Separating Income and Substitution Effects

Dakshina G. De Silva[1]

[1]Lancaster University

ECON 220

Effects of a Price Decrease

Can be broken down into two components

- Income effect
  - When the price of one goods falls, w/ other constant
  - Effectively like increase in consumers real income
  - Since it unambiguously expands the budget set
  - Income effect on demand is positive, if normal good
- Substitution effect
  - Measures the effect of the change in the price ratio
  - Holding some measure of income or well being constant
  - Consumers substitute it for other now relatively more expensive commodities
  - That is, Substitution effect is always negative
- Two decompositions: Hicks & Slutsky.

Hicks & Slutsky Decompositions

Hicks

- *Substitution Effect*: change in demand, holding utility constant
- *Income Effect*: Remaining change in demand, due to m change

Slutsky

- *Substitution Effect*: change in demand, holding real income constant
- *Income Effect*: Remaining change in demand, due to m change

Mathematics of Slutsky Decomposition

We seek a way to calculate mathematically the Income and Substitution Effects

Assume:

1. Income: m
2. Initial prices: p_1^0, p_2
3. Final prices: p_1^1, p_2

- Note that the price of good two, here, does not change

Given the demand functions, demands can be readily calculated as:

1. Initial demands: $x_i^0 = x_i(p_1^0, p_2, m)$
2. Final demands: $x_i^1 = x_i(p_1^1, p_2, m)$

Slutsky Mathematics (cont)

We need to calculate an intermediate demand that holds buying power constant

Let m_s the income that provides exactly the same buying power as before at the new price

– Thus: $m_s = p_1^1 x_1^0 + p_2 x_2^0$

The demand associated with this income is:

- $x_i^s = x_i(p_1^1, p_2, m_s) = x_i^s(p_1^1, p_2, x_1^0, x_2^0)$

Finally we have:

- Substitution Effect: $SE = x_i^s - x_i^0$
- Income Effect: $IE = x_i^1 - x_i^s$

Hicks' Mathematics

The only difference is between Hicks' and Slutsky is in the calculation of the intermediate demand

Let m_h the income that provides exactly the same utility as before at the new price

If u_0 is initial utility level, then: m_h solves
$u_0 = u(x_1(p_1^1, p_2, m_h), x_2(p_1^1, p_2, m_h))$

The demand associated with this income is:
- $x_i^h = x_i(p_1^1, p_2, m_h) = x_i^h(p_1^1, p_2, u_0)$

Finally we have:
- Substitution Effect: $SE = x_i^h - x_i^0$
- Income Effect: $IE = x_i^1 - x_i^h$

Calculating the Slutsky Decomposition

Assume that

$$u = x^{\alpha} y^{1-\alpha}$$

So the demand functions are:

$$x = \alpha \frac{m}{p_x}$$

and

$$y = (1-\alpha)\frac{m}{p_y}$$

Initial price is p_x^0 and final price is is p_x^1

$$x^0 = \alpha \frac{m}{p_x^0}$$

and

$$x^1 = \alpha \frac{m}{p_x^1}$$

Calculating the Slutsky Decomposition (cont.)

$$y^0 = y^1 = y = (1-\alpha)\frac{m}{p_y}$$

Now sub from x and y

$$m_s = p_x^1 x^0 + p_y y = p_x^1 \alpha \frac{m}{p_x^0} + p_y (1-\alpha)\frac{m}{p_y} = \left[\alpha \frac{p_x^1}{p_x^0} + (1-\alpha)\right] m$$

Calculating the Slutsky Decomposition (cont.)

since

$$m_s = \left[\alpha \frac{p_x^1}{p_x^0} + (1-\alpha)\right] m$$

we get:

$$x^s = \alpha \frac{m^s}{p_x^1} = \alpha \frac{m}{p_x^1} \left[\alpha \frac{p_x^1}{p_x^0} + (1-\alpha)\right] = \alpha^2 \frac{m}{p_x^0} + \alpha(1-\alpha) \frac{m}{p_x^1}$$

or

$$x^s = \alpha x^0 + (1-\alpha)x^1$$

Finally, we get:

$$SE = x^s - x^0 = \alpha x^0 + (1-\alpha)x^1 - x^0 = (1-\alpha)(x^1 - x^0)$$

$$IE = x^1 - x^s = x^1 - [\alpha x^0 + (1-\alpha)x^1] = \alpha(x^1 - x^0)$$

Calculating the Hicks Decomposition

We need to calculate m_h

Substituting our demand functions back into utility we get:

$$u = x^{\alpha} y^{1-\alpha} = \left(\alpha \frac{m}{p_x}\right)^{\alpha} \left((1-\alpha)\frac{m}{p_y}\right)^{1-\alpha} = \left(\frac{\alpha}{p_x}\right)^{\alpha} \left(\frac{1-\alpha}{p_y}\right)^{1-\alpha} m$$

Then m_h solves:

$$\left(\frac{\alpha}{p_x^1}\right)^{\alpha} \left(\frac{1-\alpha}{p_y}\right)^{1-\alpha} m_h = \left(\frac{\alpha}{p_x^0}\right)^{\alpha} \left(\frac{1-\alpha}{p_y}\right)^{1-\alpha} m$$

or

$$m_h = \left(\frac{p_x^1}{p_x^0}\right)^{\alpha} m$$

Calculating the Hicks Decomposition (cont.)

$$x^h = \alpha \frac{m^h}{p_x^1} = \alpha \frac{m}{p_x^1} \left(\frac{p_x^1}{p_x^0} \right)^{\alpha} = \alpha \frac{m}{\left(p_x^0\right)^{\alpha} \left(p_x^1\right)^{1-\alpha}}$$

Finally, we get:

$$SE = x^s - x^0 = x^1 \left(\frac{p_x^1}{p_x^0} \right)^{\alpha} - x^0$$

$$IE = x^1 - x^s = x^1 - x^1 \left(\frac{p_x^1}{p_x^0} \right)^{\alpha}$$

Demand Curves

We have already met the Marshallian demand curve

– It was demand as price varies, holding all else constant

There are two other demand curves that are sometimes used

- Slutsky Demand

– Change in demand holding purchasing power constant
– The function $x_i^s = x_i(p_1^1, p_2, m_s)$ we just defined

- Hicks Demand

– Change in demand holding utility constant
– The function $x_i^h = x_i(p_1^1, p_2, m_h)$ we just defined

Demand Curves (cont.)

We mentioned before that with Giffen Goods, the Marshallian demand curve slopes upward

However,

– Since the substitution effect is always negative, then

– both the Slutsky and Hicks Demands always slope downward—even with Giffen Goods