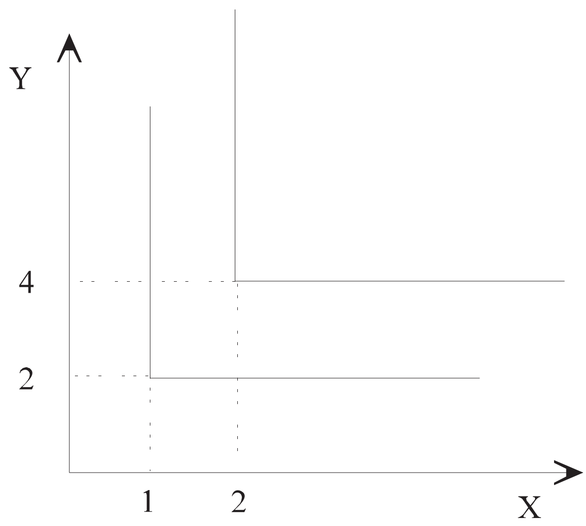
Short Questions

1. Consider an individual whose preferences for goods X and Y are given by the following set of indifference curves.



a. Is the utility function of this person Cobb-Douglas, Leontieff, or Linear Utility?

   This is a Leontieff utility function.

b. Write the mathematical expression for the utility function of *this particular* consumer.

   The utility function

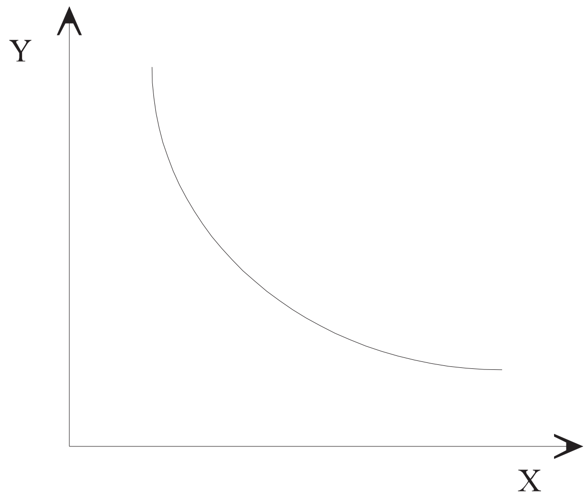
   $$U = \min\{2X, Y\}$$

corresponds to the preferences of this consumer. Of course, any increasing transformation of the above expression also corresponds to the preferences of this consumer, i.e., the function

$$U = \min\{X, 0.5Y\}$$

is also a mathematical expression for the above consumer's utility function.

2. Consider a person with the following indifference curve between goods X and Y:



What does this indifference curve indicate for this consumer's preferences for X and Y?

In particular:

a. Are the two goods perfect substitutes?

No they are not. If they were perfect substitutes, the indifference curve would have been linear.

b. Are they perfect complements?

No they are not. If they were perfect complements, the indifference curve would have been L-shaped (and the utility function would have been Leontieff).

c. Are they imperfect substitutes?

Yes. The indifference curve is downward sloping and exhibits diminishing MRS. *X* can substitute for *Y*, but the more *X* the consumer has (relative to *Y*), the greater the value of *Y* becomes (relative to the value of *X*).

Problems

1. Consider the utility function $U(X, Y) = \log(X) + 2 \log(Y)$.

a. What is the marginal utility of X? What is the marginal utility of Y?

The marginal utility of X is

$$MU_X = \frac{\partial U}{\partial X} = \frac{1}{X}$$

The marginal utility of Y is

$$MU_Y = \frac{\partial U}{\partial Y} = \frac{2}{Y}$$

b. What is $MRS_{Y,X}$? That is, how many units of X do I need to make up for a loss of one unit of Y?

$$MRS_{Y,X} = \frac{MU_Y}{MU_X}$$

$$= \frac{\frac{2}{Y}}{\frac{1}{X}}$$

$$= 2 \frac{X}{Y}$$

c. Consider next the utility function $U(X, Y) = X Y^2$. What is $MRS_{Y,X}$ for this utility function.?

$$MRS_{Y,Y} = \frac{MU_Y}{MU_X}$$

$$= \frac{2\ X\ Y}{Y^2}$$

$$= 2\ \frac{X}{Y}$$

d. Do these two utility function represent the same preferences ?

These two utility functions represent the same preferences. They have the same marginal rate of substitution between X and Y, that is, the consumer who is described by either utility function is willing to make the same trade-offs.

Another way one can see that these two functions represent the same preferences is to observe that the first one is obtained from the second one by taking logs.

2. What is $MRS_{X,Y}$ for the Cobb-Douglas utility function $U(X,Y) = X^{\alpha}Y^{\beta}$? What is the $MRS_{X,Y}$ for the Linear Utility function $U(X,Y) = \alpha\ X + \beta\ Y$? Which one of these two utility function does NOT exhibit decreasing MRS ?

The $MRS_{X,Y}$ for Cobb-Douglas is given by

$$MRS_{X,Y} = \frac{MU_X}{MU_Y}$$

$$= \frac{\alpha\ X^{\alpha-1}\ Y^{\beta}}{\beta\ X^{\alpha}\ Y^{\beta-1}}$$

$$= \frac{\alpha}{\beta}\ \frac{Y}{X}$$

Similarly, the $MRS_{X,Y}$ for the Linear Utility function is given by

$$MRS_{X,Y} = \frac{MU_X}{MU_Y}$$

$$= \frac{\alpha}{\beta}$$

The linear utility function does not exhibit decreasing MRS. The marginal rate of substitution remains constant regardless of how much Y the consumer consumes relative to X.