

Chapter 2

Budget Constraint

Consumption Choice Sets

- A consumption choice set is the collection of all consumption choices available to the consumer.
- What constrains consumption choice?

-Budgetary, time and other resource limitations.

- A consumption bundle containing x₁ units of commodity 1, x₂ units of commodity 2 and so on up to x_n units of commodity n is denoted by the vector (x₁, x₂, ..., x_n).
- ♦ Commodity prices are p₁, p₂, ..., p_n.

Q: When is a consumption bundle
 (x₁, ..., x_n) affordable at given prices
 p₁, ..., p_n?

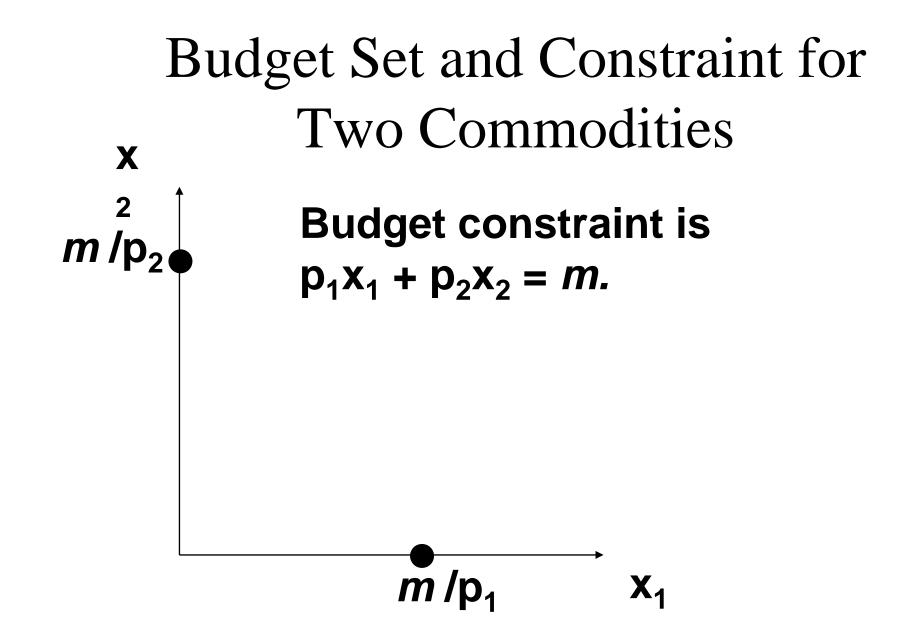
- ♦ Q: When is a bundle (x₁, ..., x_n) affordable at prices p₁, ..., p_n?
- A: When

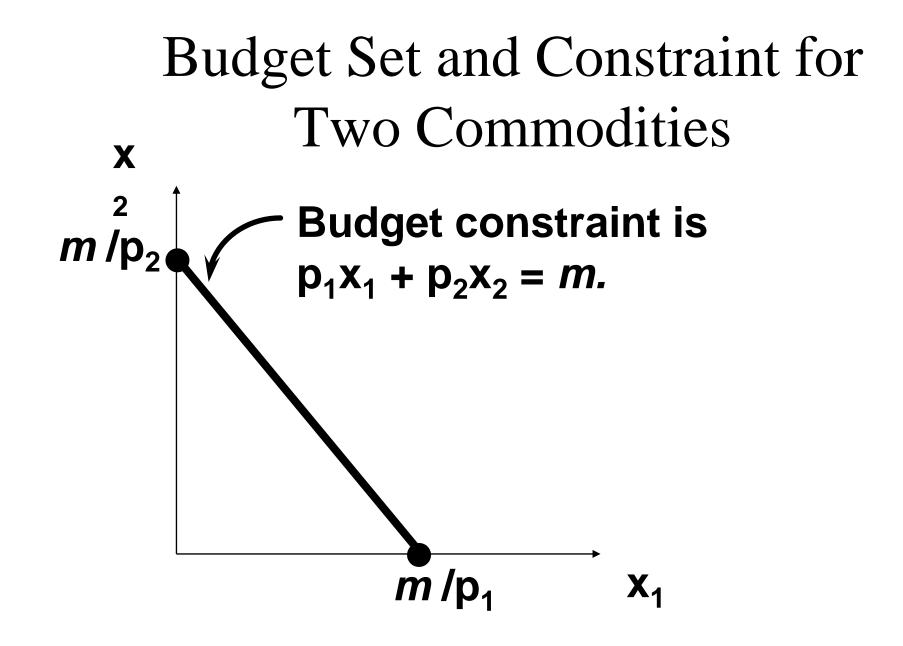
 $p_1x_1 + ... + p_nx_n \le m$ where *m* is the consumer's (disposable) income.

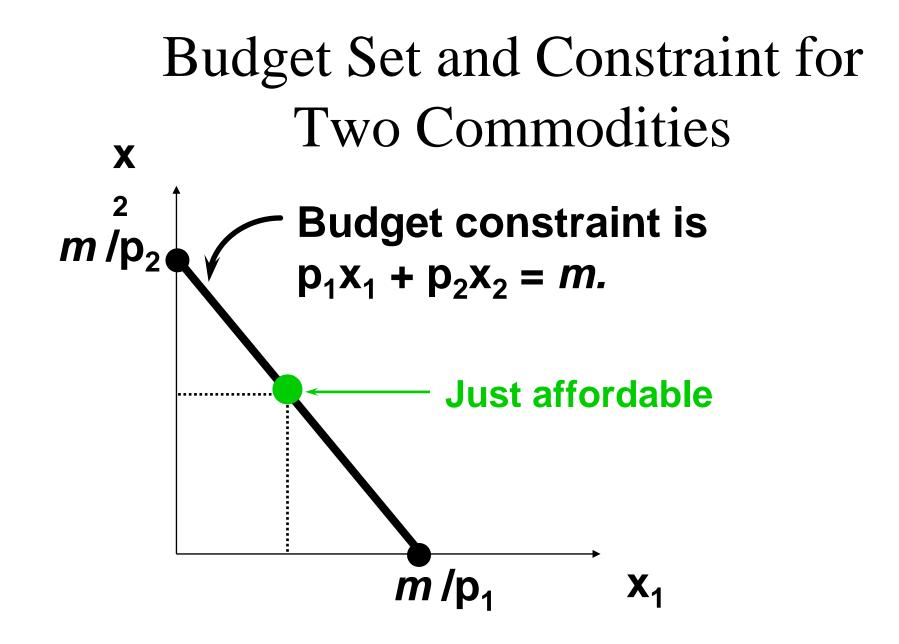
 The bundles that are only just affordable form the consumer's budget constraint. This is the set

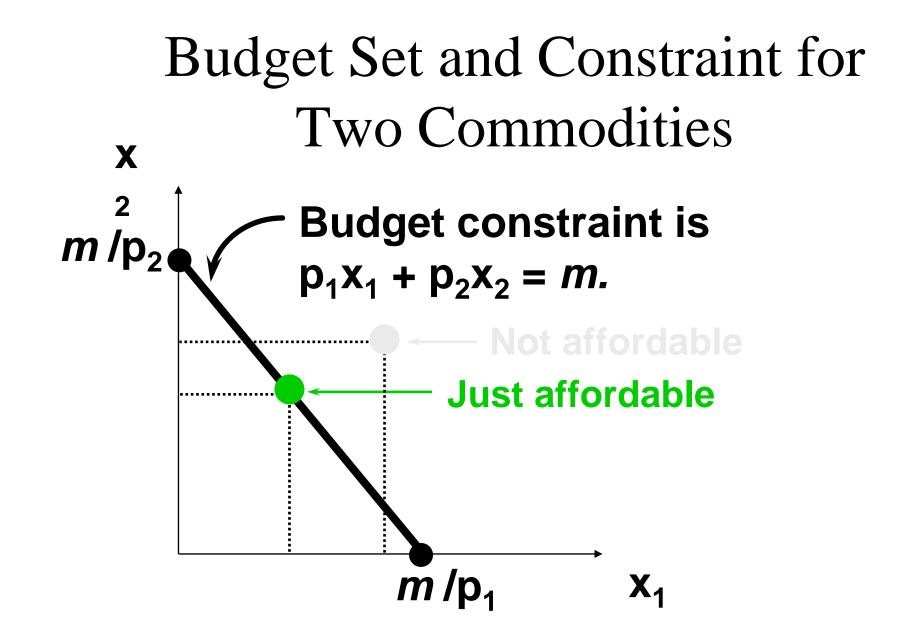
{
$$(\mathbf{x}_1,...,\mathbf{x}_n) | \mathbf{x}_1 \ge \mathbf{0}, ..., \mathbf{x}_n \ge 0$$
 and
 $\mathbf{p}_1 \mathbf{x}_1 + ... + \mathbf{p}_n \mathbf{x}_n = \mathbf{m}$ }.

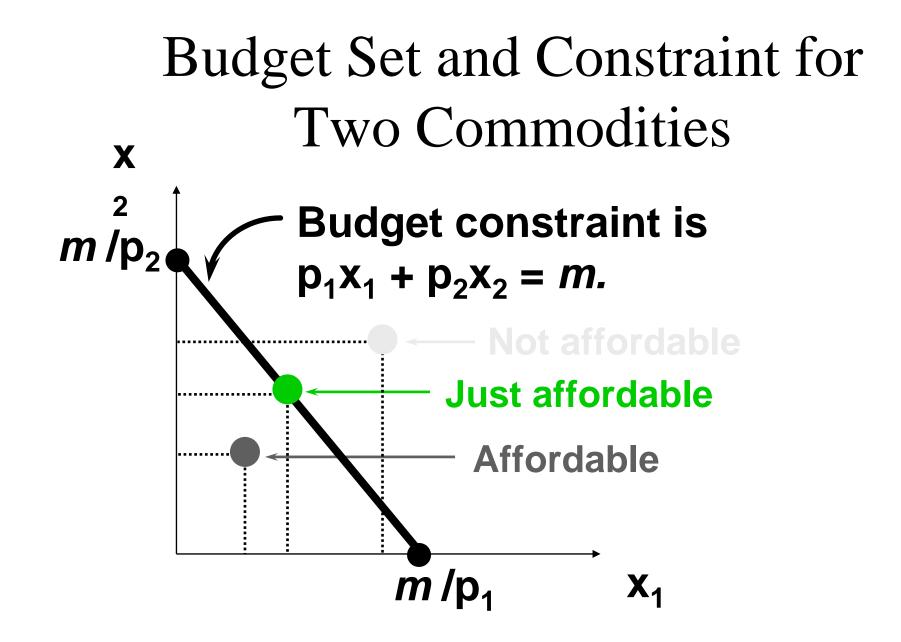
- ◆ The consumer's budget set is the set of all affordable bundles; B(p₁, ..., p_n, m) = { (x₁, ..., x_n) | x₁ ≥ 0, ..., x_n ≥ 0 and p₁x₁ + ... + p_nx_n ≤ m }
- The budget constraint is the upper boundary of the budget set.

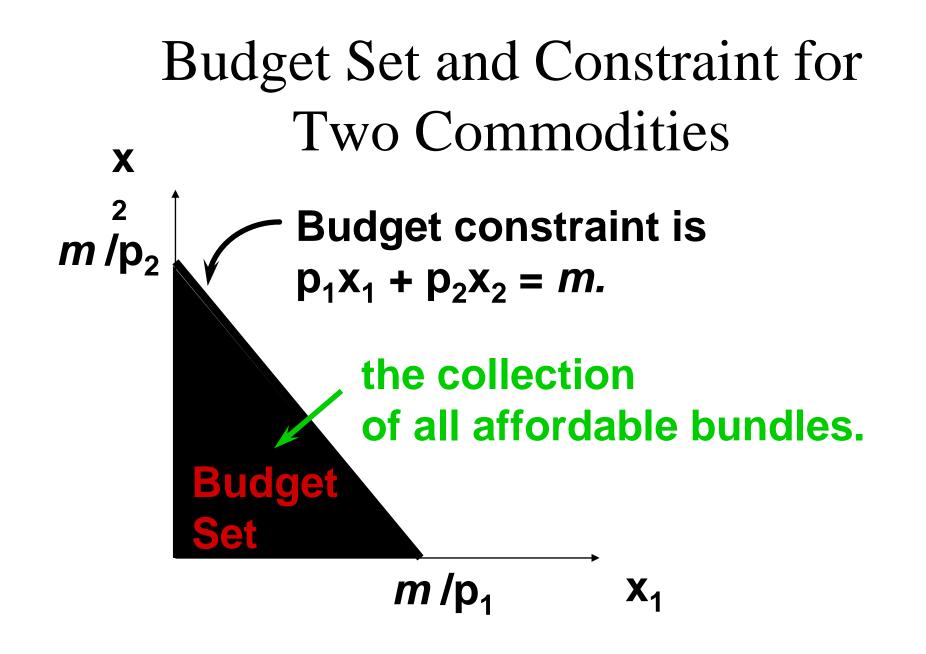


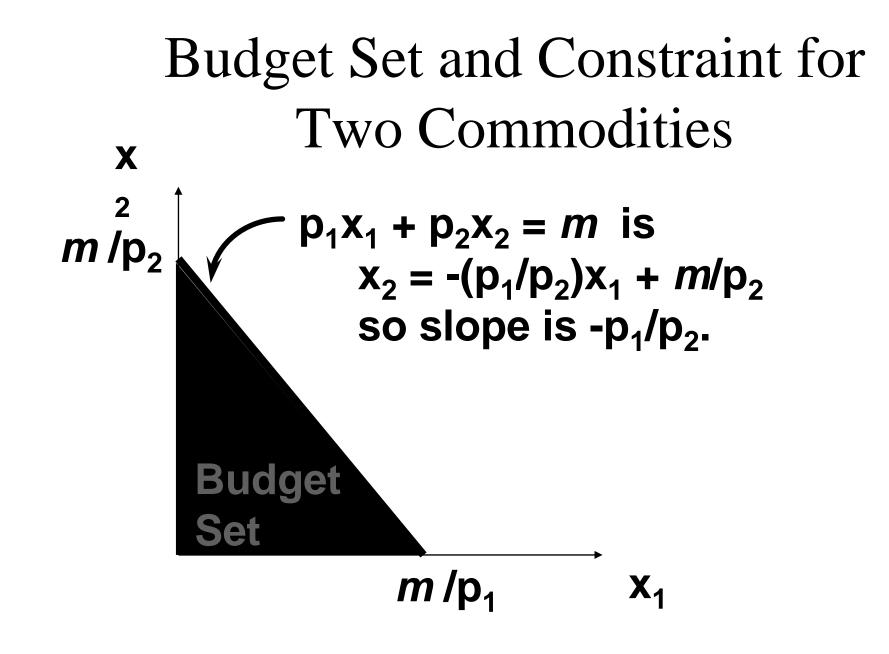






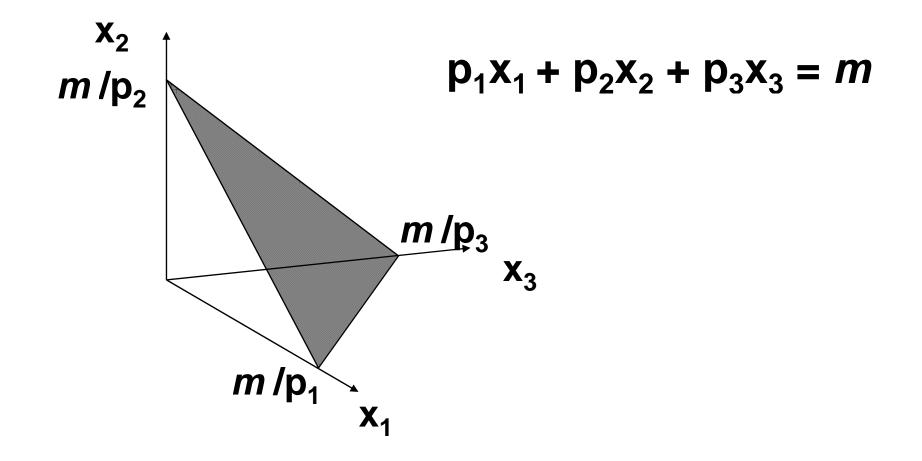


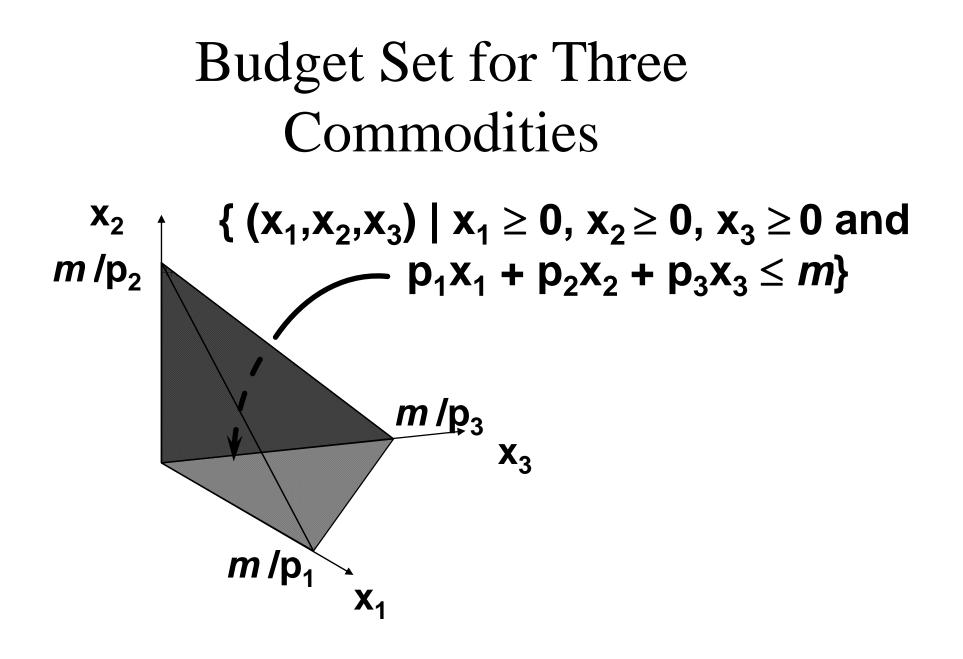




If n = 3 what do the budget constraint and the budget set look like?

Budget Constraint for Three Commodities

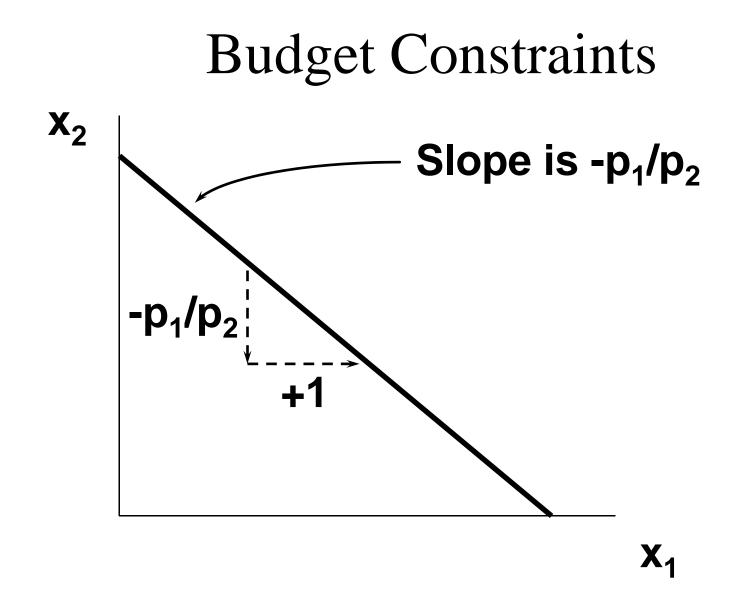


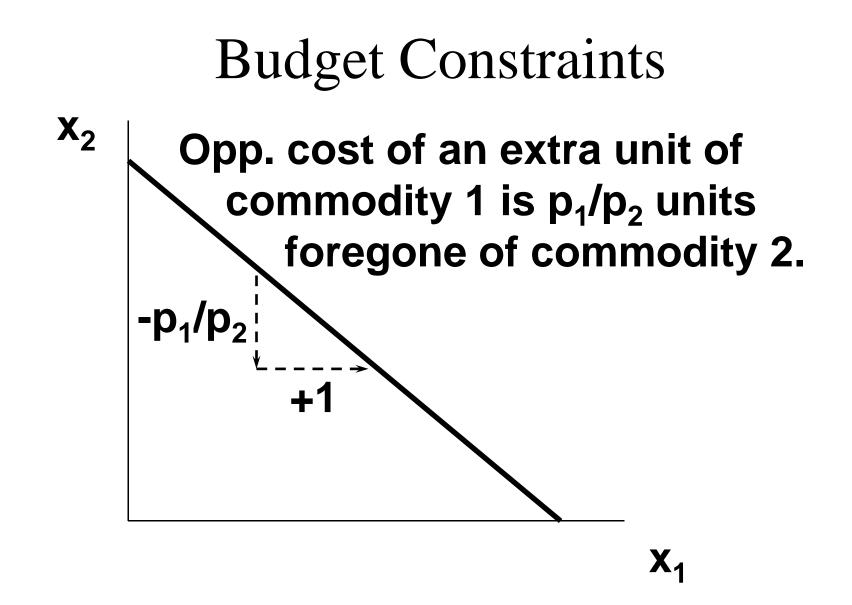


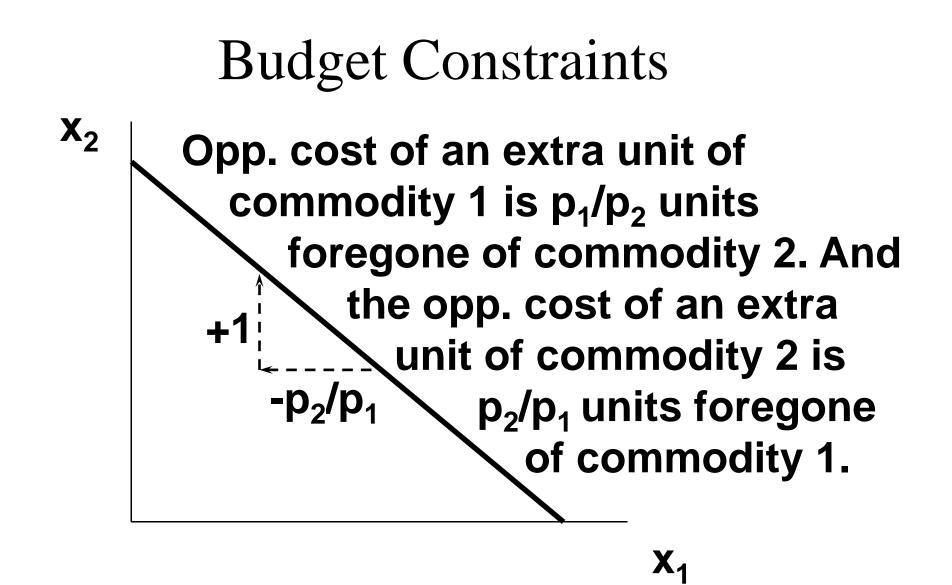
Budget Constraints For n = 2 and x₁ on the horizontal axis, the constraint's slope is -p₁/p₂. What does it mean?

Budget Constraints For n = 2 and x₁ on the horizontal axis, the constraint's slope is -p₁/p₂. What does it mean?

Increasing x₁ by 1 must reduce x₂ by p₁/p₂.

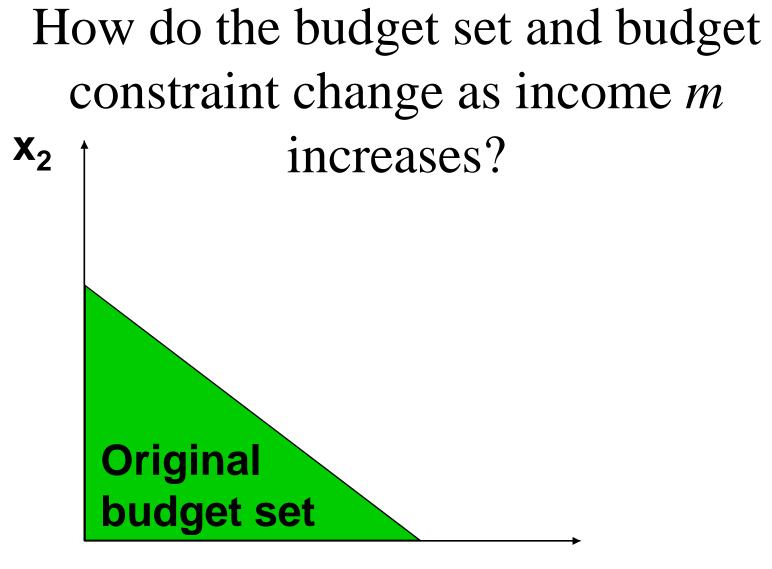




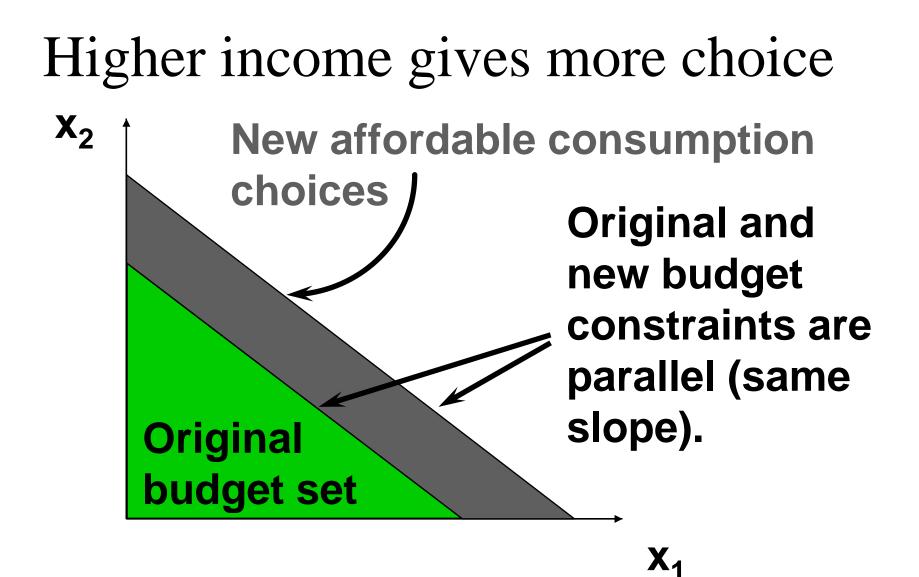


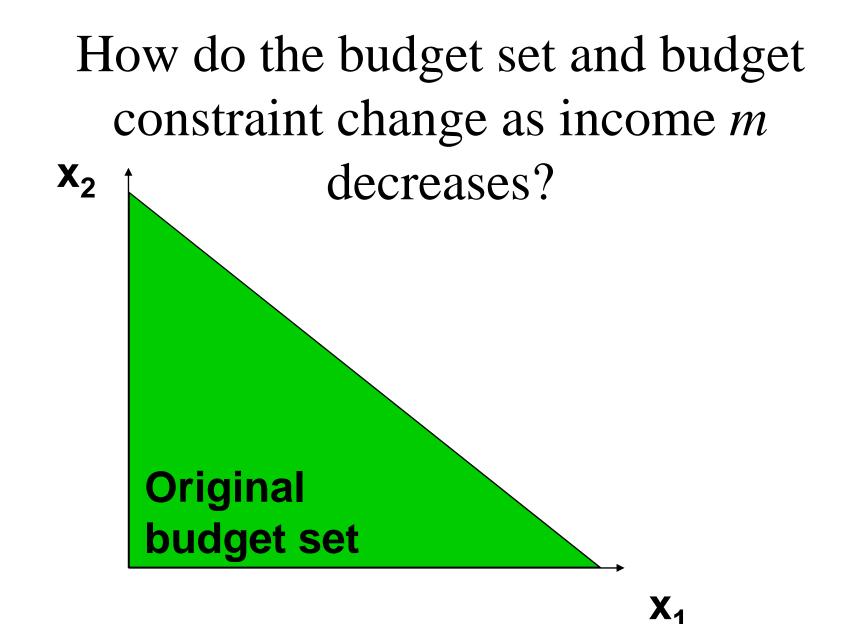
Budget Sets & Constraints; Income and Price Changes

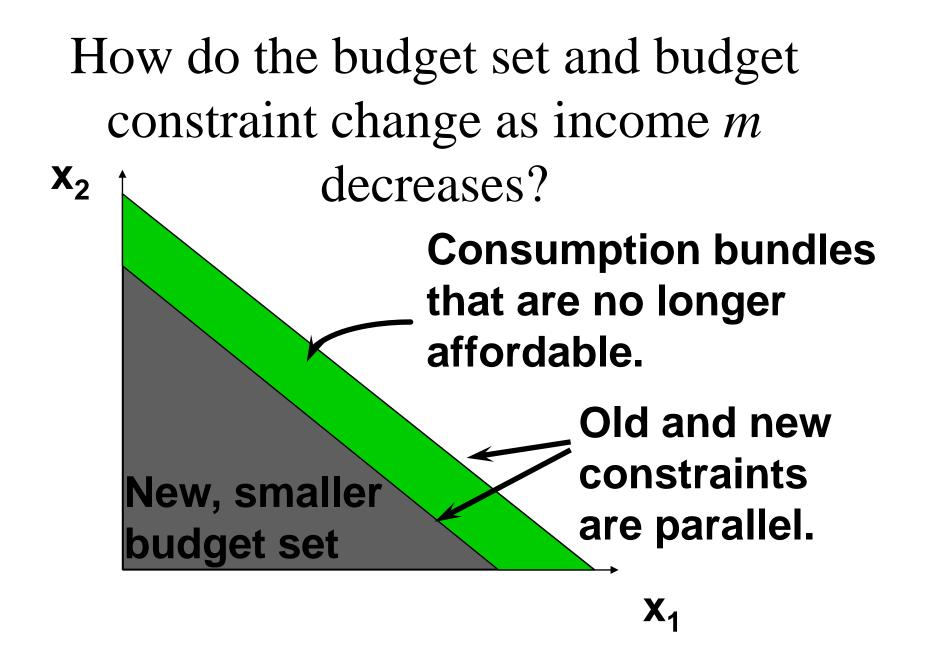
 The budget constraint and budget set depend upon prices and income. What happens as prices or income change?



X₁







Budget Constraints - Income Changes

 Increases in income *m* shift the constraint outward in a parallel manner, thereby enlarging the budget set and improving choice.

Budget Constraints - Income Changes

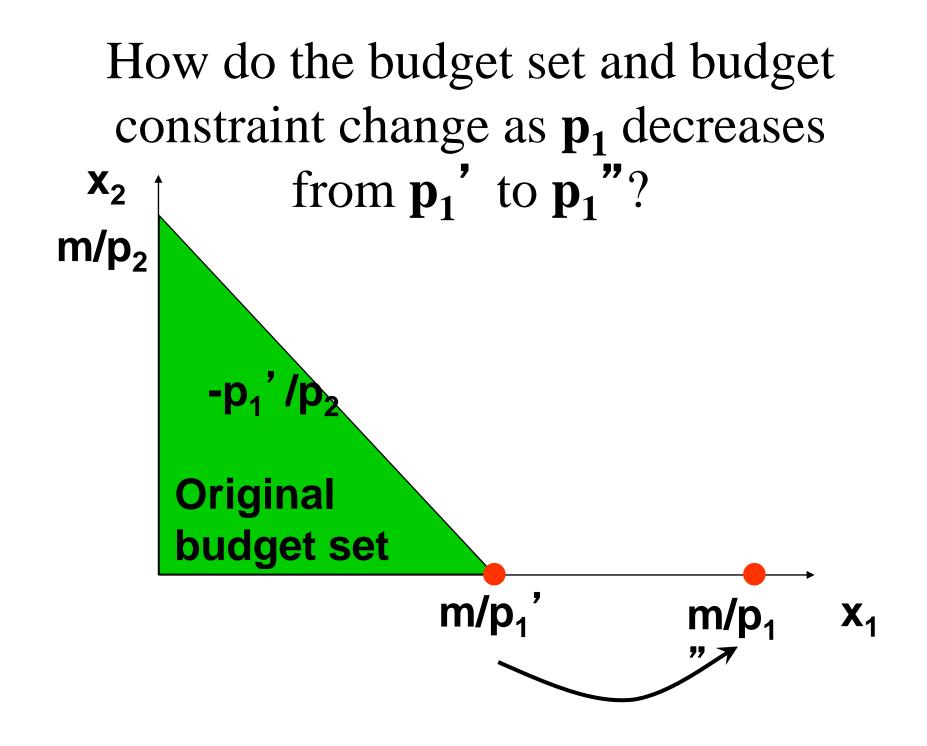
- Increases in income *m* shift the constraint outward in a parallel manner, thereby enlarging the budget set and improving choice.
- Decreases in income *m* shift the constraint inward in a parallel manner, thereby shrinking the budget set and reducing choice.

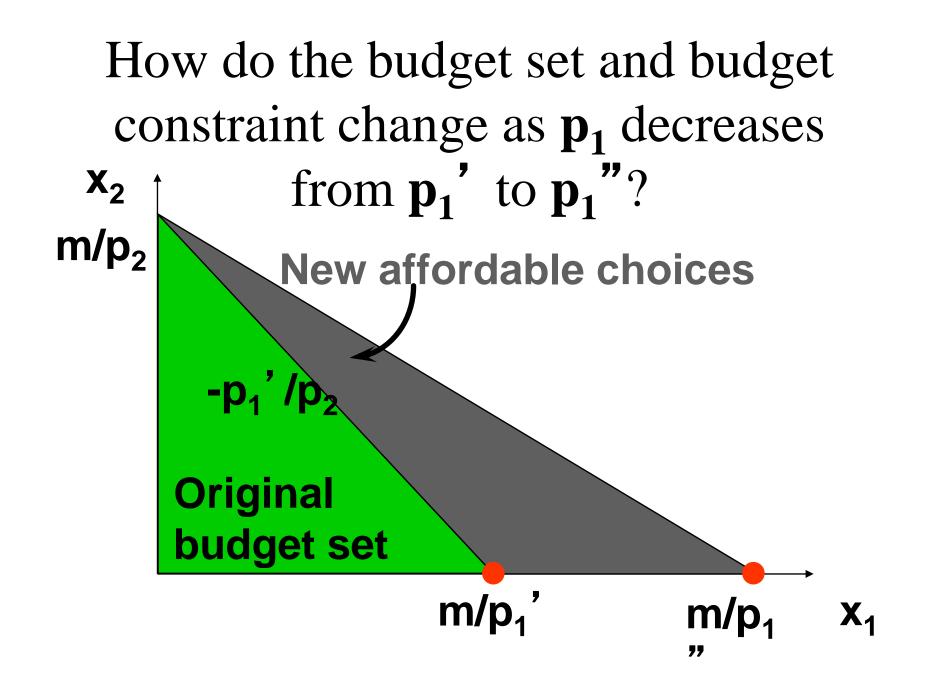
Budget Constraints - Income Changes

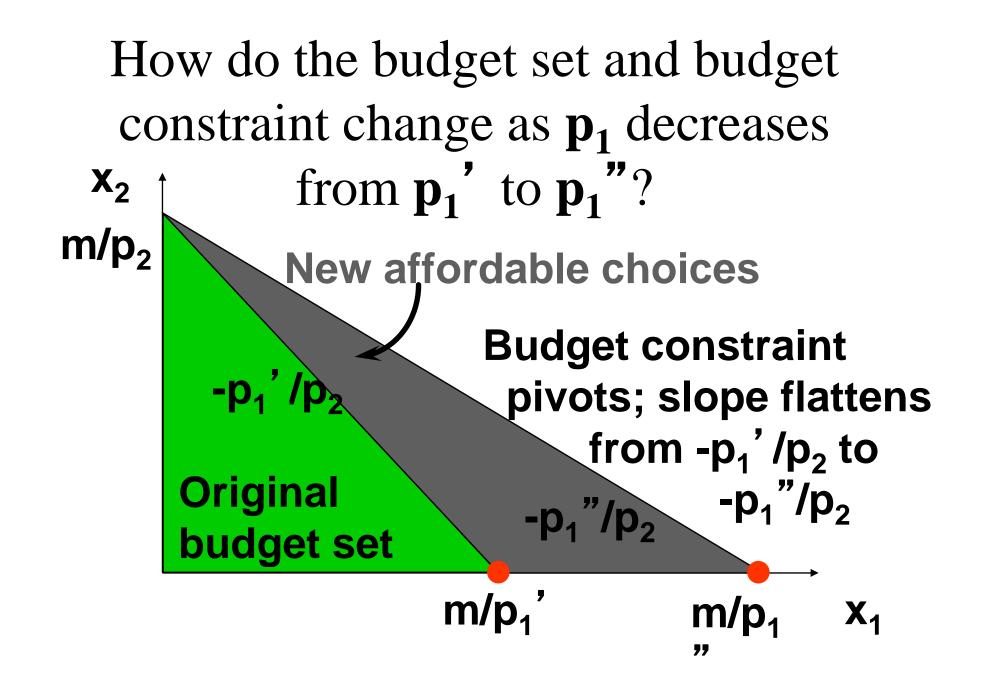
- No original choice is lost and new choices are added when income increases, so higher income cannot make a consumer worse off.
- An income decrease may (typically will) make the consumer worse off.

Budget Constraints - Price Changes

- What happens if just one price decreases?
- ♦ Suppose p₁ decreases.







Budget Constraints - Price Changes

 Reducing the price of one commodity pivots the constraint outward. No old choice is lost and new choices are added, so reducing one price cannot make the consumer worse off.

Budget Constraints - Price Changes

 Similarly, increasing one price pivots the constraint inwards, reduces choice and may (typically will) make the consumer worse off.

Uniform Ad Valorem Sales Taxes

- An ad valorem sales tax levied at a rate of 5% increases all prices by 5%, from p to (1+1 × .05)p = 1.05p.
- An ad valorem sales tax levied at a rate of t increases all prices by tp from p to (1+t)p.
- A uniform sales tax is applied uniformly to all commodities.

Uniform Ad Valorem Sales Taxes

A uniform sales tax levied at rate t changes the constraint from
 p₁x₁ + p₂x₂ = m

$$(1+t)p_1x_1 + (1+t)p_2x_2 = m$$

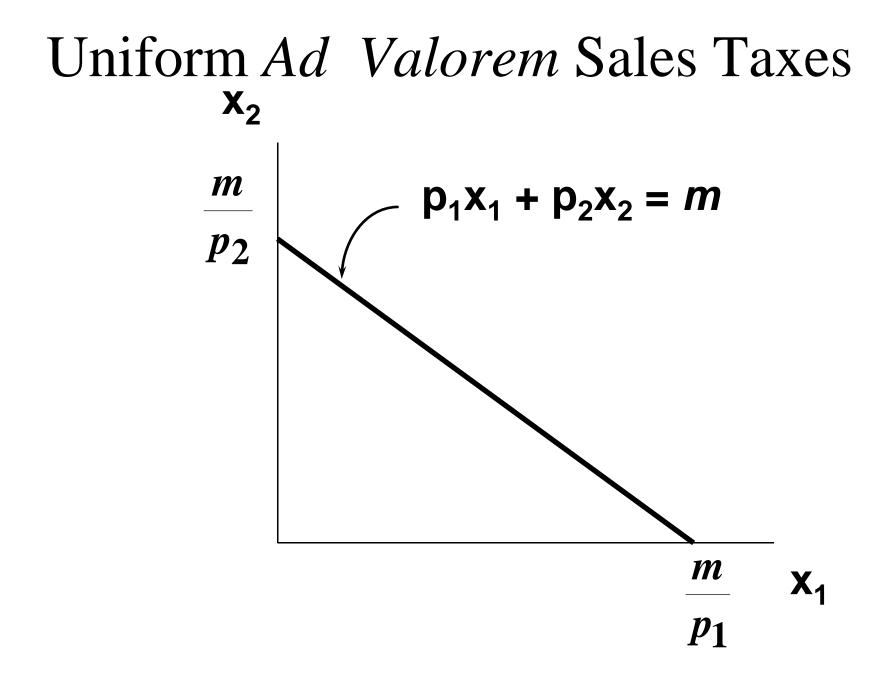
Uniform Ad Valorem Sales Taxes

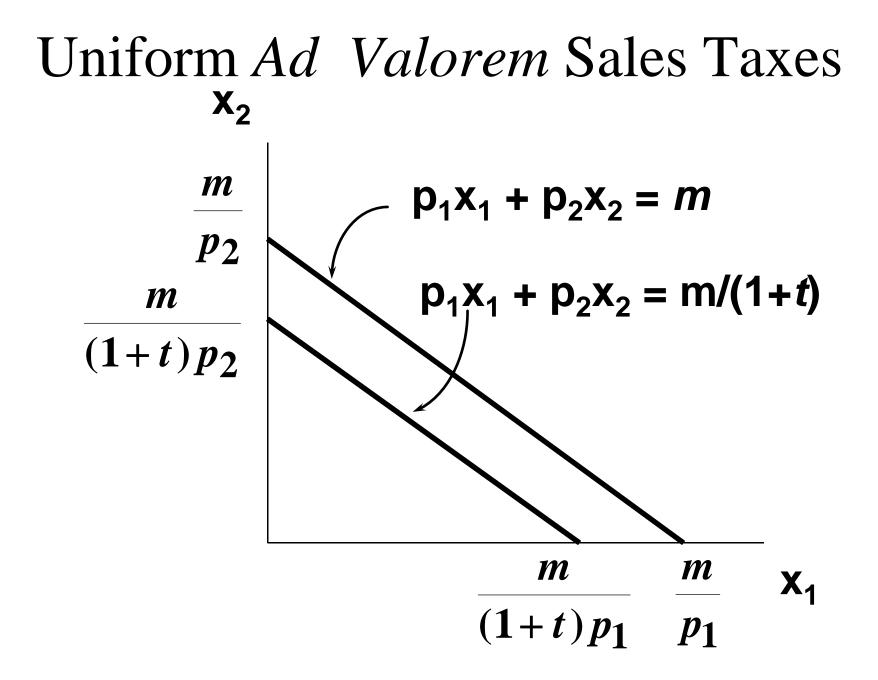
A uniform sales tax levied at rate t changes the constraint from
 p₁x₁ + p₂x₂ = m

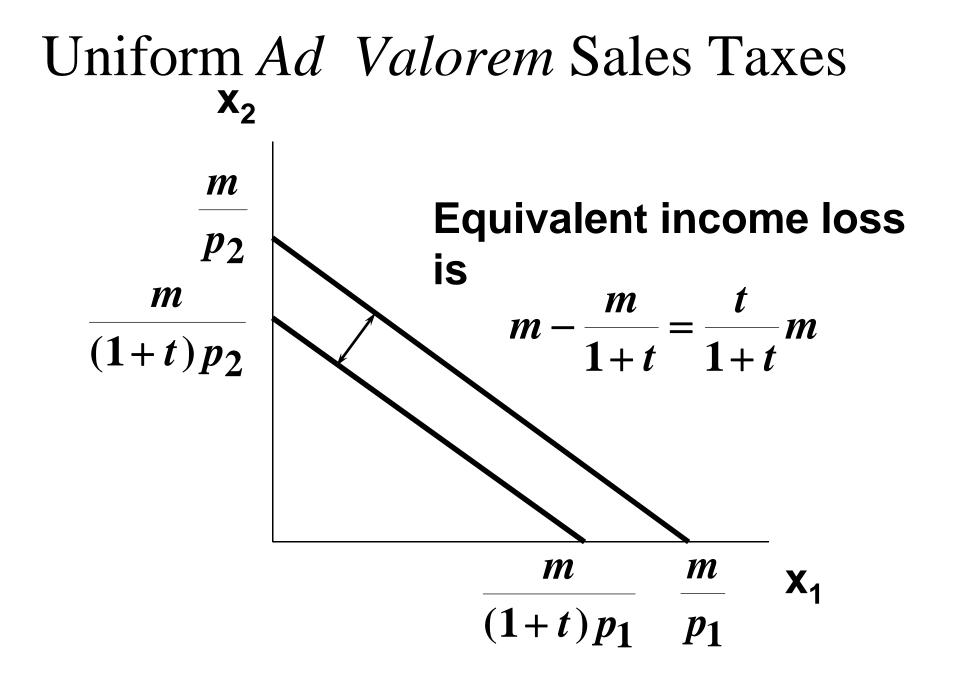
 to
 (1+t)p₁x₁ + (1+t)p₂x₂ = m

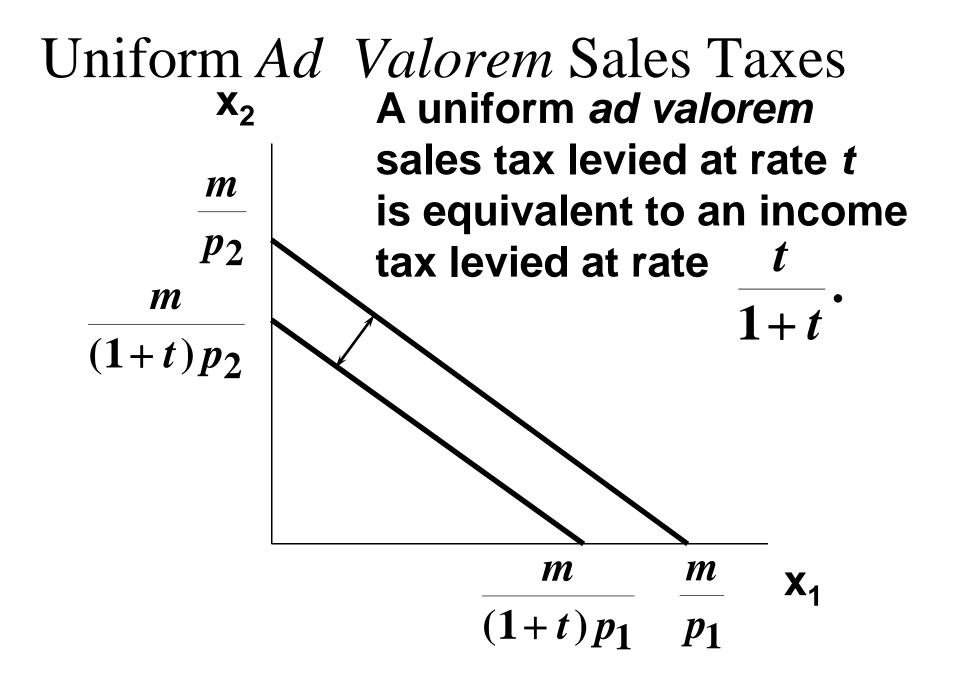
 i.e.

$$p_1x_1 + p_2x_2 = m/(1+t).$$







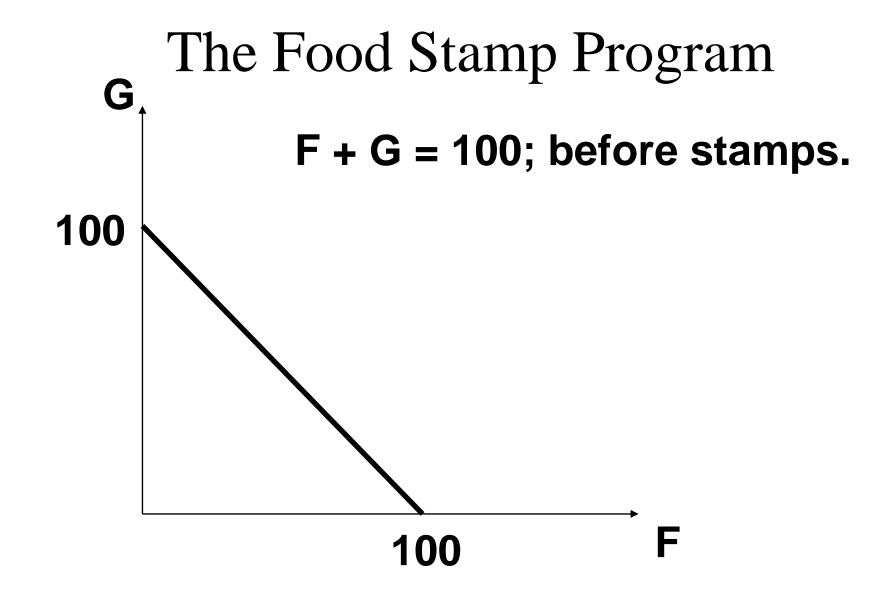


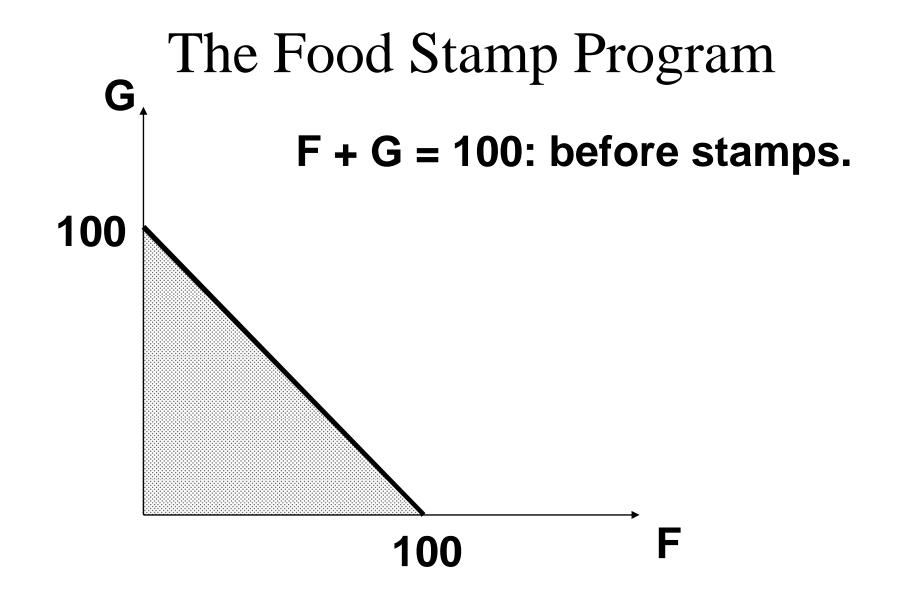
The Food Stamp Program

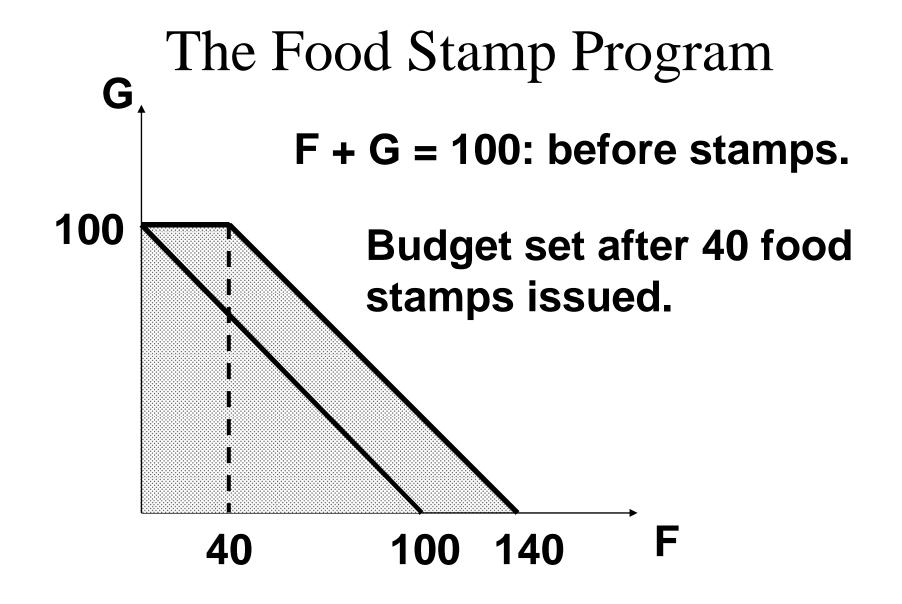
- Food stamps are coupons that can be legally exchanged only for food.
- How does a commodity-specific gift such as a food stamp alter a family's budget constraint?

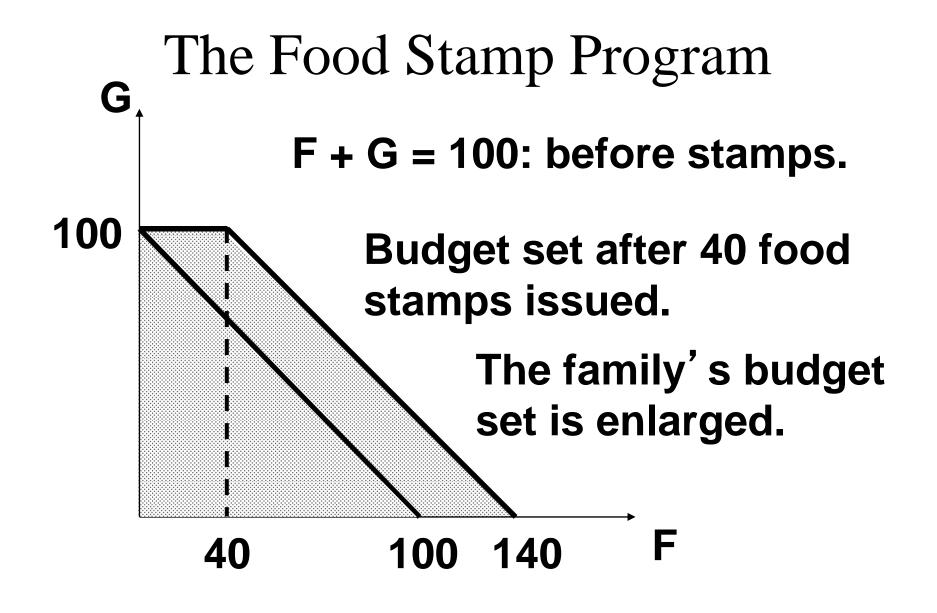
The Food Stamp Program

- Suppose m = \$100, p_F = \$1 and the price of "other goods" is p_G = \$1.
- ◆ The budget constraint is then
 F + G =100.



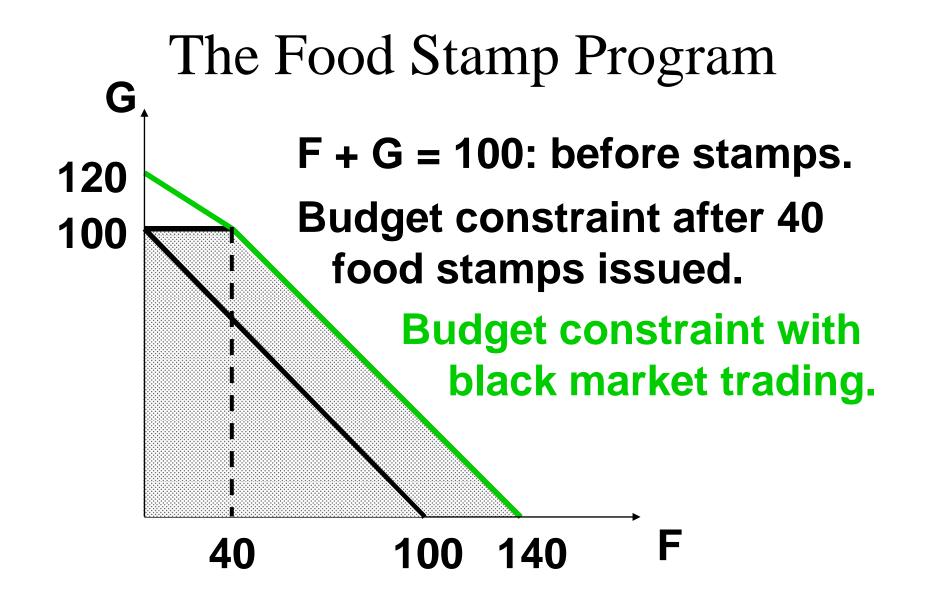


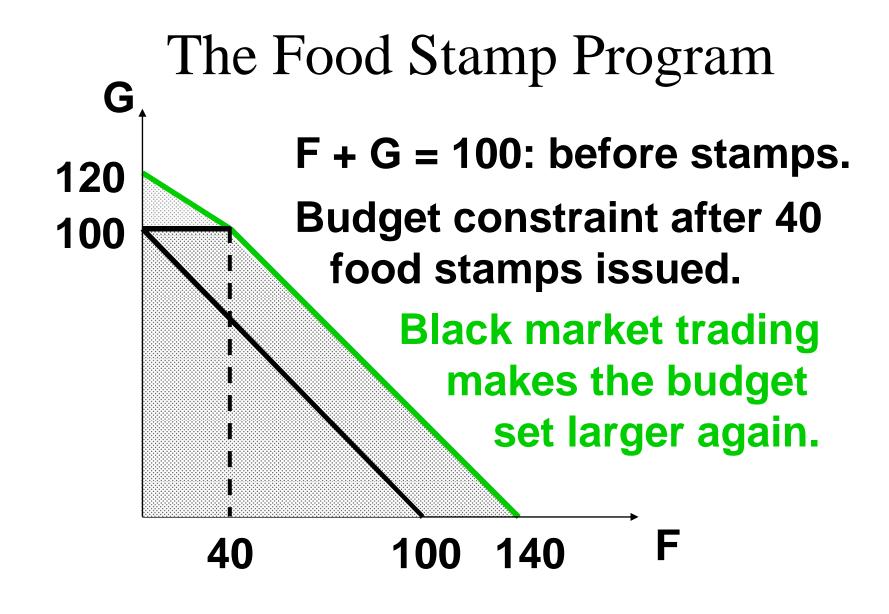




The Food Stamp Program

What if food stamps can be traded on a black market for \$0.50 each?





- "Numeraire" means "unit of account".
- Suppose prices and income are measured in dollars. Say p₁=\$2, p₂=\$3, *m* = \$12. Then the constraint is

$$2x_1 + 3x_2 = 12.$$

If prices and income are measured in cents, then p₁=200, p₂=300, *m*=1200 and the constraint is

 $200x_1 + 300x_2 = 1200,$

the same as

$$2x_1 + 3x_2 = 12.$$

 Changing the numeraire changes neither the budget constraint nor the budget set.

• The constraint for $p_1=2$, $p_2=3$, m=12 $2x_1 + 3x_2 = 12$ is also $1.x_1 + (3/2)x_2 = 6$, the constraint for $p_1=1$, $p_2=3/2$, m=6. Setting p₁=1 makes commodity 1 the numeraire and defines all prices relative to p_1 ; e.g. 3/2 is the price of commodity 2 relative to the price of commodity 1.

 Any commodity can be chosen as the numeraire without changing the budget set or the budget constraint.

- $p_1=2$, $p_2=3$ and $p_3=6 \implies$
- price of commodity 2 relative to commodity 1 is 3/2,
- price of commodity 3 relative to commodity 1 is 3.
- Relative prices are the rates of exchange of commodities 2 and 3 for units of commodity 1.

Shapes of Budget Constraints

- Q: What makes a budget constraint a straight line?
- A: A straight line has a constant slope and the constraint is

 $p_1x_1 + ... + p_nx_n = m$ so if prices are constants then a constraint is a straight line.

Shapes of Budget Constraints

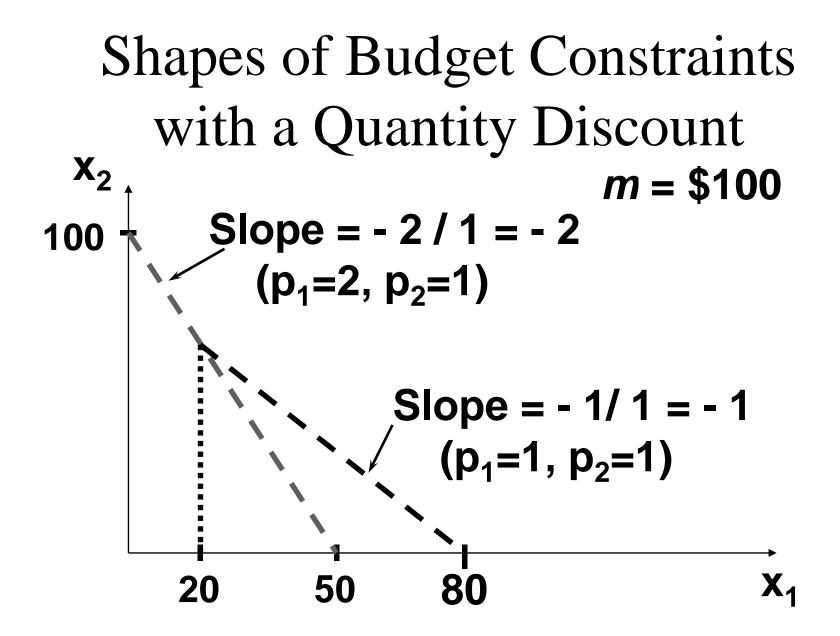
- But what if prices are not constants?
- *E.g.* bulk buying discounts, or price penalties for buying "too much".
- Then constraints will be curved.

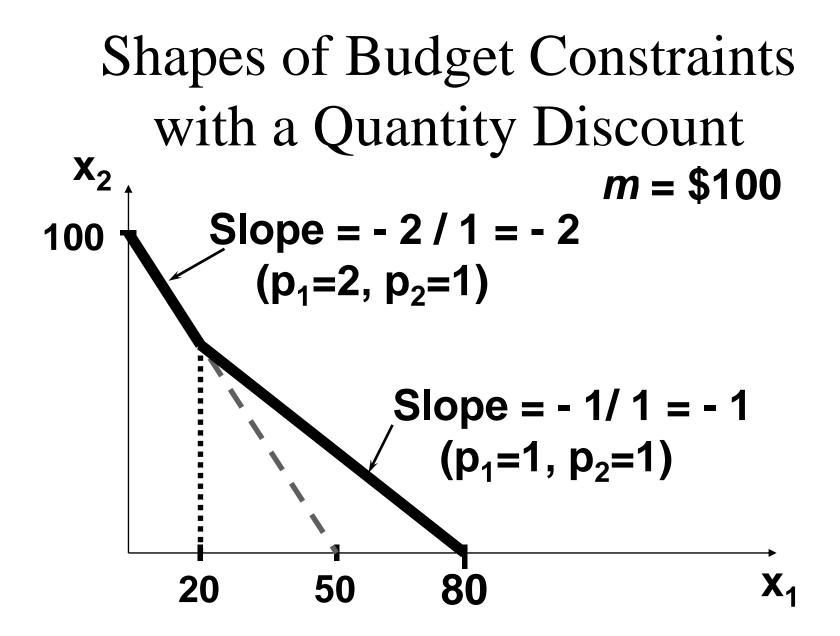
Shapes of Budget Constraints -Quantity Discounts

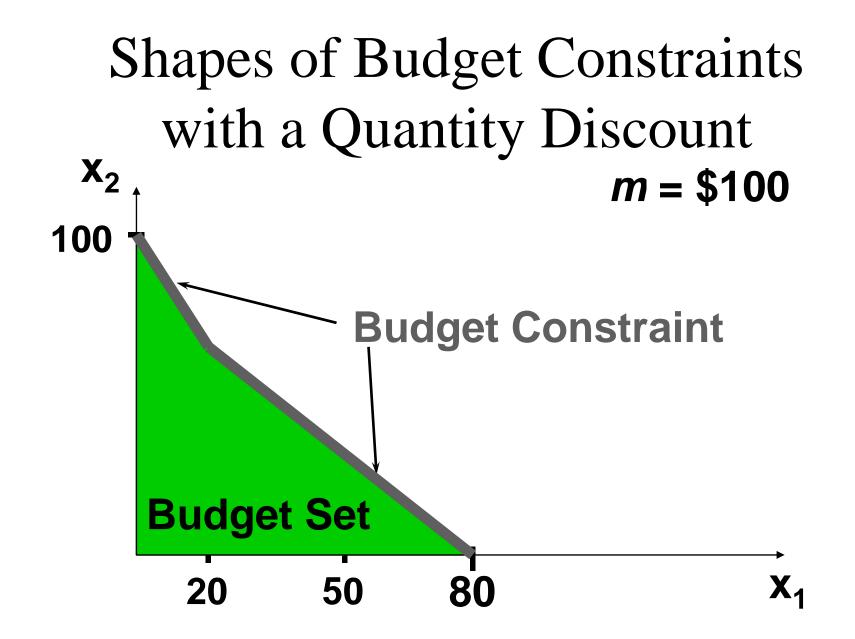
• Suppose p_2 is constant at \$1 but that $p_1=$ \$2 for $0 \le x_1 \le 20$ and $p_1=$ \$1 for $x_1>20$.

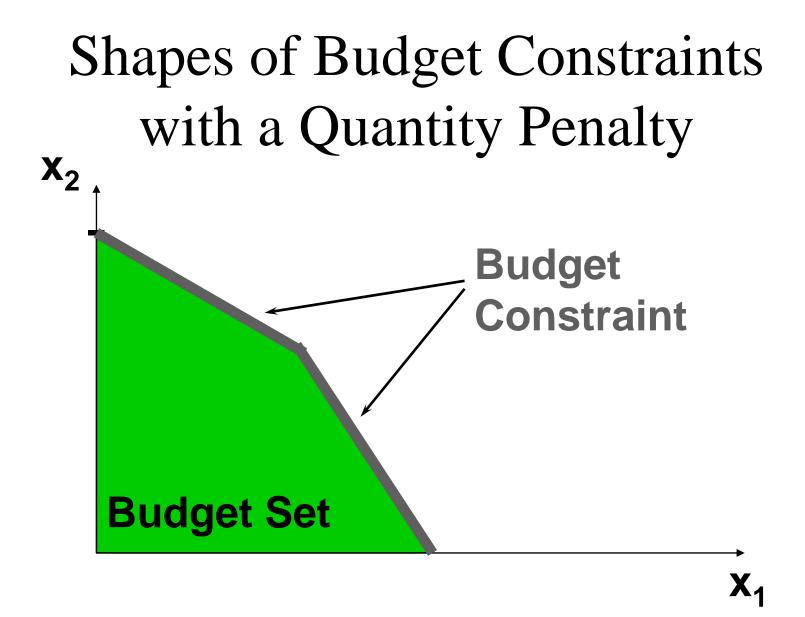
Shapes of Budget Constraints -Quantity Discounts

• Suppose p_2 is constant at \$1 but that $p_1=$ \$2 for $0 \le x_1 \le 20$ and $p_1=$ \$1 for $x_1>20$. Then the constraint's slope is $-p_1/p_2 = \begin{cases} -2, \text{ for } 0 \le x_1 \le 20 \\ -1, \text{ for } x_1 > 20 \end{cases}$ and the constraint is





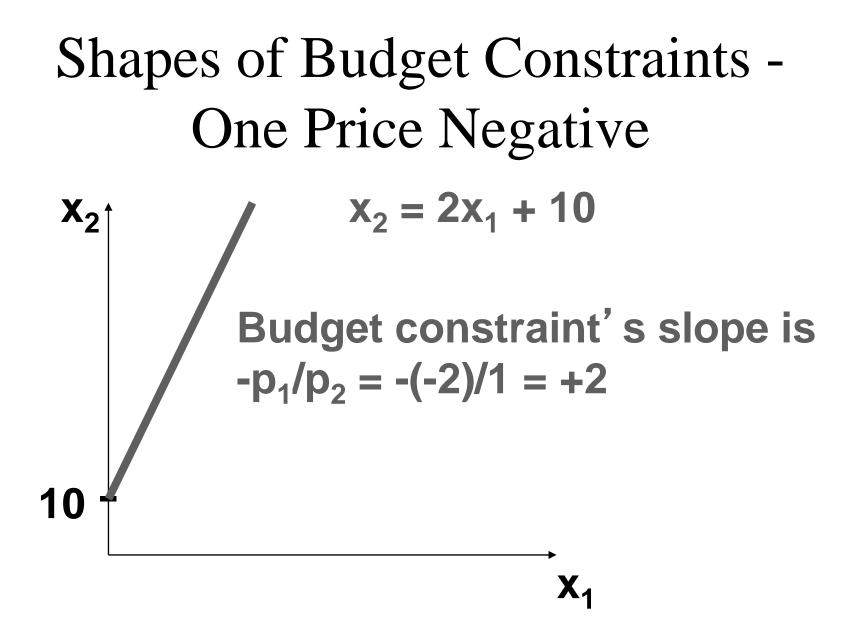




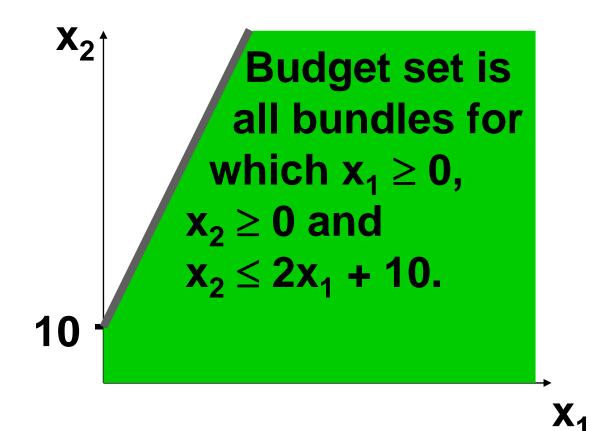
Shapes of Budget Constraints -One Price Negative

- Commodity 1 is stinky garbage. You are paid \$2 per unit to accept it; *i.e.* p₁ = \$2. p₂ = \$1. Income, other than from accepting commodity 1, is *m* = \$10.
- Then the constraint is

$$-2x_1 + x_2 = 10$$
 or $x_2 = 2x_1 + 10$.

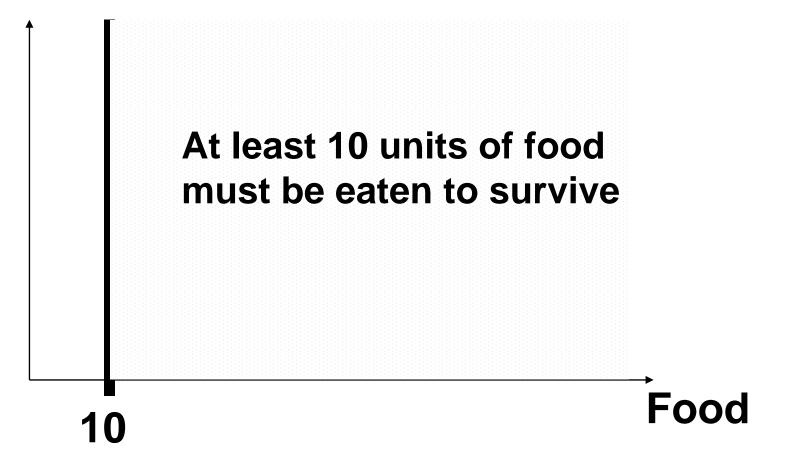


Shapes of Budget Constraints -One Price Negative



More General Choice Sets

- Choices are usually constrained by more than a budget; e.g. time constraints and other resources constraints.
- A bundle is available only if it meets every constraint.

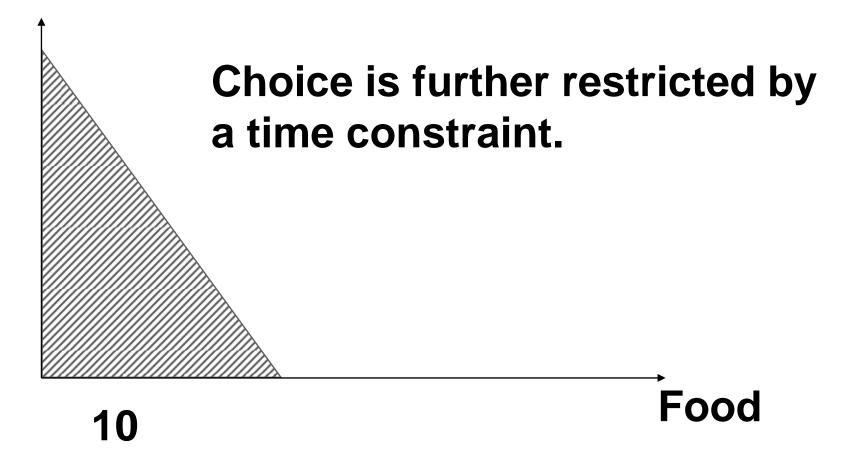




Budget Set

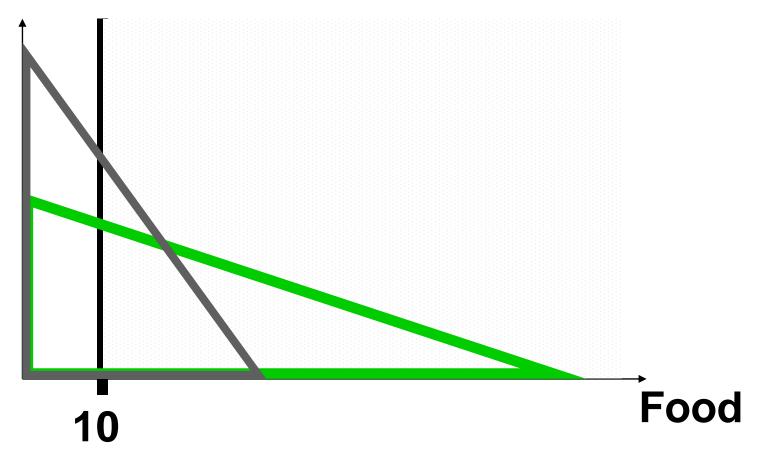
Food

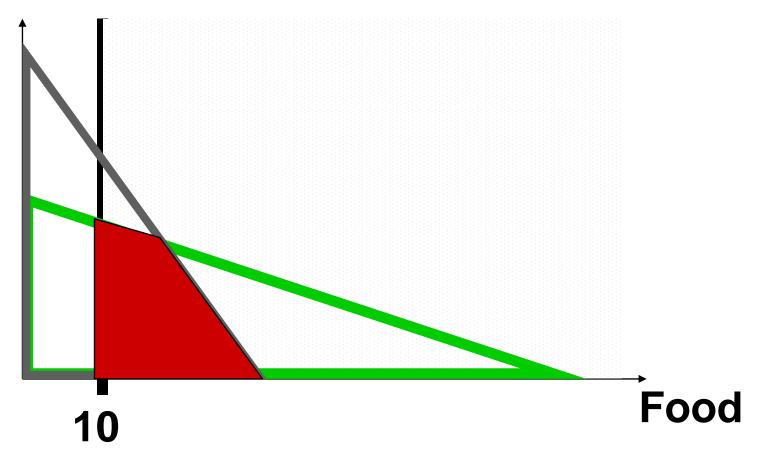
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More General Choice Sets

So what is the choice set?









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