Auctions and Bidding

How to design auctions and markets

Lollie Auction

- Jar contains a certain number of lollies
- Winner of auction will get 2 cents per lollie
- You are bidding for the money, *not the lollies*!
- Playing for real money.

Traditional Auction Experience

Wine Auctions

	Auction House			
Later Price	**Christie's London**	**Sotheby's London**	**Christie's Chicago**	**Butterfield's San Francisco**
Higher	271	143	90	20
Lower	628	430	183	41
Identical	1498	1073	226	39
Mean Ratio (2^{nd} to 1^{st} price)	.9943	.9875	.9884	.9663

US Spectrum Allocation

- Comparative hearings
  - Public interest
  - Overwhelmed by mobile phone applications
- Lotteries (Reagan)
  - Political compromise
  - ‘Unjust enrichment’ and admin probs
- Auctions (Clinton)
  - Market determined assignments
  - Regulation of allocation (‘uses’)

The RCA Transponder Auction

- A sequential auction (1981)

Order	Winning Bidder	Price Obtained
1	TLC	14,400,000
2	Billy H. Batts	14,100,000
3	Warner Amex	13,700,000
4	RCTV	13,500,000
5	HBO	12,500,000
6	Inner City	10,700,000
7	UTV	11,200,000
Total		90,100,000

FCC DBS Auction Results

- A sequential auction

	High Bidder	Final Price
Auction 7	MCI	682,500,000
Auction 8	Echostar	52,295,000

New Zealand UHF License Auction

- A simultaneous sealed-bid second price auction (1993)

Lot	Winner	High Bid	2nd Bid
1	Sky Network TV	2,371,000	401,000
2	Sky Network TV	2,273,000	401,000
3	Sky Network TV	2,273,000	401,000
4	BCL	255,124	200,000
5	Sky Network TV	1,121,000	401,000
6	Totalisator A.B	401,000	100,000
7	United Christian	685,200	401,000

Stuff-Ups

- No minimum bid price was set
- A student in Dunedin bid $1 for a TV license and got it for $0 because there was no other bid.

Australian Satellite TV Auction

- A sealed bid auction with no withdrawal penalty (1991)

Initial Winning Bid	Final Transaction Price
212,000,000	117,000,000
177,000,000	77,000,000

Money money money

Auction No.	Auction Name	Licenses Auctioned	Net High Bids ($M)
1	Nationwide Narrowband PCS 7/25/1994-7/29/1994, Nationwide	10	$617.0
4	A & B Block PCS 12/5/1994-3/13/1995, MTA	99	$7,019.4
5	C Block PCS 12/18/1995-5/6/1996, BTA	493	$9,197.5
8	DBS (110 W) 1/24/1996-1/25/1996, Nationwide	1	$682.5
10	C Block PCS Reauction 7/3/1996-7/16/1996, BTA	18	$904.6
11	D, E, & F Block PCS 8/26/1996-1/14/1997, BTA	1479	$2,517.4
17	Local Multipoint Distribution Service (LMDS) 2/18/1998-3/25/1998, BTA	986	$578.7
33	Upper 700 MHz Guard Bands 9/6/2000-9/21/2000, MEA	104	$519.9
35	C & F Block PCS 12/12/2000-1/26/2001, BTA	422	$16,857.0

Failed Thrifts

- US S&L Crisis: set up the Resolution Trust Corporations to sell off failed thrifts
- Some had negative value: allowed negative bids
- Bidders
  - Had to be of right type
  - Got lots of information (due diligence)
  - Could withdraw bid due to ‘computational error’ (I.e., never overbid)
- Has been broadly successful in recovering losses
- Suffered a little from lack of competition

Other auction settings

Auctions you may not have known were there

Babylonian Wife Auction

As described by Herodotus …

- All women had to be auctioned off as wives of the men of the land
- The women were presented sequentially. Open cry English auction.
- The high bidder married that woman
- The money collected did not go to the family of the woman sold.
- If none of the men wished to pay a positive amount to purchase one of the women as a wife, the auction proceeded in the reverse direction: the woman was offered along with a dowry of 100 drachmas to the audience.
- If no one took the woman plus one hundred drachmas then the monetary value was increased from 100 drachmas to 200 to 300 to 400 etc. until someone was willing to take that woman together with the monetary compensation that would go along with her.
- The Babylonian auction represented a kind of socialization of the value of beauty, which is unfairly distributed by fate or nature. What it meant between men and women is, of course, another story.

Google

- Google is the world's largest auction house
- Advertisers submit bids for a specific keyword (state their willingness to pay for a click)
- When user searches, received sponsored links in descending order based on bids
  - If user clicks on a bid in position 2, the advertiser is charged the bid price of the bid in position 3.
  - Actually, this is Yahoo (Google also bases the ranking and bids on estimated click through rates)

Economic Analysis of Auctions

- An auction governed by precise rules can be modelled and analysed as a formal "*game*."
- Game theory and lab experiments help predict:
  - The outcomes of alternative auctions with different rules
  - Which rules generate the highest revenue
  - The biases the auction creates for different kinds of bidders
- *Theory supported by experiments can be used to create and test auctions to advance various design goals*

Outline

- Auction Theory
  - Traditional Auction Design (Today)
  - Comparing Auctions (Thursday)
- Mini-Cases (Next Monday)
- Case (Thursday Week)
- Other Auctions
  - All-pay
  - Incentive payments and linkage
  - Multi-unit (e.g., electricity)
  - Combinatorial
  - Internet

Auction Theory

Private Values

- Bidders' willingnesses-to-pay for the item are distinct (independent)
- "I want it"
- Examples
  - JFK Golf Clubs
  - Holidays and travel
  - Trading Post

Common Values

- Bidders' willingnesses-to-pay are related to one another (correlated)
- "I am not sure I want it, but others might in the future"
- Examples
  - real estate
  - asset sales
  - oil rights

English or Oral Ascending

- Auctioneer calls out an increasing series of prices, until no bidder is willing to top the current price.
- Examples:
  - Antiques
  - Real estate
  - Art
  - Wine
  - Many Internet auctions
  - US Government timber sales

Optimal Bidding

- Independent estimates
  - Bid so long as WTP > price
- Correlated estimates
  - Need to update maximum bid depending upon what others' are bidding

Dutch or Oral Descending

- Auctioneer calls out a decreasing series of prices, until a bidder says “I’ll take it at this price.”
- Examples
  - Tulips in Holland
  - Fish in Israel

Optimal Bidding

- Independent estimates
  - Need to estimate what others' WTP is
- Correlated estimates
  - Need to consider fact that others have not bid

First-Price Sealed Bid

- Each bidder submits a bid in an envelope. The bids are opened at the same time, and the high bid wins the object and pays their bid.
- Examples:
  - Government sales of timber and offshore oil
  - Government purchases of french fries, airplanes, road construction, pencils, zero gravity toilets
  - Typically used by bureaucracies (governments or large firms) to sell or buy things

Optimal Bidding

- Independent estimates
  - Need to estimate what others' WTP is
- Correlated estimates
  - Need to consider fact that others have not bid

(Similar to Dutch auction)

Strategic Bidding

- “Would I be willing to purchase the tract of land for $1.08 billion, given what I know before submitting my bid?”

versus

- “Would I still be willing to purchase the tract of land for $1.08 billion, given what I know before submitting my bid and given the knowledge that I will be able to purchase the land only if no one else is willing to bid $1.08 billion for it?”

Winner's Curse

- "I paid too much for it, but it's worth it." (Sam Goldwyn)
- The winner's curse is the fact that the bidder who overestimates the value of the object wins the bidding.
- Should reduce bid to adjust for this, and reduce more the more bidders there are.

Winners curse

- Perhaps the most preposterous auction in history was held in 193A.D., when the Praetorian Guard placed the whole Roman Empire on the block. After having killed the preceding emperor, Pertinax, the leaders of the guard announced that they would bestow the crown upon the Roman who offered the largest donation. Didius Julianus outbid all his rivals by promising each man 6,250 drachmas and the guards declared him emperor. Just after two months in power, he was overthrown and executed by Septimius Serverus.

Second-price Sealed Bid or Vickery Auction

- Bidders submit bids in envelopes, which are opened simultaneously. High bidder pays the second highest bid.
- Examples:
  - Radio spectrum in NZ

Optimal Bidding

- Suppose know you value the object at *v*.
- *It is a (weakly) dominant strategy to bid v.*
- If the highest opposing bid of B is made by K bidders and i bids b, i's payoff is

$$\left.\begin{array}{c} v-B \\ (v-B)/(K+1) \\ 0 \end{array}\right\} \text{ if } \begin{cases} B<b \\ B=b \\ B>b \end{cases}$$

Remainder of Proof

- Notice that
  1. if $v > B$, then only bids greater than B earn the maximal payoff of v-B;
  2. if $v < B$, then only bids less than B earn the maximal payoff of 0, and
  3. if $v = B$, then all bids earn the same payoff, namely, 0.
- For all B, this function is maximised by $b = v$, and no other bid has that property. QED
- Get truthful revelation. Eliminates strategic bidding. No winner's curse

Comparing Auctions

Which auction design should you choose?

Goals of Auction Design

- Maximise seller's expected revenue
- Maximise efficiency
  - Ensure the object(s) go to the bidders who value them the most.

Comparing Revenue

- What is the best the seller could ever do?
- What is the expected highest willingness-to-pay of bidders?
- Is it a good idea for bidders to have complete information?

Bidding Strategies

Auction	Independent Estimates	Correlated Estimates
English	Bid up to WTP	Observe others' bids and adjust WTP
Dutch	Forecast others' WTP	Forecast others' and adjust WTP as price falls
First-Price Sealed Bid	Forecast others' WTP	Forecast others' WTP and adjust for winner's curse
Vickery	Bid WTP	Adjust WTP for winner's curse and bid WTP

Basic Assumptions

- Two bidders: A and B
- Independent estimates: I.e.,, an independent draw from a uniform distribution between 0 and 1
- Bidders are risk neutral
- A knows their WTP = V_A. B knows their WTP = V_B
- Suppose that $V_A > V_B$, although neither player knows this

Aside: Experimental Results

- Did people bid their values in a 2nd Price auction?
  - Average VALUE – BID = ?? and ?? (compared with ? and ? in the 1st price auction)
  - Number who had 1 or 0 difference = ? out of ?

Price in an English Auction

- When will the lowest bidder drop out on average?
  - Given V_A, B's valuation is likely to lie anywhere between 0 and V_A.
  - On average $V_B = V_A/2$.
  - This is when the lowest bidder drops out on average
- What will the highest bidder pay on average?
  - On average, the highest bidder (in this case A) will pay $V_A/2$ and receives expected surplus of $V_A/2$.

Revenue in an English Auction

- Given two draws from the distribution:
  - On average, V_B halfway between 0 and V_A
  - On average, V_A halfway between V_B and 1
  - Implies that $E[V_B] = 1/3$ and $E[V_A] = 2/3$
- Therefore, expected revenue is

$$E[V_B] = E[V_A]/2 = 1/3$$

English v. Vickery

- In Vickery auction, price will be equal to the second highest bid
- Since A and B reveal true values,

$$E[\text{price}] = E[V_B] = E[V_A]/2 = 1/3$$

- Therefore, English and Vickery auction will have the same expected revenue.

Price in a Sealed Bid Auction

- Bidders will shade bids below their true values; but by how much?
- Bidders choose bid to maximise

$$p(V_i - b_i)$$

where p is the probability of winning and b_i is the bid of i.

- Show that Nash equilibrium involves each bidder bidding half their value.

Nash Equilibrium I

- Suppose that B submits $V_B/2$ (as per NE), will A adopt the same strategy?
- A's Probability of Winning (p)

$$
\begin{aligned}
p &= \Pr[b_A > b_B] \\
&= \Pr[b_A > V_B / 2] \\
&= \Pr[V_B < 2b_A] \\
&= 2b_A
\end{aligned}
$$

Nash Equilibrium II

- A chooses bid to maximise

$$\pi_A = 2b_A(V_A - b_A) = 2b_A V_A - 2b_A^2$$

$$\frac{\partial \pi_A}{\partial b_A} = 2V_A - 4b_A = 0$$

- Therefore, $b_A = V_A/2$.
- Analysis for B proceeds the same way.

Aside: Optimal 1st Price Bid

- In our experiment, how close were people to the equilibrium bid?
  - Average V/2 – b = ? and ??
  - Number who bid an equilibrium = ? in the 2nd one
- If others are likely to overbid, should you also overbid?
  - For given bid, less chance of winning
  - Therefore, likely to bid more (can work this out mathematically)

Revenue Equivalence

- Can show first-price sealed bid and Dutch auctions have same outcome.
- Have demonstrated that, in each case, the winner's expected bid is half their value.
- Hence, expected revenue is the same over all four auction types.

More than Two Bidders

- *n* bidders
- English auction
  - Expected price is $(n-1)v/n$
- Sealed bids
  - Nash equilibrium to bid $(n-1)/n$ of true WTP
- As *n* increases, price rises.
- However, value of extra competition greatest when there are few bidders.

Correlated Estimates

- May have some common value elements
- Positive correlation or affiliation
  - As bidder's estimate of value rises so does other bidders' estimates.
  - Sellers make more from an English than first-price sealed bid auction
  - More information is available thus reducing the effect of the winner's curse in reducing bids.

Risk Aversion

- Makes bidders concerned about losses from over-bidding
  - More conservative in second-price auctions
- Also, concerned about not receiving object
  - Bid more quickly in Dutch auctions and higher in first-price sealed bid auctions
- Neither matters for English auctions – lower uncertainty

Seller Information

- Should the seller give (verifiable) private information to buyers?
- Milgrom and Weber say ‘yes’
  - Winning bidders receive more surplus if know more than rivals
  - By providing information, reduces the rents accruing to bidders from their private information

Evaluating English Auctions

- Benefits
  - More revenue
  - Economises on information gathering and bid preparation costs
- Costs
  - Require presence of bidders (or at least on-line)
  - Susceptible to bidding rings (collusion)

Reserve Prices

- Having a reserve guarantees a minimum profit if there are insufficient bidders.
- But does it turn it from an auction into a negotiation?

Financial Constraints

- What happens when bidders are financially constrained?
  - For 2nd price auctions, nothing: bid the minimum of your WTP or ATP.
  - For 1st price auctions, play similar strategy when constraints are high. In the absence of constraints, buyers bid less aggressively than 2nd price. So constraints are less likely to be a problem.
- Playing similar strategies but pay more in 1st price. Also, this means can more effectively use reserve prices in 1st price auctions.
- Revenue equivalence and efficiency no longer hold

Efficiency

- Private and Common values
  - All four auctions are efficient
- Correlated values
  - Only efficient if draw information from the same distribution and have symmetric strategies in equilibrium

Bidding to Buy or Sell?

- In auctions where parties *bid to buy* with value *V* and bid *V*, the winner's payoff is *V* - *b* and the losers get zero.
- In auctions where parties *bid to sell* goods or services at cost *c*, the winner is paid its bid amount. So, the winner gets *b* – *c* and
- losers get zero.
  - An equivalent formulation is that the parties are bidding to buy with a "value" of –*c* and a bid of –*b*.
  - As in any "bid to buy" auction, the winner's payoff is its value minus its bid, or –*c*-(-*b*)=*b*-*c*.
- Conclusion: our results about *bidding-to-buy* also apply to the case of *bidding-to-sell*.

Patent Auction Proposal

- Michael Kremer has proposed to eliminate the patent system and award inventors on the basis of the private value of the innovation.
- But how do you determine the value?
  - Answer: use an auction
  - Occasionally (with probability α) the bidder will actually have to pay for the patent.
  - When bidding will maximise

$$\alpha\left((v-b_i)\Pr[b_i > b_{-i}]\right)$$

Optimal Auctions

- Cannot improve on four auctions (for independent estimates case)
- Revelation principle: can we structure the auction so that bids tell true value.
- That is, suppose that price paid = βb; where β is some fraction.
  - Note that in a first-price sealed bid or Dutch auction $\beta = 1$.
  - In Vickery, β depends on others' bids.
- What would we choose β to be in order to maximise revenue?

Choose $\beta = ½$

- Suppose there are two bidders and the other bidder chooses $b_B = V_B$.
- Participation constraint: $(V_i - \beta V_i)\Pr[V_i > V_j] \geq 0$
  - satisfied for β less than or equal to 1.
- Incentive constraint:
  - A will choose b_A to maximise
    $$\pi_A = (V_A - \beta b_A)\Pr[b_A > b_B] = (V_A - \beta b_A)b_A$$
  - Choose $b_A = V_A/(2\beta)$
- Therefore, for truthful revelation set $\beta = ½$. This is at least as good as any other mechanism with another value of β.