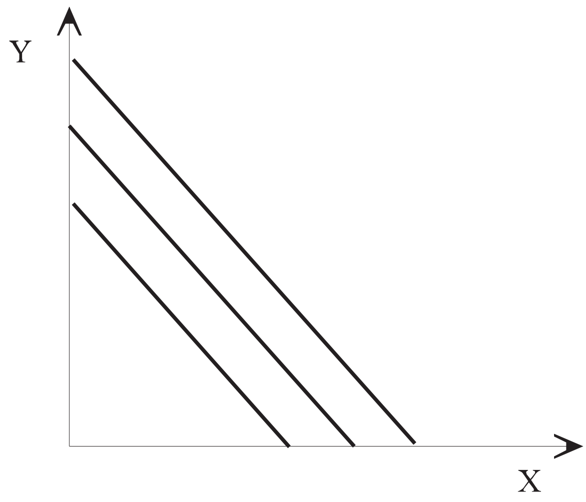
Short Questions

1. Consider an individual whose preferences for goods X and Y are given by the following set of indifference curves.



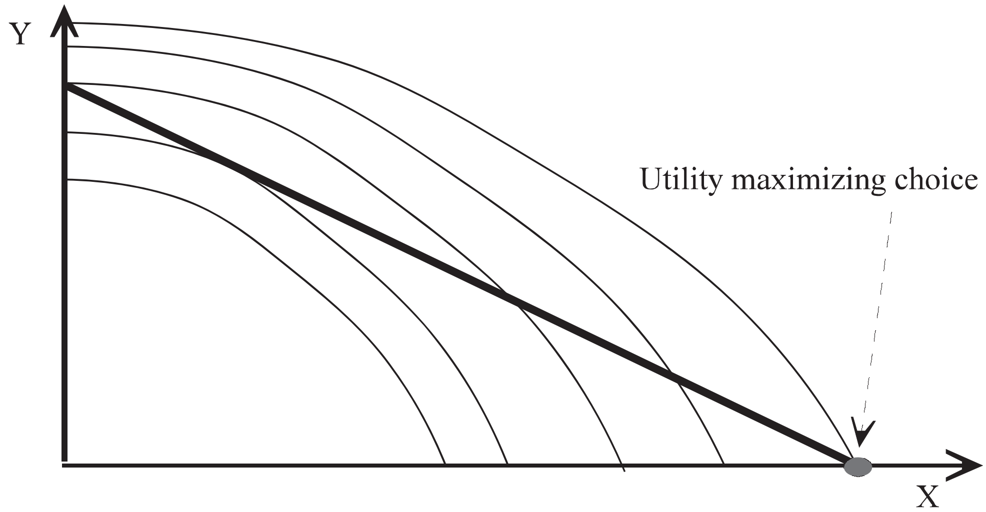
a. Is the utility function of this person Cobb-Douglas, Leontieff, or Linear Utility?

   It is a Linear Utility function.

b. Suppose the budget constraint of this individual is steeper than the indifference curves. Will this person consume only good X, only good Y, or some of both goods?

   He will consume only good Y.

2. One budget constraint (the thick straight line) and a few indifference curves (thin curved lines) are plotted in the figure below. Both goods, X and Y are desirable to the consumer, i.e., utility is higher for indifference curves that are further from the origin.



a. In the above figure, clearly indicate and label the utility maximizing choice of consumption levels of X and Y.

b. Which of the following expressions are (or is) true at the utility maximizing choice of consumption? Circle the ones that are known to be correct on the basis of the above figure.

$$\frac{MU_X}{P_X} < \frac{MU_Y}{P_Y} \qquad \frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} \qquad \boxed{\frac{MU_X}{P_X} > \frac{MU_Y}{P_Y}}$$

This is because the optimum is a corner solution in which he only consumes X , i.e., X gives more utility per dollar than Y.

$$MU_X < MU_Y \qquad MU_X = MU_Y \qquad \boxed{MU_X > MU_Y}$$

This is because the slope of the indifference curve is equal to the MRS=ratio of marginal utilities. A relatively steep indifference curve (steeper than a slope of -1) means that it takes more than one unit of Y to make up for the loss of one unit of X, and thus good X gives the consumer more satisfaction that good Y (at the margin).

$$\boxed{P_X < P_Y} \qquad P_X = P_Y \qquad P_X > P_Y$$

This is because the slope of the budget constraint is equal to the ratio of prices. In particular, a relatively flat budget constraint (i.e., one with slope flatter than -1) implies that the good in the Y axis is more expensive that the good in the X axis.

c. How would this choice of consumption change if the price of Y decreases by a little bit? Would the consumption of Y increase, decrease, or stay the same? How would the consumption of X change?

A small decrease in the price of Y will make the budget constraint a bit steeper. But if the change is small, X will still be the optimal choice. Given that the price X has remained unchanged, the consumer will be able to afford the same amount of X.

d. Answer the questions in (c) if instead the price of Y increased by a little bit?

If the price of Y increased, it would become even less desirable. The consumer would still only consume X.

Problems

1. A consumer's utility from the consumption of French and California wine is given by:

$$U(C,F) = C + F$$

a. Plot the indifference curve of this consumer that corresponds to, say, utility level 5 with quantity of California wine on the vertical axis and French wine on the horizontal axis. Label all points and slopes.

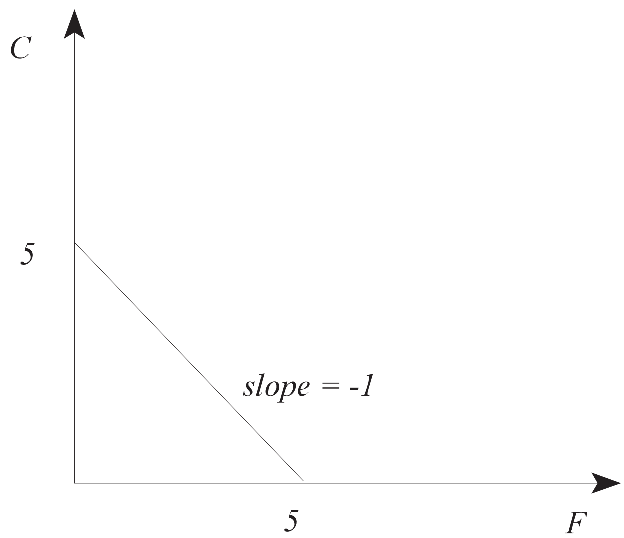
The indifference curve for utility level equal to 5 is given by the formula:

$$5 = C + F$$

If we would like to plot this indifference curve with California wine on the vertical axis we should solve the above equation for C.

$$C = 5 - F$$

The graph of this indifference curve is shown below:



b. Suppose that this consumer wants to spend 100 dollars on wine, and that French wine costs 10 dollars per bottle while California wine costs 5 dollars per bottle. Draw this consumer's budget constraint for consumption of wine with the quantity of California on the vertical axis and French wine on the horizontal axis. Label all points and slopes.

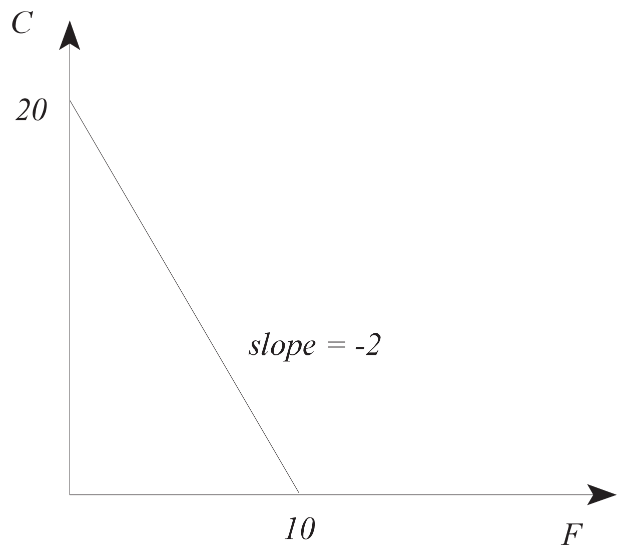
The consumer's budget constraint is given by

$$100 = 5\ C + 10\ F$$

Solving for California wine, in order to facilitate the drawing of the budget constraint, we get:

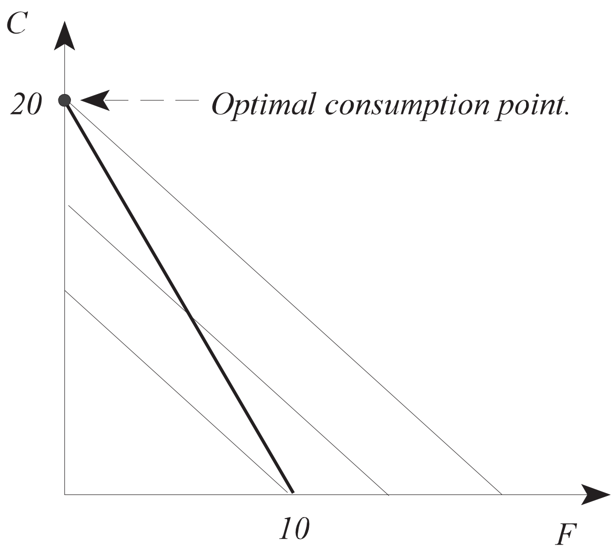
$$C = 20 - 2\ F$$

The plot of the budget constraint is given by the figure below:



c. How much California and French wine will this consumer buy ? Show your answer diagrammatically using the budget constraint and an indifference curve.

The consumer will maximize his utility subject to his budget constraint. Graphically, this means getting on the furthest out indifference curve that touches his budget set. The solution can been seen graphically in the figure below, where the indifference curves are in thin lines and the budget constraint in thicker line.



This consumer will consume 20 bottles of California wine and no French wine.

2. A consumer's utility from consumption during his working years and consumption during his retirement years is given by:

$$U(C_W, C_R) = 5 \log(C_W) + \log(C_R)$$

where C_W and C_R indicate his consumption in thousands of dollars during the work years and retirement years, respectively. The consumer earns a total of 600 thousand dollars during his work years. He can put any amount of this money in a bank that earns NO interest, and withdraw this money for consumption during his retirement years.

a. If he saves 100 in the bank during his working years, what will C_W and C_R be ? What if he saves 200 in the bank ? In general, what will his consumption be if he saves S dollars ?

If he saves 100 during his working years his consumption during his working years will be 600-100=500 thousand. His consumption in his retirement years will be 100 thousand. If he saves 200 thousand his consumption during his working years will be 600-200=400 thousand. His consumption during his retirement years will be 200 thousand. In general if he saves S thousand during his working years C_W will be 600-S and C_R will be S.

b. What will his utility be if he saves S dollars during his working years ?

Plugging the consumption in terms of the amount of savings into his utility function yields:

$$U(S) = 5 \log(600 - S) + \log(S)$$

c. What choice of savings will maximize this consumer's utility ?

If this consumer is utility maximizing he will increase his savings until his utility reaches a maximum. The optimal savings can then be found by the first order condition:

$$\frac{dU(s)}{ds} = -\frac{5}{600 - S} + \frac{1}{S} = 0 \quad \Rightarrow$$

$$\frac{5}{600 - S} = \frac{1}{S} \quad \Rightarrow$$

$$5S = 600 - S \quad \Rightarrow$$

$$6S = 600 \quad \Rightarrow$$

$$S = 100$$

d. Suppose now that the government is taxing this consumer 50 thousand dollars during his working years which it gives back to him as social security payments during his retirement years. What would C_W and C_R be now, if he saves 100 during his working years ? What if he saves 200 ? In general, what will his consumption be if he saves S dollars ?

If he saves 100 thousand his consumption during the working years will be 600-50-100=450 thousand. His consumption in the retirement years will be 100+50=150 thousand. If he saves 200 his consumption during his working years will be 600-50-200=350 thousand. His consumption during the retirement years will be 200+50=250 thousand. If he saves S thousand, then C_W=600-S-50=550-S thousand. His retirement during the retirement years would be C_R = S+50 thousand.

e. What will his utility be if he saves S dollars during his working years ? [Don't forget that he is being taxed part of his income, which he gets back as social security payments.]

Plugging in the consumption as a function of S into the utility function we get:

$$U(S) = 5 \log(550 - S) + \log(50 + S)$$

f. What choice of savings will maximize this consumer's utility ?

The utility maximizing choice of savings yields first order condition:

$$\frac{dU(S)}{dS} = -\frac{5}{550 - S} + \frac{1}{50 + S} = 0 \quad \Rightarrow$$

$$\frac{5}{550 - S} = \frac{1}{50 + S} \quad \Rightarrow$$

$$250 + 5S = 550 - S \quad \Rightarrow$$

$$6S = 300 \quad \Rightarrow$$

$$S = 50$$

g. What is the effect of the social security program to this consumer's savings ? What is the effect to this consumer's utility ?

With the social security scheme his savings will drop to 50 thousand. Without the scheme his consumption during the working years is 500 thousand and during his retirement years 100 thousand. With the social security scheme his savings drop to 50 thousand, but he is taxed an additional 50 thousand so his consumption during the working years remains constant at 500 thousand. Similarly, his consumption during the retirement years also remains constant at 100 thousand since his drop in savings is exactly balanced out by the social security income. Since his consumption in either period remains unchanged his utility will remain unchanged by the social security scheme.

3. A consumer's utility from the consumption of French and California wine is given by:

$$U(C,F) = C + F$$

a. Plot the indifference curve of this consumer that corresponds to, say, utility level 5 with quantity of California wine on the vertical axis and French wine on the horizontal axis. Label all points and slopes.

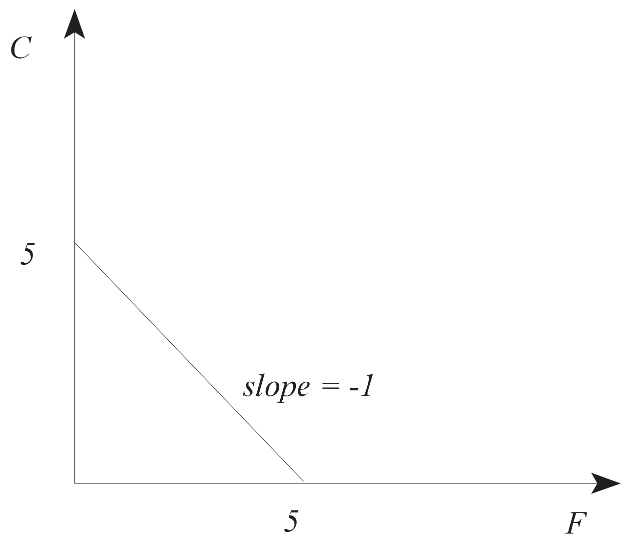
The indifference curve for utility level equal to 5 is given by the formula:

$$5 = C + F$$

If we would like to plot this indifference curve with California wine on the vertical axis we should solve the above equation for C.

$$C = 5 - F$$

The graph of this indifference curve is shown below:



b. Suppose that this consumer wants to spend 100 dollars on wine, and that French wine costs 10 dollars per bottle while California wine costs 5 dollars per bottle. Draw this consumer's budget constraint for consumption of wine with the quantity of California on the vertical axis and French wine on the horizontal axis. Label all points and slopes.

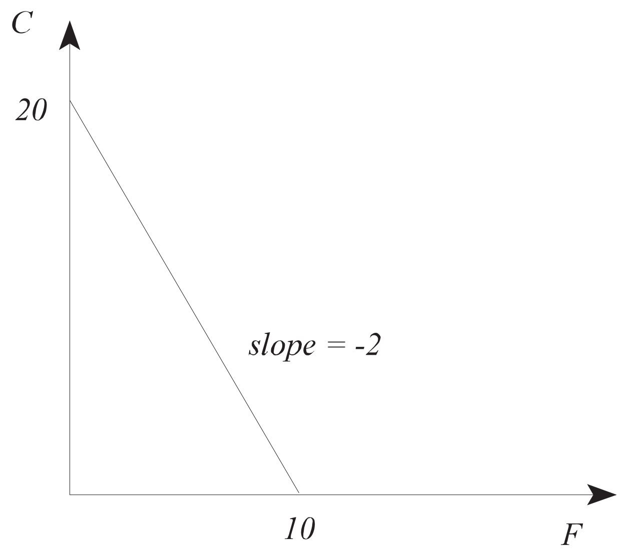
The consumer's budget constraint is given by

$$100 = 5\,C + 10\,F$$

Solving for California wine, in order to facilitate the drawing of the budget constraint, we get:

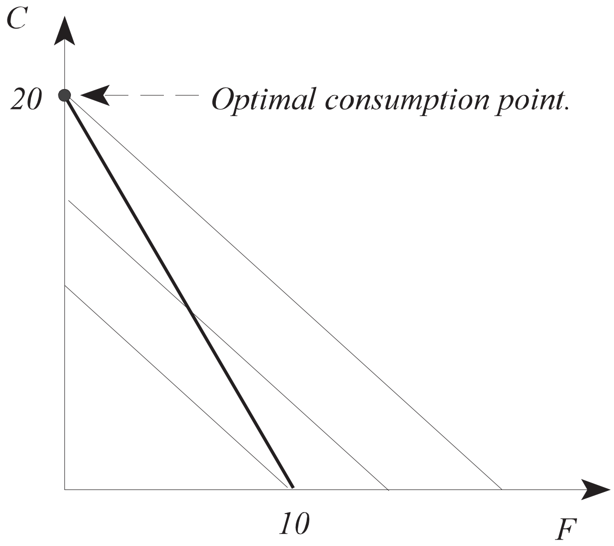
$$C = 20 - 2\,F$$

The plot of the budget constraint is given by the figure below:



c. How much California and French wine will this consumer buy ? Show your answer diagrammatically using the budget constraint and an indifference curve.

The consumer will maximize his utility subject to his budget constraint. Graphically, this means getting on the furthest out indifference curve that touches his budget set. The solution can been seen graphically in the figure below, where the indifference curves are in thin lines and the budget constraint in thicker line.



This consumer will consume 20 bottles of California wine and no French wine.

d. How high must the price of California wine go before the consumer is indifferent between California and French wines ?

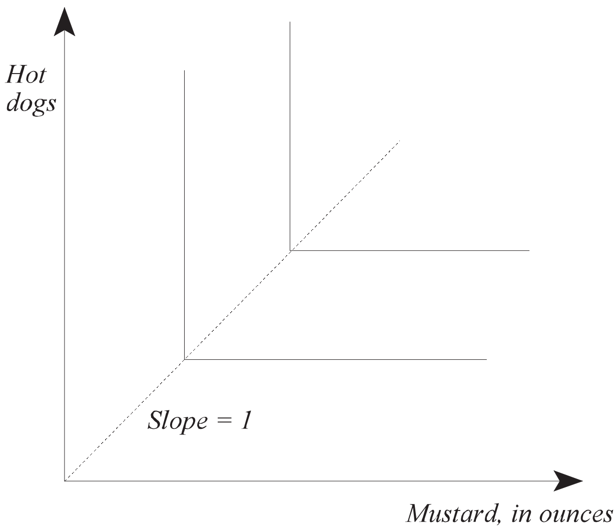
For the consumer to be indifferent between California and French wine the psychic tradeoff between the two, which is one for one, must be equal to their monetary tradeoff. This means that the price of California wine must be equal to the price French wine (10 dollars per bottle) for the consumer to be indifferent between them.

e. How much money would the consumer be willing to spend in order to avoid the price increase that was your answer in (d.) ? [You need no more than 2 lines of calculations here, and perhaps a sentence of English or two.]

At the low price of California wine, the consumer buys 20 bottles. At the high price he is indifferent between French and California. Say he decides to buy California wine. Then he will be able to buy 10 bottles. In order to avoid having to suffer the price increase, he would be willing to, at most, pay an amount of money such that at the old prices his utility would be at least as high as the utility after the price change. This means that he would be willing to pay an amount of money up to the point that he can afford to buy 10 bottles of California wine at the old prices. Since he needs 50 dollars to buy 10 bottles of California wine at the old prices, the maximum amount of money he would be willing to pay to avoid the price increase is 100 - 50 = 50 dollars.

4. Georgia always eats hot dogs together with 1 oz. of mustard. Each hot dog eaten in this way provides 15 units of utility, but any other combination of hot dogs and mustard is worthless to Georgia. [If she has an excess of one of the ingredients than these proportions, she will just throw the extra amount away.]

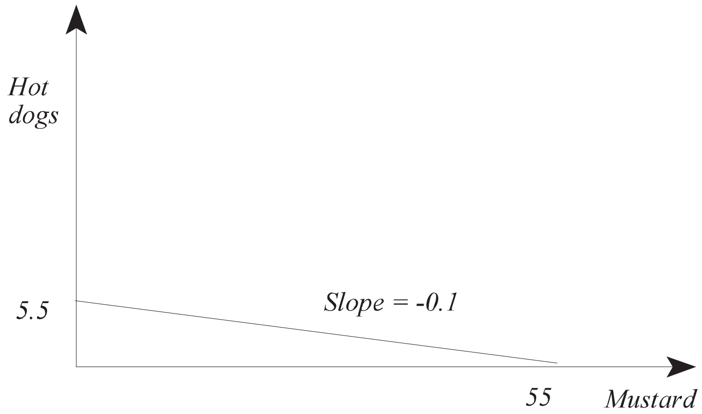
a. Draw a couple of Georgia's indifference curves for consumption of hot dogs and mustard, with the number of hot dogs on the vertical axis.



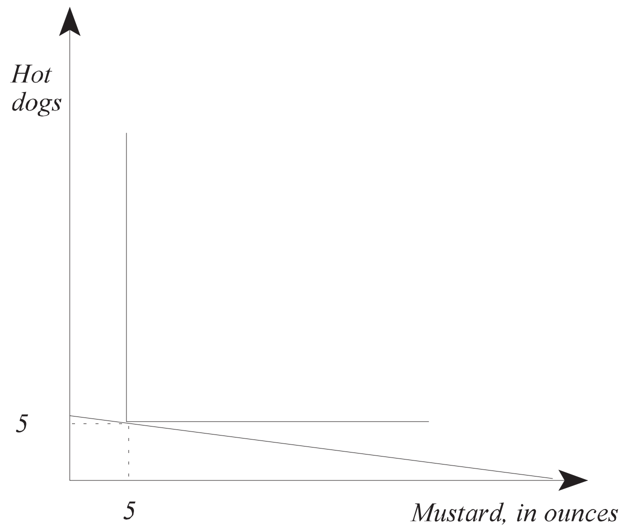
Since she wants to consume mustard and hot dogs in fixed proportions only, there is no substitutability between the two goods. This means that, starting from the optimal proportions, adding any more of either of the two goods will leave her indifferent. Therefore, her indifference curves are L-shaped, with the corner happening where the number of hot dogs is equal to the ounces of mustard, which is the optimal consumption proportions for her utility function.

All the corners of her indifference curve are on the same line from the origin, which is indicated with the dotted line on the figure above. This line has a slope equal to the desired proportion of the one good to the other.

b. Suppose she has $5.50 to spend on hot dogs and mustard, and that hot dogs cost $1 each, while mustard costs $0.10 per ounce. Draw her budget line, with the number of hot dogs indicated on the vertical axis. Label all intercepts on your graph.



c. Combine the two graphs above to show Georgia's utility maximizing consumption of these two items, given her budget constraint.



d. Algebraically derive how many hot dogs and mustard she will buy. Label this consumption bundle on the graph.

Georgia's budget constraint is:

$$0.1 \; mustard + 1 \; hot\,dogs = 5.5$$

Since she always consumes then in equal proportions, the number of hot dogs will be equal to the ounces of mustard. Substituting in for the number of hot dogs we have:

$$0.1 \; mustard + mustard = 5.5 \quad \Rightarrow$$

$$1.1 \; mustard = 5.5 \quad \Rightarrow$$

$$mustard = 5 \quad \Rightarrow$$

$$hot\,dogs = 5$$

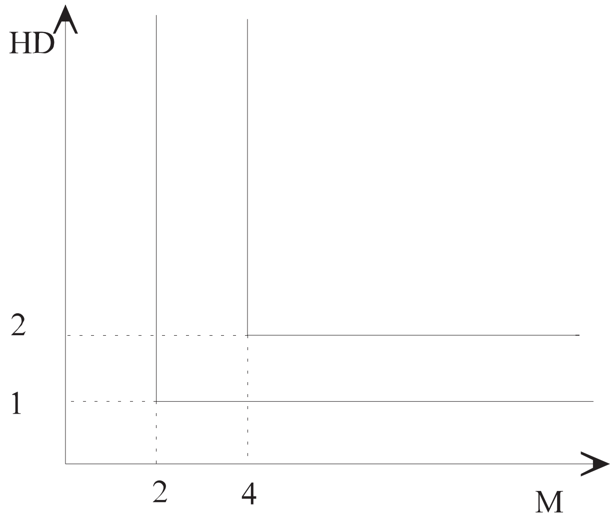
She will buy 5 hot dogs and 5 ounces of mustard. This consumption bundle is at the corner of the indifference curve shown in the graph above, where it touches on her budget constraint.

5. Georgia always eats hot dogs together with 2 oz. of mustard. Each hot dog eaten in this way provides 10 units of utility. If she has an excess of one of the ingredients than these proportions, she will just throw the extra amount away.

a. Draw a couple of Georgia's indifference curves for consumption of hot dogs and mustard, with the number of hot dogs on the vertical axis. Label your graph carefully.

Let's first draw the indifference curve that corresponds to the consumption of 2 ounces of mustard and a single hot dog. Adding more mustard gives no additional utility, so the indifference curve must be flat to the right of the (2,1) bundle. Similarly, since getting more hot dogs while keeping the level of mustard the same yields no additional utility, the indifference curve must be vertical above the (2,1) bundle.

Repeating the process for the (4,2) bundle gives the figure below.

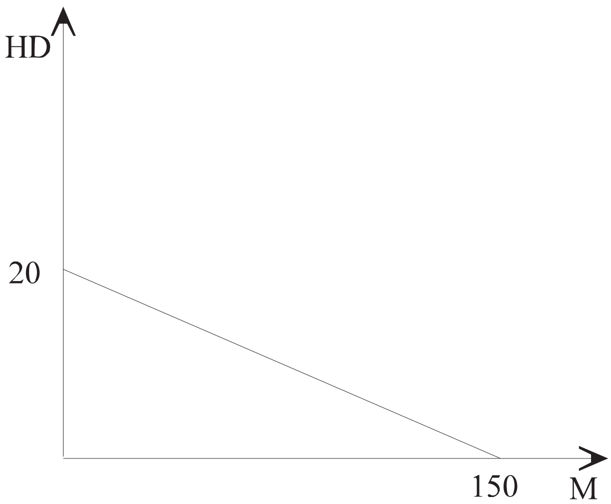


b. Suppose she has $30 to spend on hot dogs and mustard, and that hot dogs cost $1.5 each, while mustard costs $0.20 per ounce. Draw her budget line, with the number of hot dogs indicated on the vertical axis. Label all intercepts on your graph.

If Georgia spends all her money on hot dogs she will be able to buy 20 of them.

If she spends all her money on mustard she will be able to buy 150 ounces.

Therefore, the budget constraint is given by the figure below.



c. Algebraically derive how many hot dogs and mustard she will buy. [Quantities do not have to be integers.] Label this consumption bundle on a graph that combines the budget constraint and the indifference curve that corresponds to the optimal consumption bundle.

Georgia will always buy twice as many ounces of mustard as she will buy hot dogs.

Therefore,

$$M = 2\ HD$$

The budget constraint of Georgia is given by

$$30 = 0.2\ M + 1.5\ HD$$

Using the fact that she will consume twice as much mustard as hot dogs we get

$$30 = 0.2 \cdot 2\ HD + 1.5\ HD \quad \Rightarrow$$

$$30 = 1.9\ HD \quad \Rightarrow$$

$$HD = 15.79$$

It immediately follows that she will consume

$$M = 2 \cdot 15.79$$

$$= 31.58$$

6. Consider a person preparing hamburger patties for a Memorial Day barbecue. He can use either regular ground beef or extra lean ground beef. His utility function for these two types of beef is given by:

$$U = 4L + 3R$$

where L is the amount of lean ground beef he purchases and R is the amount of regular ground beef he purchases. In other words, these two types of beef are perfect substitutes, but, all things equal, he prefers lean to regular beef.

a. Does this person prefer 4 pounds of lean beef to 3 pounds of regular beef ? Show why or why not ?

If he consumes 4 pounds of lean beef his utility is 4 * 4 = 16.

If he consumes 3 pounds of regular beef his utility is 3 * 3 = 9.

Therefore, he prefers to consume 4 pounds of lean beef.

b. In terms of this person's utility function, how many units of regular beef are equivalent to one unit extra of lean beef ?

Since this person has linear utility, the tradeoff between lean and regular beef is constant, i.e., it does not depend on how many units of each he consumes.

A unit of lean beef gives increases his utility by 4 units.

To get a 4 unit increase in his utility by consuming regular beef he will need 4/3 units of it.

Therefore, 1.333 units of regular beef are equivalent to one unit of lean beef.

c. Suppose extra lean beef costs \$3.29 per pound and regular beef costs \$2.89 a pound. Will this person purchase only extra lean beef, only regular beef, or a combination of the two ?

Buying lean beef yields

$$\frac{4}{3.29} = 1.2158$$

"utils" per dollar.

Buying regular beef yields

$$\frac{3}{2.89} = 1.038$$

"utils" per dollar.

Lean beef is the better bargain for this consumer. He will only purchase lean beef.

d. Suppose he has \$14 to spend on meat. How many pounds will he purchase ? [Give your answer to 2 decimals.]

Since he only purchases extra lean beef, which costs \$3.29 per pound, his total meat purchase will be equal to

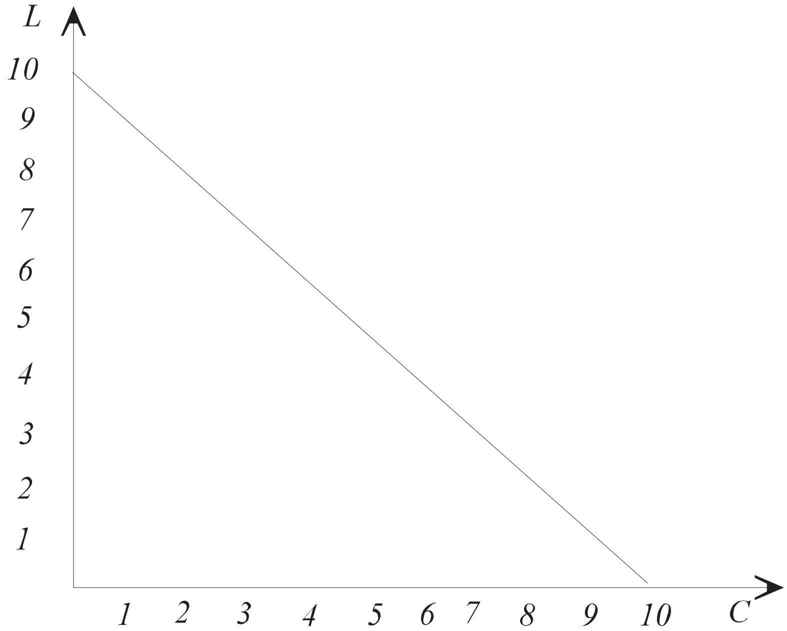
$$\frac{14}{3.29} = 4.255$$

pounds.

7. A consumer's utility of cappuccinos and lattes is given by $U(L,C) = 2\ L + 3\ C$. The price of either of the two drinks at Espresso Royale is equal to \$3, and the person spends \$30 on coffee.

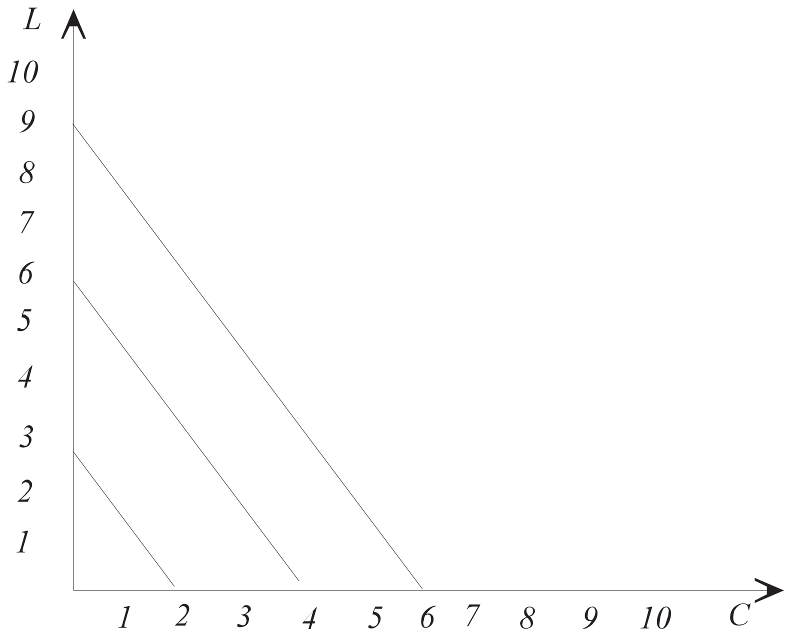
a. Draw this consumer's budget constraint in a figure with L on the vertical axis and C on the horizontal axis

This person can afford 10 cappuccinos if he spends all his money of cappuccinos, or 10 lattes if he spends all his money on lattes. Therefore, his budge constraint is given by the straight line below:


L
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10
C

b. Draw two indifference curves for this consumer with L on the vertical axis and C on the horizontal axis.

For this person, lattes and cappuccinos are perfect substitutes and the indifference curves would be straight lines. However, even though lattes and cappuccinos are perfect substitutes, he likes latter more than the former (they give him more utility per unit of consumption). In fact, 2 cappuccinos give him as much satisfaction as 3 lattes. Therefore, one indifference curve would be a straight line that connects the consumption bundle of C=2 and L=0 with the consumption bundle of C=0 and L=3. Moreover, any indifference curve would be parallel to this one. A couple are drawn below.



Note: Mathematically, one could draw an indifference curve by fixing utility to some number, then solving the utility function for L and plotting the resulting equation.

c. How many lattes and how many cappucinos will this consumer purchase?

The consumer has linear utility and is facing a linear budget constraint. Therefore, he will either consume only cappuccinos or he will consumer only lattes. Since the prices are the same and cappuccinos are more desirable, he will only drink cappuccinos. He can afford 10 of them. He will buy no lattes.

Alternative solution: The solution of the utility maximization problem of a consumer with linear utility facing a linear budget constraint is a corner solution. The consumer will only purchase the product that yields the higher (marginal) utility per dollar. Since

$$\frac{MU_L}{P_L} = \frac{2}{3}$$

while

$$\frac{MU_C}{P_C} = \frac{3}{3} = 1$$

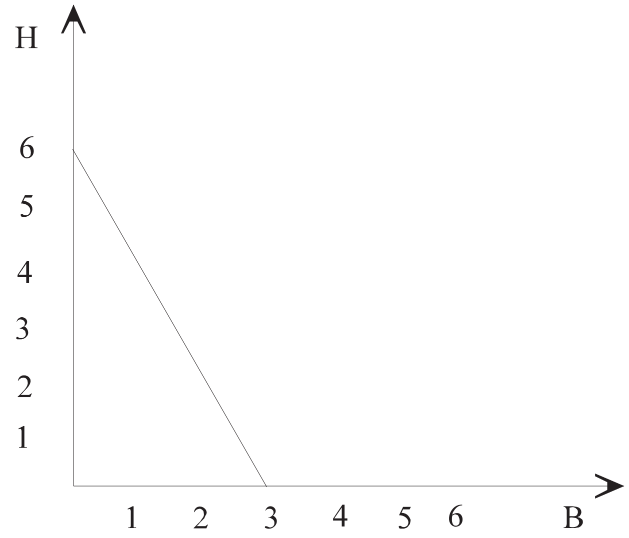
he will only buy only cappucinos.

d. Suppose the person visits Italy, and cappuccinos there cost \$4, while lattes still cost only \$3. Will the person switch to lattes? Why or why not?

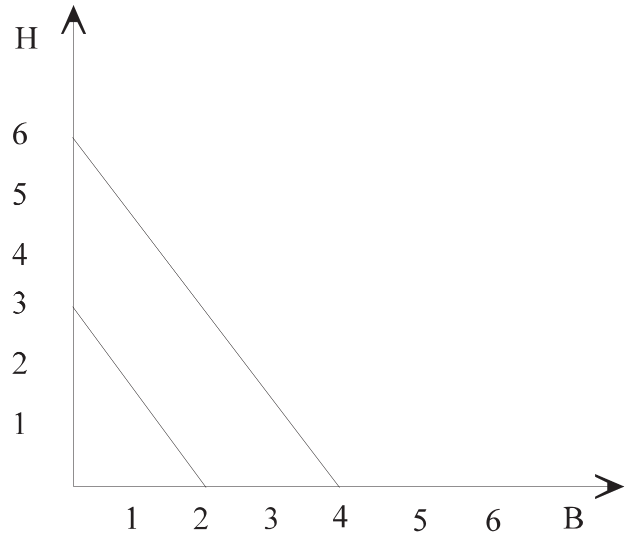
He will still drink only cappuccinos. The reason is that marginal utility of cappuccinos per dollar spent is equal to 3/4, which is still higher than the marginal utility per dollar of lattes (which remains equal to 2/3).

8. A consumer's utility of hot dogs and burgers is given by $U(H,B) = 2H + 3B$. The price of a hot dog is 2 and the price of a burger is P_B. The consumer has 12 to spend on hot dogs and burgers, that is, his (relevant) income is 12.

a. Suppose $P_B = 4$. Draw this consumer's budget constraint in a figure with *H* on the vertical axis and *B* on the horizontal axis



b. Draw two indifference curves for this consumer with H on the vertical axis and B on the horizontal axis.



c. How many burgers and how many hot dogs will this consumer purchase if $P_B = 4$?

The consumer has linear utility and is facing a linear budget constraint. Therefore, he will either consume only burgers or he will consumer only hot dogs. With the prices above, if he only buys burgers, he can afford 3 of them, and obtain utility of 9. If he only buys hot dogs, he can afford 6 of them and he will obtain utility of 12. Therefore, he will only buy hot dogs.

Alternative solution: The solution of the utility maximization problem of a consumer with linear utility facing a linear budget constraint is a corner solution. The consumer will only purchase the product that yields the higher (marginal) utility per dollar. Since

$$\frac{U_B}{P_B} = \frac{3}{4}$$

while

$$\frac{U_H}{P_H} = \frac{2}{2}$$

he will only buy hot dogs.

d. How many burgers and how many hot dogs will he purchase if the price of each hot dog increases to 3 and the price of burgers remains equal to 4?

If the price of a hot dog is 3 and he only buys hot dogs, he can afford 4 of them and obtain a utility of 8. Therefore, given the discussion in part (iii), he will choose to only buy burgers.