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Oligopoly

Oligopoly

- A monopoly is an industry consisting a single firm.
- A duopoly is an industry consisting of two firms.
- An oligopoly is an industry consisting of a few firms. Particularly, each firm's own price or output decisions affect its competitors' profits.

Oligopoly

- How do we analyze markets in which the supplying industry is oligopolistic?
- Consider the duopolistic case of two firms supplying the same product.

Quantity Competition

- Assume that firms compete by choosing output levels.
- If firm 1 produces y_1 units and firm 2 produces y_2 units then total quantity supplied is $y_1 + y_2$. The market price will be $p(y_1 + y_2)$.
- The firms' total cost functions are $c_1(y_1)$ and $c_2(y_2)$.

Quantity Competition

- Suppose firm 1 takes firm 2's output level choice y_2 as given. Then firm 1 sees its profit function as

$$\Pi_1(y_1; y_2) = p(y_1 + y_2)y_1 - c_1(y_1).$$

- Given y_2, what output level y_1 maximizes firm 1's profit?

Quantity Competition; An Example

- Suppose that the market inverse demand function is

$$p(y_T) = 60 - y_T$$

and that the firms' total cost functions are

$$c_1(y_1) = y_1^2 \quad \textbf{and} \quad c_2(y_2) = 15y_2 + y_2^2.$$

Quantity Competition; An Example

Then, for given y_2, firm 1's profit function is

$$\Pi(y_1;y_2) = (60 - y_1 - y_2)y_1 - y_1^2.$$

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$$\frac{\partial \Pi}{\partial y_1} = 60 - 2y_1 - y_2 - 2y_1 = 0.$$

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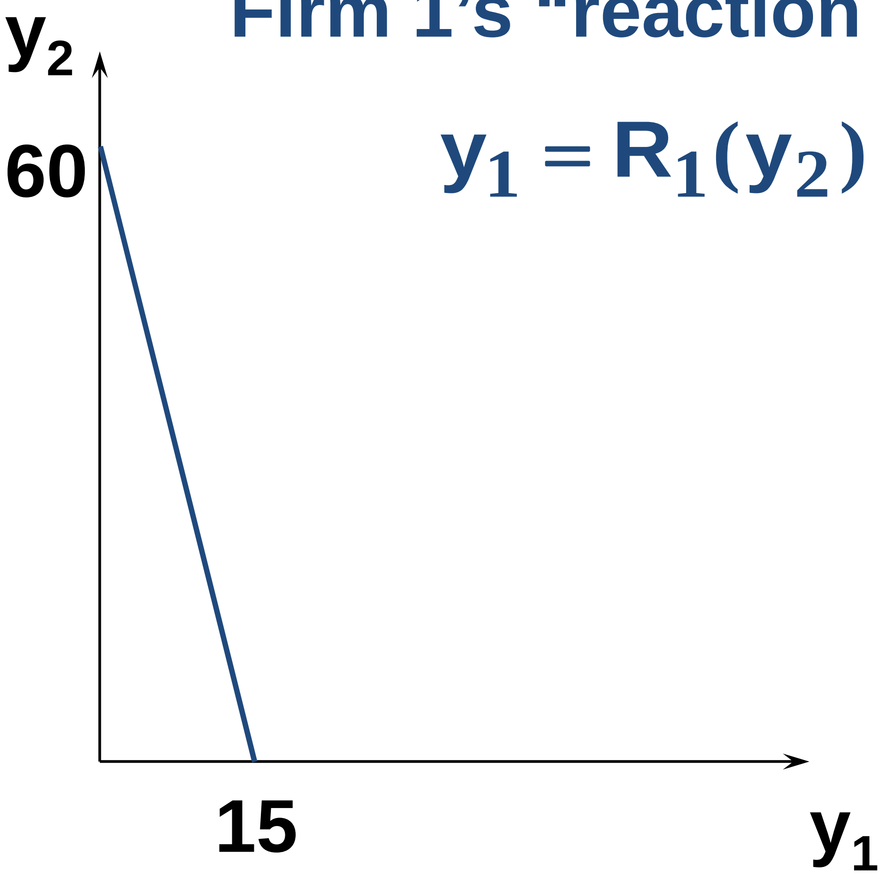
***I.e.*, firm 1's best response to y_2 is**

$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2.$$

Quantity Competition; An Example



Firm 1's "reaction curve"

$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2.$$

Quantity Competition; An Example

Similarly, given y_1, firm 2's profit function is

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$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

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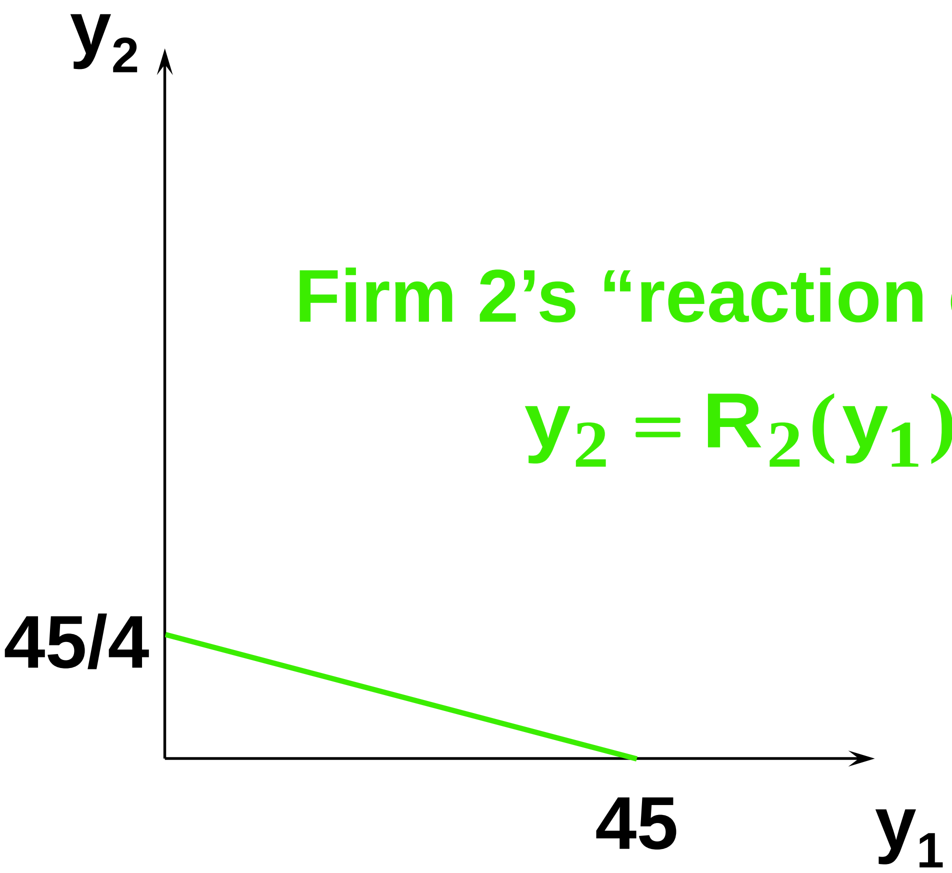

Quantity Competition; An Example

- An equilibrium is when each firm's output level is a best response to the other firm's output level, for then neither wants to deviate from its output level.
- A pair of output levels (y_1*,y_2*) is a Cournot-Nash equilibrium if

$$y_1^* = R_1(y_2^*) \quad \textbf{and} \quad \mathbf{y_2^* = R_2(y_1^*)}.$$

Quantity Competition; An Example

$$y_1^* = R_1(y_2^*) = 15 - \frac{1}{4}y_2^*$$ **and** $$y_2^* = R_2(y_1^*) = \frac{45 - y_1^*}{4}.$$

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Substitute for y_2* to get

$$y_1^* = 15 - \frac{1}{4}\left(\frac{45 - y_1^*}{4}\right)$$

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Hence $$y_2^* = \frac{45 - 13}{4} = 8.$$

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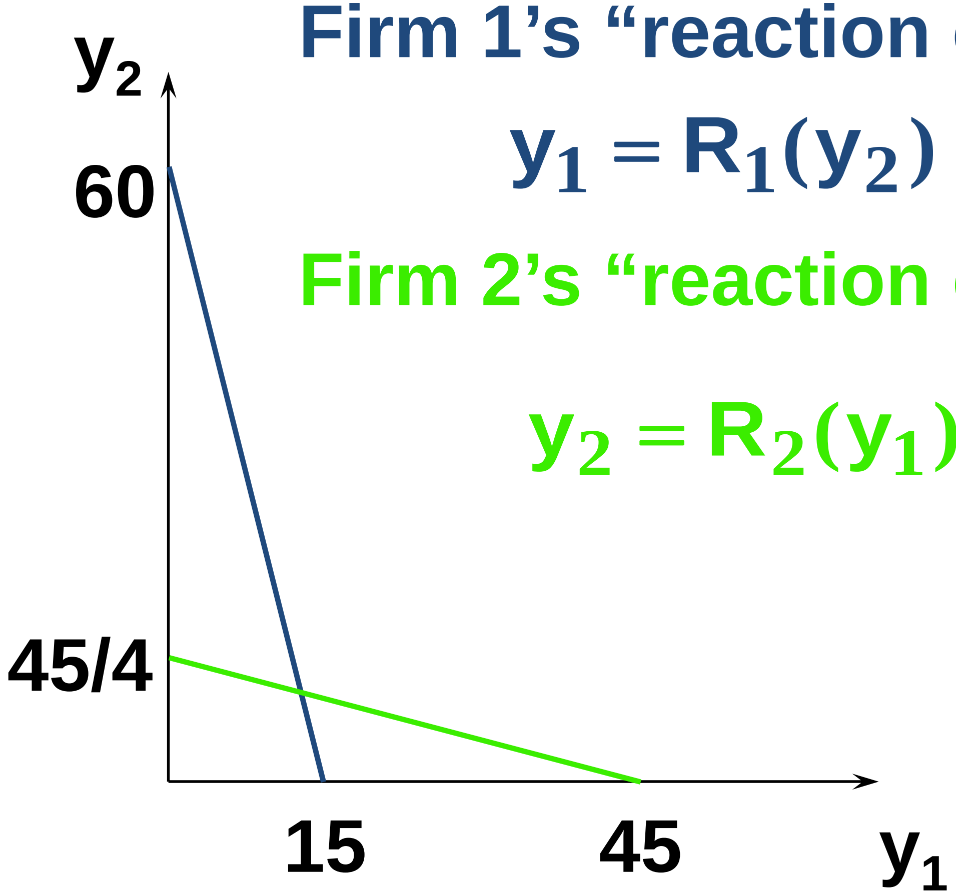
So the Cournot-Nash equilibrium is

$$(y_1^*, y_2^*) = (13, 8).$$

Quantity Competition; An Example



Firm 1's "reaction curve"

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Firm 2's "reaction curve"

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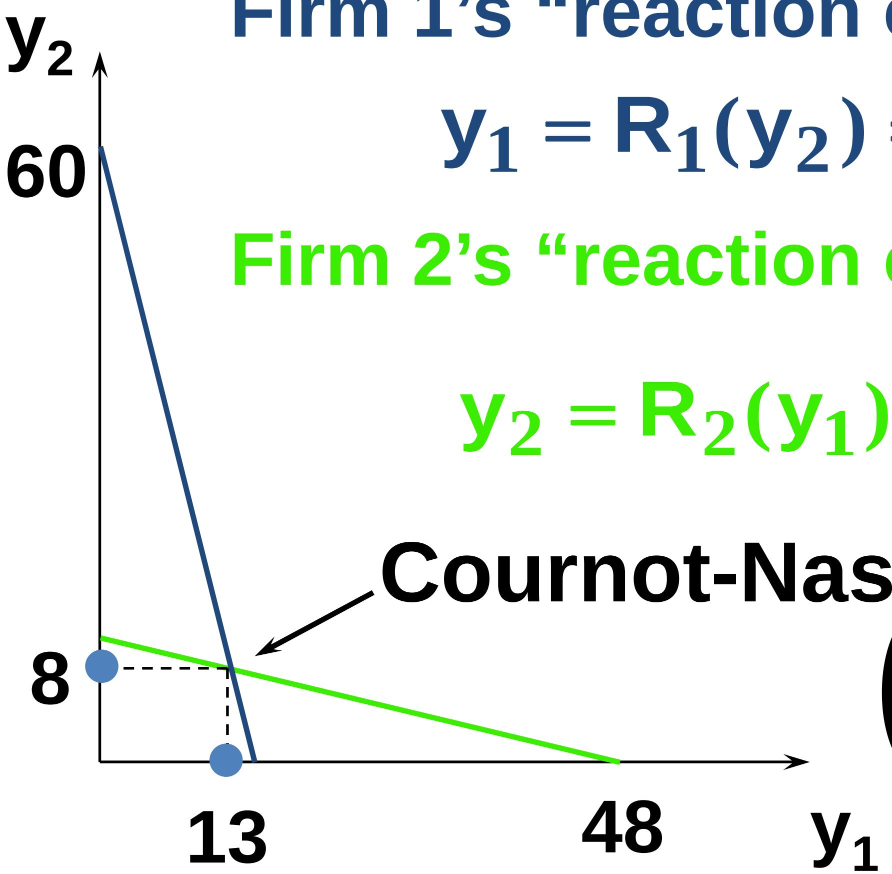

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Cournot-Nash equilibrium

$$\left(y_1^*, y_2^*\right) = (13, 8).$$

Quantity Competition

Generally, given firm 2's chosen output level y_2, firm 1's profit function is

$$\Pi_1(y_1;y_2) = p(y_1+y_2)y_1 - c_1(y_1)$$

and the profit-maximizing value of y_1 solves

$$\frac{\partial \Pi_1}{\partial y_1} = p(y_1+y_2) + y_1\frac{\partial p(y_1+y_2)}{\partial y_1} - c_1{}'(y_1) = 0.$$

The solution, $y_1 = R_1(y_2)$, is firm 1's Cournot-Nash reaction to y_2.

Quantity Competition

Similarly, given firm 1's chosen output level y_1, firm 2's profit function is

$$\Pi_2(y_2; y_1) = p(y_1 + y_2)y_2 - c_2(y_2)$$

and the profit-maximizing value of y_2 solves

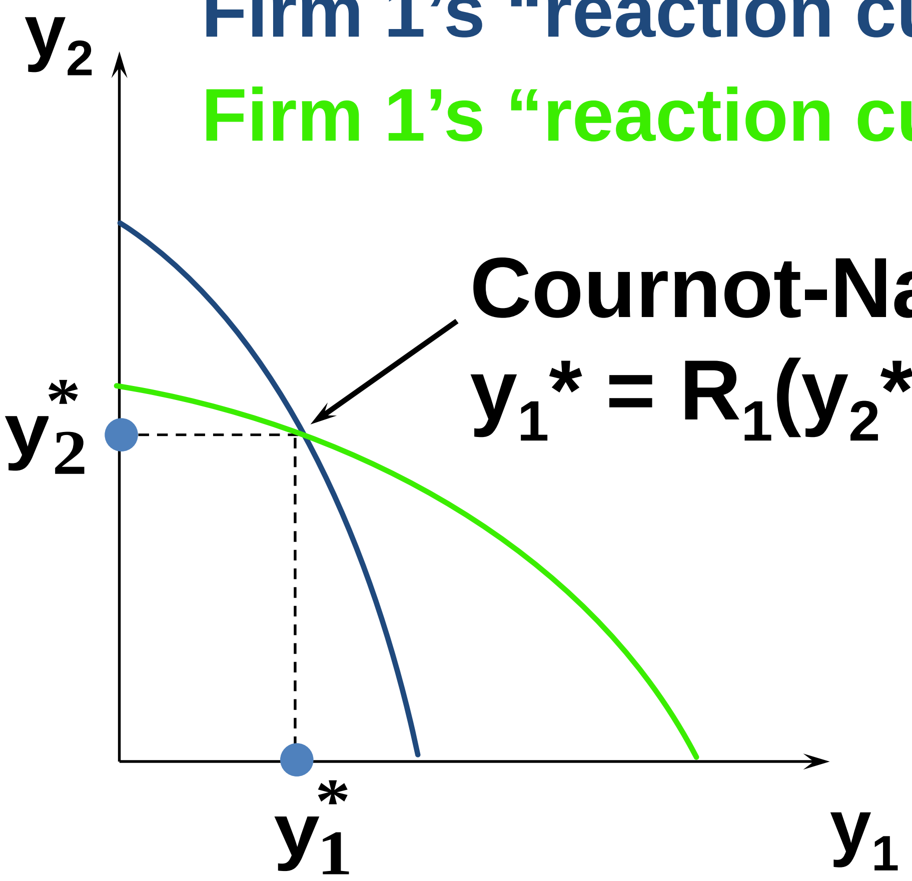
$$\frac{\partial \Pi_2}{\partial y_2} = p(y_1 + y_2) + y_2 \frac{\partial p(y_1 + y_2)}{\partial y_2} - c_2{}'(y_2) = 0.$$

The solution, $y_2 = R_2(y_1)$, is firm 2's Cournot-Nash reaction to y_1.

Quantity Competition

Firm 1's "reaction curve" $y_1 = R_1(y_2)$.

Firm 1's "reaction curve" $y_2 = R_2(y_1)$.



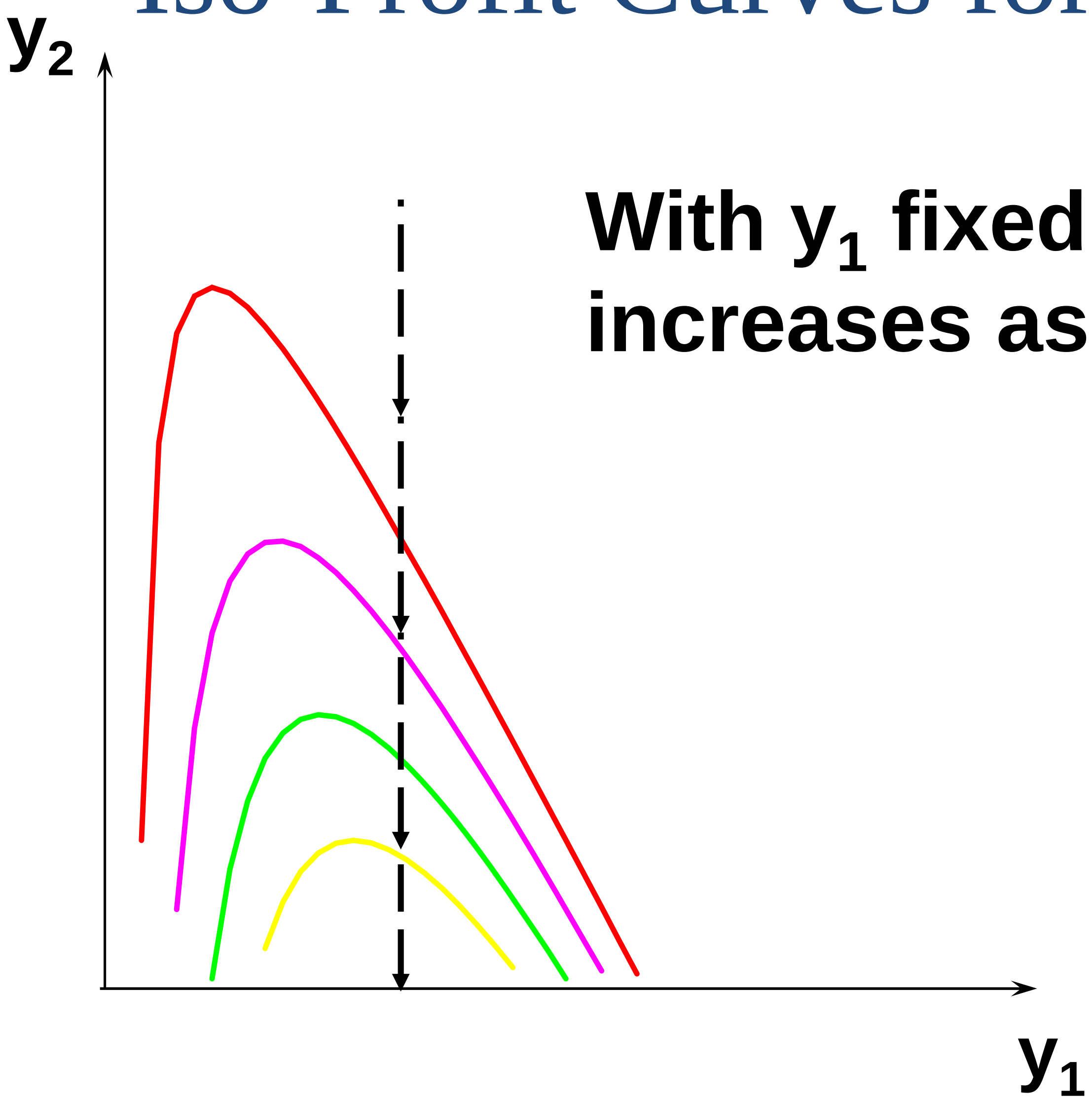
Cournot-Nash equilibrium

$y_1^* = R_1(y_2^*)$ and $y_2^* = R_2(y_1^*)$

Iso-Profit Curves

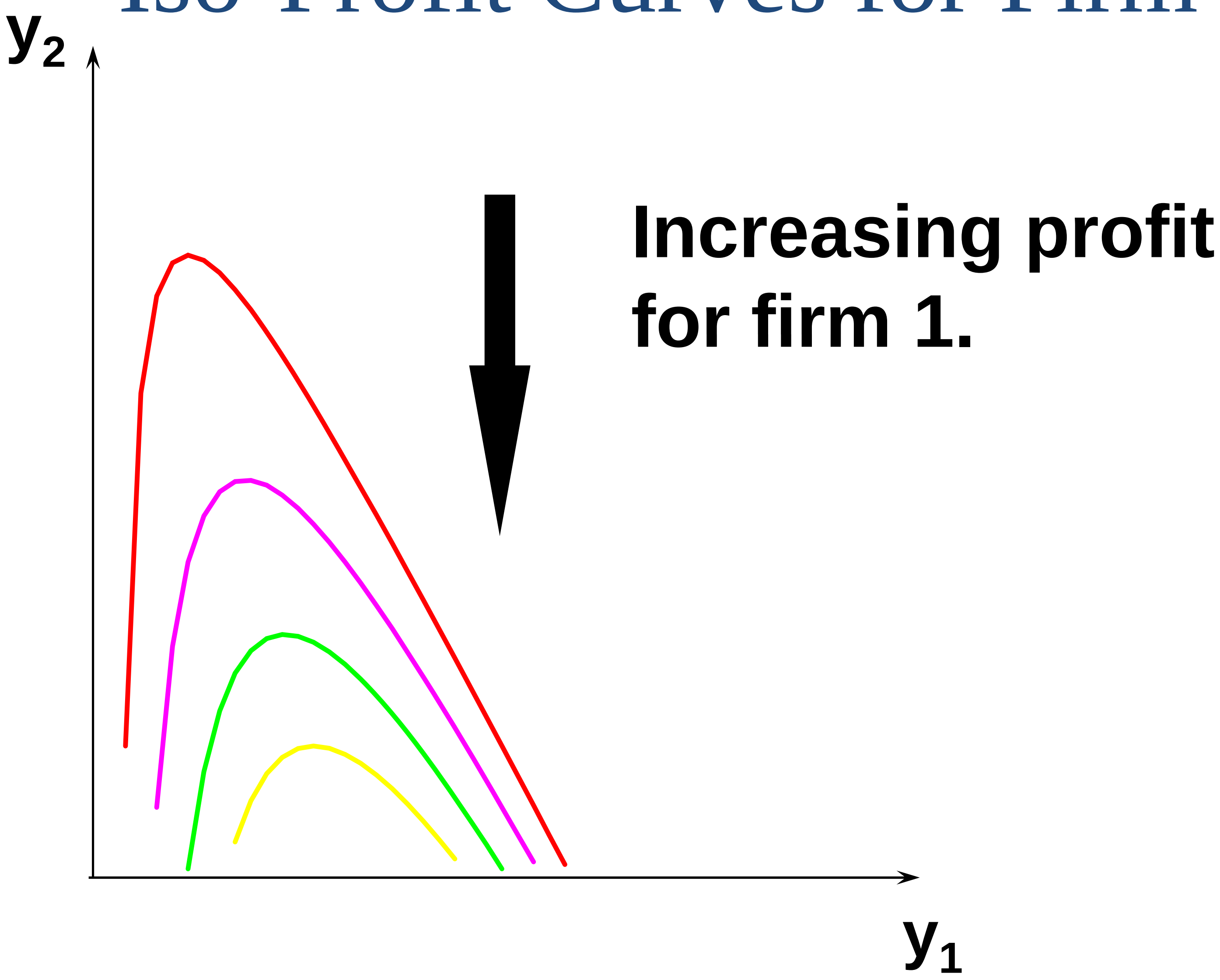
- For firm 1, an iso-profit curve contains all the output pairs (y_1,y_2) giving firm 1 the same profit level Π_1.
- What do iso-profit curves look like?

Iso-Profit Curves for Firm 1



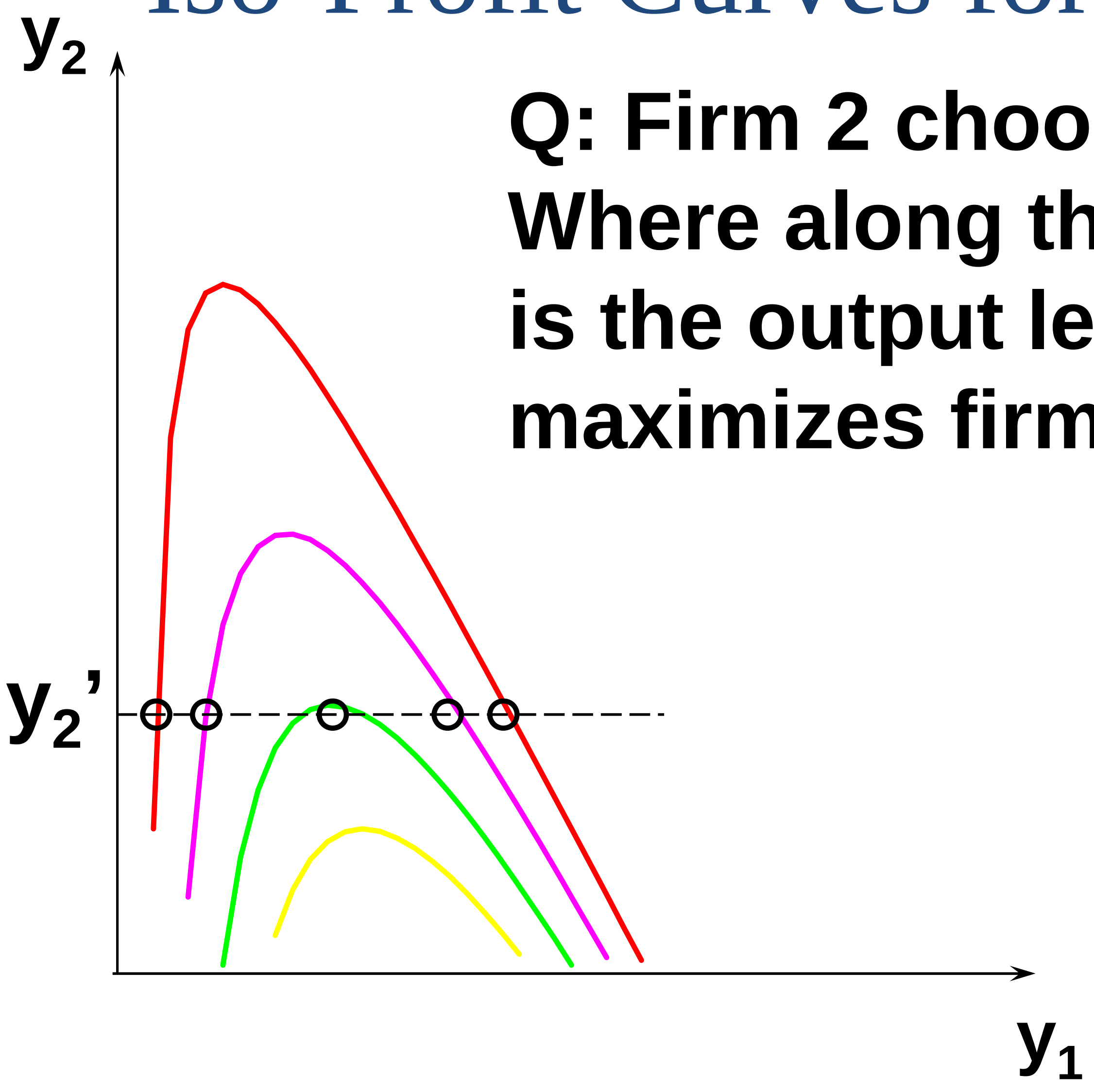
With y_1 fixed, firm 1's profit increases as y_2 decreases.

Iso-Profit Curves for Firm 1



Iso-Profit Curves for Firm 1

Q: Firm 2 chooses $y_2 = y_2'$. Where along the line $y_2 = y_2'$ is the output level that maximizes firm 1's profit?



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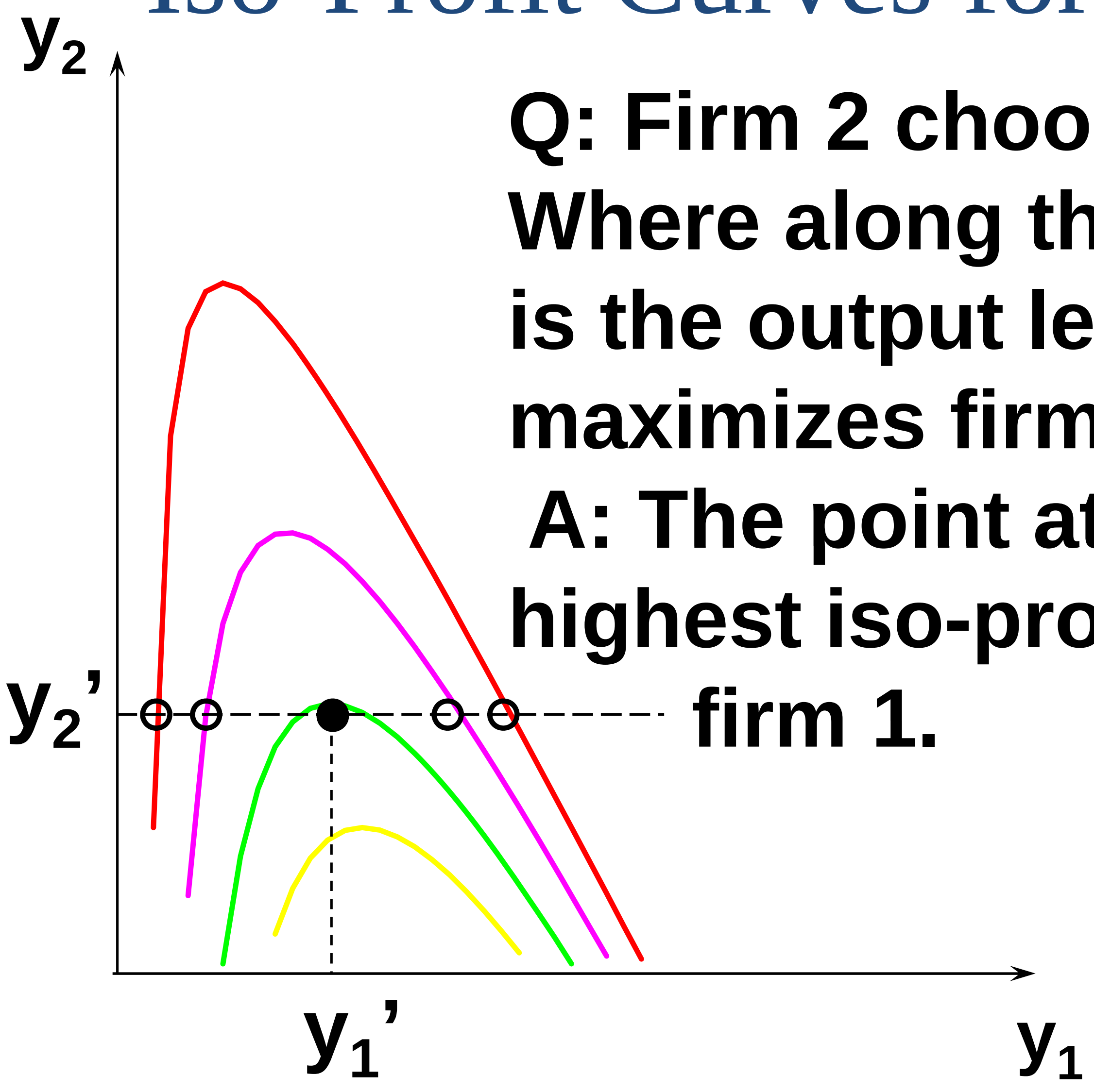

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A: The point attaining the highest iso-profit curve for firm 1.

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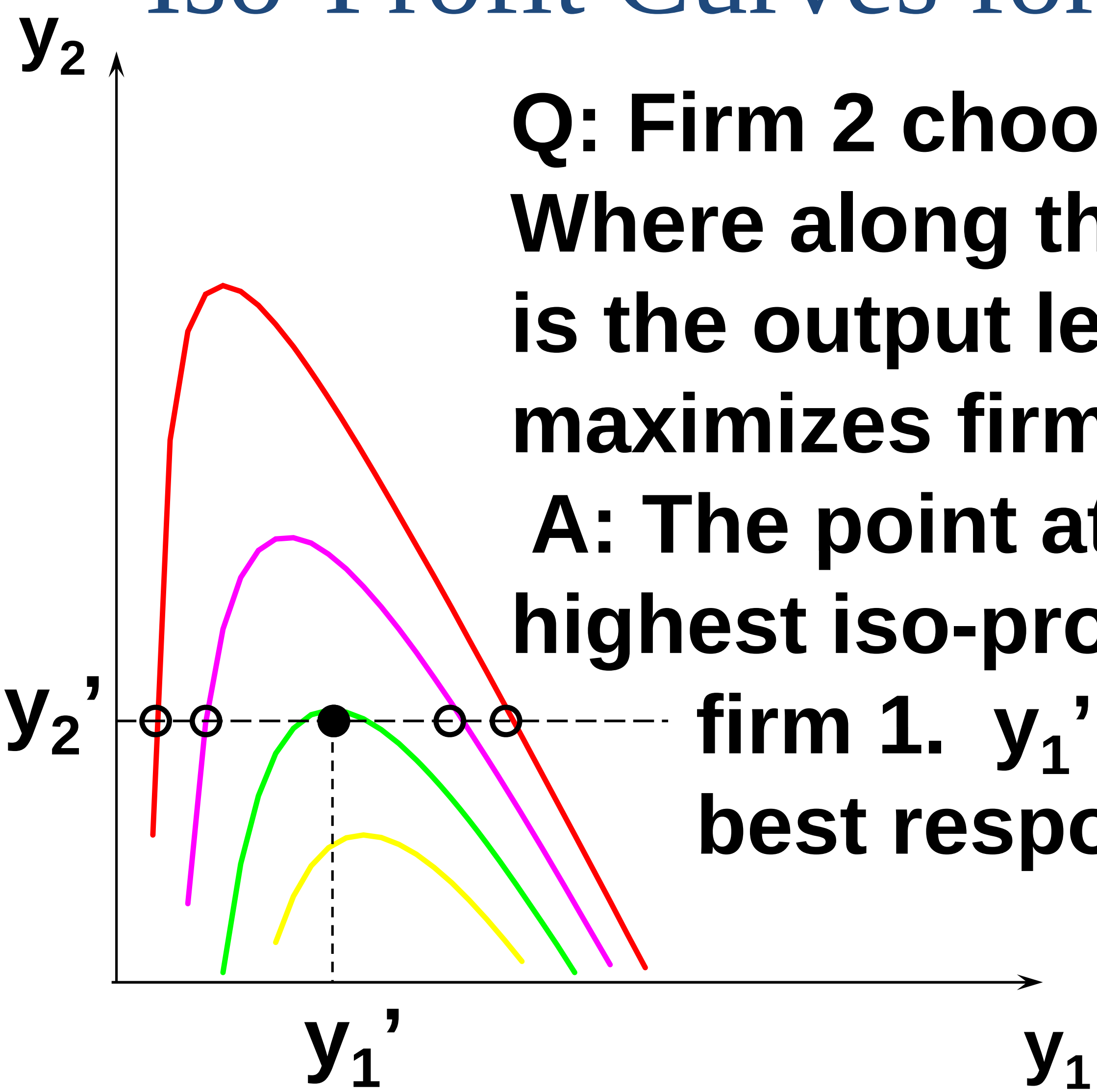

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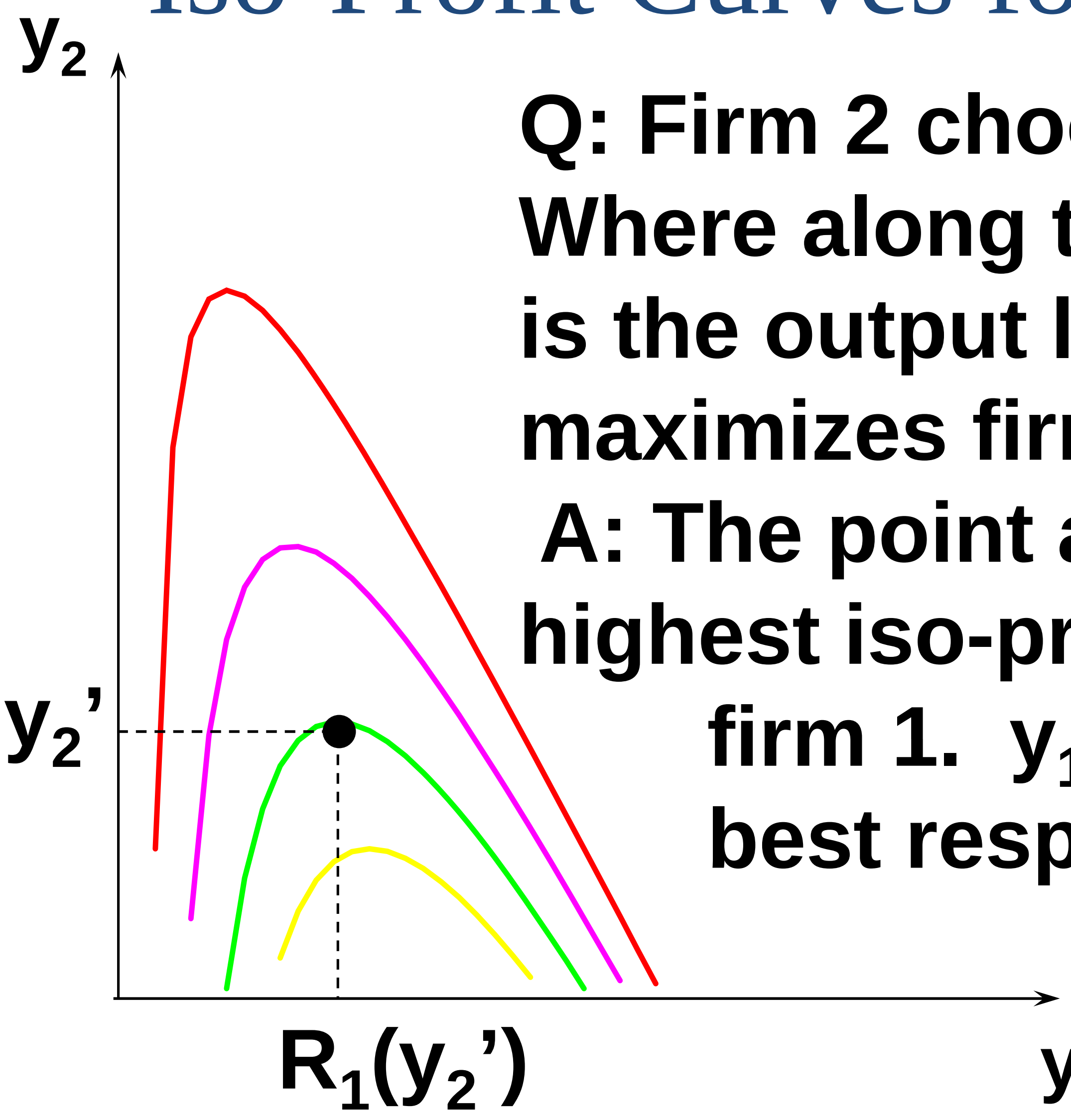

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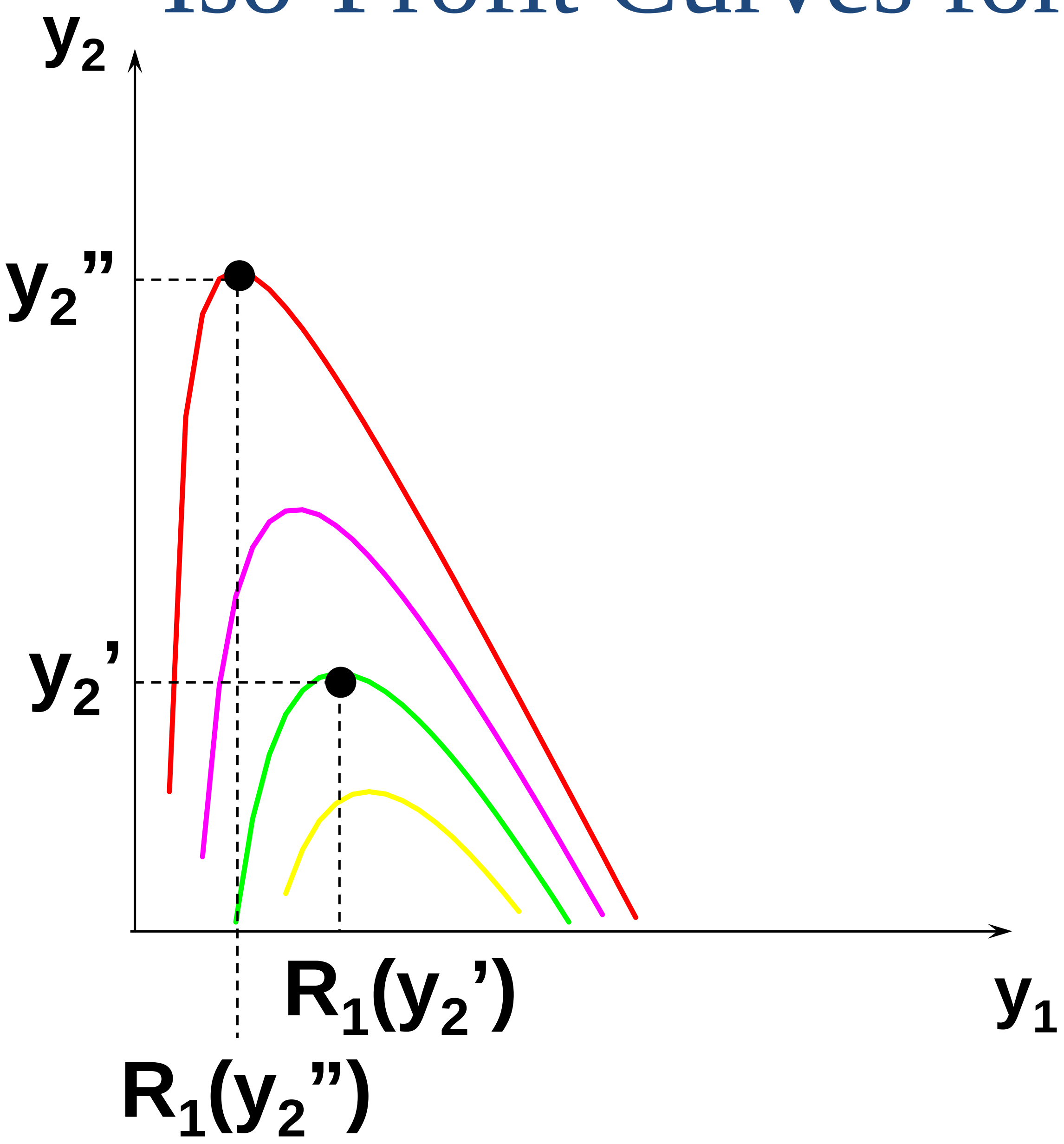

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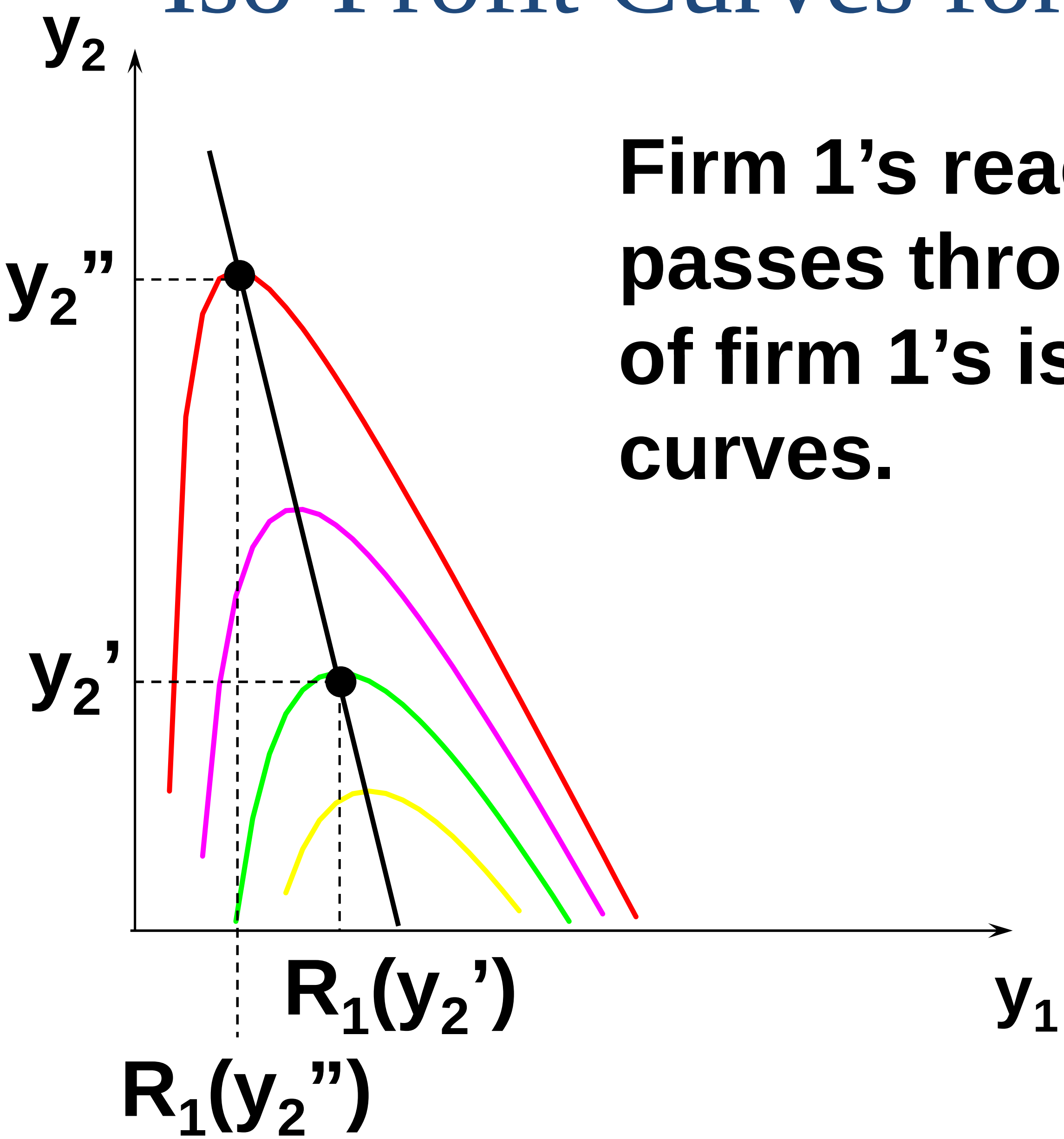
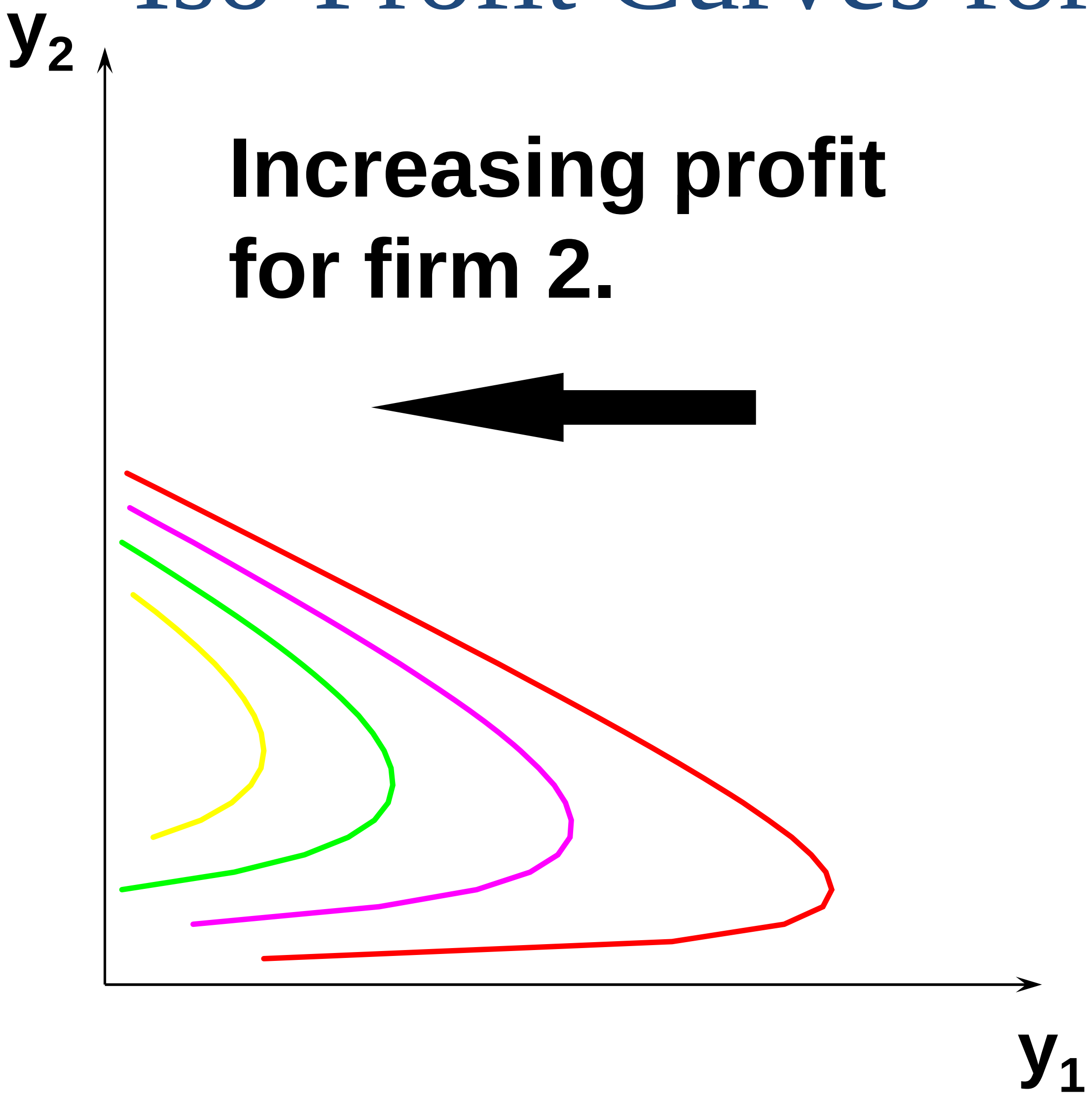


Firm 1's reaction curve passes through the "tops" of firm 1's iso-profit curves.

Iso-Profit Curves for Firm 2



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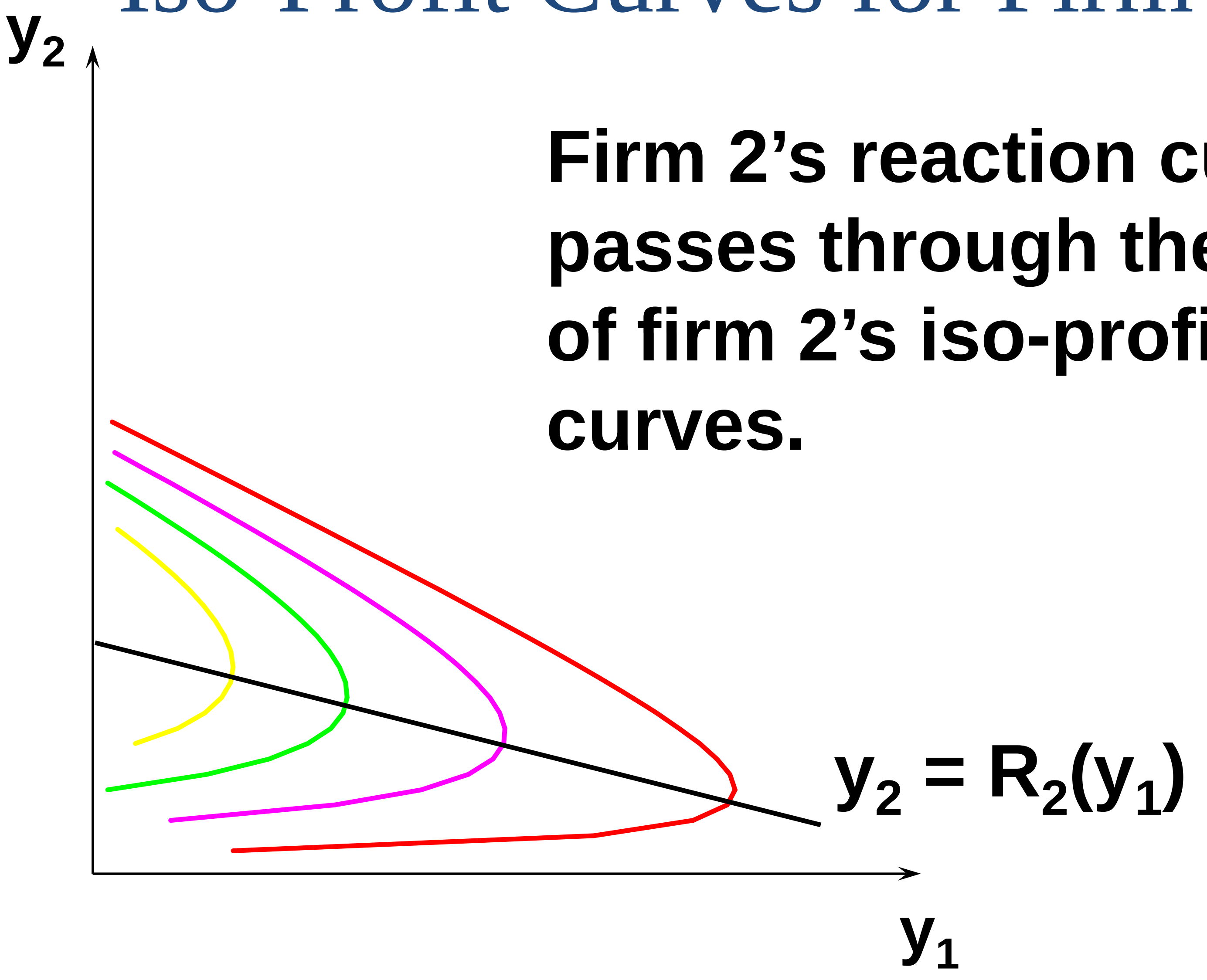


Firm 2's reaction curve passes through the "tops" of firm 2's iso-profit curves.

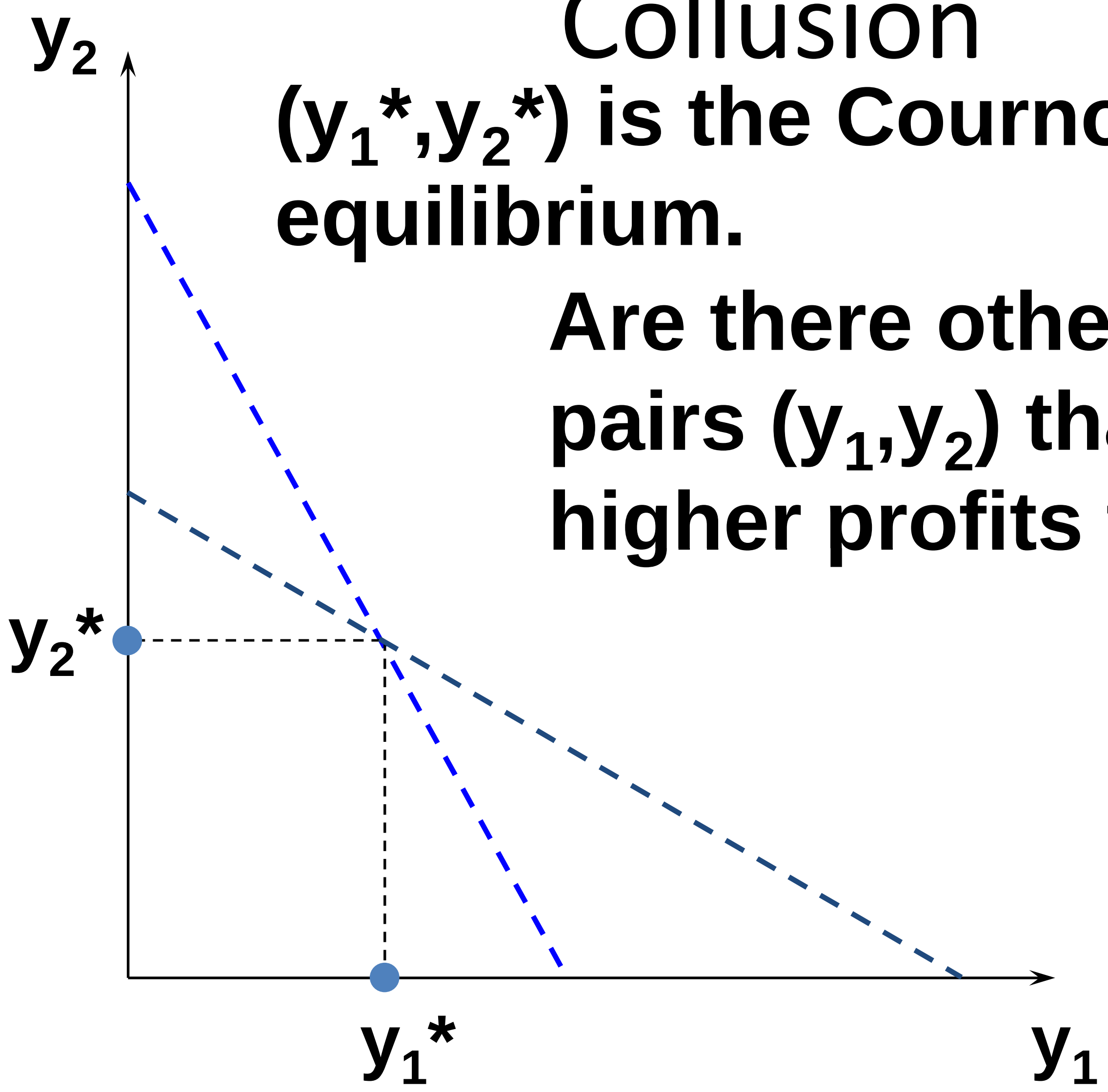
Collusion

- Q: Are the Cournot-Nash equilibrium profits the largest that the firms can earn in total?

Collusion

(y_1^*, y_2^*) is the Cournot-Nash equilibrium.

Are there other output level pairs (y_1, y_2) that give higher profits to both firms?



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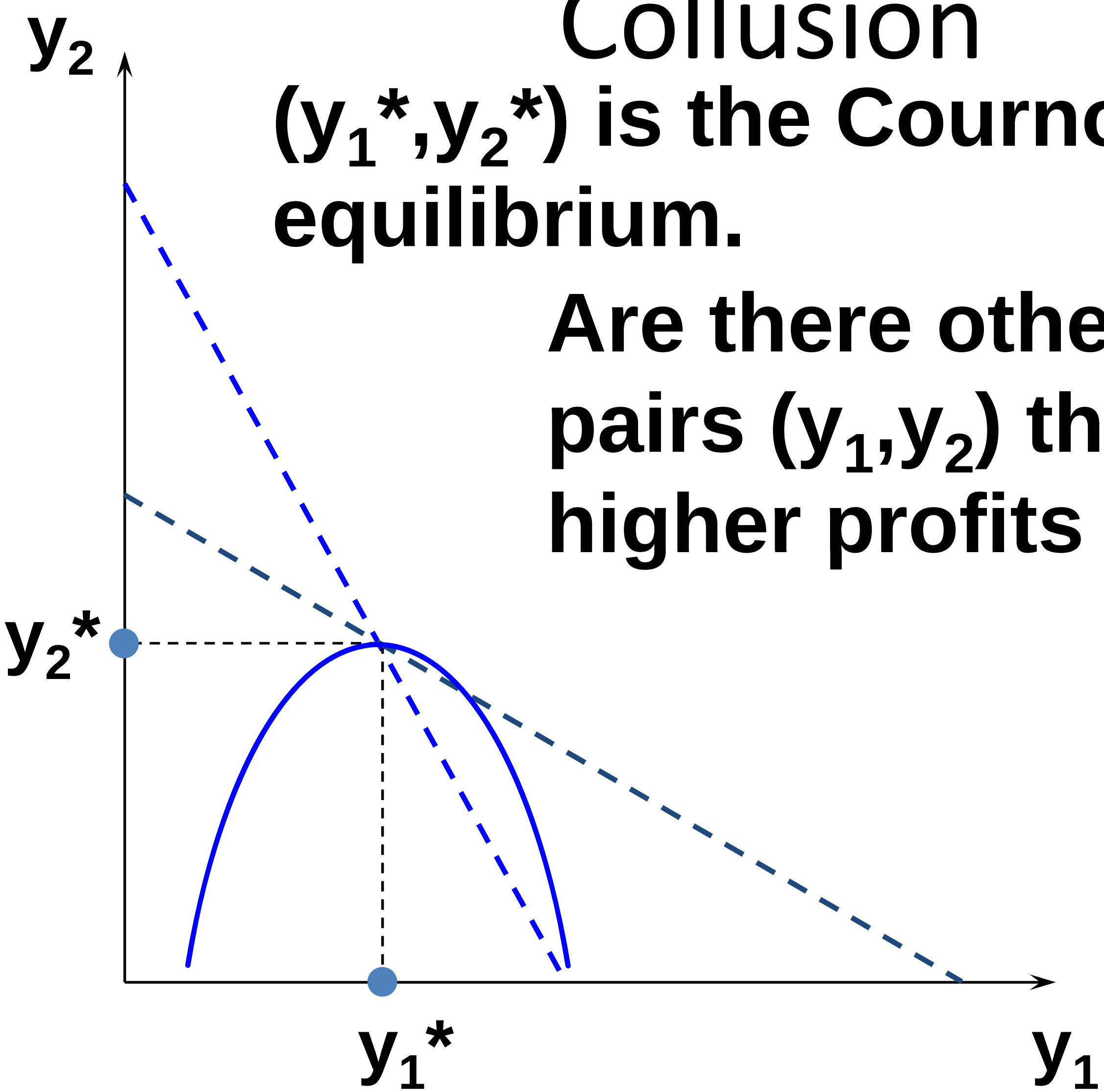

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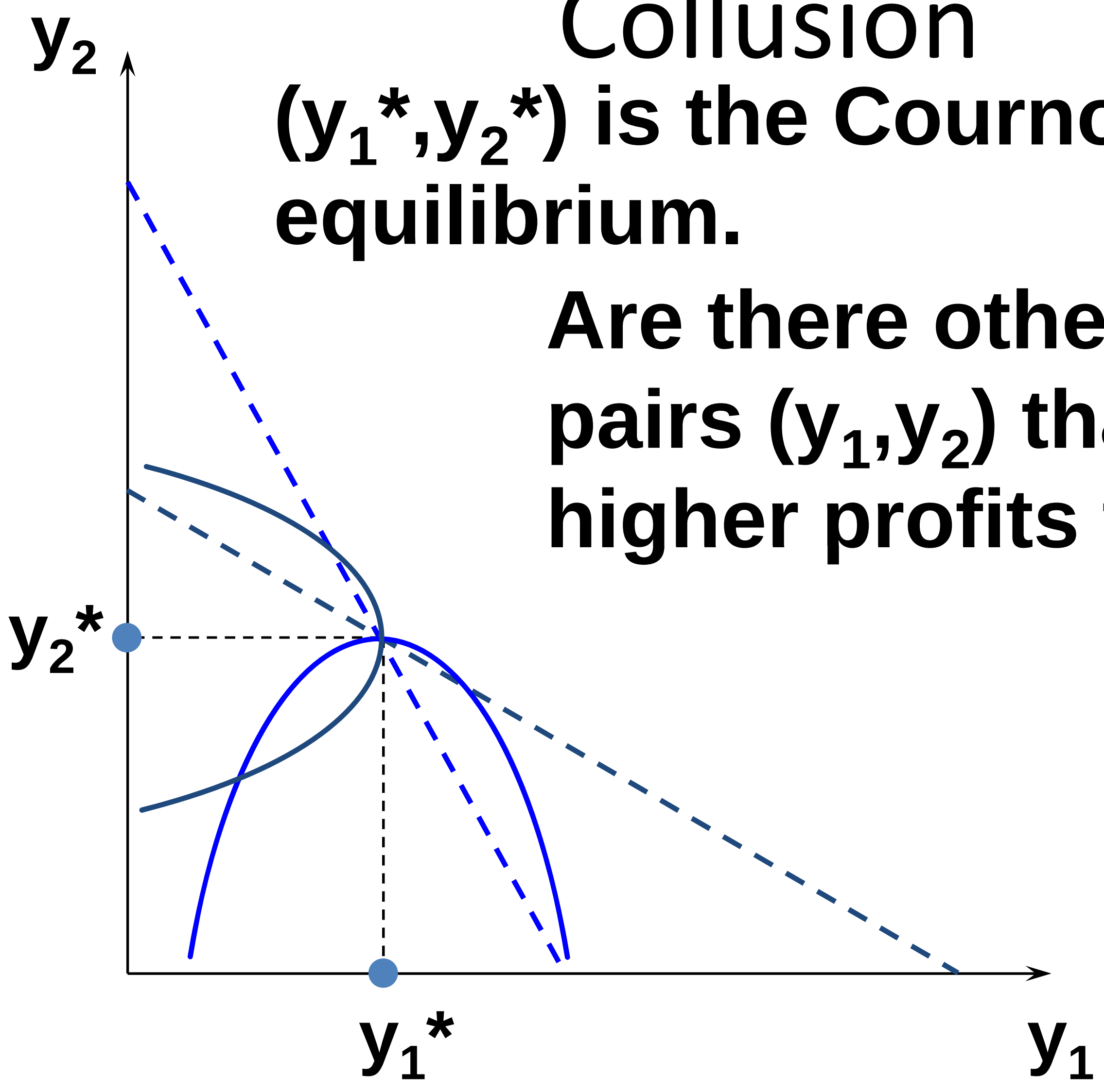

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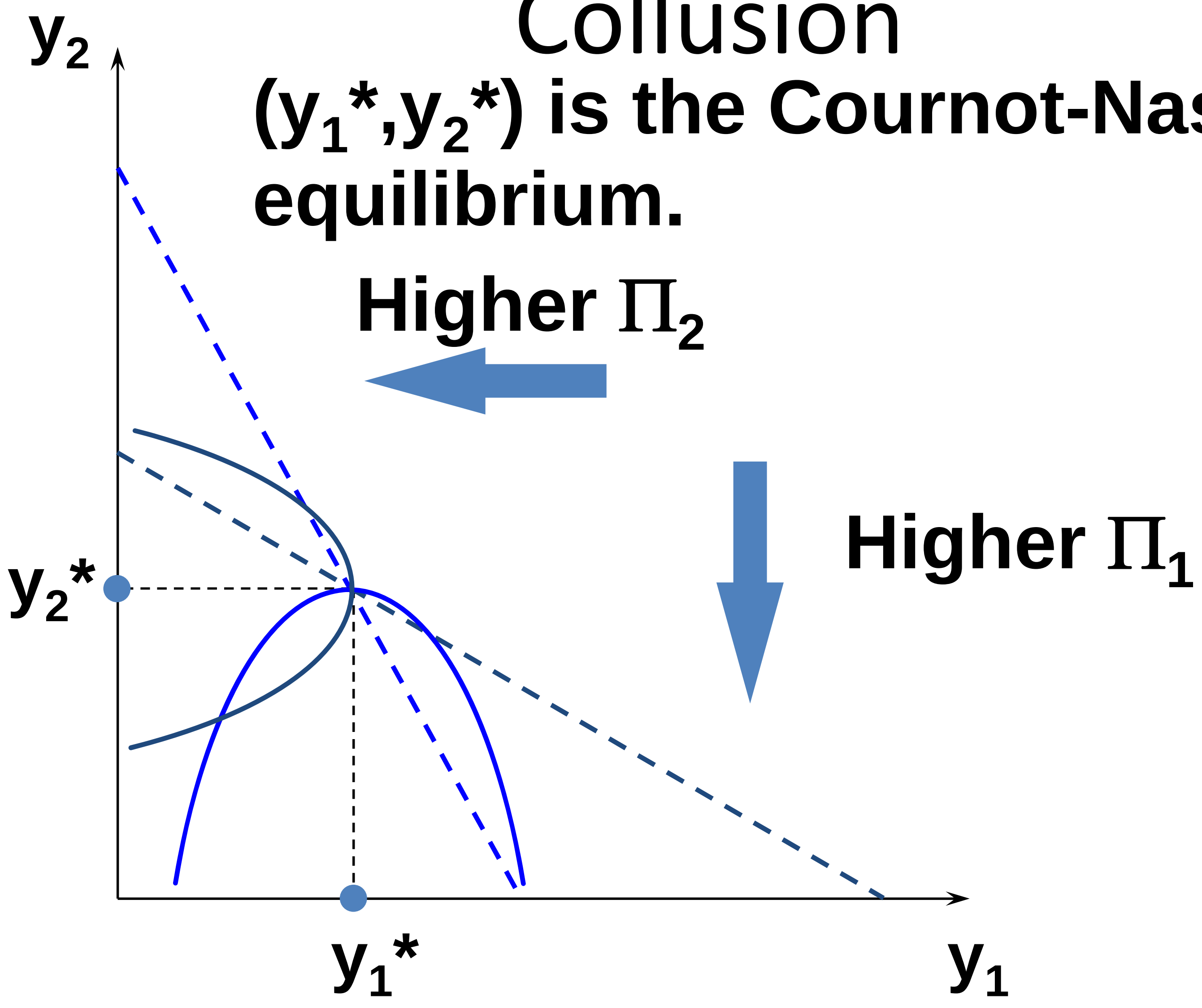

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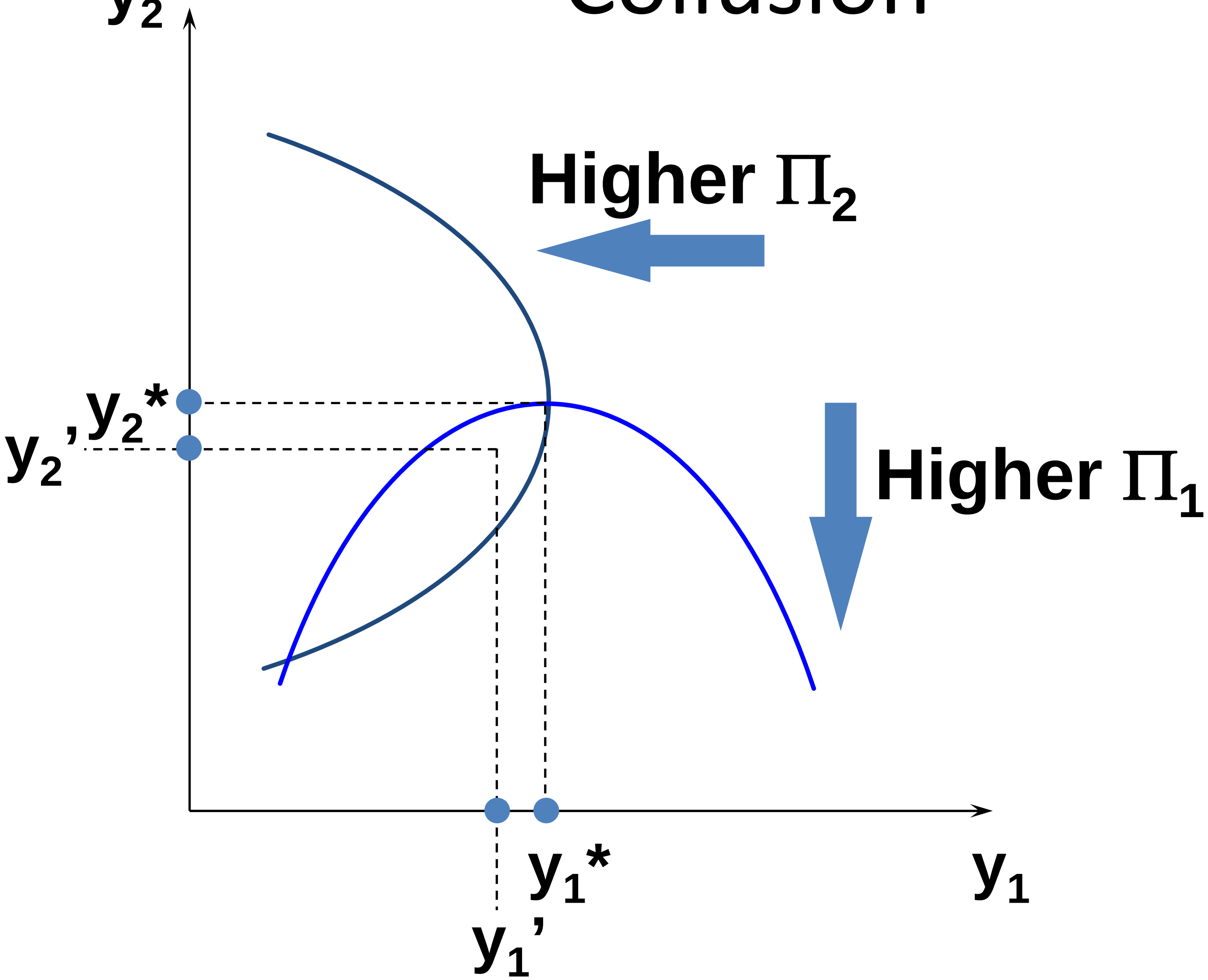

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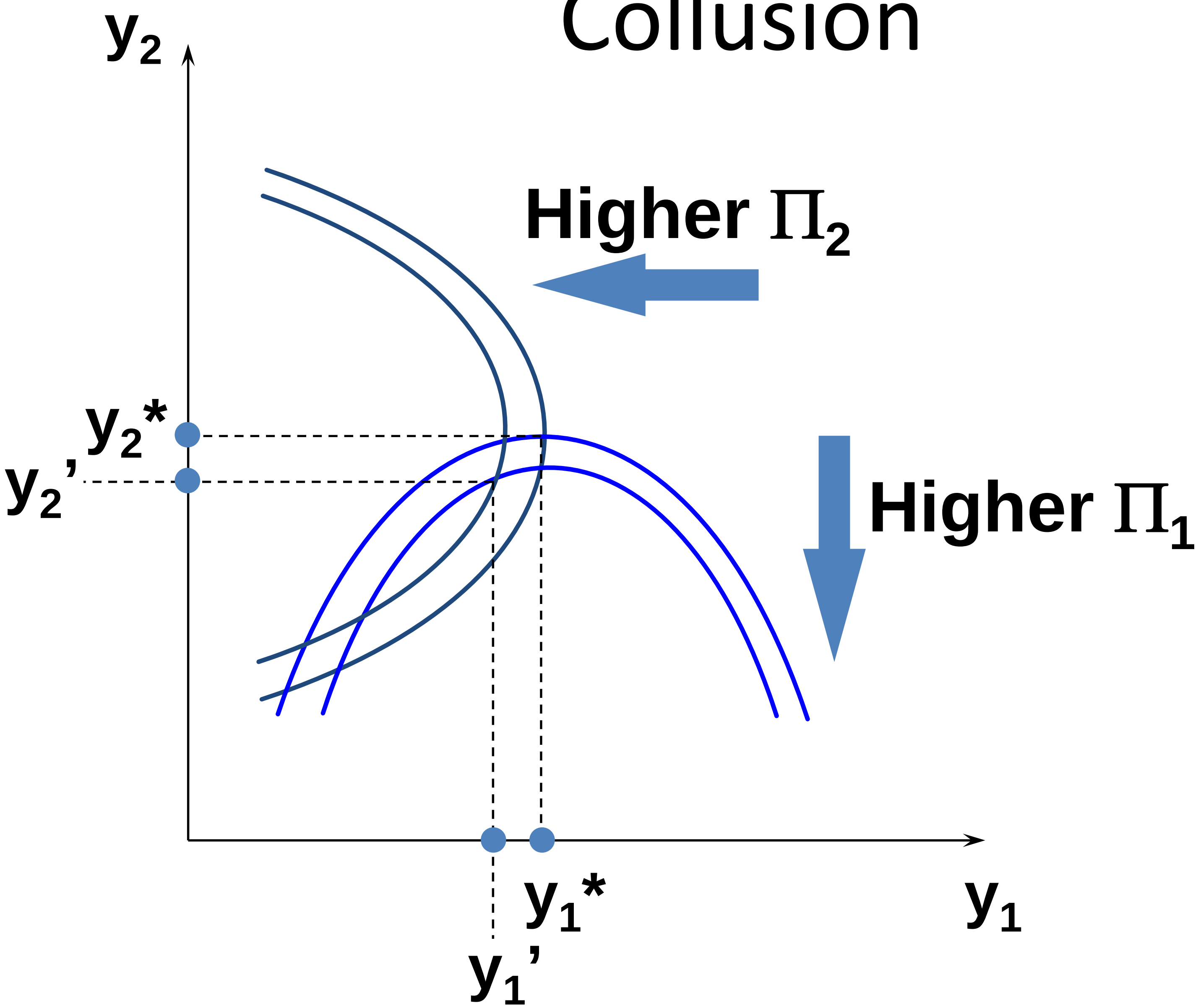

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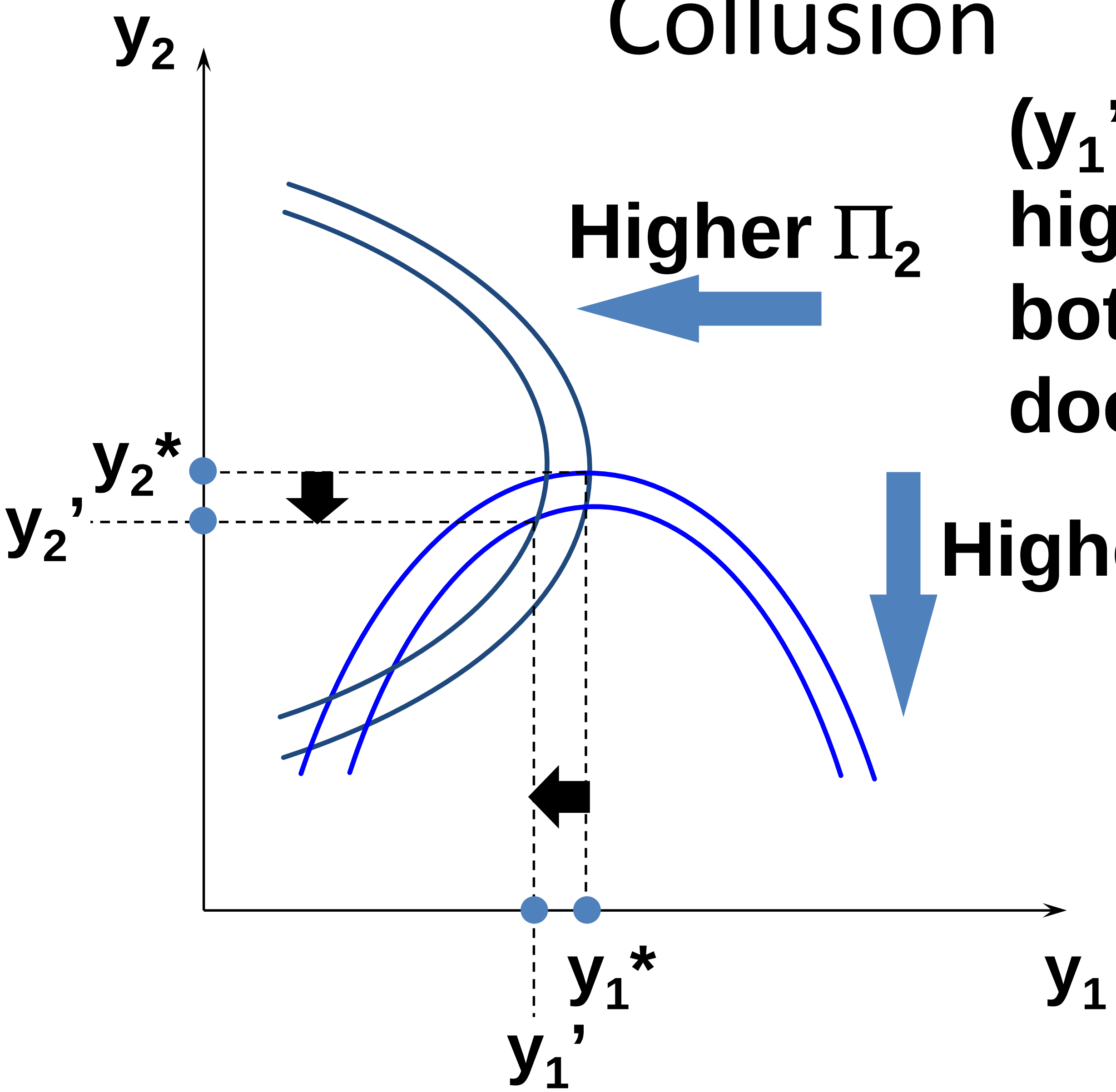


(y_1', y_2') earns higher profits for both firms than does (y_1^*, y_2^*).

Collusion

- So there are profit incentives for both firms to "cooperate" by lowering their output levels.
- This is collusion.
- Firms that collude are said to have formed a cartel.
- If firms form a cartel, how should they do it?

Collusion

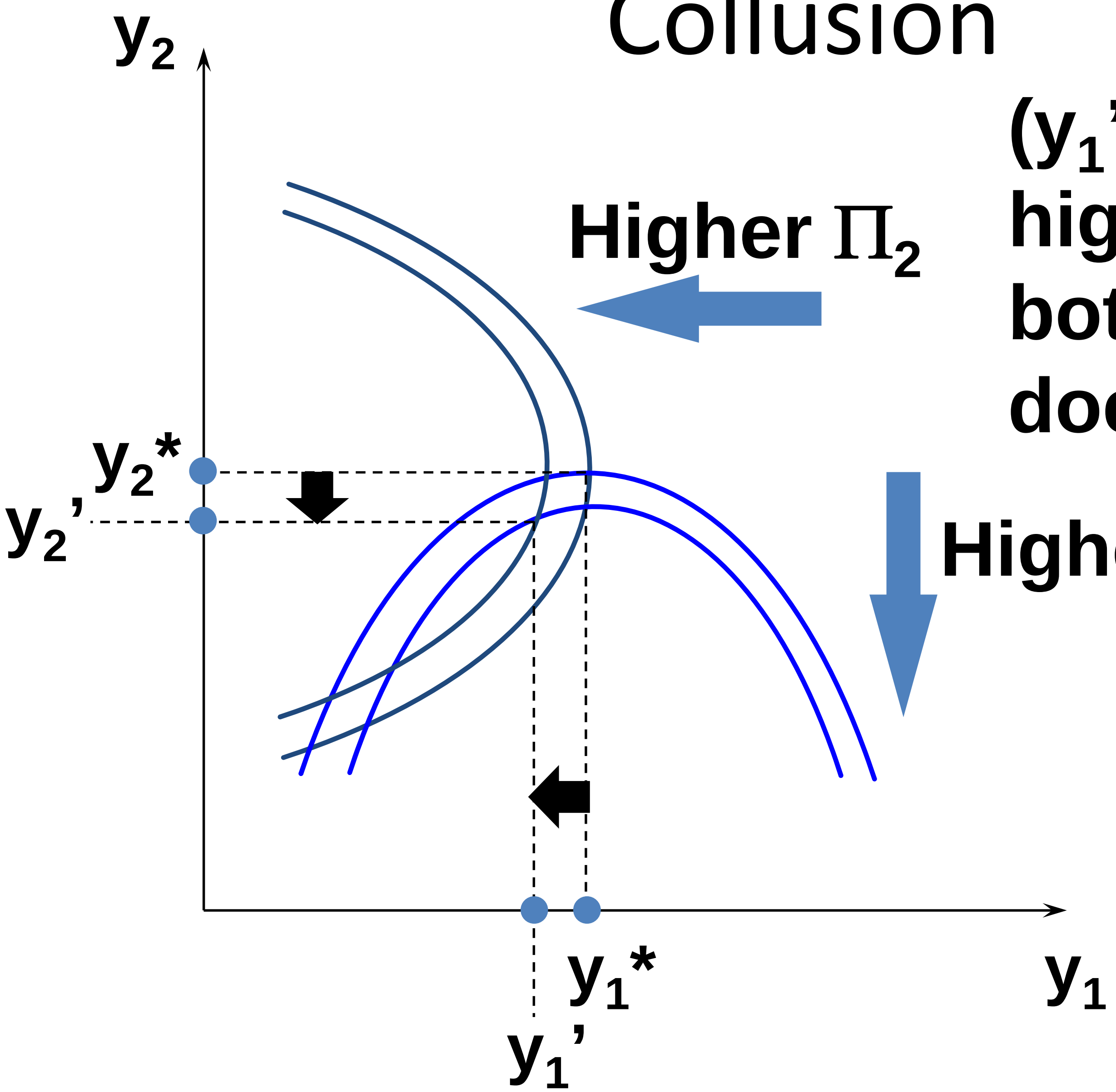
- Suppose the two firms want to maximize their total profit and divide it between them. Their goal is to choose cooperatively output levels y_1 and y_2 that maximize

$$\Pi^m(y_1, y_2) = p(y_1 + y_2)(y_1 + y_2) - c_1(y_1) - c_2(y_2).$$

Collusion

- The firms cannot do worse by colluding since they can cooperatively choose their Cournot-Nash equilibrium output levels and so earn their Cournot-Nash equilibrium profits. So collusion must provide profits at least as large as their Cournot-Nash equilibrium profits.

Collusion



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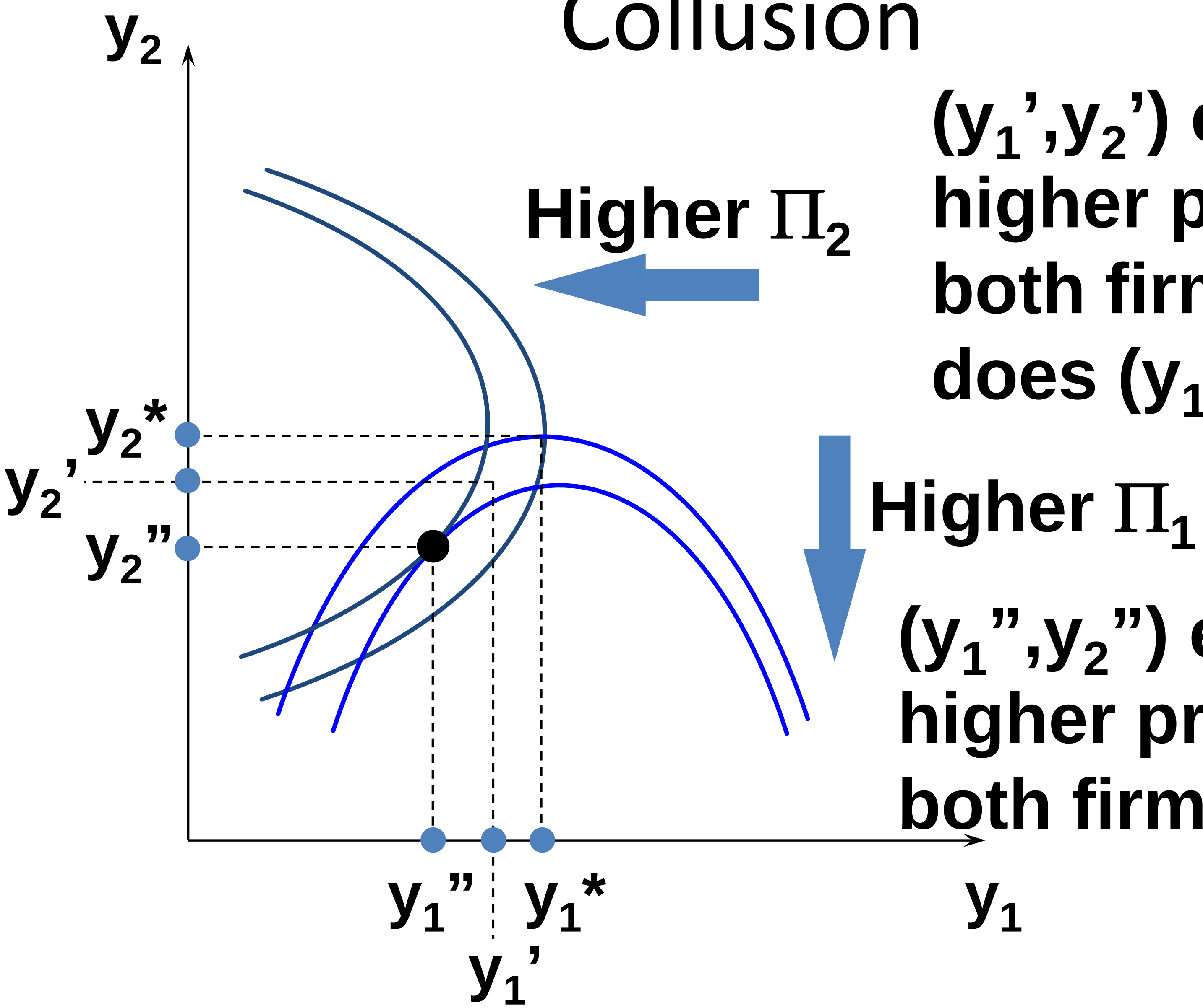
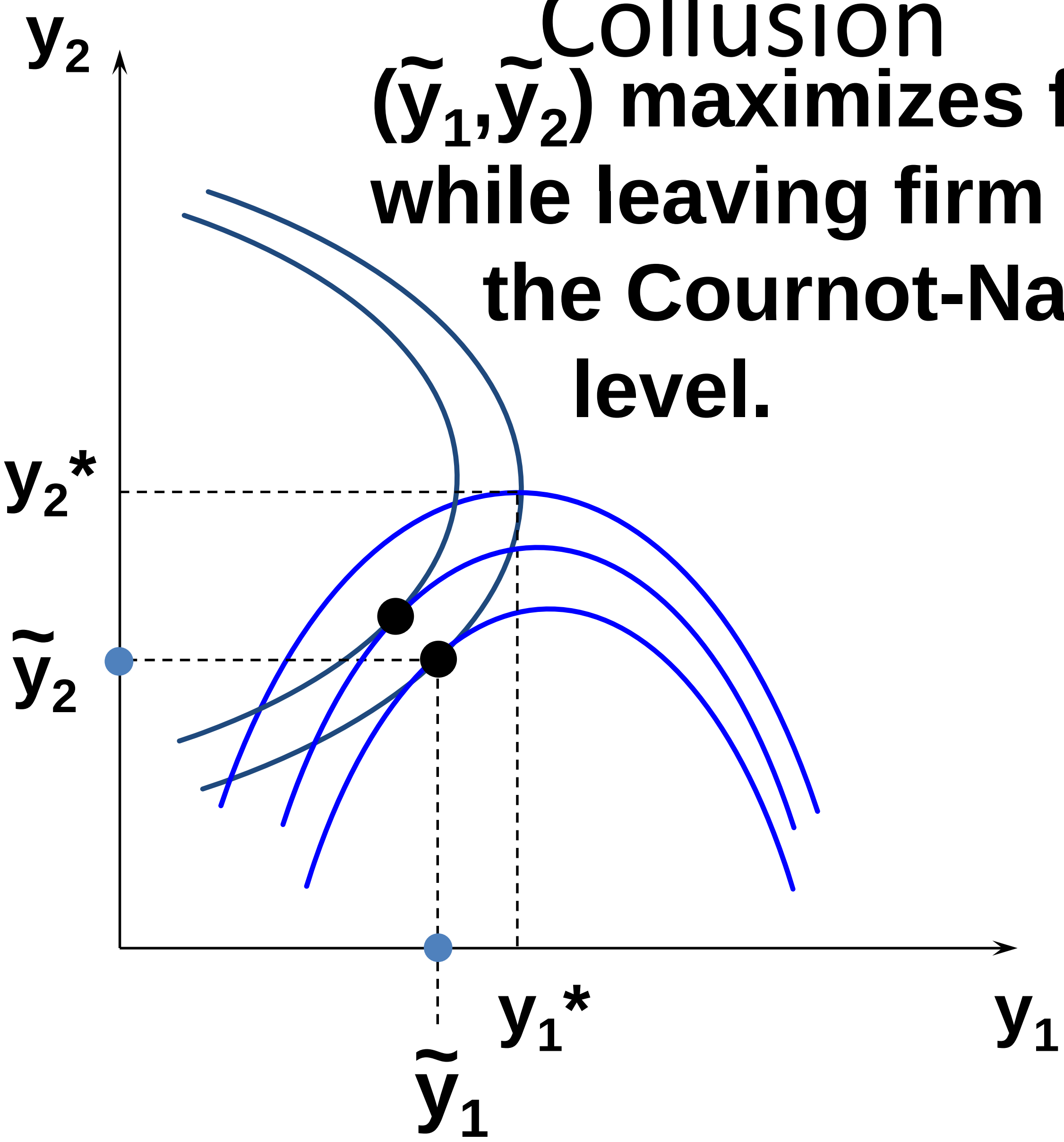


(y_1', y_2') earns higher profits for both firms than does (y_1^*, y_2^*).

(y_1'', y_2'') earns still higher profits for both firms.

Collusion

$(\tilde{y}_1,\tilde{y}_2)$ maximizes firm 1's profit while leaving firm 2's profit at the Cournot-Nash equilibrium level.



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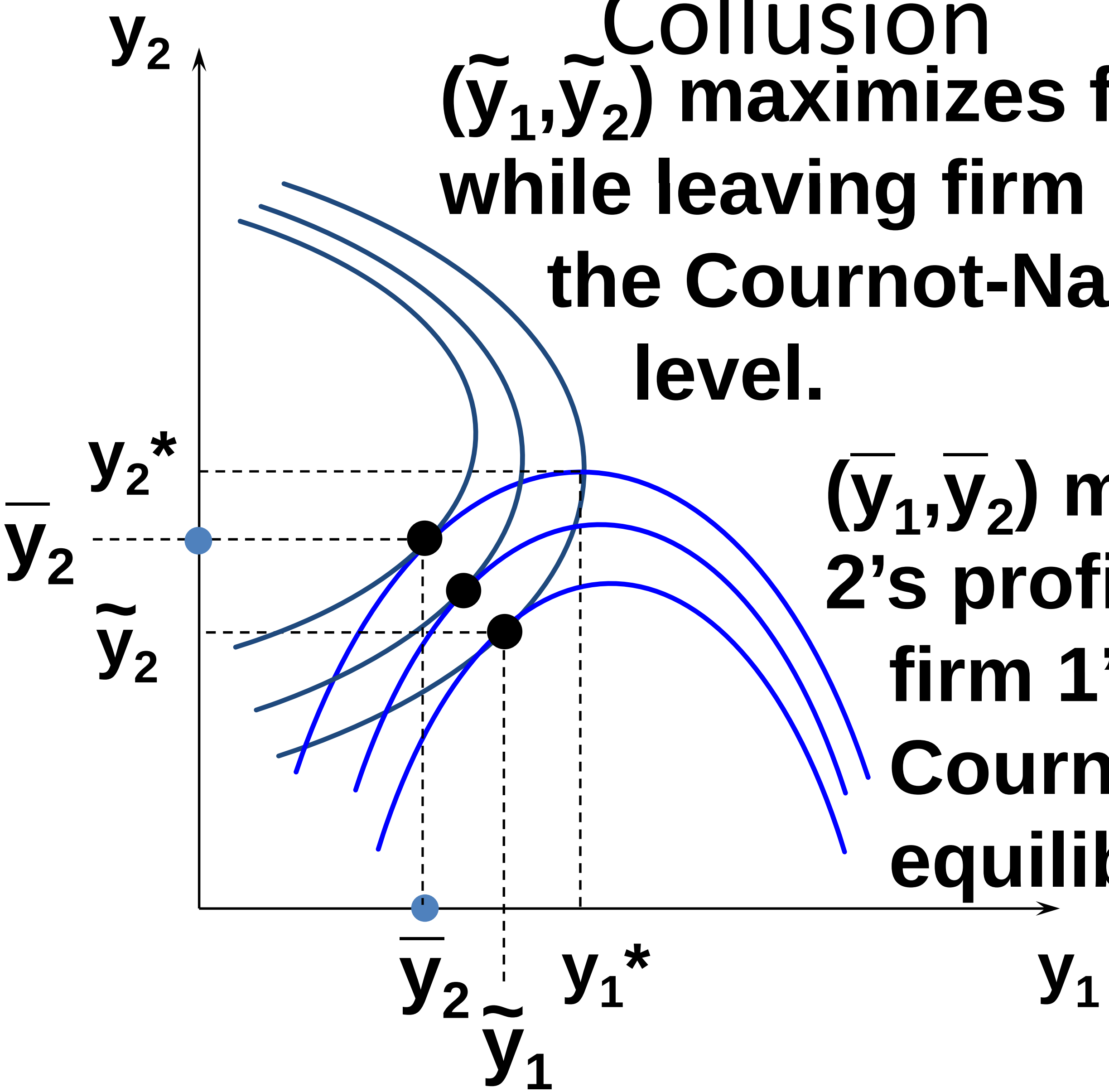
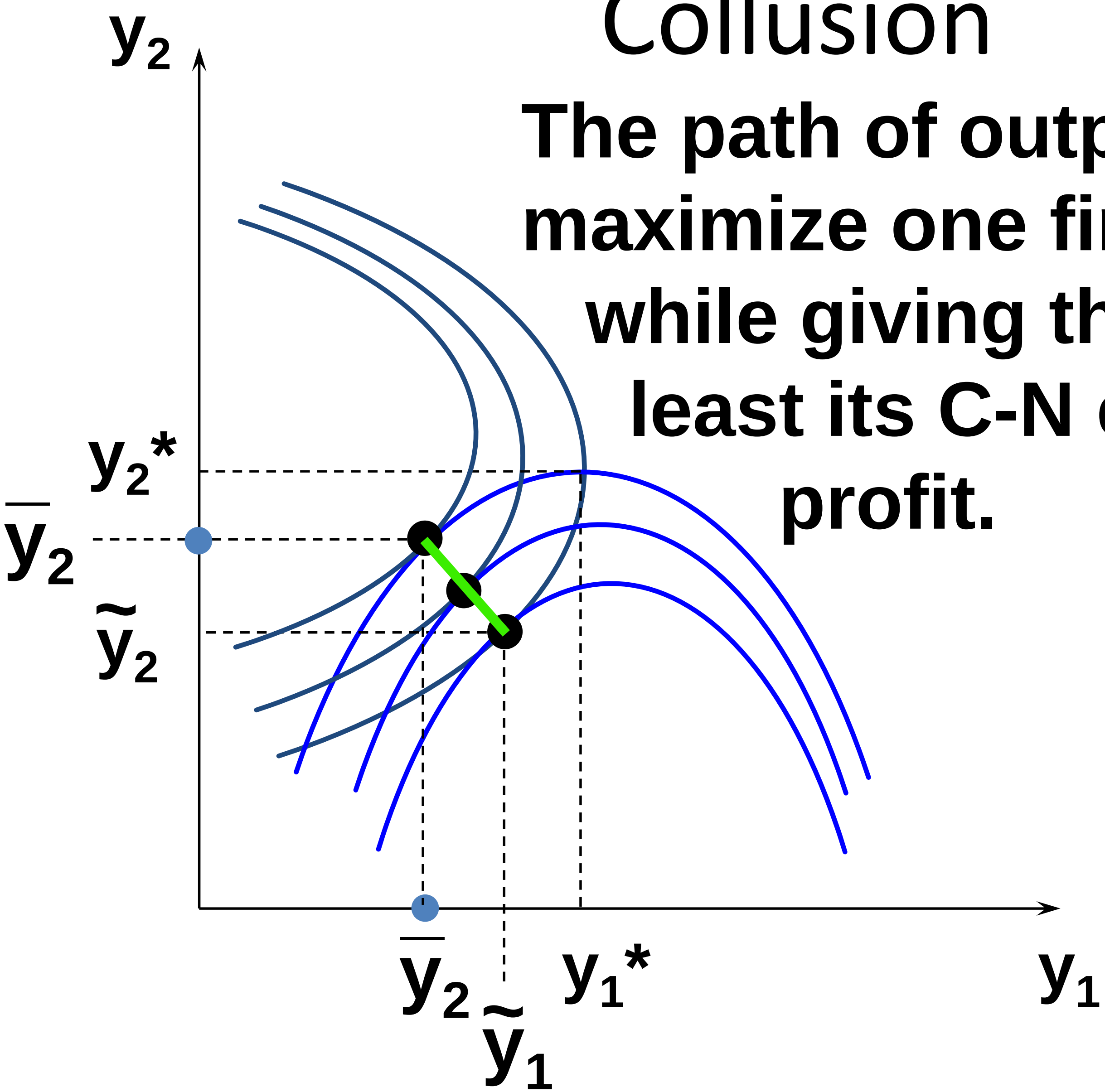


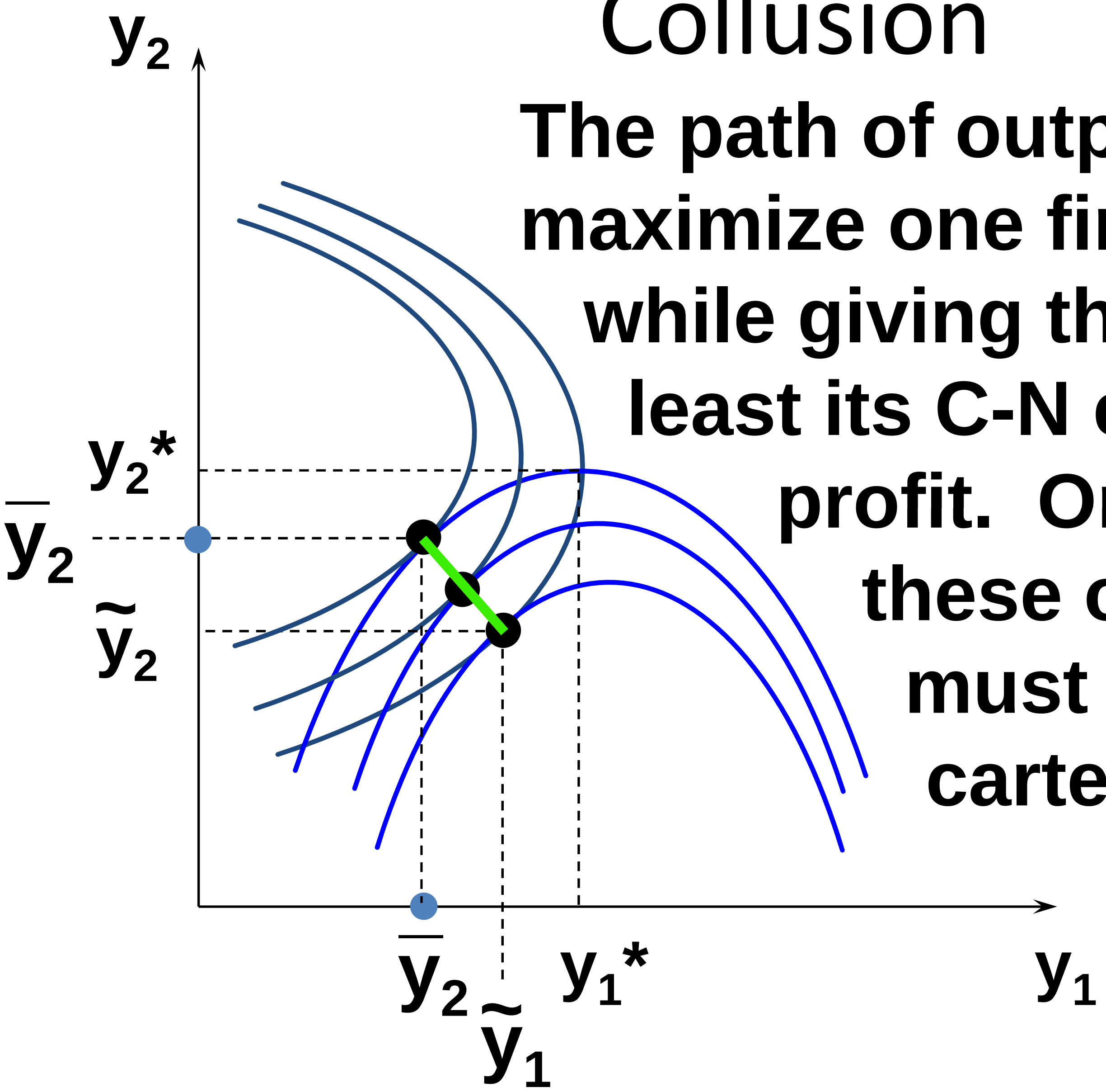
$(\bar{y}_1,\bar{y}_2)$ **maximizes firm 2's profit while leaving firm 1's profit at the Cournot-Nash equilibrium level.**

Collusion

The path of output pairs that maximize one firm's profit while giving the other firm at least its C-N equilibrium profit.

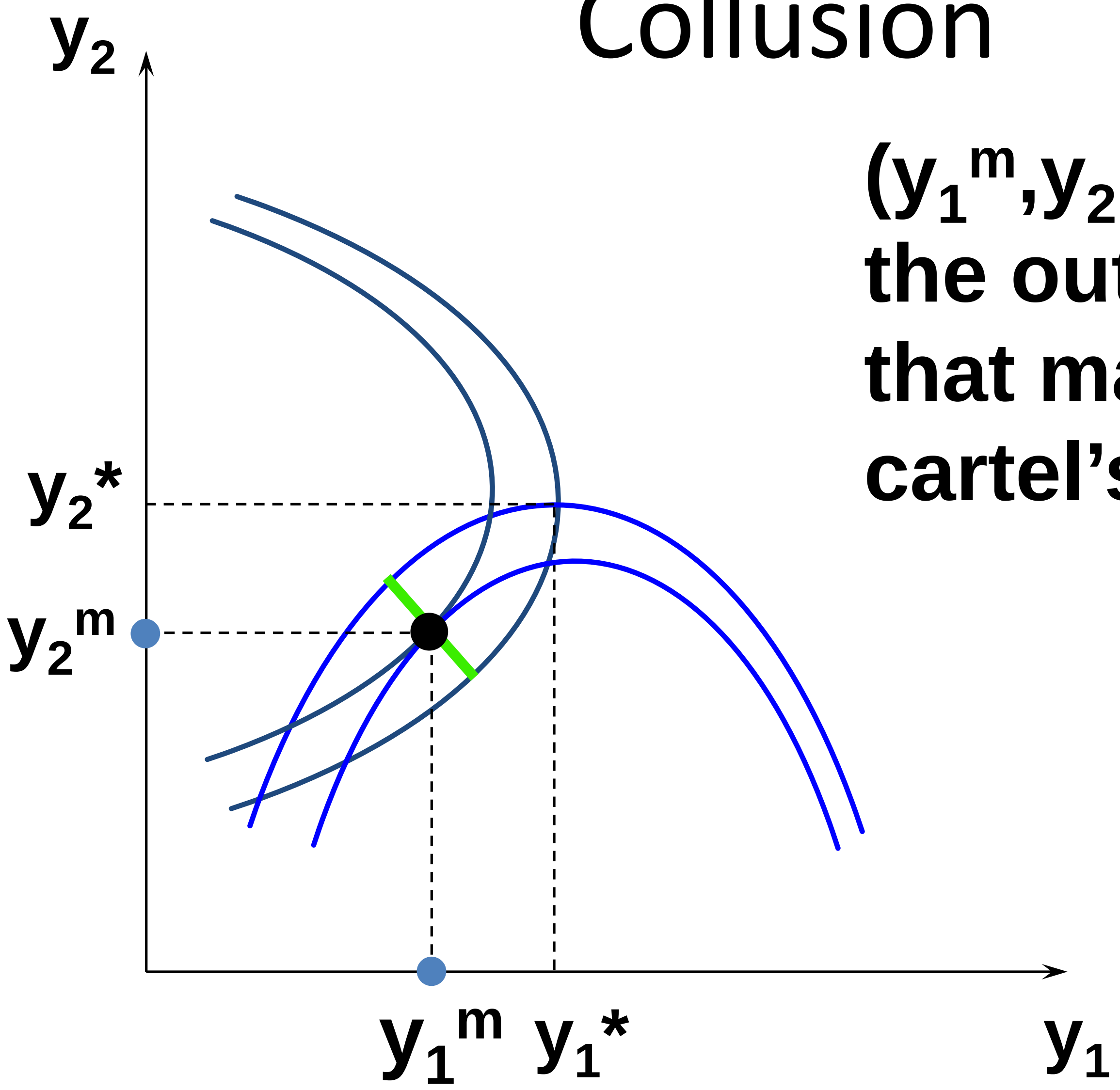


Collusion



The path of output pairs that maximize one firm's profit while giving the other firm at least its C-N equilibrium profit. One of these output pairs must maximize the cartel's joint profit.

Collusion



(y_1^m,y_2^m) denotes the output levels that maximize the cartel's total profit.

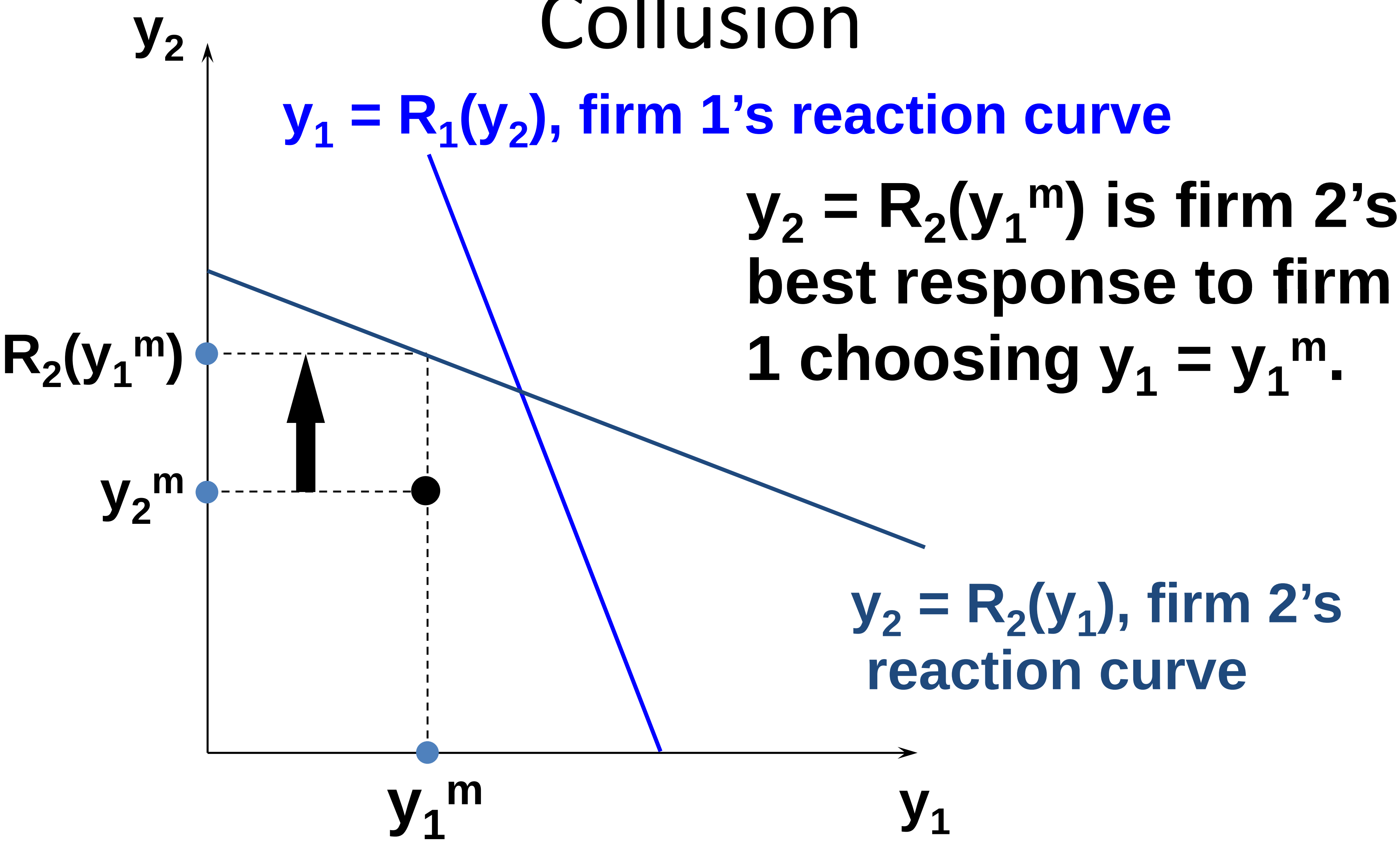
Collusion

- Is such a cartel stable?
- Does one firm have an incentive to cheat on the other?
- *I.e.,* if firm 1 continues to produce y_1^m units, is it profit-maximizing for firm 2 to continue to produce y_2^m units?

Collusion

- Firm 2's profit-maximizing response to $y_1 = y_1^m$ is $y_2 = R_2(y_1^m)$.

Collusion


y2
y1 = R1(y2), firm 1's reaction curve
y2 = R2(y1m) is firm 2's best response to firm 1 choosing y1 = y1m.
R2(y1m)
y2m
y2 = R2(y1), firm 2's reaction curve
y1m
y1

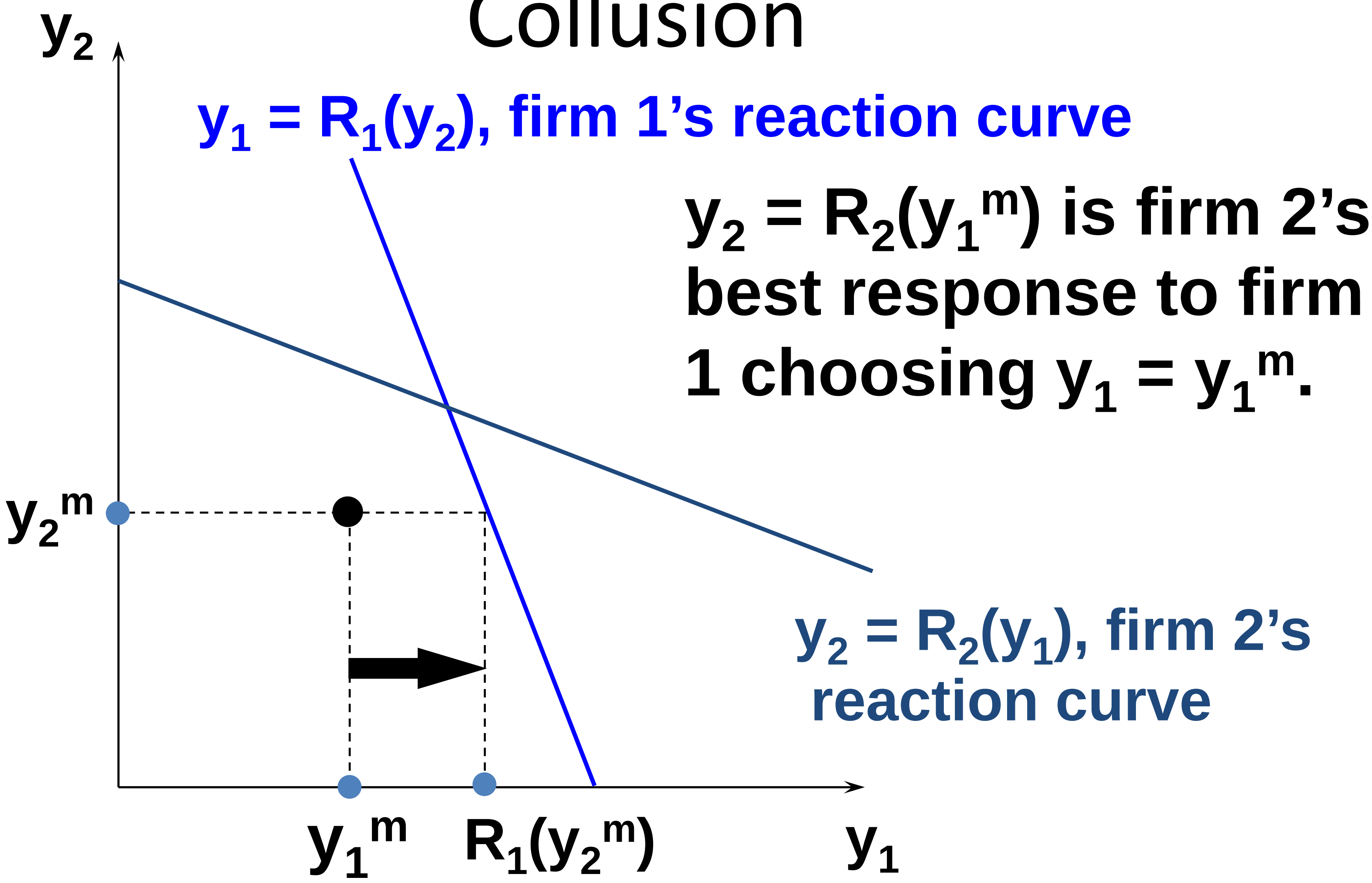
Collusion

- Firm 2's profit-maximizing response to $y_1 = y_1^m$ is $y_2 = R_2(y_1^m) > y_2^m$.
- Firm 2's profit increases if it cheats on firm 1 by increasing its output level from y_2^m to $R_2(y_1^m)$.

Collusion

- Similarly, firm 1's profit increases if it cheats on firm 2 by increasing its output level from y_1^m to $R_1(y_2^m)$.

Collusion


y_2
$y_1 = R_1(y_2)$, firm 1's reaction curve
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y_2^m
$y_2 = R_2(y_1)$, firm 2's reaction curve
y_1^m
$R_1(y_2^m)$
y_1

Collusion

- So a profit-seeking cartel in which firms cooperatively set their output levels is fundamentally unstable.
- *E.g.,* OPEC's broken agreements.

Collusion

- So a profit-seeking cartel in which firms cooperatively set their output levels is fundamentally unstable.
- *E.g.,* OPEC’s broken agreements.
- But is the cartel unstable if the game is repeated many times, instead of being played only once? Then there is an opportunity to punish a cheater.

Collusion & Punishment Strategies

- To determine if such a cartel can be stable we need to know 3 things:
  - (i) What is each firm's per period profit in the cartel?
  - (ii) What is the profit a cheat earns in the first period in which it cheats?
  - (iii) What is the profit the cheat earns in each period after it first cheats?

Collusion & Punishment Strategies

- Suppose two firms face an inverse market demand of $p(y_T) = 24 - y_T$ and have total costs of $c_1(y_1) = y^2_1$ and $c_2(y_2) = y^2_2$.

Collusion & Punishment Strategies

- (i) What is each firm's per period profit in the cartel?
- $p(y_T) = 24 - y_T$, $c_1(y_1) = y^2_1$, $c_2(y_2) = y^2_2$.
- If the firms collude then their joint profit function is
  $\pi^M(y_1,y_2) = (24 - y_1 - y_2)(y_1 + y_2) - y^2_1 - y^2_2$.
- What values of y_1 and y_2 maximize the cartel's profit?

Collusion & Punishment Strategies

- $\pi^M(y_1,y_2) = (24 - y_1 - y_2)(y_1 + y_2) - y^2{}_1 - y^2{}_2$.
- What values of y_1 and y_2 maximize the cartel's profit? Solve

$$\frac{\partial \pi^M}{\partial y_1} = 24 - 4y_1 - 2y_2 = 0$$

$$\frac{\partial \pi^M}{\partial y_2} = 24 - 2y_1 - 4y_2 = 0.$$

Collusion & Punishment Strategies

- $\pi^M(y_1,y_2) = (24 - y_1 - y_2)(y_1 + y_2) - y^2_1 - y^2_2$.
- What values of y_1 and y_2 maximize the cartel's profit? Solve

$$\frac{\partial \pi^M}{\partial y_1} = 24 - 4y_1 - 2y_2 = 0$$

$$\frac{\partial \pi^M}{\partial y_2} = 24 - 2y_1 - 4y_2 = 0.$$

- Solution is $y^M_1 = y^M_2 = 4$.

Collusion & Punishment Strategies

- $\pi^M(y_1,y_2) = (24 - y_1 - y_2)(y_1 + y_2) - y^2{}_1 - y^2{}_2$.
- $y^M{}_1 = y^M{}_2 = 4$ maximizes the cartel's profit.
- The maximum profit is therefore

  $\pi^M = \$(24 - 8)(8) - \$16 - \$16 = \$112.$
- Suppose the firms share the profit equally, getting \$112/2 = \$56 each per period.

Collusion & Punishment Strategies

- (iii) What is the profit the cheat earns in each period after it first cheats?
- This depends upon the punishment inflicted upon the cheat by the other firm.

Collusion & Punishment Strategies

- (iii) What is the profit the cheat earns in each period after it first cheats?
- This depends upon the punishment inflicted upon the cheat by the other firm.
- Suppose the other firm punishes by forever after not cooperating with the cheat.
- What are the firms’ profits in the noncooperative C-N equilibrium?

Collusion & Punishment Strategies

- What are the firms' profits in the noncooperative C-N equilibrium?
- $p(y_T) = 24 - y_T$, $c_1(y_1) = y^2{}_1$, $c_2(y_2) = y^2{}_2$.
- Given y_2, firm 1's profit function is $\pi_1(y_1;y_2) = (24 - y_1 - y_2)y_1 - y^2{}_1$.

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- The value of y_1 that is firm 1's best response to y_2 solves

$$\frac{\partial \pi_1}{\partial y_1} = 24 - 4y_1 - y_2 = 0 \quad \Rightarrow \quad y_1 = R_1(y_2) = \frac{24 - y_2}{4}.$$

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- Similarly, $$y_2 = R_2(y_1) = \frac{24 - y_1}{4}.$$

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- $$y_1 = R_1(y_2) = \frac{24 - y_2}{4}.$$
- Similarly, $$y_2 = R_2(y_1) = \frac{24 - y_1}{4}.$$
- The C-N equilibrium (y^*_1, y^*_2) solves
  $y_1 = R_1(y_2)$ and $y_2 = R_2(y_1) \Rightarrow y^*_1 = y^*_2 = 4{\cdot}8$.

Collusion & Punishment Strategies

- What are the firms' profits in the noncooperative C-N equilibrium?
- $\pi_1(y_1;y_2) = (24 - y_1 - y_2)y_1 - y^2{}_1$.
- $y^*{}_1 = y^*{}_2 = 4{\cdot}8$.
- So each firm's profit in the C-N equilibrium is $\pi^*{}_1 = \pi^*{}_2 = (14{\cdot}4)(4{\cdot}8) - 4{\cdot}8^2 \approx \$46$ each period.

Collusion & Punishment Strategies

- (ii) What is the profit a cheat earns in the first period in which it cheats?
- Firm 1 cheats on firm 2 by producing the quantity y^{CH}_1 that maximizes firm 1's profit given that firm 2 continues to produce $y^M_2 = 4$. What is the value of y^{CH}_1?

Collusion & Punishment Strategies

- (ii) What is the profit a cheat earns in the first period in which it cheats?
- Firm 1 cheats on firm 2 by producing the quantity y^{CH}_1 that maximizes firm 1's profit given that firm 2 continues to produce $y^M_2 = 4$. What is the value of y^{CH}_1?
- $y^{CH}_1 = R_1(y^M_2) = (24 – y^M_2)/4 = (24 – 4)/4 = 5$.
- Firm 1's profit in the period in which it cheats is therefore
  $\pi^{CH}_1 = (24 – 5 – 1)(5) – 5^2 = \$65$.

Collusion & Punishment Strategies

- To determine if such a cartel can be stable we need to know 3 things:
  - (i) What is each firm's per period profit in the cartel? $56.
  - (ii) What is the profit a cheat earns in the first period in which it cheats? $65.
  - (iii) What is the profit the cheat earns in each period after it first cheats? $46.

Collusion & Punishment Strategies

- Each firm's periodic discount factor is 1/(1+r).
- The present-value of firm 1's profits if it does not cheat is ??

Collusion & Punishment Strategies

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- The present-value of firm 1's profits if it does not cheat is

$$PV^{CH} = \$56 + \frac{\$56}{1+r} + \frac{\$56}{(1+r)^2} + \cdots = \$\frac{(1+r)56}{r}.$$

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- The present-value of firm 1's profit if it cheats this period is

$$PV^{M} = \$65 + \frac{\$46}{1+r} + \frac{\$46}{(1+r)^2} + \cdots = \$65 + \frac{\$46}{r}.$$

Collusion & Punishment Strategies

$$PV^{CH} = \$56 + \frac{\$56}{1+r} + \frac{\$56}{(1+r)^2} + \cdots = \$\frac{(1+r)56}{r}.$$

$$PV^{M} = \$65 + \frac{\$46}{1+r} + \frac{\$46}{(1+r)^2} + \cdots = \$65 + \frac{\$46}{r}.$$

So the cartel will be stable if

$$\frac{(1+r)56}{r} + 56 < 65 + \frac{46}{r} \quad \Rightarrow \quad r > \frac{10}{9} \quad \Rightarrow \quad \frac{1}{1+r} < \frac{9}{19}.$$

The Order of Play

- So far it has been assumed that firms choose their output levels simultaneously.
- The competition between the firms is then a simultaneous play game in which the output levels are the strategic variables.

The Order of Play

- What if firm 1 chooses its output level first and then firm 2 responds to this choice?
- Firm 1 is then a leader. Firm 2 is a follower.
- The competition is a sequential game in which the output levels are the strategic variables.

The Order of Play

- Such games are von Stackelberg games.
- Is it better to be the leader?
- Or is it better to be the follower?

Stackelberg Games

- Q: What is the best response that follower firm 2 can make to the choice y_1 already made by the leader, firm 1?

Stackelberg Games

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- A: Choose $y_2 = R_2(y_1)$.

Stackelberg Games

- Q: What is the best response that follower firm 2 can make to the choice y_1 already made by the leader, firm 1?
- A: Choose $y_2 = R_2(y_1)$.
- Firm 1 knows this and so perfectly anticipates firm 2's reaction to any y_1 chosen by firm 1.

Stackelberg Games

- This makes the leader's profit function

$$\Pi_1^s(y_1) = p(y_1 + R_2(y_1))y_1 - c_1(y_1).$$

Stackelberg Games

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- The leader chooses y_1 to maximize its profit.

Stackelberg Games

- This makes the leader's profit function

$$\Pi_1^s(y_1) = p(y_1 + R_2(y_1))y_1 - c_1(y_1).$$

- The leader chooses y_1 to maximize its profit.
- Q: Will the leader make a profit at least as large as its Cournot-Nash equilibrium profit?

Stackelberg Games

- A: Yes. The leader could choose its Cournot-Nash output level, knowing that the follower would then also choose its C-N output level. The leader's profit would then be its C-N profit. But the leader does not have to do this, so its profit must be at least as large as its C-N profit.

Stackelberg Games; An Example

- The market inverse demand function is $p = 60 - y_T$. The firms' cost functions are $c_1(y_1) = y_1^2$ and $c_2(y_2) = 15y_2 + y_2^2$.
- Firm 2 is the follower. Its reaction function is

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

Stackelberg Games; An Example

The leader's profit function is therefore

$$\Pi_1^s(y_1) = (60 - y_1 - R_2(y_1))y_1 - y_1^2$$

$$= (60 - y_1 - \frac{45 - y_1}{4})y_1 - y_1^2$$

$$= \frac{195}{4}y_1 - \frac{7}{4}y_1^2.$$

Stackelberg Games; An Example

The leader's profit function is therefore

$$\Pi_1^s(y_1) = (60 - y_1 - R_2(y_1))y_1 - y_1^2$$

$$= (60 - y_1 - \frac{45 - y_1}{4})y_1 - y_1^2$$

$$= \frac{195}{4}y_1 - \frac{7}{4}y_1^2.$$

For a profit-maximum for firm 1,

$$\frac{195}{4} = \frac{7}{2}y_1 \Rightarrow y_1^s = 13 \cdot 9.$$

Stackelberg Games; An Example

Q: What is firm 2's response to the leader's choice $y_1^s = 13 \cdot 9$**?**

Stackelberg Games; An Example

Q: What is firm 2's response to the leader's choice $y_1^s = 13 \cdot 9$?

A: $y_2^s = R_2(y_1^s) = \dfrac{45 - 13 \cdot 9}{4} = 7 \cdot 8.$

Stackelberg Games; An Example

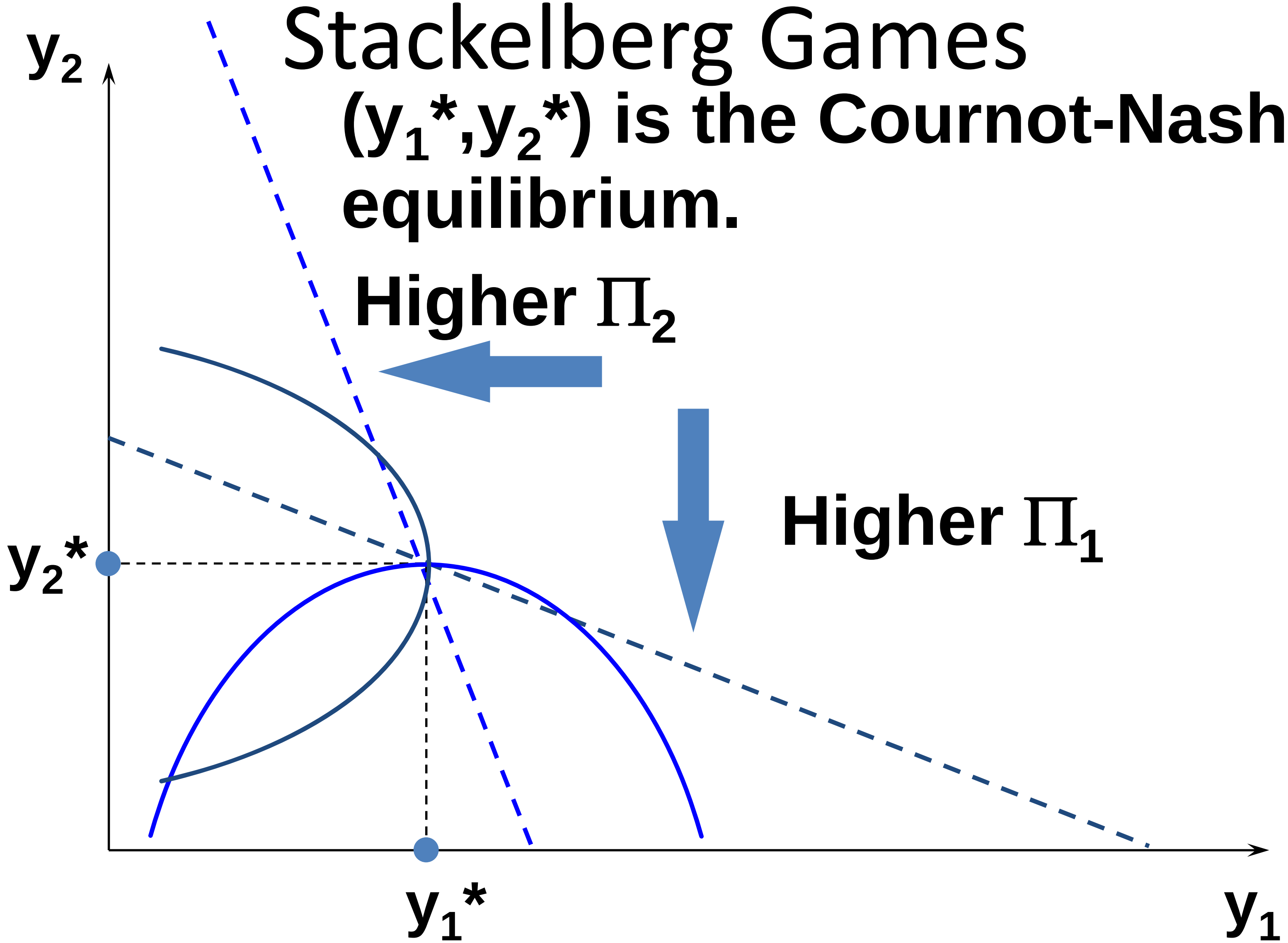
Q: What is firm 2's response to the leader's choice $y_1^s = 13 \cdot 9$?

A: $y_2^s = R_2(y_1^s) = \dfrac{45 - 13 \cdot 9}{4} = 7 \cdot 8.$

The C-N output levels are $(y_1^*, y_2^*) = (13,8)$ so the leader produces more than its C-N output and the follower produces less than its C-N output. This is true generally.

Stackelberg Games

(y_1^*, y_2^*) is the Cournot-Nash equilibrium.



Stackelberg Games

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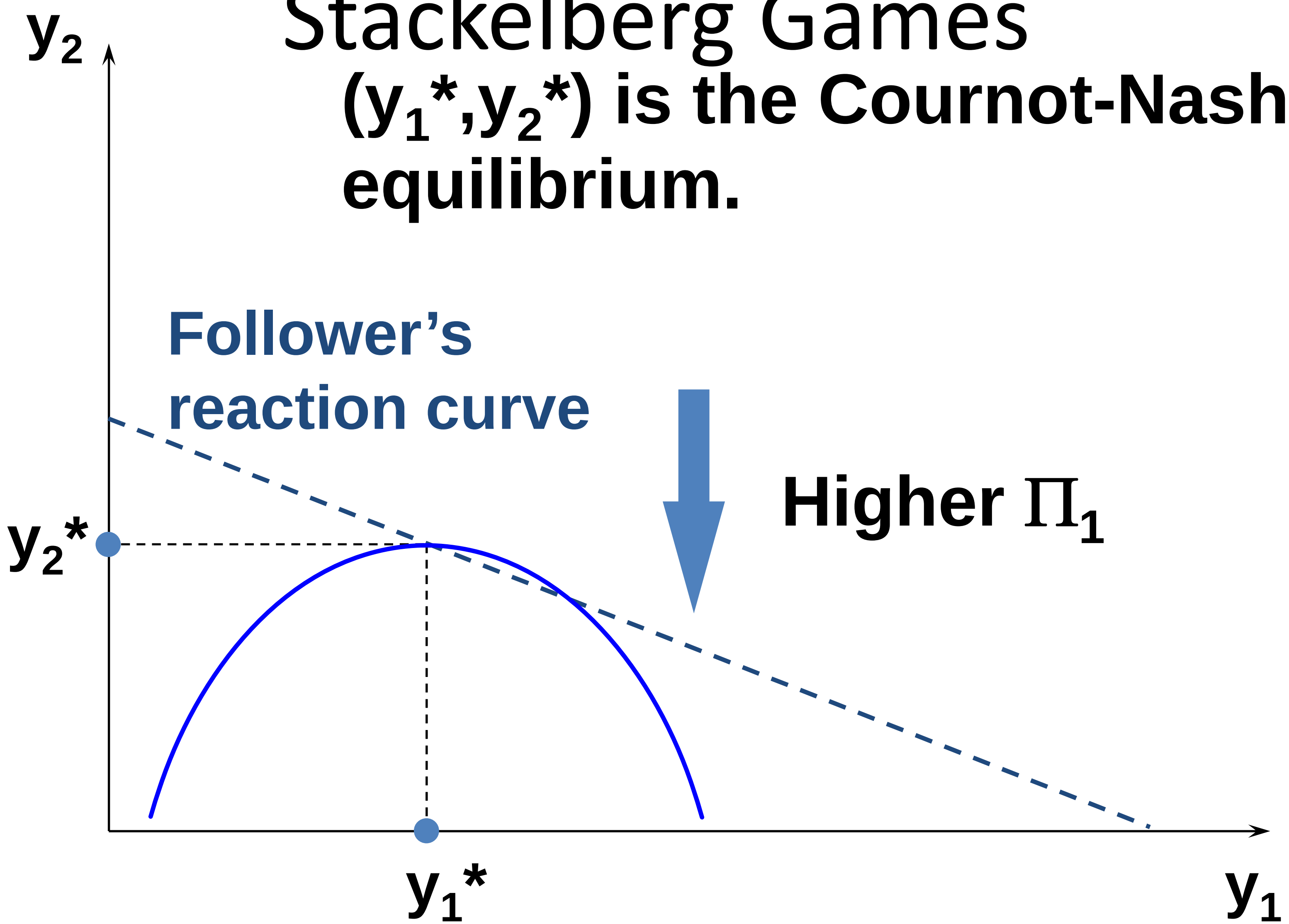
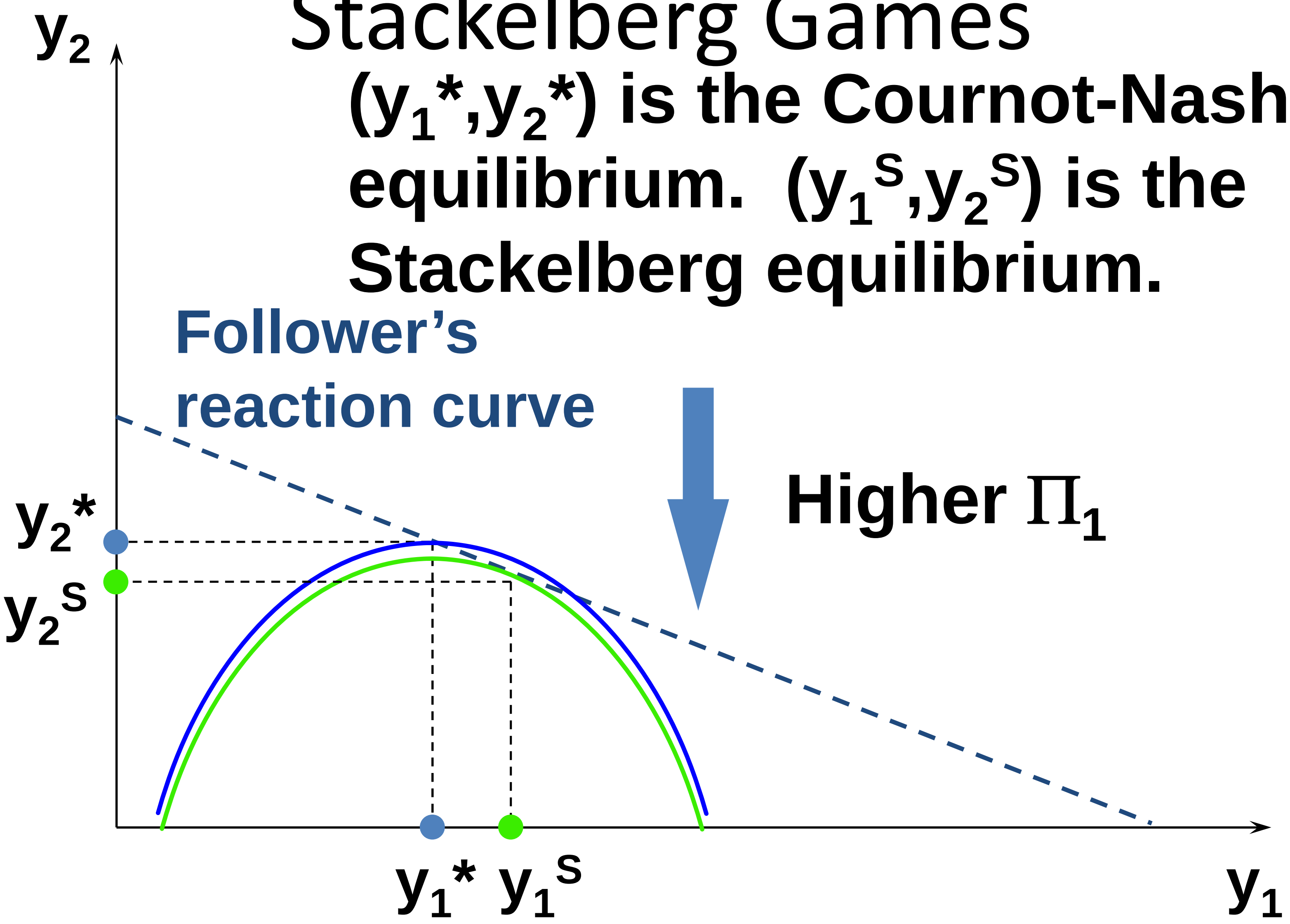


Stackelberg Games

(y_1^*, y_2^*) is the Cournot-Nash equilibrium. (y_1^S, y_2^S) is the Stackelberg equilibrium.



Stackelberg Games

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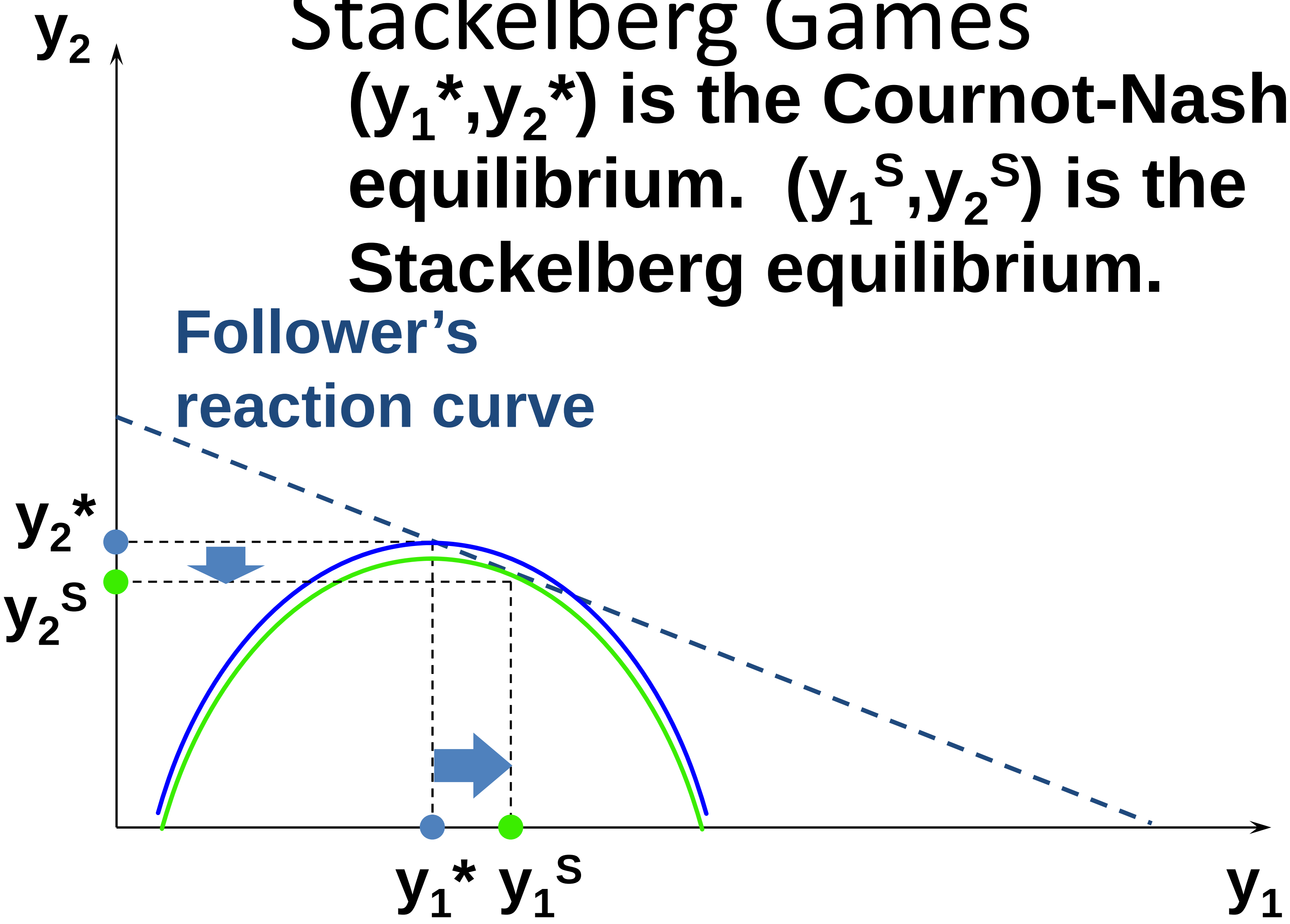

Price Competition

- What if firms compete using only price-setting strategies, instead of using only quantity-setting strategies?
- Games in which firms use only price strategies and play simultaneously are Bertrand games.

Bertrand Games

- Each firm's marginal production cost is constant at c.
- All firms set their prices simultaneously.
- Q: Is there a Nash equilibrium?

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Bertrand Games

- Each firm's marginal production cost is constant at c.
- All firms set their prices simultaneously.
- Q: Is there a Nash equilibrium?
- A: Yes. Exactly one. All firms set their prices equal to the marginal cost c. Why?

Bertrand Games

- Suppose one firm sets its price higher than another firm's price.

Bertrand Games

- Suppose one firm sets its price higher than another firm’s price.
- Then the higher-priced firm would have no customers.

Bertrand Games

- Suppose one firm sets its price higher than another firm's price.
- Then the higher-priced firm would have no customers.
- Hence, at an equilibrium, all firms must set the same price.

Bertrand Games

- Suppose the common price set by all firm is higher than marginal cost c.

Bertrand Games

- Suppose the common price set by all firm is higher than marginal cost c.
- Then one firm can just slightly lower its price and sell to all the buyers, thereby increasing its profit.

Bertrand Games

- Suppose the common price set by all firm is higher than marginal cost c.
- Then one firm can just slightly lower its price and sell to all the buyers, thereby increasing its profit.
- The only common price which prevents undercutting is c. Hence this is the only Nash equilibrium.

Sequential Price Games

- What if, instead of simultaneous play in pricing strategies, one firm decides its price ahead of the others.
- This is a sequential game in pricing strategies called a price-leadership game.
- The firm which sets its price ahead of the other firms is the price-leader.

Sequential Price Games

- Think of one large firm (the leader) and many competitive small firms (the followers).
- The small firms are price-takers and so their collective supply reaction to a market price p is their aggregate supply function $Y_f(p)$.

Sequential Price Games

- The market demand function is D(p).
- So the leader knows that if it sets a price p the quantity demanded from it will be the residual demand

$$L(p) = D(p) - Y_f(p).$$

- Hence the leader's profit function is

$$\Pi_L(p) = p(D(p) - Y_f(p)) - c_L(D(p) - Y_f(p)).$$

Sequential Price Games

- The leader's profit function is

$$\Pi_L(p) = p(D(p) - Y_f(p)) - c_L(D(p) - Y_F(p))$$

so the leader chooses the price level p* for which profit is maximized.

- The followers collectively supply $Y_f(p^*)$ units and the leader supplies the residual quantity $D(p^*) - Y_f(p^*)$.