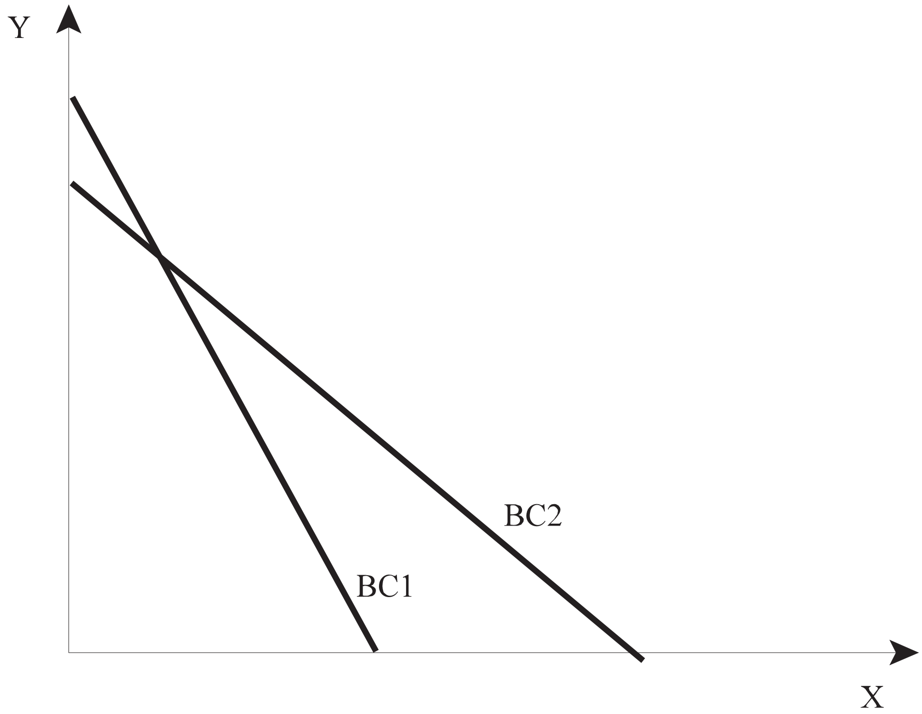
Short Questions

1. Consider the figure with the following two budget constraints, BC1 and BC2.



Consider next the following possibilities:

A. Price of X increases and income of the consumer also increases.

B. Price of Y decreases and income of the consumer also decreases.

C. Price of Y increases and income of the consumer decreases.

D. Price of X decreases and income of the consumer remains unchanged.

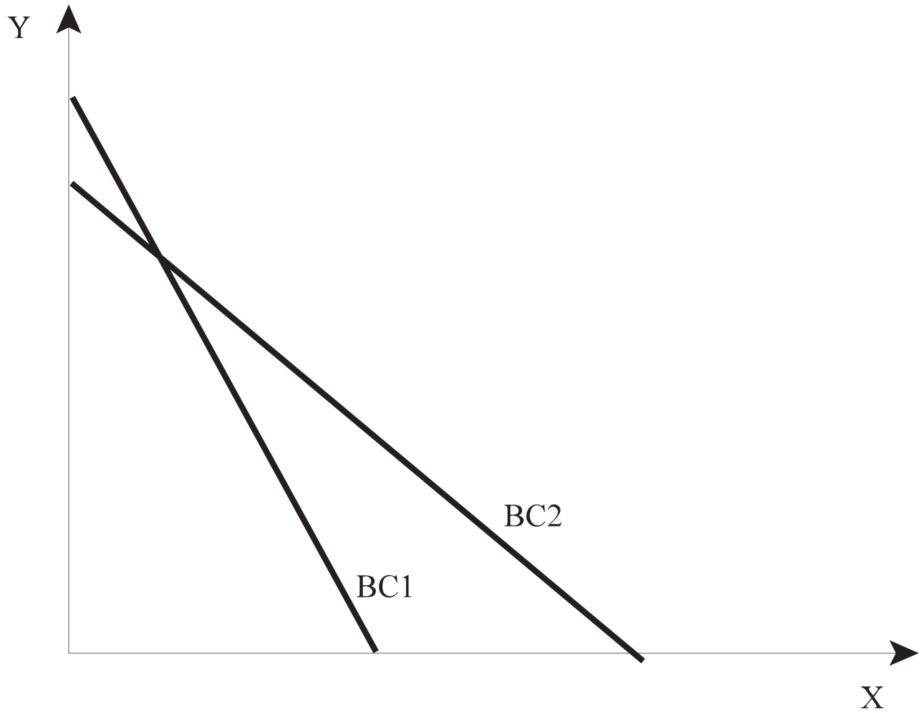
E. Price of X decreases and income of the consumer increases.

F. Price of X decreases and income of the consumer also decreases.

Which of the above possibilities can lead to a movement of the budget constraint from BC2 to BC1? (Write below the letters that correspond to each scenario that can have the effect stated above.)

Since the budget constraint becomes steeper, then if the price of Y stays the same, it must be that the price of X increases. Therefore, D, E, and F **cannot** be correct. If the price of X stays the same, it must be the price of Y goes down. Therefore, C **cannot** be correct. The statement A can be correct (notice the Y intercept is further out for BC1 than for BC2, implying an increase in income). The statement B can also be correct (notice that the X intercept is further in for BC1 than BC2, implying a decrease in income).

2. Consider the figure with the following two budget constraints, BC1 and BC2.



Consider next the following possibilities:

A. Price of X increases and income of the consumer also increases.

B. Price of Y decreases and income of the consumer also decreases.

C. Price of Y increases and income of the consumer decreases.

D. Price of X decreases and income of the consumer remains unchanged.

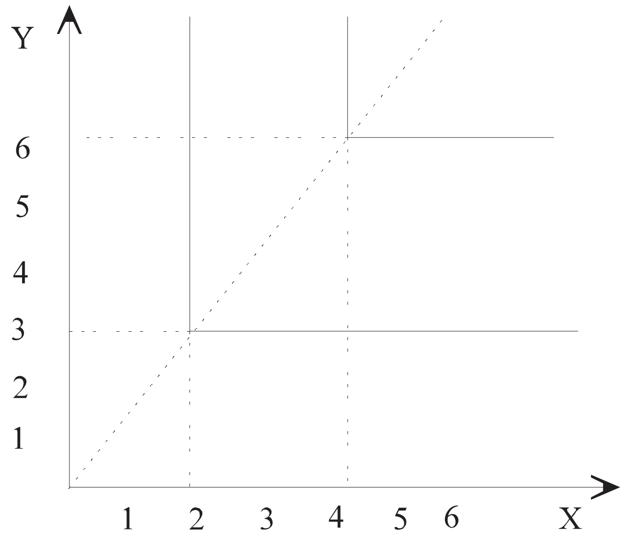
E. Price of X decreases and income of the consumer increases.

F. Price of X decreases and income of the consumer also decreases.

Which of the above possibilities can lead to a movement of the budget constraint from BC1 to BC2? (Write below the letters that correspond to each scenario that can have the effect stated above.)

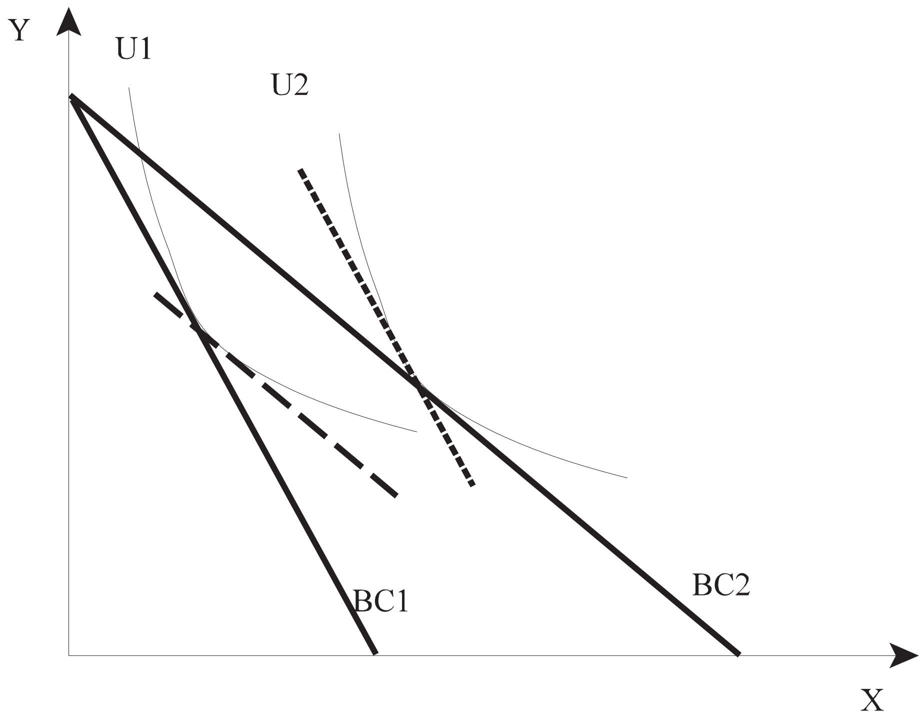
Since the budget constraint become flatter, it must be that X becomes cheaper relative to Y. Hence, A and B **cannot** be right. C cannot be right, because the X intercept shifts outwards (implying more income). D cannot be right since the Y intercept is not unchanged. E cannot be right since the Y intercept shifts in (implying an decrease in income). The only correct statement is F.

3. A consumer always purchases 2 units of X with every 3 units of Y regardless of the prices of X and Y.



In the above figure, draw two of this consumer's indifference curves.

4. Consider the figure below, which has two indifference curves, U1 and U2 (U2 corresponds to a high level of utility) and two budget constraints, BC1 and BC2. The dashed line is parallel to BC2 and tangent to U1, while the dotted line is parallel to BC1 and tangent to U2.



Answer the following questions with regards to this figure.

a. The shift from budget constraint BC2 to BC1 represents an increase in the price of product Y, decrease in income, a decrease in the price of X, or none of the above?

   None of the above. It represents an increase in the price of X.

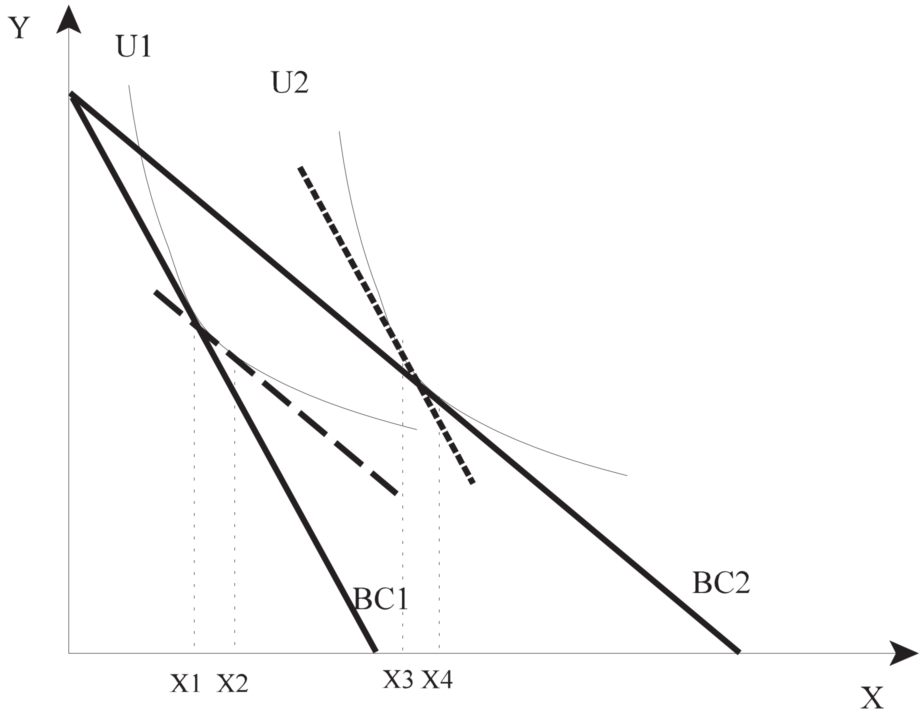
b. In the range of prices and real income changes shown in the figure above, is good Y normal or inferior? Explain why (in one sentence or two at most).

   It is inferior because the income effect of the price increase of X is to increase the demand for Y, while the income effect of a price decrease of X is to decrease the demand for Y.

c. Good X is Giffen. Is this statement true, false, or we can't tell on the basis of the above figure. Explain in a single sentence.

   It is false, because a decline in the price of X leads to an increase in the demand for X.

5. Consider the figure below, which has two indifference curves, U1 and U2 (U2 corresponds to a high level of utility) and two budget constraints, BC1 and BC2. The dashed line is parallel to BC2 and tangent to U1, while the dotted line is parallel to BC1 and tangent to U2.



Answer the following questions with regards to this figure.

a. The change in the consumption of X due to the substitution effect of the budget constraint shift from BC1 to BC2 is given by (i) X2-X1, (ii) X3-X1, (iii) X3-X2, (iv) X4-X2, or (v) X4-X3?

   It is given by X2-X1.

b. In the range of prices and real income changes shown in the figure above, is good X normal or inferior? Explain why (in one sentence or two at most).

It is normal, because comparing the consumption of X for budget constraints that correspond to different incomes but the same prices, we see that more X is purchased as income goes up. In particular, X4>X2 and X3>X1.

c. Good Y is Giffen. Is this statement true, false, or we can't tell on the basis of the above figure. Explain in a single sentence.

Good Y is inferior, which suggests that it might be Giffen, but we cannot tell for sure because the above figure does not involve changes in the price of Y.

6. The demand for a good X is given by

$$X = \frac{P_Y}{P_X}\sqrt{I}$$

Answer the following questions. Support your assertions using algebra.

a. Is good X normal? Why or why not?

Yes, because the derivative with respect to *I* is positive (an increase in *I* increases *X*).

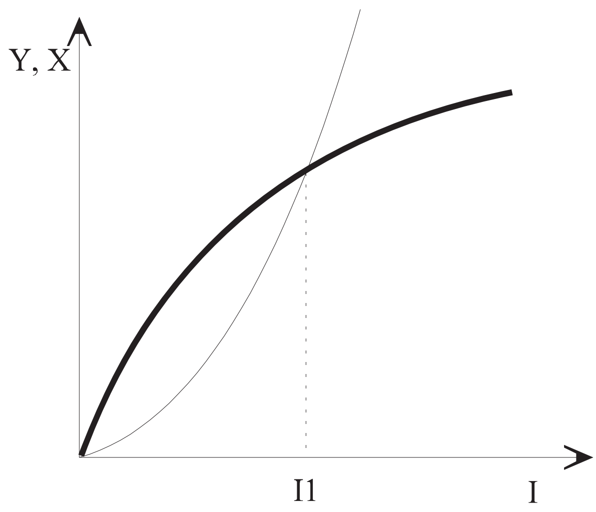
b. Is good X a luxury? Why or why not?

No, it is not a luxury, because an increase in *I* increases *X* less than proportionately (for example, doubling *I* leads to less than a doubling in *X*).

c. Is good X Giffen? Why or why not?

It is not Giffen because the derivative of X with respect to its price is negative. Increasing the price of X reduces the demand for X.

7. Consider the following two Engel curves for goods X and Y. The Engel curve for Y is in bold.



Which of the following statements are true and why. [They are not mutually exclusive, so more than one of them can be true. You have enough information to answer all of them.]

a. This consumer consumes more Y than X when his income is less than I1.

   True.

b. Y is a normal good.

   True (the Engel curve is upward sloping).

c. X is an inferior good.

   False (the Engel curve is upward sloping).

d. X is a luxury good.

   True (the Engle curve is increasing faster than income)

e. X is a luxury only for incomes that exceed I1.

   False (the Engle curve is increasing faster than income for all income levels)

f. Y is a Giffen good.

False. Giffen goods must be inferior, and Y is a normal good.

8. Consider the following statement:

"A good may be a luxury to some people but a necessity to others."

Is this statement true or false. Explain why in the space provided below. [You only need 2 or 3 sentences for this.]

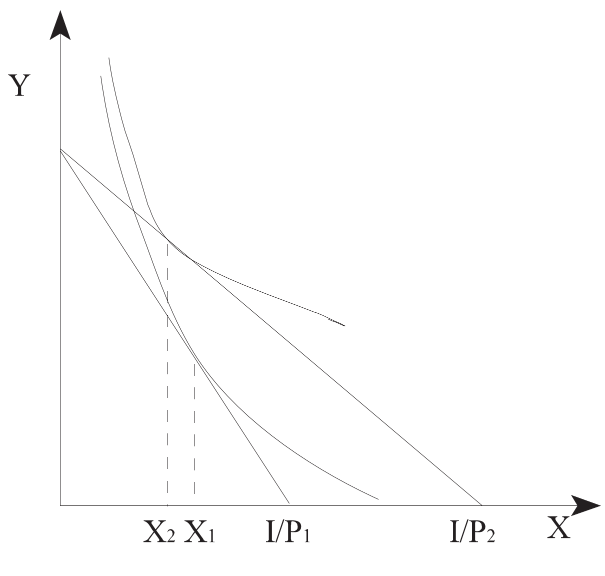
Even for the same person, a good may be a luxury at low income levels and a necessity at high income levels. Clearly then, this statement can be true for two people with the same preferences but different incomes.

9. Consider two goods, X and Y, that are perfect complements. [In other words, consumers have Leontieff preferences for them.] Consider an increase in the price of X. Show in a diagram how big is the change in the consumption of X that is due to the *substitution* effect.

No portion of the change in the consumption of X is due to the substitution effect (see lecture notes).

10. What is a Giffen good ? Illustrate the concept of a Giffen good by using a figure with indifference curves and budget constraints. Label your figure carefully.

A Giffen good is a good the demand for which is reduced when its price decreases. This concept is illustrated in the figure below, where good X is assumed to be a Giffen good.



11. What goods are known in economic terminology as 'luxuries' ?

Luxuries are the goods the demand for which goes up by more than 1% when income increases by 1%.

Problems

1. The Kreatofagos family is having a barbecue. They send their son, Bobiras, to the local store to buy meat. The parents don't know the prices of meat in the store. They give Bobiras $40 and tell him to spend all the money on beef and chicken, but buy twice as many pounds of beef than chicken *regardless of the prices* in the store.

a. Do the instructions of the parents imply that beef and chicken perfect substitutes or do they imply that they are perfect complements ? Why ?

The instructions of the parents imply that the goods are perfect complements. The proportion in which they want to purchase the two types of meat are independent of their relative prices. Had the two types of meat been perfect substitutes, they would only want to buy one of them: the one that was the best value.

b. What is the budget constraint of Bobiras as he heads into the store ?

$$P_{Beef}\ Q_{Beef} + P_{Chicken}\ Q_{Chicken} = 40$$

c. If the price of chicken is $2.7 per pound and the price of beef $4.0 per pound, how many pounds of each will he purchase ?

At these prices his budget constraint becomes:

$$4\ Q_{Beef} + 2.7\ Q_{Chicken} = 40$$

Using the fact that he needs to purchase twice as much beef as chicken, we can write

$$4\ 2\ Q_{Chicken} + 2.7\ Q_{Chicken} = 40 \quad \Rightarrow$$

$$10.7\ Q_{Chicken} = 40 \quad \Rightarrow$$

$$Q_{Chicken} = 3.738$$

Since he purchases twice as much beef,

$$Q_{Beef} = 7.476$$

d. Suppose that, on the way to the check out counter, Bobiras finds out a coupon that entitles him to 25% off on any beef. [It is today's special!] Will Bobiras buy more beef as a result? If so, how much more ?

With the coupons, the price of beef becomes \$3. The budget constraint at these prices is

$$3\ Q_{Beef} + 2.7\ Q_{Chicken} = 40$$

Repeating the steps in part (c) we get

$$3\ 2\ Q_{Chicken} + 2.7\ Q_{Chicken} = 40 \quad \Rightarrow$$

$$8.7\ Q_{Chicken} = 40 \quad \Rightarrow$$

$$Q_{Chicken} = 4.6$$

and

$$Q_{Beef} = 9.2$$

Therefore, he will purchase 1.724 more pounds of beef.

e. Will he also buy more chicken ? If so, how much more ?

He will also purchase 0.862 more pounds of chicken.

2. A consumer always consumes products X and Y in fixed proportions regardless of prices: 2 units of X are consumed with 3 units of Y. The price of X is P_X and the price of Y is P_Y. The consumer's income is I.

a. How many units of X and how many units of Y will this consumer purchase? (The answer will be a function of prices and income.)

Since the consumer always consumes X and Y in a ratio of 2 to 3 regardless of prices, his preferences for X and Y are Leontieff, with the optimal allocation of funds to X and Y satisfying

$$\frac{X}{Y} = \frac{2}{3} \Rightarrow$$

$$X = \frac{2}{3} Y$$

The optimal consumption levels of X and Y are obtained by combining the above relationship with the budget constraint

$$P_X X + P_Y Y = I$$

Substituting in for X from the above expression and solving for Y we get

$$P_X + \frac{2}{3} Y + P_Y Y = I \Rightarrow$$

$$Y \left(P_X \frac{2}{3} + P_Y \right) = I \Rightarrow$$

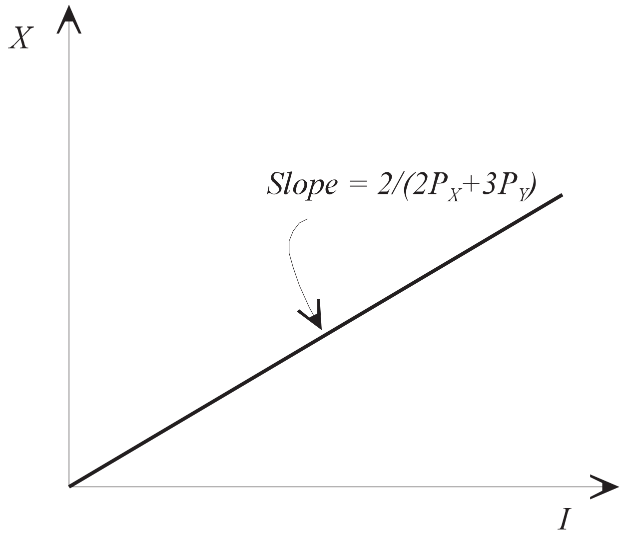
$$Y = \frac{I}{\frac{2}{3} P_X + P_Y} \Rightarrow$$

$$Y = \frac{3 I}{2 P_X + 3 P_Y}$$

Substituting this into the expression for X above and simplifying we get:

$$X = \frac{2 I}{2 P_X + 3 P_Y}$$

b. In the space below, draw the Engel curve for *X*. Make sure you label the graph carefully, including giving the intercept and the slope of the Engel curve.



The Engel curve is a straight line, starting from the origin, and with a slope of $2/(2P_X+3P_Y)$.

3. Consider a consumer with a utility function for goods *X* and *Y* given by

$$U = 5Y - \frac{10}{X}$$

The price of *Y* is equal to 5 and the price of *X* is equal to 10. The consumer has income *I*.

a. How many units of *X* and how many units of *Y* will the consumer purchase if *I*=5?

If the consumer's income is equal to 5, the budget constraint is given by

$$5Y + 10X = 5$$

Before we proceed to calculate the optimal choice of *X* and *Y*, let us ensure that this utility function has diminishing MRS. Solving the utility function for *Y*, we obtain

$$Y = \frac{U}{5} + \frac{2}{X}$$

This yields the equation for the indifference curve that corresponds to utility U. The slope of the indifference curve (times -1) equals $MRS_{X,Y}$. This is given by

$$MRS_{X,Y} = -\frac{dY}{dX}$$

$$= \frac{2}{X^2}$$

Clearly, as the consumer has more and more units of X (holding utility constant), the value of X goes down: For higher values of X, it takes fewer units of Y to make up for the loss of one units of X. Thus, this utility function does exhibit diminishing MRS.

Since this is a differentiable function with diminishing MRS, we will employee the condition

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$$

and the budget constraint to find the optimal values of X and Y (this assumes that the consumer consumes positive quantities of X and Y, but we can easily check whether the answer to the problem consists of positive quantities!)

This condition yields

$$\frac{5}{5} = \frac{\frac{10}{X^2}}{10} \quad \Rightarrow$$

$$1 = \frac{1}{X^2} \quad \Rightarrow$$

$$X = 1$$

Substituting into the budget constraint, we get

$$5\,Y + 10 \cdot 1 = 5 \quad \Rightarrow$$

$$5\,Y = -5 \quad \Rightarrow$$

$$Y = -1$$

The solution yields a negative value for *Y*! Since the consumer cannot possibly consume negative quantities of a good, this suggests a corner solution: the optimal value of *Y* is zero, and the consumer will spend all his money on *X*.

Thus, the optimal consumption of *X* is given by

$$X = \frac{I}{P_X} = \frac{5}{10} = \frac{1}{2}$$

and the optimal consumption of *Y* is zero.

b. How many units of *X* and how many units of *Y* will the consumer purchase if *I*=20?

The only difference between part (a) and part (b) is in the budget constraint (income has increased to 20). Substituting $X = 1$ into the new budget constraint we obtain

$$5Y + 10 \cdot 1 = 20 \quad \Rightarrow$$

$$5Y = 10 \quad \Rightarrow$$

$$Y = 2$$

Both of these quantities are positive; thus the consumer will purchase one unit of X and two units of Y.

c. Derive and plot the Engel curve for good *Y*. Label your graph carefully.

To derive the Engel curve, we need to compute the consumption for *Y* as a function of income, *I*. The budget constraint, treating income as a parameter, is given by

$$5Y + 10X = I$$

Plugging in $X = 1$ we get

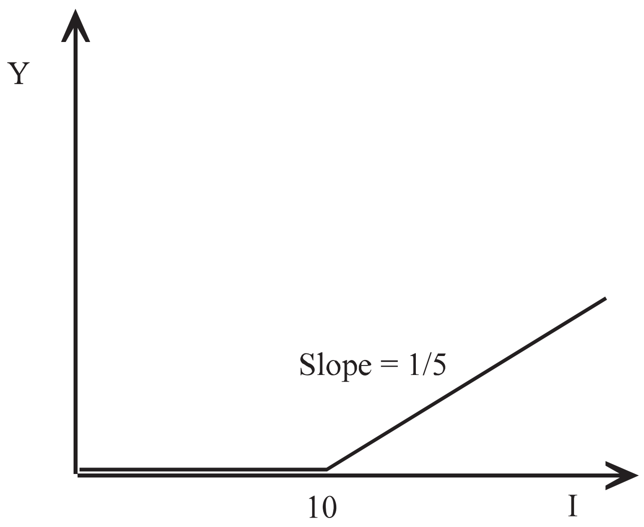
$$5Y + 10 \cdot 1 = I \quad \Rightarrow$$

$$5Y = I - 10 \quad \Rightarrow$$

$$Y = \frac{I}{5} - 2$$

When income is less than or equal to 10, the consumer will consume no Y. [For low values of I, the expression above yields negative numbers.] For income greater than 10, Y will be positive and increasing by 1/5 units for every one unit increase in income.

Therefore, the Engel curve for Y is given by the figure below.



4. A consumer always consumes soda (S) and chips (C) in fixed proportions regardless of prices: 5 ounces of S are consumed with 2 ounces of C. The price of chips is always equal to 0.2 per ounce, while the price of soda varies from time to time, as it is often on sale, and is denoted by P_S. The consumer's income budget for soda and chips is equal to I.

a. How many units of soda (S) and how many units of chips (C) will this consumer purchase? (The answer will be a function of the price of soda and the budget for soda and chips).

Because the consumer buys 5 units of S for every 2 units of C regardless of prices, the quantities of S and C are related by the equation

$$2S = 5C \quad \Rightarrow$$

$$S = \frac{5}{2}C$$

Another way to see that this relationship holds is to observe that 5 S for every 2 C means that for unit of C, he will consume 5/2 units of S, thus

$$S = \frac{5}{2} C$$

The consumer has an income I. His budget constraint, then, is given by

$$P_S S + 0.2 C = I$$

Substituting in the relationship between S and C we get

$$P_S \frac{5}{2} C + \frac{1}{5} C = I$$

Solving for C gives the optimal consumption of chips.

$$\left(P_S \frac{5}{2} + \frac{1}{5} \right) C = I \Rightarrow$$

$$C = \frac{I}{2.5 P_S + 0.2}$$

Plugging back into the relationship between S and C above, we obtain the optimal consumption of soda to be

$$S = \frac{5}{2} \frac{I}{2.5 P_S + 0.2}$$

$$= \frac{I}{P_S + \frac{0.2}{2.5}}$$

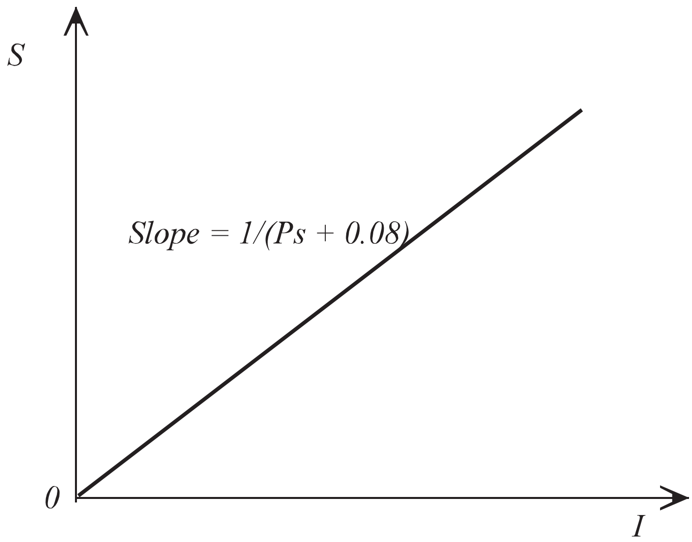
$$= \frac{I}{P_S + 0.08}$$

b. Does the consumption of chips change when soda is on sale, and if so, it what way? Can you explain in plain, clear English why the price of soda has this effect on the consumption of chips?

Yes, when soda is on sale, i.e., when P_S goes down, the consumption of chips goes up. This is because soda and chips are perfect complements: a consumer can only enjoy any additional soda that he buys (because it is on sale) if he also consumes chips along with it.

c. In the space below, draw the relationship between the consumption of soda and the consumer's budget for soda and chips, with *I* in the horizontal axis and the consumption of soda in the vertical axis. Make sure you label the graph carefully, including labeling the slope and intercept.

Plotting the expression obtained in part (a) with respect to I we obtain the following figure.



As the price of soda goes up, the slope of this line goes down. With soda being more expensive, the consumer buys less of it given any level of budget for soda and drinks. The converse is true when the price of soda goes down.

5. Consider a consumer with a utility function for goods X and Y given by

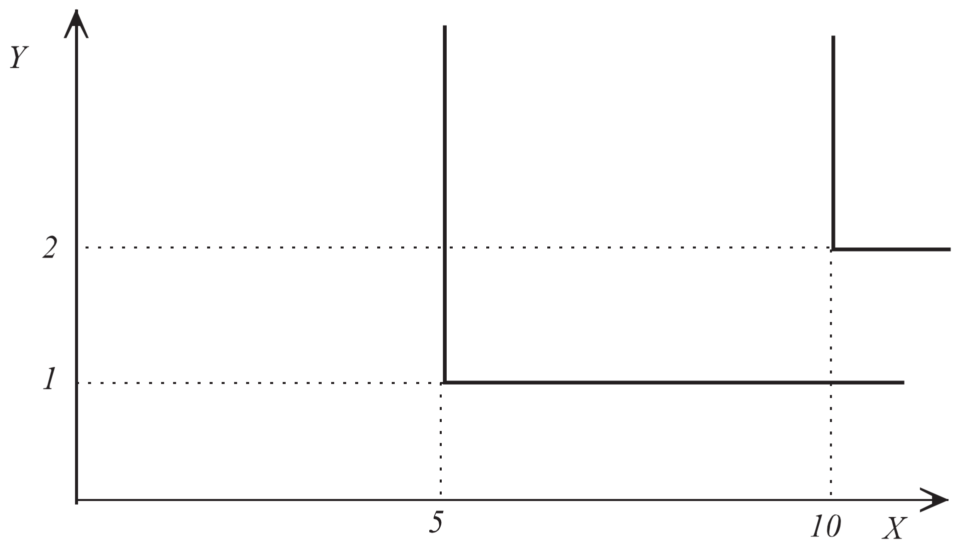
$$U = \min\{5Y, X\}$$

The price of Y is equal to 5 and the price of X is equal to 10. The consumer has income I.

a. Draw two indifference curves for this consumer with Y on the vertical axis and X in the horizontal axis. It does not matter which two indifference curves you draw, but you must label your graph clearly.

Recall that the indifference curves of the Leontief (perfect complements) utility function are L-shaped. The corner of the "L" has values of Y and X that satisfy $5Y = X$, that is, it has values such that both terms of the utility function yield the same value.

If we set Y to 1 and 2, the corresponding values of X would be 5 and 10. Thus, two indifference curves of this utility function can be represented in the figure below.



b. How many units of X and how many units of Y will the consumer purchase if I=110?

The optimal consumption level of the consumer will be at the corner of one of the indifference curves and will, thus, satisfy the equation $5Y = X$. This equation gives the optimal proportion of X and Y. To obtain the actual level of X and Y, we need to incorporate the budget constraint, which is given by the equation

$$10X + 5Y = 110$$

Substituting the expression of X into the budget constraint we get

$$10\ 5Y + 5Y = 110.$$

Solving for Y we obtain

$$55Y = 110 \quad \Rightarrow$$

$$Y = 2$$

The optimal consumption of X is obtained by observing that the consumer will choose five units of X for every one unit of Y. That is,

$$X = 10$$

c. Compute the optimal consumption of Y as a function of income I. Plot your answer in a graph with income in the horizontal axis and Y in the vertical axis. Label your graph carefully.

Regardless of the level of income the consumer has, the optimal proportion of X and Y is given by the equation $5Y = X$. However, if the consumer's income is I, the budget constraint is given by

$$10X + 5Y = I$$

Substituting the expression of X into the budget constraint we get

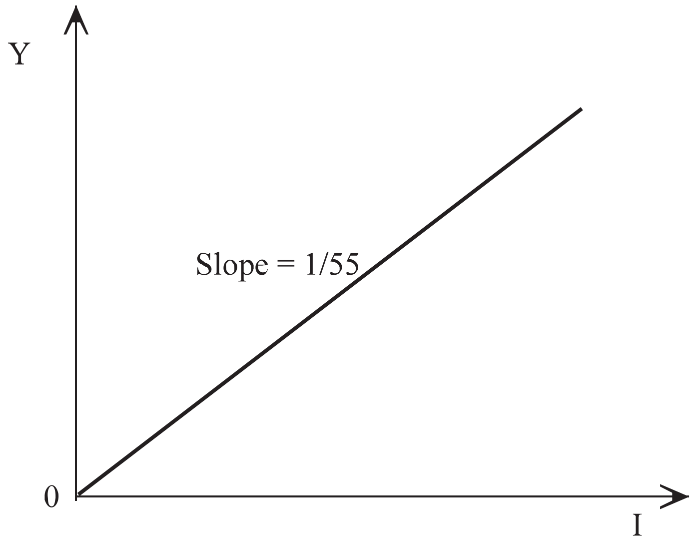
$$10\ 5Y + 5Y = I.$$

Solving for Y we obtain

$$55Y = I \quad \Rightarrow$$

$$Y = \frac{I}{55}$$

Observe that this gives us the equation of a straight line with an intercept of 0, and a slope of 1/55. Therefore, the graph of this equation with Y on the vertical axis and I on the horizontal axis is as given below.


Y
Slope = 1/55
0
I