Oligopoly (Game Theory)

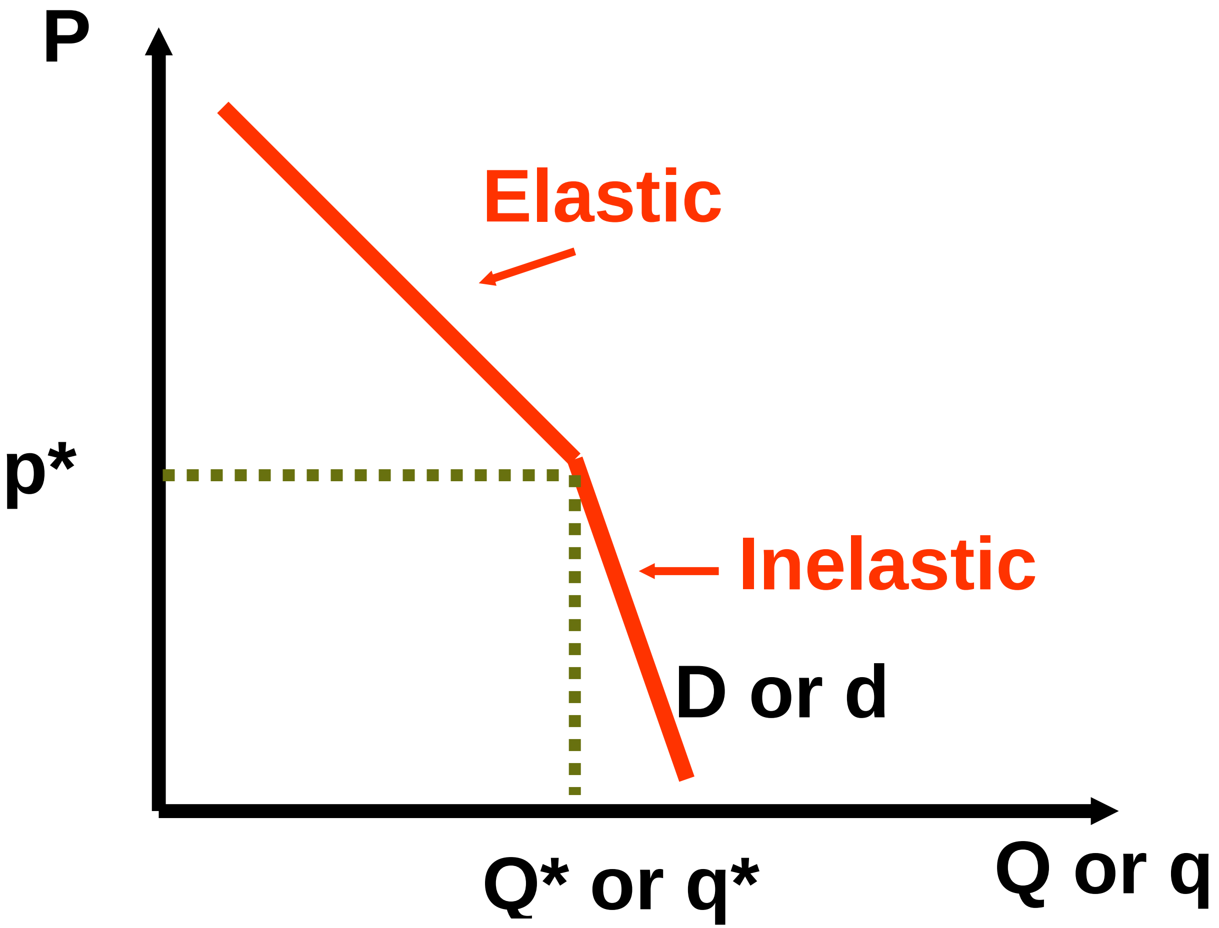
Oligopoly: Assumptions

- Many buyers
- Very small number of major sellers (actions and reactions are important)
- Homogeneous product (usually, but not necessarily)
- Perfect knowledge (usually, but not necessarily)
- Restricted entry (usually, but not necessarily)

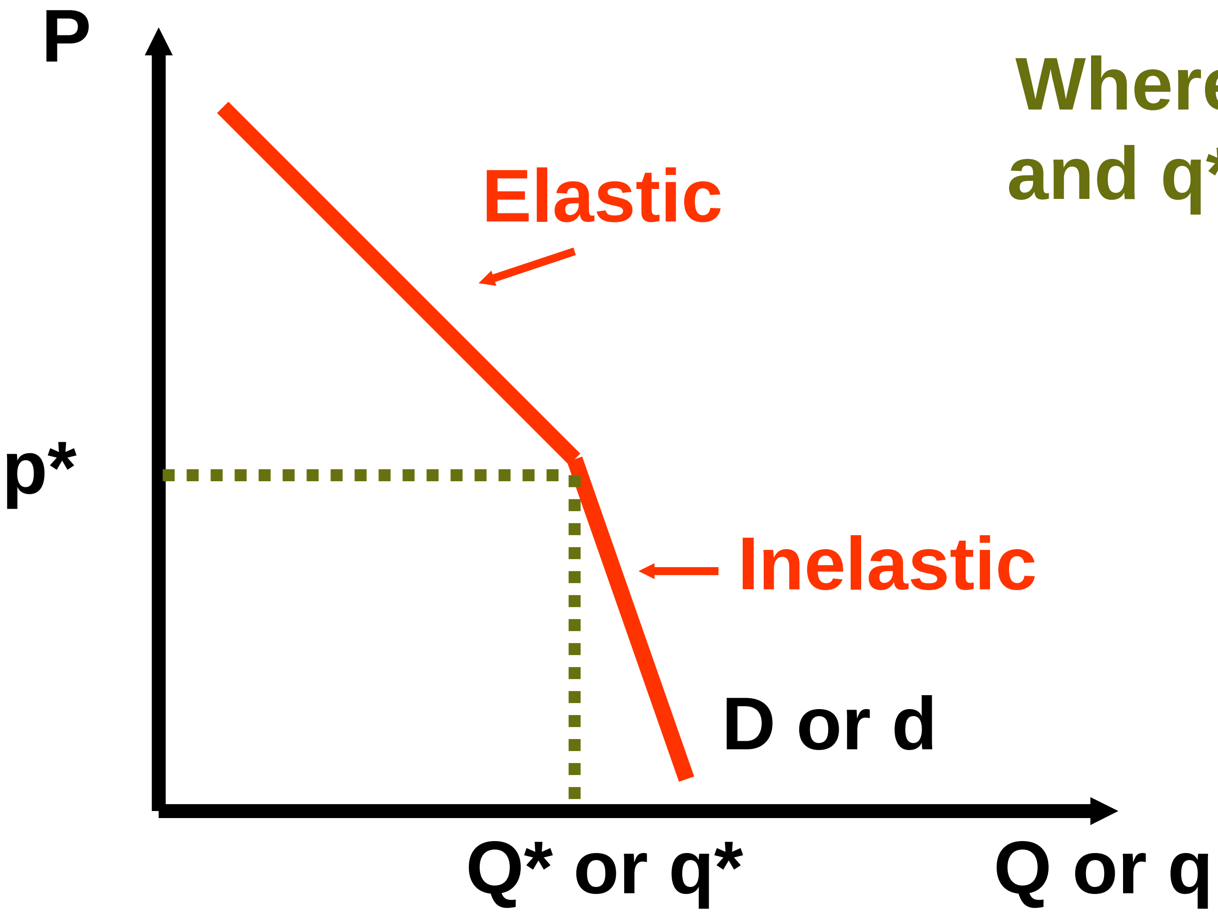
Oligopoly Models

1. **“Kinked” Demand Curve**
2. **Cournot (1838)**
3. **Bertrand (1883)**
4. **Nash (1950s): Game Theory**

“Kinked” Demand Curve



“Kinked” Demand Curve



Where do p* and q* come from?

Cournot Competition

- **Assume two firms with no entry allowed and homogeneous product**
- **Firms compete in quantities (q_1, q_2)**
- **$q_1 = F(q_2)$ and $q_2 = G(q_1)$**
- **Linear (inverse) demand, $P = a – bQ$ where $Q = q_1 + q_2$**
- **Assume constant marginal costs, i.e.**

  $TC_i = cq_i$ for $i = 1,2$
- **Aim: Find q_1 and q_2 and hence p, i.e. find the equilibrium.**

Cournot Competition

Firm 1 (w.o.l.o.g.)

Profit $= TR - TC$

$\Pi_1 = P.q_1 - c.q_1$

[$P = a - bQ$ and $Q = q_1 + q_2$, hence

$P = a - b(q_1 + q_2)$

$P = a - bq_1 - bq_2$]

Cournot Competition

$$\Pi_1 = Pq_1 - cq_1$$

$$\Pi_1 = (a - bq_1 - bq_2)q_1 - cq_1$$

$$\Pi_1 = aq_1 - bq_1^2 - bq_1q_2 - cq_1$$

Cournot Competition

$$\Pi_1 = aq_1 - bq_1^2 - bq_1q_2 - cq_1$$

To find the profit maximising level of q_1 for firm 1, differentiate profit with respect to q_1 and set equal to zero.

$$\frac{\partial \Pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - c = 0$$

Cournot Competition

$$a - 2bq_1 - bq_2 - c = 0$$

$$-2bq_1 - bq_2 = c - a$$

$$2bq_1 + bq_2 = a - c$$

$$2bq_1 = a - c - bq_2$$

$$q_1 = \frac{a - c - bq_2}{2b}$$

Firm 1's "Reaction" curve

Cournot Competition

$$q_1 = \frac{a - c - bq_2}{2b}$$

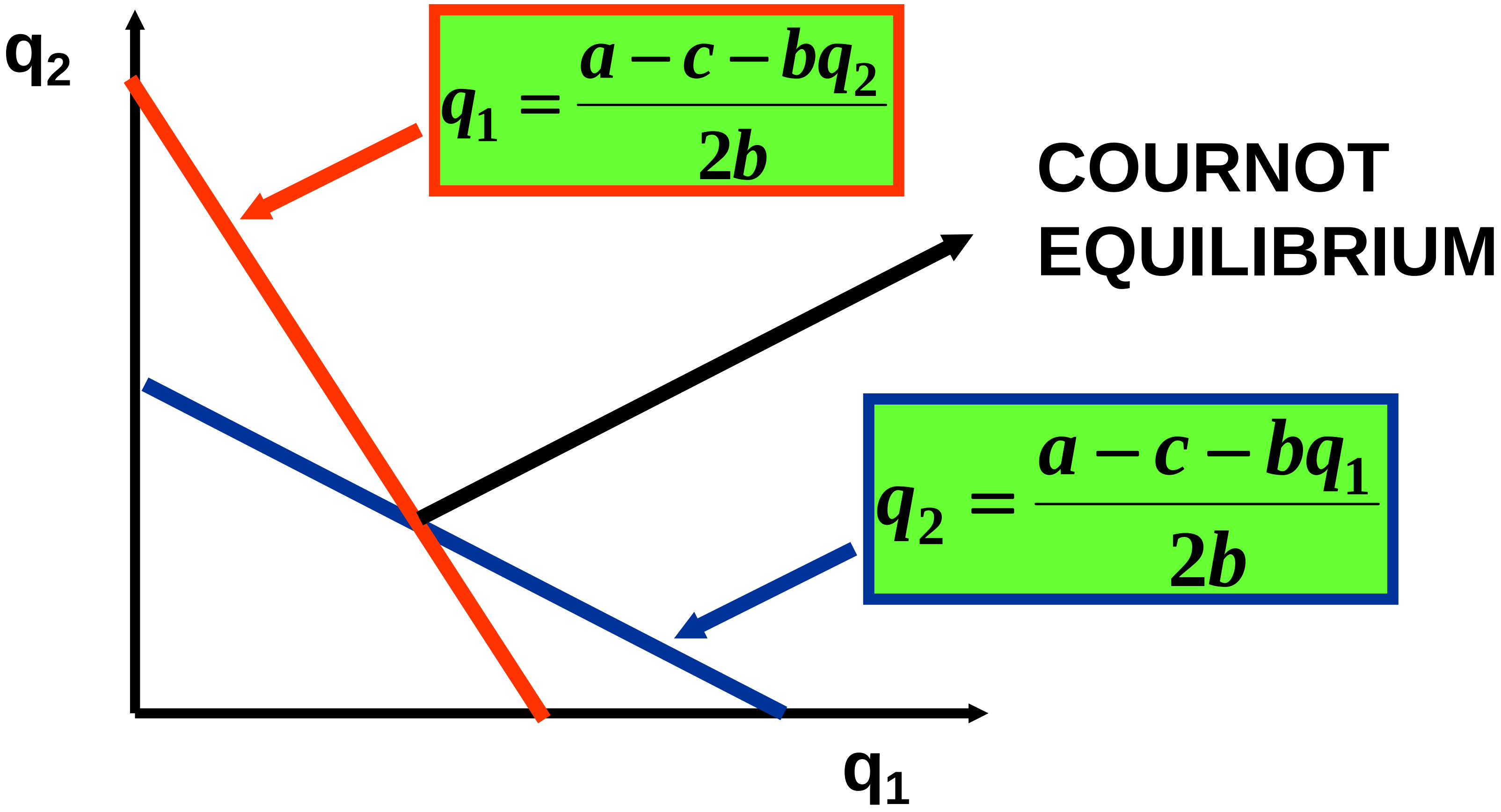
Next graph with q_1 on the horizontal axis and q_2 on the vertical axis

Do the same steps to find q_2

$$q_2 = \frac{a - c - bq_1}{2b}$$

Note: We have two equations and two unknowns so we can solve for q_1 and q_2

Cournot Competition



Cournot Competition

$$q_1 = \frac{a - c - bq_2}{2b}$$

$$q_2 = \frac{a - c - bq_1}{2b}$$

Step 1: Rewrite q_1

$$q_1 = \frac{a}{2b} - \frac{c}{2b} - \frac{bq_2}{2b}$$

Step 2: Cancel b

$$q_1 = \frac{a}{2b} - \frac{c}{2b} - \frac{q_2}{2}$$

Step 3: Factor out 1/2

$$q_1 = \frac{1}{2}\left(\frac{a}{b} - \frac{c}{b} - q_2\right)$$

Step 4: Sub. in for q_2

$$q_1 = \frac{1}{2}\left(\frac{a}{b} - \frac{c}{b} - \left(\frac{a - c - bq_1}{2b}\right)\right)$$

Cournot Competition

$$q_1 = \frac{1}{2}\left(\frac{a}{b} - \frac{c}{b} - \left(\frac{a - c - bq_1}{2b}\right)\right)$$

Step 5: Multiply across by 2 to get rid of the fraction

$$2q_1 = 1\left(\frac{a}{b} - \frac{c}{b} - 1\left(\frac{a - c - bq_1}{2b}\right)\right)$$

Step 6: Simplify

$$2q_1 = \frac{a}{b} - \frac{c}{b} - \frac{a}{2b} + \frac{c}{2b} + \frac{q_1}{2}$$

Cournot Competition

$$2q_1 = \frac{a}{b} - \frac{c}{b} - \frac{a}{2b} + \frac{c}{2b} + \frac{q_1}{2}$$

Step 7: Multiply across by 2 to get rid of the fraction

$$4q_1 = \frac{2a}{b} - \frac{2c}{b} - \frac{2a}{2b} + \frac{2c}{2b} + \frac{2q_1}{2}$$

Step 8: Simplify

$$4q_1 = \frac{2a}{b} - \frac{2c}{b} - \frac{a}{b} + \frac{c}{b} + q_1$$

Cournot Competition

$$4q_1 = \frac{2a}{b} - \frac{2c}{b} - \frac{a}{b} + \frac{c}{b} + q_1$$

Step 9: Rearrange and bring q_1 over to LHS.

$$4q_1 - q_1 = \frac{2a}{b} - \frac{a}{b} - \frac{2c}{b} + \frac{c}{b}$$

Step 10: Simplify

$$3q_1 = \frac{a}{b} - \frac{c}{b}$$

Step 11: Simplify

$$q_1 = \frac{a-c}{3b}$$

Cournot Competition

$$q_1 = \frac{a-c}{3b}$$

Step 12: Repeat above for q_2

$$q_2 = \frac{a-c}{3b}$$

Step 13: Solve for price (go back to demand curve)

$$P = a - bQ$$

Step 14: Sub. in for q_1 and q_2

$$P = a - b\left(\frac{a-c}{3b} + \frac{a-c}{3b}\right)$$

Cournot Competition

$$P = a - b\left(\frac{a-c}{3b} + \frac{a-c}{3b}\right)$$

Step 14: Simplify

$$P = a - \left(\frac{a-c}{3} + \frac{a-c}{3}\right)$$

$$P = a - \left(\frac{a}{3} - \frac{c}{3} + \frac{a}{3} - \frac{c}{3}\right)$$

Cournot Competition

$$P = a - \left(\frac{a}{3} - \frac{c}{3} + \frac{a}{3} - \frac{c}{3} \right)$$

$$P = a - \frac{1}{3}a - \frac{1}{3}a + \frac{1}{3}c + \frac{1}{3}c$$

$$P = a - \frac{2}{3}a + \frac{2}{3}c$$

$$P = \frac{1}{3}a + \frac{2}{3}c$$

$$P = \frac{a + 2c}{3}$$

Cournot Competition: Summary

$$q_1 = \frac{a-c}{3b}$$

$$q_2 = \frac{a-c}{3b}$$

$$Q = \frac{a-c}{3b} + \frac{a-c}{3b}$$

$$Q = \frac{2}{3}\left(\frac{a-c}{b}\right)$$

Cournot v. Bertrand

Cournot Nash (q_1, q_2): Firms compete in quantities,

i.e. Firm 1 chooses the best q_1 given q_2 and

Firm 2 chooses the best q_2 given q_1

Bertrand Nash (p_1, p_2): Firms compete in prices,

i.e. Firm 1 chooses the best p_1 given p_2 and

Firm 2 chooses the best p_2 given p_1

Nash Equilibrium (s_1, s_2): Player 1 chooses the best s_1 given s_2 and Player 2 chooses the best s_2 given s_1

Bertrand Competition: Bertrand Paradox

Assume two firms (as before), a linear demand curve, constant marginal costs and a homogenous product.

Bertrand equilibrium: $p_1 = p_2 = c$

(This implies zero excess profits and is referred to as the Bertand Paradox)

Perfect Competition v. Monopoly v. Cournot Oligopoly

Given

$$P = a - bQ \;\; and \;\; TC_i = cq_i$$

Perfect Competition

$$P = MC \Rightarrow P = C \Rightarrow Q^{pc} = \frac{a-c}{b}$$

Monopoly

$$\Pi = TR - TC$$

$$\Pi = PQ - CQ$$

$$\Pi = (a - bQ)Q - CQ$$

$$\Pi = aQ - bQ^2 - CQ$$

Perfect Competition v. Monopoly v. Cournot Oligopoly

$$\Pi = aQ - bQ^2 - CQ$$

$$\frac{\partial \Pi}{\partial Q} = a - 2bQ - C = 0$$

$$2bQ = a - C$$

$$Q^m = \frac{a - C}{2b}$$

$$P = a - bQ$$

$$P = a - b\left(\frac{a - C}{2b}\right)$$

$$P^M = \frac{a + C}{2}$$

Perfect Competition v Monopoly v Cournot Oligopoly

$$Q^{CO} = \frac{2}{3}\left(\frac{a-c}{b}\right)$$

$$P^{CO} = \frac{a+2c}{3}$$

$Q^{m} < Q^{co} < Q^{PC}$

$P^{m} > P^{co} > P^{pc}$