

LECTURE 9: TEMPORAL ASPECTS OF PRODUCTION FUNCTIONS

ANSWERS AND SOLUTIONS

True/False Questions

True_ If two firms have the same technology but differ in their capital assets, their short-run capability to produce output will also differ.

False_ Technological progress is graphically depicted by shifting out the isoquants of a production function.

Short Questions

1. A firm production function is characterized by fixed proportions of capital (workstations) and labor (workers who man these workstations). Because the firm runs two shifts, the fixed proportions are 2 workers per workstation. Each workstation coupled with the two workers produces 5 units of output.

a. Write down the production function of this firm.

Denote the number of workstations by K and the number of workers by L . For each unit of output, the firm needs $1/5$ workstations and $2/5$ workers. Thus, the production function is given by the equation

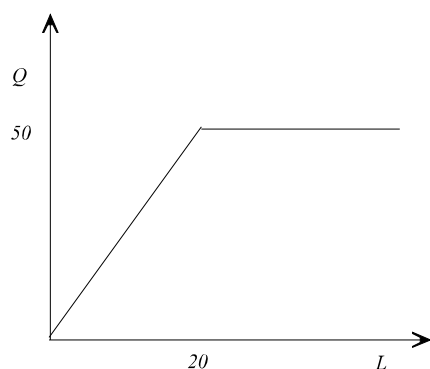
$$q = \min\left\{5K, \frac{5}{2}L\right\}$$

b. Suppose the firm's capital (workstations) is fixed in the short run to 10 units. Graph this firm's short run production function with output on the vertical axis and workers in the horizontal axis.

If the capital is fixed to 10 units, the production function becomes

$$q = \min\left\{50, \frac{5}{2}L\right\}$$

When $2.5 * L$ is less than 50 (or $L < 20$), the output is constrained by the number of workers, and is thus given by $q = 2.5 * L$. When $2.5 * L > 50$ ($L > 20$), then the output is constrained by the number of workstations, and it will be equal to 50.



Problems

1. A firm's production function uses skilled and unskilled labor (denoted by S and U, respectively). A skilled worker can do the work of two unskilled workers, and this ratio does not depend on the number of the two types of workers that are employed by the firm. The elasticity of scale of this firm is 0.5.

- a. Write down this firm's production function. [Note: there is one unspecified parameter! you will not be able to express it solely in terms of S, U, and numbers but need to use some additional parameter to specify the firm's productivity.]

This is a linear production function, since the MRTS is independent of the ratio of the inputs. Given that the productivity ratio is one skilled worker for two unskilled workers, and the elasticity of scale equal to 0.5, the production function is given by

$$Q = A (2 S + U)^{0.5}$$

where A is a parameter that indicates the firm's productivity.

- b. Suppose that one unskilled worker working by himself can produce 3 units of output. Now, you should be able to write down this firm's production function solely in terms of S, U, and numbers. What is it?

Imputing S=0, U=1, and Q=3, the above equation becomes

$$3 = A (1)^{0.5} \Rightarrow A = 3$$

Therefore, the production function can be written as

$$Q = 3 (2 S + U)^{0.5}$$

- c. Suppose the firm employs 10 skilled workers, the number of which cannot change in the short-run because skilled workers work for long-term contracts (cannot be quickly fired) and are hard to find (cannot be quickly hired). What is this firm's short-run production function? Plot it with output on the vertical axis and unskilled workers on the horizontal axis.

When we fix S to 10, the production function becomes

$$Q = 3 (20 + U)^{0.5}$$

Plotting this looks like this

