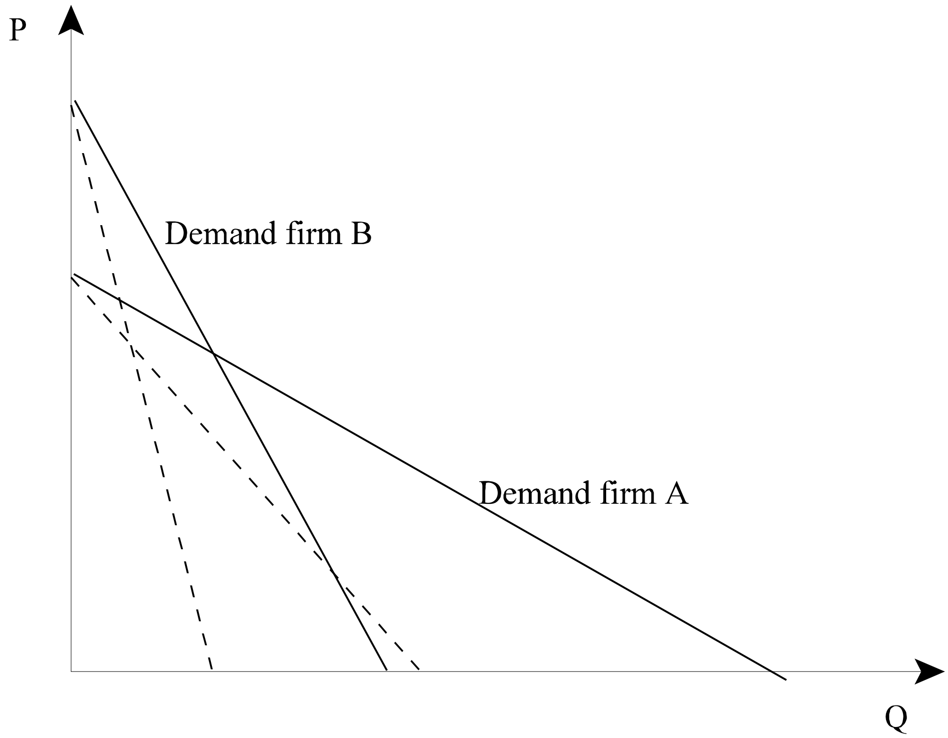
B. Suppose the price of materials goes down. In the above graph, indicate what would be the new optimal choice of *L* and *M*, if the firm were to continue producing q_0 units of output. Label the new levels of *L* and *M* by L_1 and M_1. Also draw the corresponding isocost using a dashed line.

Notice that the new isocost is flatter, because the slope of the isocost is $-w_M/w_L$ and that the firm is using more materials and less labor.

C. After the decrease in the price of materials, the firm finds it optimal to change its output from q_0 to q_2. In the above graph, draw the isoquant that corresponds to the new output level.

Notice that the firm is producing more output after the price of materials goes down.

2. Two cement companies, A and B, operate in two geographically distinct markets. [Cement is not easily transportable, so that two markets can be considered as being totally isolated.] The demand and marginal revenue functions for the two firms are given below. [Marginal revenue functions are with dashed lines.]



The marginal cost of production is constant, that is, it does not depend on output, and is the <u>same</u> for the two firms. Consider the following statements:

i. The profit maximizing output of firm B is smaller than the profit maximizing output of firm A.

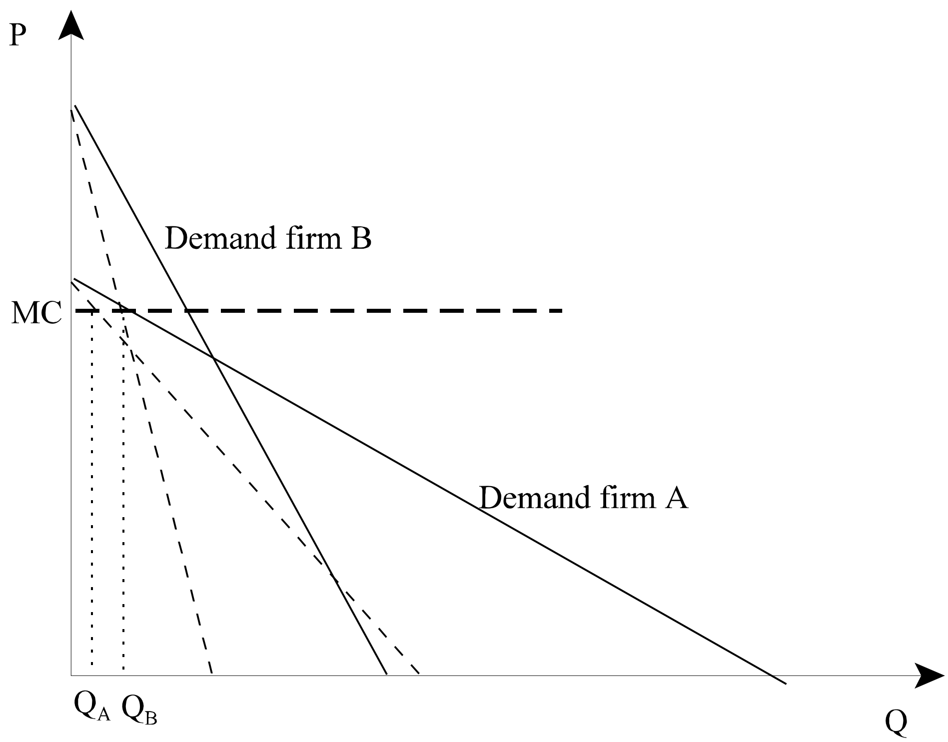
ii. If the profit maximizing output of firm A and firm B is the same, then firm A will charge a higher price than firm B.

iii. If the two firms are observed to charge the same price, then at least one of them is not maximizing profits.

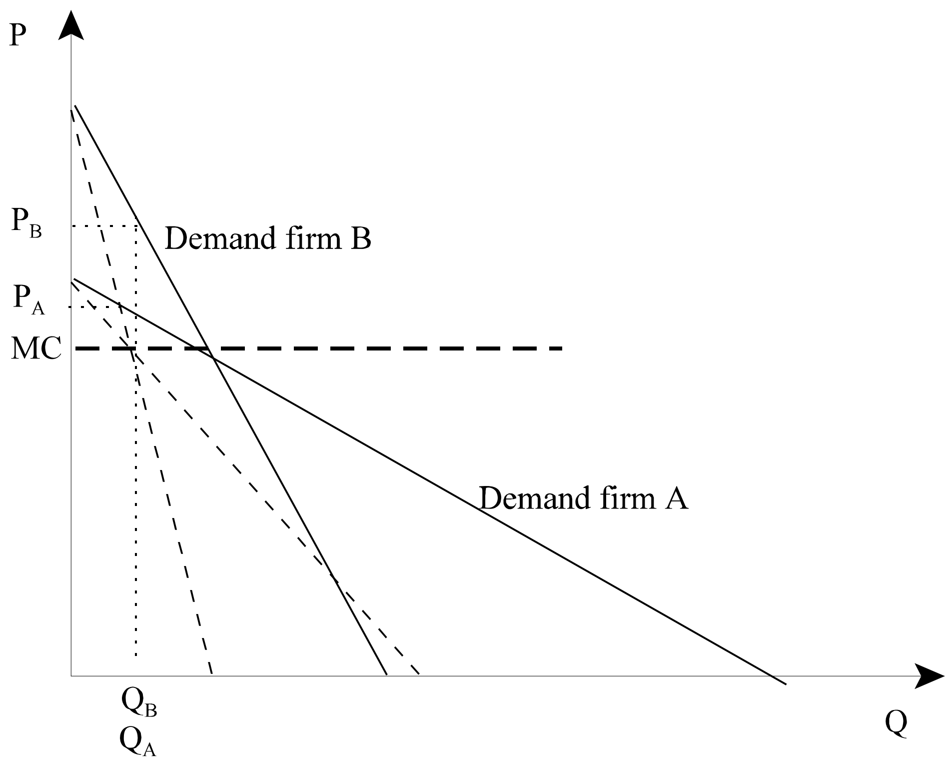
iv. If the marginal cost goes up for both firms (by an equal amount), then the output of firm B will be reduced by less than the output of firm A.

Indicate which of these statements are known to be true given the information above.

Statement (i) is not true. Optimal output is give by the intersection of the marginal cost curve with the marginal revenue curve. For high enough marginal costs, firm B will produce more than firm A. For a demonstration, consider the following counter-example:



Statement (ii) is also not true. At the optimal output, marginal revenue is equal to marginal cost. Therefore, if the optimal output of the two firms is the same, then it must be that the marginal cost line goes through the intersection of the two firms' marginal revenue lines.

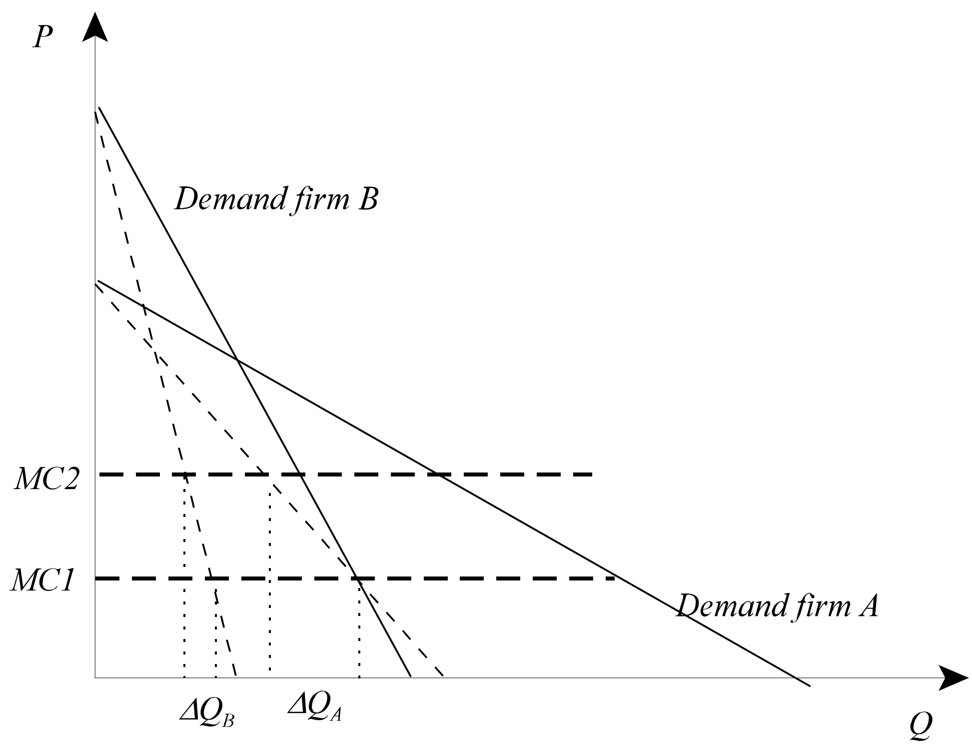


The corresponding optimal price is read-off each firm's demand curve. It can be readily seen that in that case firm B will charge a higher price than firm A.

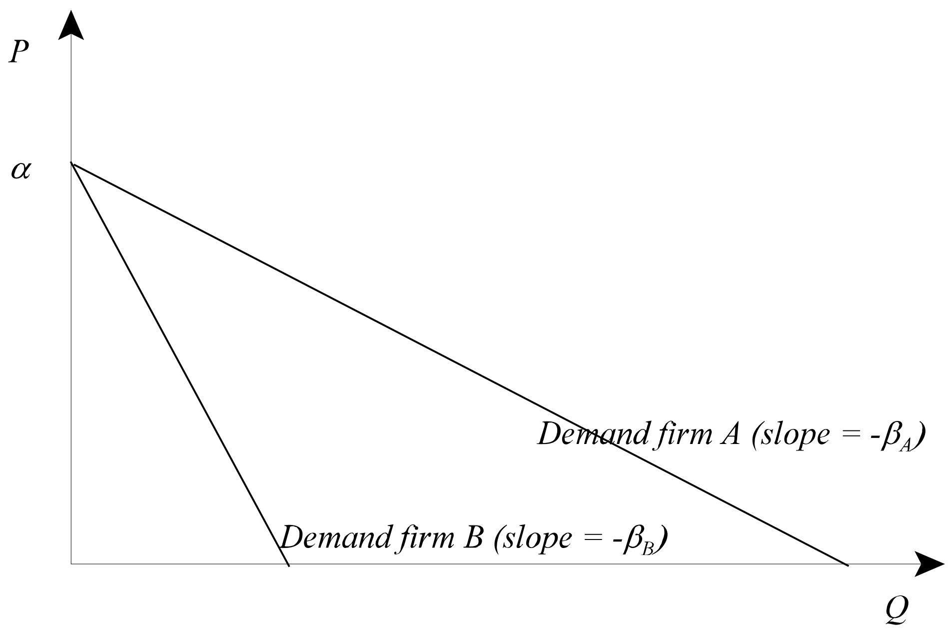
Statement (iii) is true. If you carefully consider the figure above and vary the level of the common marginal cost, it will be clear that the optimal price of firm B is always higher than the optimal price of firm A. Therefore, if the price of firm B is the same as the price of firm A, then one or both firms are mis-pricing their product.

Statement (iv) is true. Because the marginal revenue function of firm B is steeper than that of firm A, and increase in the vertical direction by the same amount leads to a lesser decrease in the horizontal direction (output).

This can be seen in the figure below, where MC1 is the initial marginal cost and MC2 is the new (higher) marginal cost, ΔQ_A is the change in the optimal output of firm A (the gap between the two right-most dotted lines) and ΔQ_B is the change in the optimal output of firm B (the gap between the two left-most dotted lines).



3. Two cement companies, A and B, operate in two geographically distinct markets. [Cement is not easily transportable, so that two markets can be considered as being totally isolated.] The demand functions for the two firms are shown below. [The demand functions are linear, have the same intercept but different slopes.]



For both firms, the marginal cost of production is constant, that is, it does not depend on output. However, the marginal cost of firm B is <u>higher</u> than that of firm A.

Consider the following statements:

i. Firm B will charge a higher price than firm A.

ii. Firm B will produce less output than firm A.

iii. If the marginal cost of firm A were to increase to the same level as that of firm B, the two firms would charge the same price.

iv. If the marginal cost of firm A were to increase to the same level as that of firm B, the two firms would produce the same level of output.

Indicate which of the above statements are true. [Some algebra is needed to determine the answer to these questions.]

The first three statements are true and the fourth one is false. It is easier to show this with a bit of algebra. Consider a linear demand curve $P = \alpha - \beta Q$. The revenue function associated with this demand function is $R(Q) = (\alpha - \beta Q) Q$. Thus, the marginal revenue of such demand curve is $MR = \alpha - 2 \beta Q$. If the marginal cost of the firm is constant at $MC = c$, then the optimal output is given by the equation

$$\alpha - 2 \beta Q = c \Rightarrow$$

$$Q = \frac{\alpha - c}{2 \beta}$$

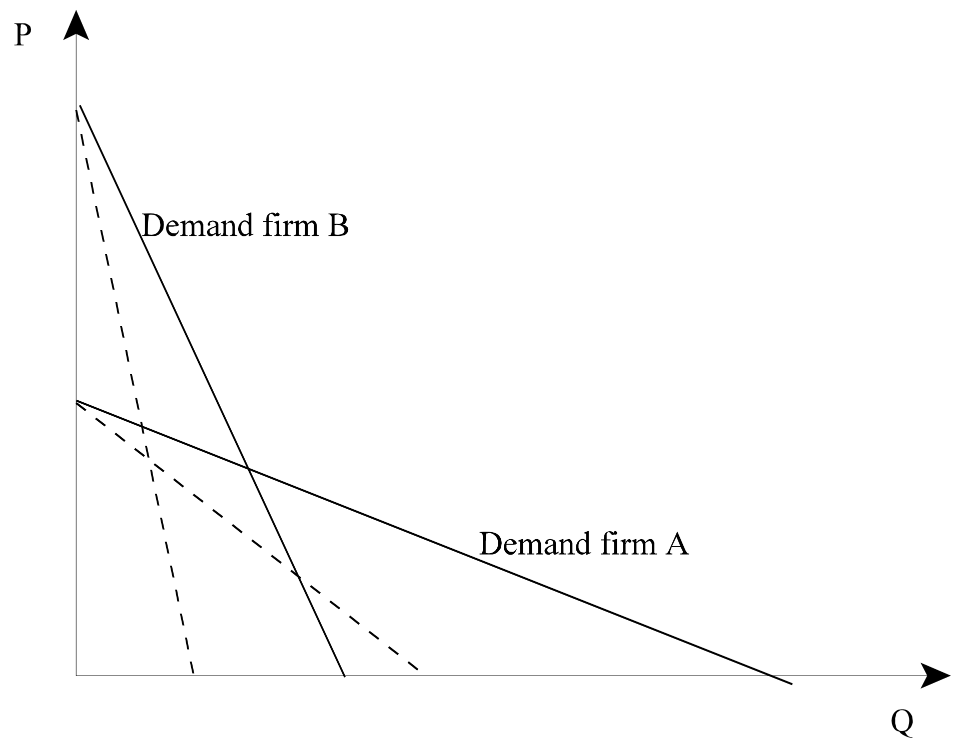
The corresponding price is

$$\begin{aligned} P &= \alpha - \beta \frac{\alpha - c}{2\beta} \\ &= \alpha - \frac{\alpha}{2} + \frac{c}{2} \\ &= \frac{\alpha + c}{2} \end{aligned}$$

It can seen that regardless of the slopes of the two demand curves, the firm with the higher cost (firm B) will charge the higher price. It can also be seen that since the slope of the demand curve of firm B is steeper and its costs higher, firm B will produce less output.

If the costs were the same, then the two firms would choose the same price, but firm B would still produce less output (this can be seen both from the graph and from the math).

4. Two cement companies, A and B, operate in two geographically distinct markets. [Cement is not easily transportable, so that two markets can be considered as being totally isolated.] The demand and marginal revenue functions for the two firms are given below. [Marginal revenue functions are with dashed lines.]



The marginal cost of production is constant, that is, it does not depend on output, it is the <u>same</u> for the two firms, and equal to *c*. Consider the following statements:

i. Suppose c=0. Then, the profit maximizing output of firm B is the same as the profit maximizing output of firm A.

ii. Suppose c>0. If the profit maximizing output of firm A and firm B is the same, then firm A will charge a lower price than firm B.

iii. Regardless of what the value of c is, firm B will charge a higher price than firm A.

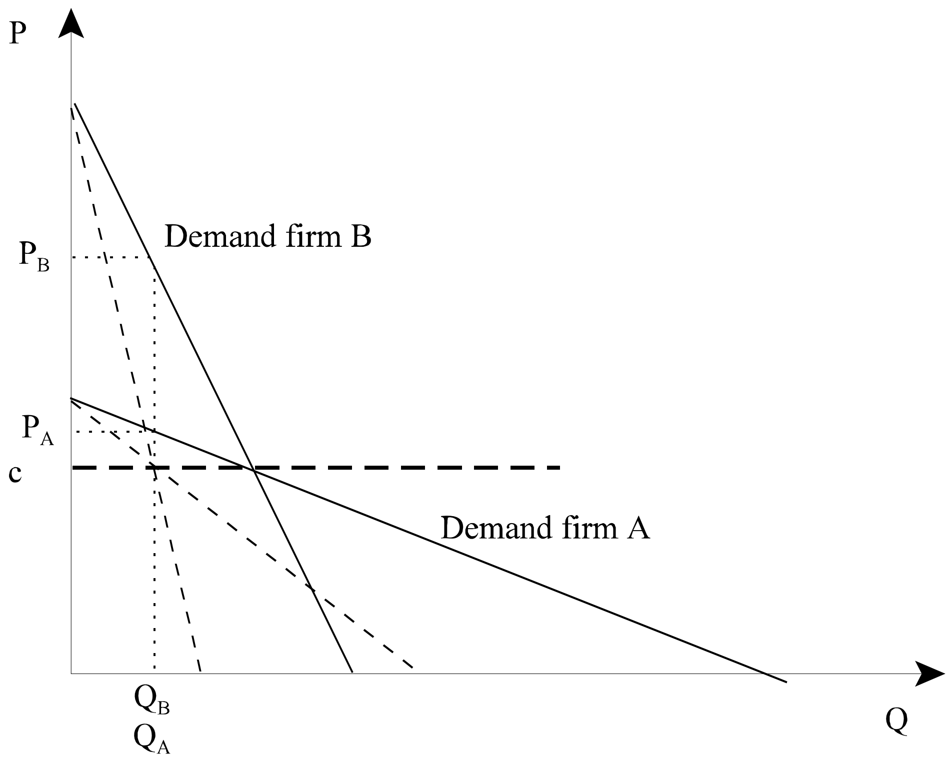
iv. If the marginal cost goes down for both firms (by an equal amount), then the output of firm B will be increased by less than the output of firm A.

v. Suppose that c>0. Now, suppose that the marginal cost of firm B drops to zero, while that of firm A stays at c. Then, the price of firm B may drop below the price of firm A.

Indicate which of these statements are known to be true given the information above.

Statement (i) is false. Optimal output is given by the intersection of the MC and MR curves. Clearly, when marginal cost is zero, optimal output for firm A exceeds that of firm B (the MR of firm B hits zero at a lower output level than the output at which the MR of firm A hits zero).

Statement (ii) is true. If the profit maximizing output is the same, this means that the marginal cost must intersect the two MR lines at the same output level. Since the marginal cost is the same for the two firms, this means that this intersection must take place where the two MR lines cross (see graph below).

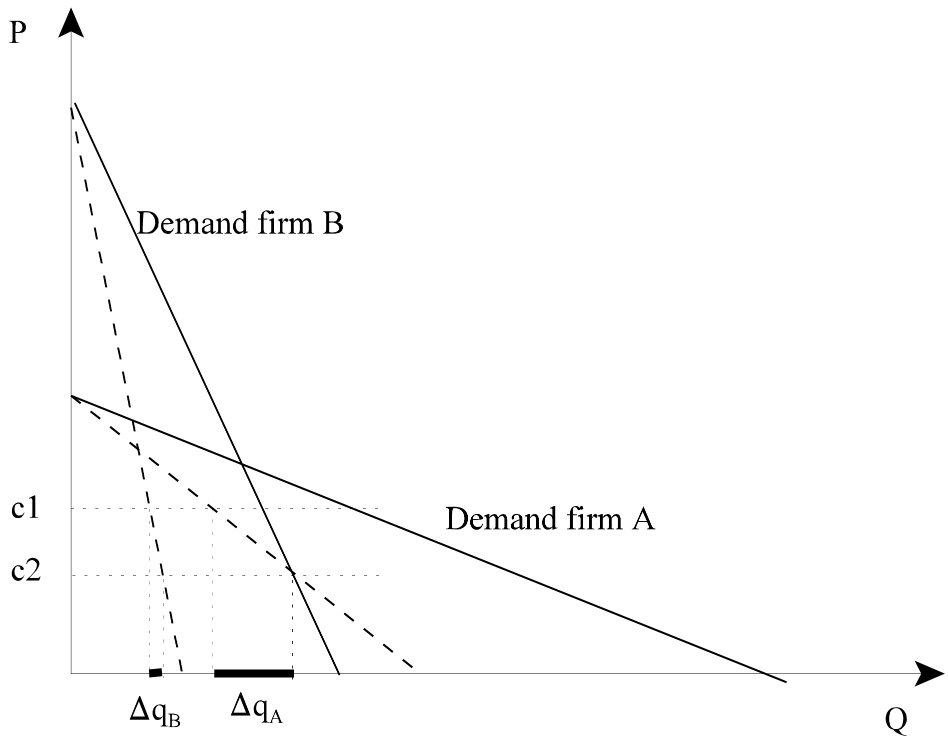


It is apparent for the output level that corresponds to this intersection, the price of firm A is higher than the price of firm B.

Statement (iii) is true. You can just convince yourself of this by plotting a few values of *c*.

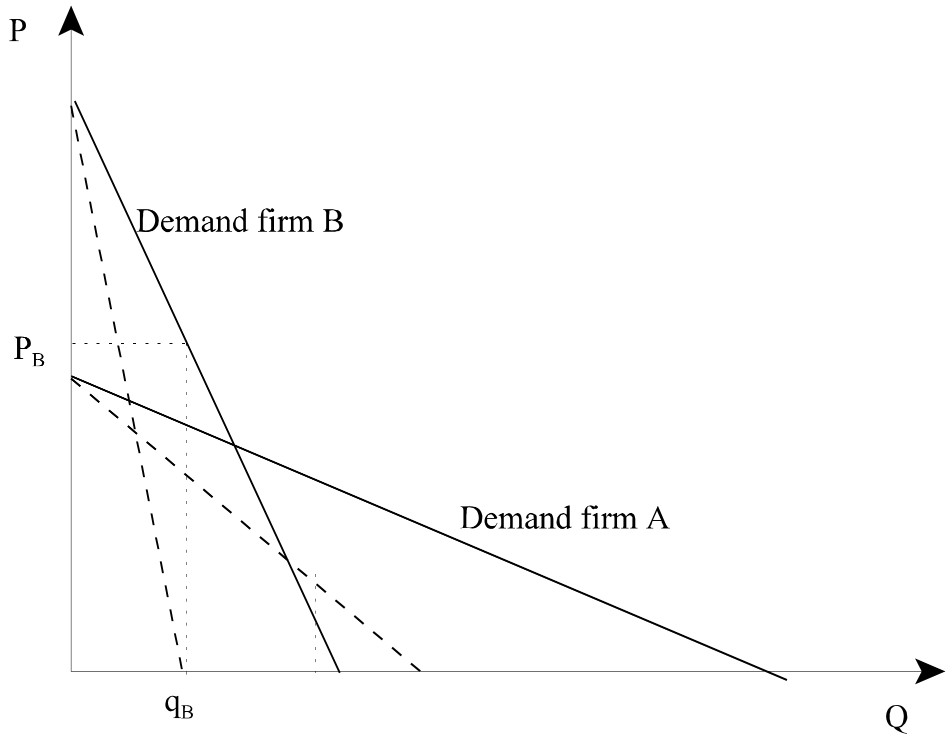
Statement (iv) is true. For any given decrease in *c*, the increase in the output of each firm is given by how steep the marginal revenue line is. If the marginal revenue line is very steep, then decreasing marginal cost will not result in a large increase in output. If the marginal revenue line is relative flat, then decreasing marginal cost will result in a large increase in output.

This is demonstrated by the example below, using the demand curves of the two firms, and shown a decline in marginal cost from c1 to c2. The change in the optimal output of the two firms is shown by the thick lines on the x-axis.



It can readily be seen that the output of firm A will increase by more than the output of firm B.

Statement (v) is false. Suppose marginal cost of firm B drops to zero. Then, the optimal output and price of firm B can be seen from the figure below.



It can be seen that the optimal price of firm B is higher than the maximum possible price of firm A (the price of firm B exceeds the y-axis intercept of the demand of firm A).

Problems

1. A firm producing hockey sticks has a production function given by:

$$q = 2\sqrt{K\,L}$$

In the short run, the firm's amount of capital equipment is fixed at $K = 100$. The cost of capital is $r = \$2$, and the wage rate for L is $w = \$2$.

a. What is the firm's short-run demand for labor as a function of output produced ?

In the short-run, when capital is fixed at K=100, output is given by the short-run production function:

$$\begin{aligned} q &= 2\sqrt{100\,L} \\ &= 20\sqrt{L} \end{aligned}$$

Since labor is the only input that the firm can vary in the short-run, there is a unique amount of labor that would allow the firm to produce any required level of output. This can be found by simply solving the short run production function for L.

$$q = 20\sqrt{L} \quad \Rightarrow$$

$$q^2 = 400\,L \quad \Rightarrow$$

$$L = \frac{q^2}{400}$$

b. Calculate the firm's short-run total cost curve.

The short-run total cost is given by:

$$SRTC = w\,L + r\,\bar{K}$$

Substituting in for w, r, and $\bar{K}$ we get:

$$SRTC = 2 \ \frac{q^2}{400} + 2 \quad 100$$

$$= \frac{q^2}{200} + 200$$

c. Calculate the short-run average cost curve.

$$SRAC = \frac{SRTC}{q}$$

$$= \frac{\frac{q^2}{200} + 200}{q}$$

$$= \frac{q}{200} + \frac{200}{q}$$

d. What is the firm's short-run marginal cost function ?

$$SRMC = \frac{d\,SRTC}{d\,q}$$

$$= 2 \ \frac{q}{200}$$

$$= \frac{q}{100}$$

e. Which output level minimizes the short run average cost ? [The answer should be a number.]

The fastest way to find the answer is to note that the marginal cost is equal to the average cost when average cost is at its minimum. Equating SRMC with SRAC we get:

$$SRMC = SRAC \quad \Rightarrow$$

$$\frac{q}{100} = \frac{q}{200} + \frac{200}{q} \quad \Rightarrow$$

$$\frac{2q}{200} = \frac{q}{200} + \frac{200}{q} \quad \Rightarrow$$

$$\frac{q}{200} = \frac{200}{q} \quad \Rightarrow$$

$$q^2 = 200^2 \quad \Rightarrow$$

$$q_{AC_{\min}} = 200$$

f. What would be the firm's profit maximizing choice of output if output price is equal to 5 ? How much labor would the firm hire ?

Profit maximization requires a price taking firm to increase its output until marginal cost is equal to the price. For this firm and for price equal to 5, setting MC=P gives us:

$$\frac{q}{100} = 5$$

which implies that the profit maximizing choice of output is:

$$q^* = 500$$

At this level of output, the firm will hire

$$L = \frac{500^2}{400} = 625$$

units of labor.

2. A firm has production function:

$$Q = K^{0.3} L^{0.5}$$

a. What is the marginal product of labor ? What is the marginal product of capital?

$$MP_L = \frac{\partial Q}{\partial L} = 0.5 K^{0.3} L^{-0.5}$$

$$MP_K = \frac{\partial Q}{\partial K} = 0.3 K^{-0.7} L^{0.5}$$

b. If the cost of labor is 2 and the cost of capital is 3, in what proportion should the firm employ capital and labor in order to minimize the cost of producing a given amount of output? [That is, how much capital will the firm hire in terms of the amount of labor hired?]

In order to minimize costs the firm will chose the proportions of capital and labor so as to equate the "bang for the buck" of the marginal units of labor and capital. That is, the ratio of marginal products to input prices will be equal for the two inputs.

$$\frac{MP_L}{w} = \frac{MP_K}{r} \quad \Rightarrow$$

$$\frac{0.5 K^{0.3} L^{-0.5}}{2} = \frac{0.3 K^{-0.7} L^{0.5}}{3} \quad \Rightarrow$$

$$0.25 K = 0.1 L \quad \Rightarrow$$

$$K = \frac{2}{5} L$$

c. How much labor and capital will the firm hire in order to produce output q in the lowest possible cost?

Substituting into the production function the cost minimizing quantity of capital in terms of labor and fixing the output to q, we get:

$$q = \left(\frac{2}{5}L\right)^{0.3} L^{0.5} \quad \Rightarrow$$

$$q = \left(\frac{2}{5}\right)^{0.3} L^{0.8}$$

Solving for labor will yield the demand for labor as a function of output.

$$q = 0.760 \ L^{0.8} \quad \Rightarrow$$

$$L = 0.760^{-1.25} \ q^{1.25} \quad \Rightarrow$$

$$L \approx 1.409 \ q^{1.25}$$

Taking this and plugging it into our expression for *K* in terms of *L* that we computed in part (b) we get:

$$K = \frac{2}{5} \ 1.409 \ q^{1.25}$$

$$\approx 0.564 \ q^{1.25}$$

d. What is this firm's cost function ? [i.e. the lowest possible cost of producing output *q*.]

The cost of production is:

$$C = w \ L + r \ K$$

Therefore,

$$C(q) = 2 \ 1.409 \ q^{1.25} + 3 \ 0.564 \ q^{1.25}$$

$$= 2.818 \ q^{1.25} + 1.692 \ q^{1.25}$$

$$= 4.510 \ q^{1.25}$$

e. What is this firm's marginal cost?

$$MC = \frac{\partial C(q)}{\partial q}$$

$$= 4.510 \ 1.25 \ q^{0.25}$$

$$= 5.6375 \ q^{0.25}$$

f. What is this firm's average cost?

$$AC = \frac{C(q)}{q}$$

$$= 4.510 \ q^{0.25}$$

h. How much output will this firm produce if the market price is equal to *P*?

Profit maximization requires the firm to increase its output until price equals marginal cost. Therefore, we have:

$$P = MC \quad \Rightarrow$$

$$P = 5.6375 \ q^{0.25} \quad \Rightarrow$$

$$P^4 \approx 1010 \ q \quad \Rightarrow$$

$$q \approx 0.00099 \ P^4$$

i. How much labor will this firm hire as a function of the market price ?

Taking our answer for part (h) and substituting it in our answer for the labor demand in part (c) we get:

$$L = 1.409 \ \left(0.00099 \, P^4\right)^{1.25}$$

$$\approx 0.001395 \ P^5$$

3. Suppose a firm has a production process that is given by the equation $q = 2G + 3E$ where G is the amount of natural gas used and E is the amount of electricity used. Let the price of electricity be equal to 4 and the price of natural gas 3. The firm is facing a demand given by $P = \alpha - q$.

a. How much electricity and how much natural gas should the firm use to produce 12 units of output at lowest cost?

Given that the MRTS of the linear production function is constant, the cost minimizing choice of inputs will be a corner solution. The firm will choose to use the input that yields the higher marginal product per dollar. For electricity, the ratio of marginal product over its price is 3/4, while for natural gas it is 2/3. Electricity yields the higher bang-for-the-buck and therefore the firm will use only electricity and no natural gas.

The required electricity to produce 12 units of output is given by the equation

$$12 = 3E \Rightarrow E = 4$$

b. In general, how much electricity and how much natural gas should the firm use to produce q units of output at lowest cost?

Following the discussion in part (a), the firm will choose to use only electricity to produce any level of output. The amount of electricity used is given by the equation

$$q = 3E \Rightarrow E = \frac{q}{3}$$

c. What is this firm's cost function?

Since the firm will only use electricity, it's cost function is given by the price of electricity times the amount of needed electricity, or

$$C(q) = 4\frac{q}{3}$$

d. How many units of electricity and how many units of natural gas would the firm demand to produce the profit maximizing level of output? How are they affected by an increase in the demand for the product? (i.e., by an increase in the α?)

The firm's profit maximizing output is obtained by equating marginal revenue with marginal cost, or (given that the firm is facing a linear demand curve) by

$$\frac{4}{3} = \alpha - 2q$$

Therefore, the profit maximizing output is

$$2q = \alpha - \frac{4}{3} \Rightarrow$$

$$q = \frac{\alpha}{2} - \frac{2}{3}$$

This implies that the demand for electricity would be

$$E^* = \frac{\alpha}{6} - \frac{2}{9}$$

An increase in demand increases the demand for electricity.