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Auctions

Who Uses Auctions?

- Owners of art, cars, stamps, machines, mineral rights etc.
- Q: Why auction?

Who Uses Auctions?

- Owners of art, cars, stamps, machines, mineral rights etc.
- Q: Why auction?
- A: Because many markets are imperfect and it is hard to discover potential buyers' true valuations of your asset. Auctions help to discover this information.

Types of Auctions

- English auction:
  - bids are public announcements
  - bid price rises until there are no further bids
  - highest bid wins
  - winner pays his bid.

Types of Auctions

- Sealed-bid first-price auction:
  - bids are private information
  - bids are made simultaneously
  - highest bid wins
  - winner pays his bid.

Types of Auctions

- Sealed-bid second-price auction:
  - bids are private information
  - bids are made simultaneously
  - highest bidder wins
  - winner pays second-highest bid
  - also known as a Vickrey auction.

Types of Auctions

- Dutch auction:
  - auctioneer announces a high bid and then gradually lowers the bid
  - first buyer to accept wins and pays that price.

Reserve Price

- A seller specified bid level below which no sale is made.

Economists' Classification of Auctions

- Private-value auctions:
  - every potential buyer knows for sure her own valuation of the item for sale
  - all these individual valuations are independent of each other.

Economists' Classification of Auctions

- Common-value auctions:
  - item for sale has the same value to every potential buyer
  - potential buyers differ in their own estimates of this common value.

Auction Design

- Goals:
  - Pareto efficiency
  - maximization of the seller's profit.

Auction Design

- Pareto efficiency:
  - the item must sell to the buyer with the highest valuation of the item.
- Which auctions are Pareto efficient?

Auctions and Efficiency

- English auction with no reserve price must be efficient since, if a buyer with a low valuation was about to buy, the highest valuation buyer would bid higher.

Auctions and Efficiency

- English auction with a reserve price need not be efficient since if the reserve price is set above the (unknown to the seller) highest buyer valuation, then there will be no sale and so no gains-to-trade.

Auctions and Efficiency

- Dutch auction need not be efficient. No buyer knows other buyers' valuations, so the highest valuation buyer may delay too long and lose to another bidder.

Auctions and Efficiency

- Sealed-bid first-price auction need not be efficient. No buyer knows other buyers' valuations, so the highest valuation buyer may bid too low and lose to another bidder.

Auctions and Efficiency

- Sealed-bid second-price auction is Pareto efficient even though no buyer knows the other buyers' valuations (more on this later).

Why Use a Reserve Price?

- Suppose there are 2 buyers.
- The seller believes each buyer's valuation is $20 with chance 1/2 and $50 with chance 1/2.
- I.e. with chance 1/4 each, the seller believes she faces buyer valuations ($20,$20), ($20,$50), ($50,$20) and ($50,$50).

Why Use a Reserve Price?

- I.e. with chance 1/4 each, the seller believes she faces buyer valuations ($20,$20), ($20,$50), ($50,$20) and ($50,$50).
- Use an English auction.
- Bids must be raised by at least $1.
- With chance 1/4 each, winning bids will be $20, $21, $21 and $50 if there is no reserve price.

Why Use a Reserve Price?

- With chance 1/4 each, winning bids will be $20, $21, $21 and $50 if there is no reserve price.
- Seller's expected revenue is
  ($20 + $21 + $21 + $50)/4 = $28
  with no reserve price.

Why Use a Reserve Price?

- With chance 1/4 each, the seller believes she faces buyer valuations ($20,$20), ($20,$50), ($50,$20) and ($50,$50).
- Set a reserve price of $50.
- With chance 1/4 there will be no sale.
- With chance 3/4 the winning bid will be $50.

Why Use a Reserve Price?

- Set a reserve price of $50.
- With chance 1/4 there will be no sale.
- With chance 3/4 the winning bid will be $50.
- Seller's expected revenue is

$$\frac{3}{4} \times \$50 + \frac{1}{4} \times \$0 = \$37.50 > \$28.$$

Reserve Price and Efficiency

- The reserve price causes an efficiency loss since, with chance 1/4, there is no trade.

Second-Price, Sealed-Bid Auction

- bids are private information
- bids are made simultaneously
- highest bidder wins
- winner pays second-highest bid
- also known as a Vickrey auction.

Second-Price, Sealed-Bid Auction

- No bidder knows any other bidder's true valuation of the item for sale.
- Yet, it is individually rational for each bidder to state truthfully his own valuation. Why?
- E.g. two bidders with true valuations v_1 and v_2.

Second-Price, Sealed-Bid Auction

- E.g. two bidders with true valuations v_1 and v_2.
- Bids are b_1 and b_2.
- Expected gain to bidder 1 is

$$(v_1 - b_2)\Pr(\mathbf{win}) + 0 \times \Pr(\mathbf{lose})$$
$$= (v_1 - b_2)\Pr(b_1 \geq b_2).$$

Second-Price, Sealed-Bid Auction

- Expected gain to bidder 1 is

$$(v_1 - b_2)\Pr(b_1 \geq b_2).$$

- How is this maximized?
- If $v_1 > b_2$, then maximize the prob. of winning; i.e. set $b_1 = v_1$.
- If $v_1 < b_2$, then minimize the prob. of winning; i.e. set $b_1 = v_1$.
- Either way, telling the truth is best!

Second-Price, Sealed-Bid Auction

- Since truth-telling is best for every bidder, the highest valuation bidder will win.
- Hence the second-price, sealed-bid auction is Pareto-efficient.

Common-Value Auctions

- The item for sale has the same value to every potential buyer.
- Potential buyers differ in their own estimates of this common value.
- Bidder i's estimate is $v_i = v + \varepsilon_i$ where v is the common value and ε_i is bidder i's estimation error.

Common-Value Auctions

- Bidder i's estimate is $v_i = v + \varepsilon_i$ where v is the common value and ε_i is bidder i's estimation error.
- If every bid is truthful, the winner is the bidder with the largest estimation error ε_i
- so a truthful winner on average pays more than the true value v -- the winner's curse.

Common-Value Auctions

- If every bid is truthful, the winner is the bidder with the largest estimation error ε_i -- call it $\varepsilon_i^{\max}$
- If $\varepsilon_i^{\max} > 0$ then a truthful winner on average pays more than the true value v -- the winner's curse.
- So bids should on average be less than v in a common-value auction.