THE JOURNAL OF INDUSTRIAL ECONOMICS 0022-1821
Volume LI September 2003 No. 3

AN EMPIRICAL ANALYSIS OF ENTRANT AND INCUMBENT BIDDING IN ROAD CONSTRUCTION AUCTIONS*

DAKSHINA G. DE SILVA†, TIMOTHY DUNNE,‡ AND GEORGIA KOSMOPOULOU‡

This paper explores differences in the bidding patterns of entrants and incumbents in road construction auctions. We find that entrants bid more aggressively and win auctions with significantly lower bids than incumbents. The differences in their bidding patterns are consistent with a model of auctions in which the distribution of an entrant's costs exhibits greater dispersion than that of an incumbent's and relations of stochastic dominance in the distributions do not persist for the entire range of estimated costs. We also find that more efficient firms bid, on average, more aggressively and firms with greater backlogs bid less aggressively.

I. INTRODUCTION

THIS PAPER COMPARES the bidding patterns of entrant and incumbent firms in road construction auctions that took place between January of 1997 and August of 2000 in the state of Oklahoma. In any market, entrants or the threat of entry should act to increase competition. The benefits of competition can be even greater in procurement auctions where in practice there is a considerable history of collusion.[1] Larger participation can reduce substantially the returns of any bidding ring. However, entering firms may be at a significant disadvantage relative to incumbents in these auctions. Entrants may face higher uncertainty in the development of a bid, since they lack bidding and production experience. They may also have access to less information than incumbent bidders regarding the pricing and cost of various bid components. On the other hand, incumbents facing entrants may be faced with a potential bidder they know little about, and hence, their bidding may be influenced by the presence of entrants. Such asymmetries have not been emphasized in the auction literature.

*The authors wish to thank George Deltas, two anonymous referees, the editor and the participants of the seminar series of the University of Missouri-Columbia for providing useful comments. We are indebted to Brad Hartronft for providing us with useful information about ODOT auctions.

†Authors' affiliations: Department of Economics, Texas Tech University, Lubbock, TX 79409-1014, USA.
email: dakshina.de-silva@ttu.edu
‡Department of Economics, University of Oklahoma, Norman, OK 73019-2103, USA.
email: tdunne@ou.edu
email: georgiak@ou.edu

[1] See, for example, the work by Porter and Zona [1993], Bajari [2001 and Bajari and Ye [2001, 2002] for analyses of bid-rigging and collusive behavior in procurement auctions.

Our paper documents differences in the bidding patterns and the winning bids between entrants and incumbents in first price sealed bid auctions. We find that entrants bid more aggressively than incumbent bidders and win auctions with significantly lower bids than incumbents do. We also show that entrants bid very aggressively in the lower tail of the bid distribution (low bids representing aggressive bids). Our data provide an opportunity to test some of the theoretical results by Maskin and Riley [2000b] who explore properties of bidding distributions in asymmetric auctions. The results indicate that more efficient firms are bidding more aggressively on average, and bidders who face tougher rivals (rivals with proven past winning records in auctions) generally bid more aggressively and win with a lower bid.

In the theoretical section, we restate some key results from the asymmetric auction literature and concentrate on the behavior of entrants and incumbents in auctions; we identify differences in the bidding distributions through a model that introduces common and private cost components. Our framework accommodates informational asymmetries due to a differential level of experience and efficiency and provides some explanation for the observed patterns.

The remainder of the paper is organized as follows. Section II describes the modeling framework. Section III provides a description of the data while section IV reports the results. Section V provides some summary comments.

II(i). *The Model*

The theoretical literature has only recently explored aspects of bidding behavior in asymmetric auctions. Lebrun [1996] considers a first price auction with independent private values. He proves existence of equilibrium in the general asymmetric case with n bidders. Lebrun [1998 and 1999] and Maskin and Riley [2000b] characterize the equilibrium strategies and concentrate on asymmetries that can create advantages likely to justify stochastic dominance in the distribution of values. Pesendorfer [2000] applies this framework of analysis to cartel asymmetries and explains the properties of the bidding equilibrium.

Lizzeri and Persico [1995] and Maskin and Riley [2000a] establish existence in asymmetric auctions with affiliated private values. In a framework with affiliated values made up as a product of private and common value components, Wilson [1998] derived the bidding strategies for a sequential equilibrium of an ascending-price auction of a single item. The information released during the auction about the set of bidders who drop out and the prices at which they drop out is critical in the determination of bidding strategies.

We consider first price auctions of construction contracts and concentrate on the behavior of entrants and incumbents in these auctions. We provide a characterization of bidding distributions in the neighborhood of low costs based on characteristics of the valuation distribution. Our framework accommodates informational asymmetries due to a differential level of experience and efficiency and provides some explanation for the observed patterns. We first describe existing results in the independent private value model and then provide a generalization to a simple model that introduces common and private cost components.

Consider a first price sealed bid auction in which two risk neutral bidders compete for a government contract.[2] The cost of the contract c_i to bidder i is drawn from a known distribution F_i with support $[c_L, c_H]$. F_i is twice continuously differentiable and has a density f_i that is strictly positive on the support. Each firm chooses a bid b to maximize its expected profit

$$\pi_i(b, c_i) = (b - c_i)(1 - F_j(b_j^{-1}(b))$$

where $b_j^{-1}(b)$ is j's inverse bid function. When the bidders' types are distributed independently, Lebrun [1999] and Maskin and Riley [2000a,b] have shown[3] that in equilibrium the bid functions are increasing and differentiable so that, for each firm i, an inverse exists and is differentiable. We let $b_i^{-1}(b) = \phi_i(b)$ thereafter.

The equilibrium to this model can be characterized as the solution to a system of differential equations with boundary conditions. This solution is unique and constitutes a pair of inverse bid functions. In particular, for each i $(i \neq j)$:

$$(1) \qquad \frac{f_j(\phi_j(b))}{1 - F_j(\phi_j(b))}\phi_j'(b) = \frac{1}{[b - \phi_i(b)]}$$

where every $\phi_i(b)$ is evaluated at b for all b in $[b_*, b^*]$. These differential equations should satisfy the following boundary conditions:

$$(2) \qquad F_j(\phi_j(b_*)) = 0, \ b^* = \phi_j(c_H)\forall j.$$

If the distribution of private costs of one bidder stochastically dominates the distribution of private costs of the other, then bidding carries the qualitative properties of the results in Maskin and Riley [2000b]. Notice that, a distribution F_j *first order* stochastically dominates another distribution F_i if and only if $F_i(c) \geq F_j(c)$ for all values of the cost, c. Stochastic dominance is likely in an environment in which the opportunity cost of completing a

[2] In this paper, we differentiate between groups of bidders with emphasis on entrants and incumbents. That is why the simplifying assumption of two bidders (which one can think of as two groups of bidders) is appropriate. In fact, Lebrun [1998] shows that the characterization results he generates assuming two bidders with asymmetric distributions generalize to the case of n bidders with no more than two different probability distributions.

[3] Their results are describing a framework in which the bidder with the highest value wins the auction. We are making here the appropriate changes in the objective function and the conclusions to fit the framework of construction contracts.

project is different among contractors. This opportunity cost depends partially on the technology available and the level of managerial efficiency. If the distribution of private costs of a 'weak' bidder stochastically dominates the distribution of private costs of a 'strong' bidder, Maskin and Riley showed (in their Proposition 3.3) that the equilibrium bid distribution should also exhibit stochastic dominance. Evidence of strength in our case can be provided by looking at the ratio of past wins relative to the number of bids submitted. Proposition 3.5 of the same paper establishes that: 1) a strong bidder will submit a bid that is further above his cost compared to a weak bidder, 2) if a weak bidder faces a strong bidder rather than another weak bidder, he will bid more aggressively (closer to his true cost), and symmetrically, 3); if a strong bidder faces a weak bidder rather than another strong bidder, he will bid less aggressively[4].

These results generalize to a more realistic model in which the cost of the contract c_i to bidder i exhibits both private and common value characteristics. When bidders receive multiple signals, they must combine different pieces of information into a summary statistic. Consideration of multidimensional types poses substantial technical difficulties with the most important the establishment of existence at some level of generality. At the core of this problem is the ability to order signals in multiple dimensions. One environment in which we can overcome the problem of existence of a solution is the following: Suppose that bidder i gets an estimate of his private cost, t_i, and receives a signal, s_i, which is an unbiased estimate of the common cost S. The estimates of the private cost and the signals of the common cost are independently distributed across bidders. Let $c_i = \alpha s_i + t_i + (1-\alpha)\Sigma_j s_j/(n-1)$ be the total estimated cost of a contract to bidder i,[5] where α is a way to parameterize the degree of uncertainty a bidder faces in the calculation of the common cost. The parameter α is common knowledge to all bidders. In a purely private value model $\alpha = 1$. In an affiliated environment, in which bidders view symmetrically the common component, $\alpha = 0$. In order to ensure monotonicity and existence within this framework, we need to make the assumption that the densities of the t_i's and the s_i's are logconcave.[6] Goeree and Offerman [1999] derived the equilibrium bidding functions in the symmetric case. In the appendix, we extend this analysis to asymmetric

[4] Proposition 3.3, in fact, also establishes that when an inexperienced bidder faces an experienced bidder rather than another inexperienced bidder he responds with a more aggressive bidding distribution in the sense of stochastic dominance, and symmetrically, when an experienced bidder faces an inexperienced bidder rather than another experienced bidder, he responds with a less aggressive bid distribution.

[5] For more details on this modeling framework and a discussion of its advantages, see Bikhchandani and Riley [1991], Alberts and Harstad [1991], Vincent [1995], Klemperer [1998], Bulow, Huang and Klemperer [1999], and Goeree and Offerman [1999].

[6] Many commonly used densities such as the uniform, normal, chi-square and exponential densities satisfy this assumption.

auctions. We derive a pair of inverse bid functions similar to (1), with the only difference that F_i is now the distribution of the combined signal, $\alpha s_i + t_i$, that bidder i receives. The result that follows can thus be established in this more general framework.

II(ii). *Characterization of the Equilibrium Bid Distributions for Low Estimates of the Cost*

In this section, we concentrate on differences between entrants and incumbents. Entrants are bidders with no prior bidding experience. The distribution of cost estimates for those firms is likely to exhibit a much greater dispersion on average relative to that of incumbents, reflecting lack of experience and larger variation in managerial efficiency. As a result, it may not satisfy stochastic dominance for every value of cost and the characterization of relative bids by Maskin and Riley [2000b] may no longer apply. In such an environment, it is not possible to establish differences in the bidding patterns that do not depend upon the parameters of the distributions of cost estimates. Nevertheless, the stochastic relation among distributions for low values of the estimated cost could allow us to predict the bidding patterns at the low end of the distribution. The following proposition shows that, if the distribution of cost estimates of entrants stochastically dominates that of incumbents in the neighborhood of c_L, entrants will bid more aggressively relative to their cost estimates than incumbents will in the neighborhood of b_* and vice versa.

Proposition. If $f_E(\phi_I(b_*)) < f_I(\phi_I(b_*))$ then $\phi_E(b) > \phi_I(b)$ for any $b \in [b_*, b_* + \varepsilon]$. Conversely, if $f_E(\phi_I(b_*)) > f_I(\phi_I(b_*))$ then $\phi_E(b) < \phi_I(b)$ for any $b \in [b_*, b_* + \varepsilon]$.

Proof. We will first prove that if $f_E(\phi_I(b_*)) < f_I(\phi_I(b_*))$ then $\phi_E(b) > \phi_I(b)$ for any $b \in [b_*, b_* + \varepsilon]$. Since the lower bound of the distribution is the same for both bidders, $\phi_I(b_*) = \phi_E(b_*)$. Furthermore, $f_E(\phi_I(b_*)) < f_I(\phi_I(b_*))$ implies that $F_E(x) < F_I(x)$ in the right neighborhood of $\phi_I(b_*)$.

From the equilibrium condition, we have:

$$\text{(3)} \qquad \frac{f_E(\phi_E(b_*))}{1 - F_E(\phi_E(b_*))}\phi'_E(b_*) = \frac{1}{b_* - \phi_I(b_*)} = \frac{1}{b_* - \phi_E(b_*)} = \phi'_I(b_*)\frac{f_I(\phi_I(b_*))}{1 - F_I(\phi_I(b_*))}.$$

It follows from (2) and (3) that $\phi'_I(b_*) < \phi'_E(b_*)$. Therefore, in the neighborhood of b_*, $\phi_I(b) < \phi_E(b)$. In words, if the distribution of cost estimates of an entrant stochastically dominates that of an incumbent, then for every bid submitted in the neighborhood of b_*, the associated cost for the entrant will be higher than the cost for the incumbent. Hence, entrants will be more aggressive in their bids. The second part of the statement can be proved following similar arguments. *QED*

Although one would expect entrants to have generally higher costs than incumbents, some entrants due to lack of experience may think that they have lower costs than they really have. The cost estimates are likely to span the same range of values for both types of firms but exhibit different variance. As a result, the predictions by Lebrun [1999] and Maskin and Riley [2000b] do not apply directly to this environment. Nevertheless, the relation of stochastic dominance at the low end of the distribution of costs allows us to infer that entrants with low costs will be bidding more aggressively than incumbents will. This interpretation is consistent with the results in Tables V and VI which suggest that the occurrence of low bids (for example in the lowest 10th or 25th percentile of all bids) is much greater for entrants than for incumbents.

III. DATA

The data used in our analysis come from the Oklahoma Department of Transportation (ODOT). The data contain information on all road construction projects offered for bid letting by the State of Oklahoma from January, 1997 to August, 2000. These projects include road construction and paving projects, traffic signal projects, bridge construction and maintenance projects as well as smaller drainage and clearance type projects.[7] Projects are auctioned off on a monthly basis and the state uses a sealed-bid auction where the low bid is awarded the contract. The state will reject the low bid when it is 7% above the state's engineering cost estimate for the project.[8] For most projects, individual bidders must be pre-qualified. Pre-qualification involves the submission of certified financial statements to the state department of transportation. The pre-qualification process determines the size of the projects a firm can bid on and is related to the level of working capital available to the firm and their past success rate in completing projects. Firms can be removed from the pre-qualification list if they fail to complete contracts successfully. Finally, bidders must include a payment of 5% of the value of project on submission of the bid.[9]

[7] Highway construction auctions have been examined in a number of papers including Thiel [1988], Porter and Zona [1993], Jofre-Bonet and Pesendorfer [2000] and Bajari and Ye [2002].

[8] There have been some exceptions to this rule mostly due to underestimation of the cost by the state.

[9] In general, these requirements establish some barriers to entry for new firms. Firms must have sufficient liquidity to post a bond, they must provide audited financial accounts and they are limited to bidding on certain size projects based on their working capital.

TABLE I
SUMMARY STATISTICS OF OKLAHOMA ROAD CONSTRUCTION AUCTIONS

Variable	Auction Statistics for Full Sample: 1997:1-2000:8	Auction Statistics for Second Sample: 1998:7-2000:8
Number of Auctions	1734	951
Number of Auctions w/ Winners	1409	770
Number of Firms	284	213
Number of Plans Purchased	9526	5240
Number of Bids	5272	2782
Average Number of Plan holders per Auction	5.494	5.510
	(3.061)	(3.087)
Average Number of Bidders per Auction	3.303	3.154
	(1.684)	(1.609)
Number of Plans Purchased by Entrants	–	186
Number of Bids by Entrants	–	70
Number of Wins by Entrants	–	17

Note: Standard Deviations are in parentheses.

The auction data that we utilize include information on the identity of the firms that purchase the plans for a project – 'the plan holders' – the identity and the bids of all bidders for a project, and the winning bid if the contract is awarded. Hence, we have information on the set of firms considering making a bid, the bidders and the winner for each project auctioned off. Furthermore, for each project the state provides the location of the project, a description of the project (e.g., bridge construction, asphalt paving, etc), the details of the project (e.g., the length and depth of the paving surface, the type of asphalt or concrete product to utilize, the amount of excavation, etc), how long the project will take (calendar days), and the engineering estimate of the project's total cost. Table I provides summary statistics on the number auctions, average number of plan holders per auction and average number of bidders per auction. During our period of analysis, there were 1,734 auctions with an average of 5.5 plan holders per auction and 3.3 bidders per auction. Of the 1,734 auctions, 1,409 were awarded contracts. In total, 284 different firms held plans while 218 firms bid on projects and 144 different firms won contracts.[10]

Throughout this analysis, we utilize a specific definition of entry to distinguish between our entering and incumbent firms. We divide our sample of auctions into two time periods – January, 1997 to June, 1998 and July, 1998 to August, 2000. The first period is used to identify incumbent bidders. Any firm that bids during the period 1/97 through 6/98 will be considered an incumbent during the 7/98-8/00 time period. The first time we observe a new firm bidding in the 7/98-8/00 time period, we consider that firm as an entrant

[10] There are several firms in our data sets that purchase plans, bid and win frequently. The maximum number of bids we observe by one firm is 218 and the maximum number of wins by a firm is 59 wins.

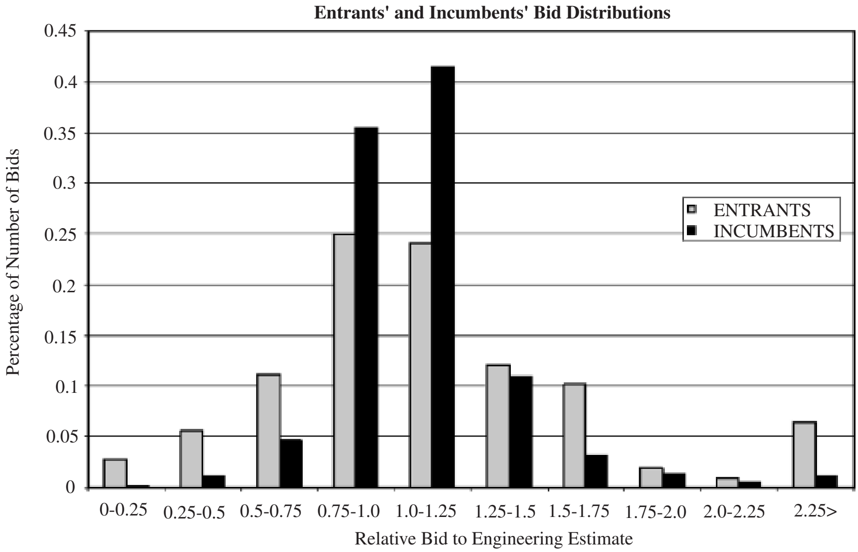

Figure 1
Entrants' and Incumbents' Bid Distributions

when it makes its initial bid. If that firm bids again in the 7/98-8/00 period, it is classified as an incumbent when we consider that and all subsequent bids.[11] The second column of Table I reports auction statistics for the period from July, 1998 through August, 2000. In this subsample, there were 951 auctions. On average, there were 5.5 plan holders and 3.2 bidders in each auction. The bidding statistics look quite similar in the subsample as compared to the overall sample. Entrants make up a relatively small number of the plan holders and bidders. Out of the 5,240 plans purchased in that period, only 186 were purchased by entrants who eventually submitted 70 bids. However, the number of auctions with entrants is somewhat higher. Out of the 951 auctions under study, 138 contain entrants.

Figure 1 presents the probability density function of the bids (each normalized by the project's engineering cost estimate) of entrants versus incumbents bidders. A low relative bid represents an aggressive bid in this figure. The mean relative bid across all auctions in the period 7/98 to 8/00 is 1.118. The figure shows that entrants place more aggressive relative bids than incumbents do. This is particularly true at the lower tail of the distribution where the probability mass under the entrants' distribution is

[11] We verified the robustness of our results to the choice of entry threshold by dividing the time period in a different fashion. We defined as an incumbent any firm that appeared in 1997 or 1998 and defined entrants in the 1999–2000 time period. The results that follow are consistent across both definitions.

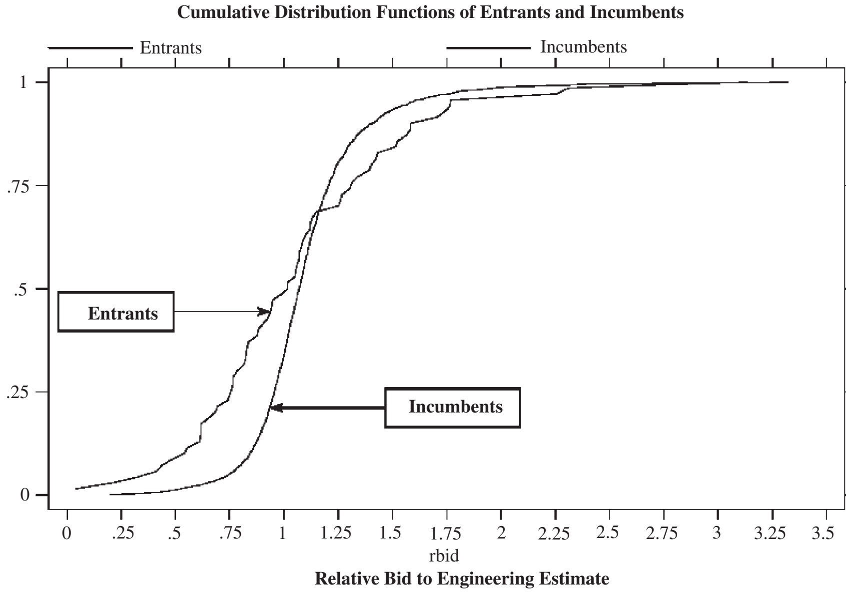

Figure 2
Cumulative Distribution Functions of Entrants and Incumbents

larger. The picture also suggests that the variance of the bids of entrants is higher and leads to a distribution that does not stochastically dominate that of incumbents for all range of relative bids (the tails are thicker for the distribution of the entrants' bids than for the incumbents' bids). This pattern could be observed if the distribution of costs of entrants did not stochastically dominate that of incumbents for all range of values. Figure 2 makes this point clear by presenting the corresponding cumulative density functions of relative bids. Figure 2 shows that the probability that an entrant submits a low bid is higher and that is consistent with the predictions of the theory. The cumulative density functions cross, which makes it more obvious that the relation of stochastic dominance does not persist for all range of values. While Figures 1 and 2 suggest that entrants place a larger number of low bids, we also need to be cautious in interpreting them. There are yet no controls for differences in project types, the numbers of competitors or the characteristics of rivals faced by bidders. Our next section presents some basic regression models that will be used to describe more fully the differences between entrant and incumbent bidders.

IV. EMPIRICAL ANALYSIS

In this section, we empirically model differences in bidding behavior between entrant and incumbent bidders. The sources of asymmetries in auctions have been explored empirically by a number of authors. Hendricks and Porter [1988] examine the role of asymmetric information among bidders in Outer Continental Shelf (OCS) drainage lease auctions. They find that informed bidders (bidders that neighbor a particular tract) earn higher profits in drainage lease auctions and interpret their findings as being in concordance with predictions of models of common value auctions with asymmetric information. More recent papers by Jofre-Bonet and Pesendorfer [2000, 2001] examine asymmetries in bidding behavior for highway procurement contracts in California. This work focuses on asymmetries in costs due to differences in backlogs. They find that bidders that have large fractions of their capacity committed have, on average, higher cost than bidders with little capacity committed. When all bidders are capacity constrained, the resulting low bid is higher than that when all bidders are unconstrained. Bajari and Ye [2001, 2002] also examine road construction auctions but in the upper Midwest. The overall objective of these papers is to identify bid-rigging in procurement auctions but they also examine two forms of bidder asymmetries: differences due to the distance from the bidder to the project and differences in capacity utilization. They estimate a reduced form bidding equation and find that bidders distance and capacity utilization are both positively related to the submitted bid.

Our focus here will be on the differences between entrant and incumbent bidding. As we discussed above, the stochastic relationship between the estimated cost distributions of entrants and incumbents for low relative costs may result in entrants winning some auctions with very low bids. However, the theory is mute on the differences between the average bid of the incumbent and entrants. Our empirical approach will be first to describe the basic differences between incumbent and entrant bidding using a simple regression model that focuses on the respective roles of auction, bidder and rival characteristics in determining bidding patterns. We will then examine how entrant and incumbent bidding varies across the bid distribution by employing quantile regression analysis. The basic structure of the regression model is as follows:

$$y_i = X\mathrm{B} + Z\Gamma + W\Phi + \varepsilon_i$$

We use two dependent variables to model the bidding patterns in these auctions – the log of the bid and the relative bid – throughout our analysis. The relative bid is measured as in Figure 1 as the ratio of the bid to the engineering cost estimate. The independent variables include three sets of controls: the X's control for auction level variables, the Z's control for bidder characteristics and the W's control for rival characteristics.

In modeling the auction characteristics (XB), we take two alternative approaches. One approach includes a set of auction fixed effects. This approach relies only on within-auction variation to estimate the parameters in Γ and Φ and is similar to the approach taken in Bajari and Ye [2001, 2002] and Porter and Zona [1999]. Alternatively, we estimate a model that directly controls for auction characteristics. In this case, we do not include auction fixed effects but instead include variables that directly control for differences across auctions. Under this approach, we allow across-auction variation to estimate the parameters in Γ and Φ. The auction characteristics that are included in the models vary somewhat across econometric specifications. In models where the dependent variable is the log of the bid, we include the log of the state's estimate of the engineering cost of the project as an independent variable. The engineering cost estimate for each project is constructed by the state by pricing each feature outlined in the design and then deriving an overall cost estimate for the project. Over the time period of our analysis, the engineering cost estimates were generally not revealed to bidders prior to the bid letting. We also include a set of six dummy variables to control for broad classes of project types: asphalt projects, clearance and bank protection, bridge work, grading and draining, concrete work, signals and lighting projects, and miscellaneous work such as intersection modification, parking lots, and landscaping. While the engineering cost estimate should control for project specific differences in cost, certain project classes have different pre-qualification standards, and the pool of potential bidders may differ somewhat across project types. The model also contains a variable that measures the log of the number of bidders in the auction. We expect that, as the number of bidders rise, the auctions will be more competitive and bids should be more aggressive. In addition, we include a variable to allow for potential differences in bidding behavior when at least one of the rival firms is an entrant. This is a dummy variable that is set equal to one when a bidder faces an entrant in the auction.

With respect to the bidder characteristics in Z, we include four variables that describe differences across bidders. In order to distinguish entrants from incumbents, we simply include a dummy variable (1 = entrant, 0 = incumbent) to capture whether a bidder is an entrant or not. Again, our definition of an entrant is a firm that bids for the first time during the 7/98-8/00 period. Firms that actively bid in the period 1/97–6/98 are considered incumbents. An entrant becomes an incumbent after the first observed bid(s).[12] In addition to the entrant-incumbent variable, we also include a variable that accounts for past success in auctions. This variable is constructed as the ratio of the past number of wins to the past number of bids. It provides information on the previous success of a firm and is included to control for differences in efficiencies across producers. We

[12] An entrant that submits multiple bids in auctions let on the same date is considered an entrant in all these auctions. In subsequent auctions, the entrant will be converted to incumbent status. In addition, we examined how changes in the period in which we define incumbents affected our results. These are reported later in the paper.

include two proxy variables to control for differences in costs across bidders. The variables are similar to the cost variables used in Bajari and Ye [2001, 2002] and Jofre-Bonet and Pesendorfer [2000, 2001]. The first variable measures the current project backlog of a bidder. This is a proxy for whether the firm is facing capacity constraints. The backlog variable is constructed as follows. For each project awarded, both the value of the contract and the length of the contract in days are given. We assume that a project is completed in a uniform fashion over the length of the contract. A contract backlog is constructed in each month by summing across the remaining value of all existing contracts for a firm. As projects are completed, the backlog of a firm goes to zero unless new contracts are won. We include the log of backlog ($\log(backlog + 1)$) in all our regressions. The other cost variable measures the distance between the bidder's location and the location of the project. Our distance measure is constructed as the distance between the county centroid the firm resides in and county centroid of the project. It is expected that as the distance between the bidder's location and the project increases so will the cost of the project to the bidder.

The matrix W includes three variables that describe the characteristics of the rivals a bidder faces in an auction. First, we utilize past information on rivals' bidding success to summarize the competitiveness of the potential set of rivals. The information that is provided in the plan holder list identifies the rivals for a particular auction. Recall that, a bidder must be a plan holder in order to participate in an auction and that the plan holder list is made available to all potential bidders prior to the auction. The measure of rivals' past average success in auctions is constructed as the average across rivals of the ratio of past wins to past number of plans held. This variable incorporates two aspects of past rival bidding behavior. It incorporates both the probability of a rival bidding given they are a plan holder and the probability the rival wins an auction given that they bid. We also include the rivals' minimum distance to the project and the minimum backlog of the rivals. These latter two rival variables are similar to those used by Bajari and Ye [2001, 2002].

The samples of data we employ will vary across our dependent variables. To examine the bids, we utilize all bidders' data for all auctions between 7/98 and 8/00 where a contract was awarded. When examining the winning bid, we use data from the winning bid record, as well as data at the auction level. There are 770 auctions that have awarded winners. Table II provides summary statistics on the variables used in the regression analysis.

Table III presents the first set of regression results. We estimate the models using least squares reporting White-corrected standard errors to correct for heteroscedasticity in models without auction effects. When including the auction effects, a standard fixed effects approach is taken. The first three columns report the results from the sample that includes all bids. The first column includes direct controls for auction characteristics. In this

TABLE II
SUMMARY STATISTICS OF REGRESSION VARIABLES

Variable	Mean (Standard Deviation)
Log of Bids	13.075 (1.646)
Relative Bids	1.118 (.399)
Log of Winning Bids	12.805 (1.653)
Relative Winning Bids	.970 (.231)
Log of Engineer's Estimate	13.003 (1.657)
Log of Number of Bidders in an Auction	1.279 (.474)
Entrant Bid Dummy	.025 (.157)
Dummy Variable for Bidders Facing Entrants	.155 (.362)
Firm's Winning to Bidding Ratio	.261 (.139)
Average Rivals Winning to Plan Holder Ratio	.156 (.064)
Log of Firm's Backlog	10.825 (6.364)
Log of Firm's Distance to the Project Location	4.104 (1.548)
Log of Firm's Closest Rival's Distance to the Project Location	3.049 (1.723)
Log of Firm's Rivals Minimum Backlog	3.221 (5.702)

Note: Standard Deviations are in parentheses.

regression, the results indicate that entrants bid, on average, more aggressively than incumbents. This is not surprising given the bid distributions of entrants and incumbents presented in Figure 1. In addition, bidders that have a history of higher than average past winning tend to bid lower. We interpret the results on the prior-winning variable as mainly picking out differences in efficiencies across bidders. The backlog variable is positive and statistically significant indicating that as project backlogs rise, firms bid less aggressively. However, the bidder's distance to the project is not statistically significant in the bid regression. This differs from other studies of road construction auctions (e.g., Bajari and Ye [2001, 2002] and Jofre-Bonet and Pesendorfer [2000, 2001]) that find that the distance between the project and the bidder's location appears to increase firms' bids. Both of these studies examined auctions that cover larger geographic areas (California and the upper Midwest) while ours is limited to Oklahoma where most of the bidders are centrally located. With regard to the rivals' variables in the column 1 regression, we get mixed results. The variable that measures rivals' past success in auctions indicates that the more competitive the set of rivals a firm faces, the more aggressively the firm bids. This is in agreement

TABLE III
REGRESSION RESULTS FOR BIDS AND WINNING BIDS

Independent Variable	Bid Regressions			Winning Bid Regressions	
	OLS	Fixed Effects		OLS	
	Log of Bids	Log of Bids (with Auction Fixed Effects)	Relative Bids (with Auction Fixed Effects)	Log of Winning Bids	Relative Winning Bids
Log of Engineer's Estimate	.953*			.990*	
	(.006)			(.009)	
Log Number of Bidders	– .008			– .050*	– .063*
	(.011)			(.016)	(.019)
Bidders Facing Entrants	– .016			– .028	– .007
	(.018)			(.034)	(.034)
Entrant Bid Dummy	– .191*	– .092*	– .120*	– .385*	– .194*
	(.070)	(.030)	(.047)	(.173)	(.072)
Firm's Winning to	– .329*	– .218*	– .339*	.053	.018
Bidding Ratio	(.047)	(.046)	(.073)	(.064)	(.084)
Log of Firm's Backlog	.004*	.003*	.003*	.003*	.002
	(.001)	(.001)	(.001)	(.001)	(.001)
Distance to the Project	– .005	– .001	– .008	– .005	– .003
Location	(.004)	(.005)	(.007)	(.007)	(.006)
Average Rivals Winning to Plan Holder Ratio	– .217*	.245	.358	– .425*	– .297*
Closest Rival's Distance to the	(.109)	(.168)	(.266)	(.134)	(.132)
Project Location	.002	– .011	– .017	.003	.001
	(.003)	(.010)	(.014)	(.006)	(.006)
Rivals Minimum Backlog	.000	.002	.001	.002	.001
	(.001)	(.002)	(.003)	(.001)	(.001)
Number of Observations	2782	2782	2782	770	770
Adj-R^2	.9736	.9870	.4457	.9788	.0461

*Denotes 95% significance.
Regressions in columns 1, 4 & 5 include six project class dummy variables.

with the theoretical results by Maskin and Riley [2000b] presented in II(i). However, the rival variables that measure rival backlog and rival distance are not statistically significant. In fact, across all our regressions, the rival backlog and the rival distance variables are never statistically significant.[13] The variable indicating that a firm faces an entrant in the auction is not statistically significant in this regression or, for that matter, in any other specification. Hence, firms do not appear to bid more or less aggressively when an entrant is present in the auction.[14] Finally, the engineering cost estimate has the expected impact on the bid while the number of bidders does not appear to affect the log of the bids.[15]

The next two columns of Table III report the estimates from the bid regression including auction fixed effects. Column 2 provides the results when the dependent variable is measured as the log of the bids while column 3 provides the results when the dependent variable is measured as the relative bid – the bid normalized by the engineering cost estimate. Entrants appear to bid more aggressively, a greater backlog increases a firm's bid and past winners bid more aggressively in both regressions. These findings are consistent with the results reported in column 1. The main difference between the fixed-effects and OLS regressions appears in the rival variables. In particular, past rival winning is no longer statistically significant and changes sign. A joint statistical test of the three rival variables indicates that we cannot reject the null hypothesis (at the 5% level) that rival variables do not matter.

One needs to be cautious in interpreting the results on the rival variables in the fixed-effects regressions for two reasons. First, there is a substantial reduction in the variation in the rival variables in the fixed effects setting. Recall that the rival variables are measuring the closest rival to the project, the minimum backlog of the rivals, and the average past winning history of the group; while these variables will vary across bidders in an auction, their variation will be small.[16] Second, the interpretation of the rival past winning variable differs greatly between the OLS and fixed-effect regressions. In the

[13] Bajari and Ye [2002] also find that rival distance and rival backlog are not statistically significant in reduced form bid equations.

[14] The bidding behavior in an auction depends upon among other things on the mix of participants and their characteristics. The presence of an entrant may not necessarily be the dominant factor determining the bidding behavior in these auctions if there is a sufficient number of incumbents present.

[15] One issue is that the actual number of bidders is most likely endogenous. This point is raised in Hendricks, Porter and Boudreau [1987] and has been recently examined by Porter and Zona [1999] and by Bajari [2001]. Later in this paper, we report on the estimates of a model that uses the expected number of bidders calculated from past information on the number of plan holders and the probability of participation.

[16] Within an auction, the set of rivals each bidder faces is slightly different. For example, when considering the average past winning history of the group, each bidder has only one component in the rival information set that differs from other participants.

TABLE IV
SUMMARY STATISTICS BY QUANTILE

	Incumbents	Entrant
Total Number of Bids	2712	70
Number of Bids by the group in the bottom 25% of all bids	662	33
Number of Bids by the group in the bottom 10% of all bids	253	25
Proportion of Bids by the group in the bottom 25% of all bids	24.4	47.1
Proportion of Bids by the group in the bottom 10% of all bids	9.3	35.7

OLS regressions, the rival variable is capturing differences in the absolute level of rival past winning. In the fixed-effects regressions, the comparison is relative and actually measures the relative weakness of the bidder vs. the set of potential rivals. Thus, the rival's past winning history variable in the fixed-effects setting identifies the relative weakness of bidders. The results show that relatively weak bidders bid less aggressively, though the effect is not statistically significant.

The last two columns in Table III report the results for the winning bid regressions. The results show that entrants win with much more aggressive bids as compared to incumbents' bids. However, in the winning bid regression, the prior-winning rate has no effect on the level of the winning bid. Thus, while firms with strong prior winning histories do bid lower, on average, relative to other firms (as is evident from the first column of Table III), they do not win with disproportionately below average bids. The winning bid is substantially less when the winner faces rivals with strong previous winning records. A winner with a larger backlog wins with a somewhat higher bid while again distance to the project does not influence either the log of the winning bid or the relative bid. Finally, the rival distance and the rival backlog variables do not influence the winning bid.

The fact that entrants bid aggressively in the lower tail is more fully documented in Table IV. Table IV presents information on bidding patterns of incumbents and entrants in the lower end of the distribution (lowest 25th and lowest 10th percentiles). The table indicates that entrants place a much larger proportion of their bids in the bottom 25% than incumbents. The difference between the proportions of bids in the two groups is even more pronounced when you consider the 10% of lowest bids.

These simple observations can be formalized in the analysis of the quantile regression model (see Koenker and Bassett [1982]) that follows. This model allows us to estimate differences in the distribution of bids between entrants and incumbents more accurately while taking into account other factors that contribute to the variability of bids. We restrict the estimation to five quantiles: .10, .25, .50, .75 and .90. The results of these estimations are presented in Table V. The dependent variable in all regressions is the relative bids. Quantile regressions using the log of the bids and including the engineering cost estimate on the right hand side yield qualitatively similar

TABLE V
QUANTILE REGRESSION RESULTS FOR RELATIVE BIDS

Independent Variable	Quantile				
	.1	.25	.5	.75	.9
Log Number of Bidders	.006	−.006	−.020*	−.033*	−.059*
	(.011)	(.010)	(.009)	(.014)	(.028)
Bidders Facing Entrants	−.111*	−.024	.004	.026	.055
	(.015)	(.013)	(.011)	(.027)	(.032)
Entrant Bid Dummy	−.240*	−.185*	−.093*	.077	−.022
	(.036)	(.030)	(.026)	(.041)	(.078)
Firm's Winning to Bidding Ratio	−.247*	−.180*	−.197*	−.224*	−.284*
	(.038)	(.033)	(.030)	(.051)	(.111)
Log of Firm's Backlog	.008*	.003*	.001*	.001	−.004
	(.000)	(.001)	(.000)	(.001)	(.002)
Distance to the Project Location	−.002	−.001	.002	.003	−.001
	(.004)	(.003)	(.003)	(.004)	(.008)
Average Rivals Winning to Plan Holder Ratio	−.147	−.094	−.162*	−.045	.361
	(.097)	(.075)	(.066)	(.111)	(.243)
Closest Rival's Distance to the Project Location	.006	−.002	.000	−.004	−.014
	(.003)	(.003)	(.002)	(.004)	(.007)
Rivals Minimum Backlog	.001	.001	.000	−.002	−.005*
	(.001)	(.000)	(.001)	(.001)	(.002)
Number of Observations	2782	2782	2782	2782	2782
R^2	.0923	.0442	.0278	.0405	.0866

*Denotes 95% significance.
Regressions include six project class dummy variables.

results. The analysis employs similar specifications to those reported in column 3 of Table III (without the fixed effects) and emphasizes the difference in the bidding patterns between the entrant and incumbent bidders. The coefficient on the dummy variable on entry varies substantially in the quantiles. The bids of entrants are smaller than those of incumbents by a larger margin at the .10 quantile than at the .25 quantile or the .50 quantile, holding everything else constant. The difference becomes smaller and statistically insignificant beyond the .50 quantile. These results are in agreement with the theoretical findings in section II(ii). The differences in their bidding patterns could be consistent with an asymmetric model of auctions in which the distribution of an entrant's costs stochastically dominates that of incumbents for low estimates of the cost and at the same time exhibit greater dispersion. As a result, stochastic dominance is evident in the 10^{th}, 25^{th}, and 50^{th} percentile but does not persist beyond that point.

When the relative winning bid is considered as the dependent variable then the differences in the two groups become more pronounced (see Table VI). In particular, holding everything else constant, the relative bids of entrants are lower than those of incumbents by the greatest at the .10 quantile. The difference between entrants and incumbent relative bids narrows substantially by the .75 quantile but there is still a statistical difference. Clearly, the quantile regressions indicate that entrants' bids are particularly aggressive in the lower tail of the bid distributions.

TABLE VI
QUANTILE REGRESSION RESULTS FOR RELATIVE WINNING BIDS

Independent Variable	Quantile				
	.1	.25	0.5	0.75	0.9
Log Number of Bidders	– .039*	– .041*	– .036*	– .052*	– .050*
	(.019)	(.018)	(.014)	(.014)	(.025)
Bidders Facing Entrants	– .092*	– .026	– .011	– .003	.041
	(.030)	(.028)	(.020)	(.021)	(.036)
Entrant Bid Dummy	– .294*	– .191*	– .207*	– .110*	– .031
	(.059)	(.065)	(.046)	(.046)	(.078)
Firm's Winning to Bidding Ratio	– .153*	– .119	– .060	– .049	.009
	(.061)	(.066)	(.050)	(.052)	(.082)
Log of Firm's Backlog	.007	.004*	.000	.001	.001
	(.001)	(.002)	(.001)	(.001)	(.002)
Distance to the Project Location	– .006	.004	.000	– .002	– .003
	(.008)	(.006)	(.004)	(.004)	(.009)
Average Rivals Winning to Plan Holder Ratio	– .684*	– .454*	– .268*	– .210	– .162
	(.159)	(.153)	(.103)	(.112)	(.177)
Closest Rival's Distance to the Project Location	.019*	.003	.000	.000	– .009
	(.007)	(.006)	(.004)	(.004)	(.009)
Rivals Minimum Backlog	.003	.002	.002	.001	– .000
	(.002)	(.002)	(.001)	(.001)	(.002)
Number of Observations	770	770	770	770	770
R^2	.1852	.0971	.0634	.0430	.0403

*Denotes 95% significance.
Regressions include six project class dummy variables.

To check on the robustness of our main results, we estimate several alternative specifications. First, we estimate a model that includes a set of firm effects for large firms that bid frequently.[17] Firm effects will control for bidder heterogeneity that is not controlled for by observable characteristics of bidders. Porter and Zona [1999] include bidder fixed effects in their analysis of Ohio milk auctions and Jofre-Bonet and Pesendorfer [2000, 2001] include them in their analysis of California road construction auctions. We cannot include a full set of firm dummies because many entrants and some incumbents only bid once during our sample period. The approach that we take is similar to Bajari and Ye [2001, 2002]. Bajari and Ye include a partial set of firm effects that allow for differences in the bidding behavior of the largest firms. We re-estimate the regressions in Table III with firm effects (a set of 42 firm dummies for incumbent firms that bid frequently). The results for the entrant variable across our alternative specifications (in Table III) are robust to this change in specification. Entrants bid aggressively and win with very aggressive bids. However, the bidder's own characteristics such as backlog and the winning-to-bidding ratio no longer remain statistically significant in these regressions.

[17] The supplementary regressions discussed here can be found at the *Journal's* editorial Web site.

Second, we redefine our period of analysis from July 1998–October 2000 to January 1999–October 2000. This redefinition of the sample redefines our entrant group. Here, an entrant is defined as a new bidder that appears in the 1/99–8/00 period. Under this definition, we use two complete prior years (1997 & 1998) of bidding data to help identify incumbents and build up bidding histories. Given that both our number of entrants and number of auctions are reduced under this criterion, it is not surprising that we lose some efficiency as compared to the original results reported in Table III. Overall the results hold up reasonably well. The coefficients on the entry variable are quite similar to those reported in Table III. However, the effect of entry in the winning bid is not statistically significant at the 5% level but it is at the 10% level.

Third, a recent concern in the empirical auction literature is the endogeneity of the number of bidders. We replace the number of bidders with a measure of the expected number of bidders and re-estimate the models. The expected number of bidders variable is constructed using historical information on individual bidder participation rates. For entrants that have no such history, we use the estimate of average participation across all auctions. The results for the entry, own and rival characteristics are invariant to the use of expected versus actual number of bidders. The main difference is that the expected number of bidder's variable is not statistically significant in the winning bids regressions.[18] Alternatively, we also estimated a model that replaced the number of bidders with the number of plan holders. The number of plan holders is known to all bidders prior to the auction and is an upper bound on the number of bidders. The results of the regression models and the quantile regressions are qualitatively similar except that the number of plan holders is not statistically significant in the winning bid regressions.

V. SUMMARY

This paper examines the patterns of bidding by incumbent and entrant firms in road construction procurement auctions. We found that entrant bidders bid more aggressively, and win with lower bids. On a theoretical level, we considered an asymmetric model of auctions with emphasis on the

[18] We also check the sensitivity of the results to the removal of extreme bids. We re-estimate the models using samples in which very low relative bids and very high relative bids were omitted. The results show that entrants still win with lower bids when extreme observations are deleted from the sample. However, the entry results on overall bids vary across specification. When fixed effects are used, the entry results are negative but not statistically significant. However, in models that include auction characteristics as opposed to fixed effects (Column 1 of Table 3), the entrant effect is still negative and statistically significant. We chose to keep all bids in our sample since many of the observed low bids were actually awarded contracts and some of these low winning bids were entrants.

characteristics of these groups. Entrant bidders can differ from incumbent bidders in a number of respects, though entrants are not expected to be more efficient than incumbents. In fact, if one examines subsequent entrant participation in these auctions, entrants bid much less frequently. Only 42% of entrants return to bid for a second time at these auctions as opposed to 96% of incumbents. More striking is the fact that only 20% of entrants that win in their initial auction go on to win another auction, compared to 80% of incumbents.[19] Entrants may also have less experience than incumbent bidders in production; they may be less certain of their own costs for completing a project than incumbents are. Both of these factors contribute to a greater dispersion in their cost estimates. Lower efficiency but greater dispersion imply that the distribution of costs of entrants will not stochastically dominate that of incumbents for all estimates of the cost as in Maskin and Riley [2000b] but it is likely to dominate it for low estimates of the cost. In a theoretical framework that provides enough flexibility to allow for these asymmetries, we produced testable predictions: entrants with low cost estimates will bid more aggressively relative to the engineering estimate than incumbents.

Our study also documents a number of other patterns in the bid data. Bidders who have a history of past winning at auctions have a tendency to bid lower but do not win with overly aggressive bids. The greater the firm's backlog, the less aggressively the firm bids. This agrees with other recent studies. However, we did not find a strong relationship between the distance of a firm to a project and the bid. With respect to rival variables, we found that the tougher the average rival is, the lower the bid and the lower the winning bid, though the significance of the bid result is sensitive to differences in econometric specification. These results are generally consistent with the theoretical predictions on bidding patterns in asymmetric auctions by Maskin and Riley [2000b].

APPENDIX

Let $c_i = \alpha s_i + t_i + (1-\alpha)\Sigma_j s_j/(n-1)$. The density of the common cost component is $g(s)$ with support $[s_L, s_H]$. Similarly, a bidder's part of the cost that is purely private is drawn from a distribution $h_i(t)$ with support $[t_L, t_H]$ where $t_L \geq 0$. In order to ensure monotonicity and existence within this framework, we will make the assumption that the densities $h_i(t)$ and $g(s)$ are logconcave. It follows from Lemma 1 in Goeree and Offerman [1999] that the distribution of $w_i = \alpha s_i + t_i$, F_{wi},will also be logconcave and both $E(s|w_i \geq x)$ and $E(s|w_i = x)$ will be monotonic in x.

Goeree and Offerman [1999] solved for the equilibrium inverse bid functions in a symmetric auction environment. We characterize the equilibrium in this first price

[19] In order to obtain these statistics, we found the number of entrants and incumbents that submitted bids between July 1998 and August 2000. Then, we extended the data set (until February 2001) to investigate whether these bidders submitted bids a second time around.

asymmetric sealed bid auction with two bidders. Notice that the bid is a monotonic function of $w_{i.}$ Taking this into account, consider a bidder's expected payoff from participation:

$$\pi_i(b) = [b - \alpha s_i - t_i - (1-\alpha)E[s_j|w \geq B_j^{-1}(b)]][1 - F_{w_j}(B_j^{-1}(b))].$$

Differentiating the expected payoff with respect to b and evaluating the expression at the optimal choice we have:

$$\begin{aligned}
\pi_i'(b) = &- \left[b - \alpha s_i - t_i - (1-\alpha)E[s_j|w \geq B_j^{-1}(b)]\right] f_{w_j}(B_j^{-1}(b))B_j^{-1'}(b) \\
&+ [1 - F_{w_j}(B_j^{-1}(b))]\left[1 + (1-\alpha)E[s_j|w = B_j^{-1}(b)]\,\frac{f_{w_j}(B_j^{-1}(b))}{1 - F_{w_j}(B_j^{-1}(b))}B_j^{-1'}(b)\right. \\
&\left. -(1-\alpha)E[s_j|w \geq B_j^{-1}(b)]\frac{f_{w_j}(B_j^{-1}(b))}{1 - F_{w_j}(B_j^{-1}(b))}B_j^{-1}(b)\right] \\
= &\left[-b + \alpha s_i + t_i + (1-\alpha_i)E[s_j|w = B_j^{-1}(b)]\right] f_{w_j}(B_j^{-1}(b))B_j^{-1'}(b) \\
&+ 1 - F_{w_j}(B_j^{-1}(b)) = 0
\end{aligned}$$

where $B_j^{-1}(b) = \alpha s_i + t_i$ is defined over $[\alpha s_L + t_L, \alpha s_H + t_H]$.

It follows that for each j $(j \neq i)$:

$$\frac{f_{w_j}(B_j^{-1}(b))}{1 - F_{w_j}(B_j^{-1}(b))}B_j^{-1'}(b) = \frac{1}{[b - B_i^{-1}(b)]}$$

where every $B_i^{-1}(b)$ is evaluated at b for all b in $[b_*, b^*]$. These differential equations should satisfy the following boundary conditions:

$$F_j(B_j^{-1}(b_*)) = 0, \quad b^* = B_j^{-1}(\alpha s_H + t_H)\,\forall j.$$

REFERENCES

Albers, W. and Harstad, R. M. A., 1991, '*Framing Effect Observed in a Market Game,' Game Equilibrium Models II. Methods*, Morals, and Markets, pp. 308–36 Publication: With contributions by D. Abreu et al., New York.

Bajari, P. and Ye, L., 2002, 'Deciding Between Competition and Collusion,' Working paper, Stanford University.

Bajari, P. and Ye, L., February 2001, 'Competition Versus Collusion in Procurement Auctions: Identification and Testing,' Working paper, Stanford University.

Bajari, P., 2001, 'Comparing Competition and Collusion: A Numerical Approach', *Economic Theory*, 18, pp. 187–205.

Bikhchandani, S. and Riley, J. G., 1991, 'Equilibria in Open Common Value Auctions', *Journal of Economic Theory*, 53(1), pp. 101–130.

Bulow, J. and Klemperer, P., 1999, 'Toeholds and Takeovers', *Journal of Political Economy*, 107(3), pp. 427–454.

Goeree, J. K. and Offerman, T., 1999, 'Competitive Bidding in Auctions with Private and Common Values,' Working Paper #11/99, University of Virginia.

Hendricks, K. and Porter, R., 1988, 'An Empirical Study of an Auction with Asymmetric Information', *American Economic Review*, 78(5), pp. 865–883.

Hendricks, K., Porter, R. and Boudreau, B., 1987, 'Information, Returns, and Bidding Behavior in OCS Auctions: 1954–1969', *Journal of Industrial Economics*, 35, pp. 517–542.

Jofre-Bonet, M. and Pesendorfer, M., 2000, 'Bidding Behavior in a Repeated Procurement Auction: A Summary', *European Economic Review*, 44, pp. 1006–1020.

Jofre-Bonet, M. and Pesendorfer, M., 2001, 'Estimation of a Dynamic Auction Game,' Working Paper, Yale University.

Klemperer, P., 1998, 'Auctions with Almost Common Values: The "Wallet Game" and Its Applications', *European Economic Review*, 42(3–5), pp. 757–769.

Koenker, R. and Bassett Jr., G., 1982, 'Robust Tests for Heteroscedasticity Based on Regression Quantiles', *Econometrica*, 50(1), pp. 43–61.

Lebrun, B., 1999, 'First Price Auctions in the Asymmetric *N* Bidder Case', *International Economic Review*, 40, pp. 125–142.

Lebrun, B., 1998, 'Comparative Statics in First Price Auctions', *Games and Economic Behavior*, 25(1), pp. 97–110.

Lebrun, B., 1996, 'Existence of an Equilibrium in First Price Auctions', *Economic Theory*, 7(3), pp. 421–443.

Lizzeri, A. and Persico, N., March 1995, 'Existence and Uniqueness of Equilibrium in First Price Auction and War of Attrition With Affiliated Values,' Working Paper # 1120, Northwestern University, Center for Mathematical Studies in Economics and Management Science.

Maskin, E. and Riley, J., 2000a, 'Equilibrium in Sealed High Bid Auctions', *Review of Economic Studies*, 67(3), pp. 439–454.

Maskin, E. and Riley, J., 2000b, 'Asymmetric Auctions', *Review of Economic Studies*, July 2000b, 67(3), pp. 413–438.

Pesendorfer, M., 2000, 'A Study of Collusion in First-Price Auctions', *Review of Economic Studies*, 67(3), pp. 381–411.

Porter, R. H. and Zona, J. D., 1999, 'Ohio School Milk Markets: An Analysis of Bidding', *RAND Journal of Economics*, 30(2), pp. 263–288.

Porter, R. H. and Zona, J. D., 1993, 'Detection of Bid Rigging in Procurement Auctions', *The Journal of Political Economy*, 101(3), pp. 518–538.

Thiel, S. E., 1988, 'Some Evidence on the Winner's Curse', *The American Economic Review*, 78(5), pp. 884–895.

Vincent, D. R., 1995, 'Bidding Off the Wall: Why Reserve Prices May Be Kept Secret', *Journal of Economic Theory*, 65(2), pp. 575–584.

Wilson, R., 1998, 'Sequential Equilibria of Asymmetric Ascending Auctions: The Case of Log Normal Distributions', *Economic Theory*, 12(2), pp. 433–440.