31

Exchange

Exchange

- Two consumers, A and B.
- Their endowments of goods 1 and 2 are

$$\omega^A = (\omega_1^A, \omega_2^A) \text{ and } \omega^B = (\omega_1^B, \omega_2^B).$$

- E.g. $\omega^A = (6,4)$ **and** $\omega^B = (2,2)$.
- The total quantities available

are $\omega_1^A + \omega_1^B = 6 + 2 = 8$ **units of good 1**

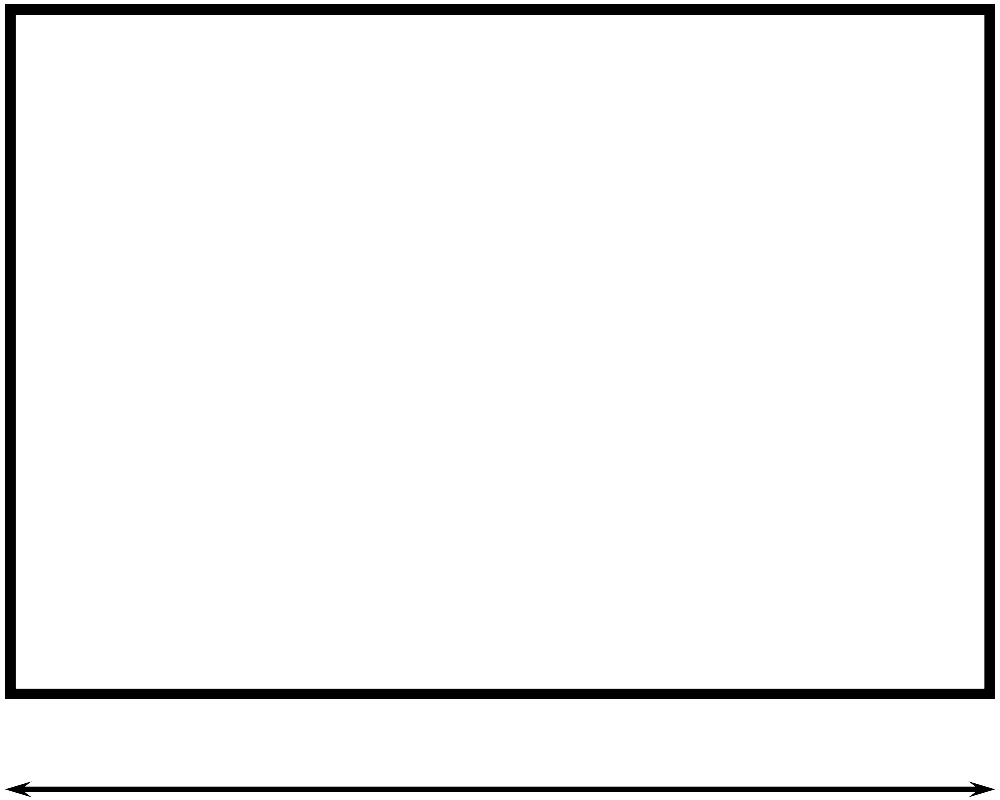
and $\omega_2^A + \omega_2^B = 4 + 2 = 6$ **units of good 2.**

Exchange

- Edgeworth and Bowley devised a diagram, called an Edgeworth box, to show all possible allocations of the available quantities of goods 1 and 2 between the two consumers.

Starting an Edgeworth Box

Starting an Edgeworth Box



Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

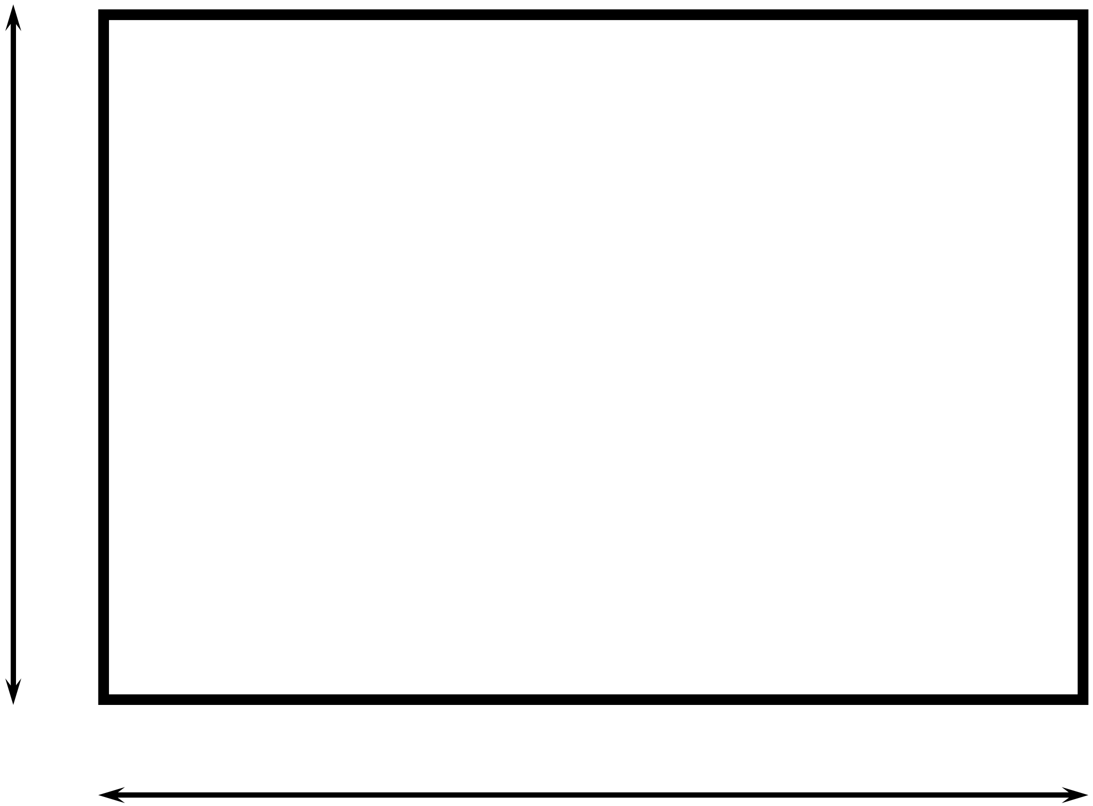
Starting an Edgeworth Box

Height =

$\omega_2^A + \omega_2^B$

$= 4 + 2$

$= 6$



Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

Starting an Edgeworth Box

Height =

$\omega_2^{\mathbf{A}} + \omega_2^{\mathbf{B}}$

$= 4 + 2$

$= 6$

The dimensions of the box are the quantities available of the goods.

Width = $\omega_1^{\mathbf{A}} + \omega_1^{\mathbf{B}} = 6 + 2 = 8$

Feasible Allocations

- What allocations of the 8 units of good 1 and the 6 units of good 2 are feasible?
- How can all of the feasible allocations be depicted by the Edgeworth box diagram?

Feasible Allocations

- What allocations of the 8 units of good 1 and the 6 units of good 2 are feasible?
- How can all of the feasible allocations be depicted by the Edgeworth box diagram?
- One feasible allocation is the before-trade allocation; i.e. the endowment allocation.

The Endowment Allocation

The endowment allocation is

$\omega^{A} = (6,4)$

and

$\omega^{B} = (2,2).$

Height =

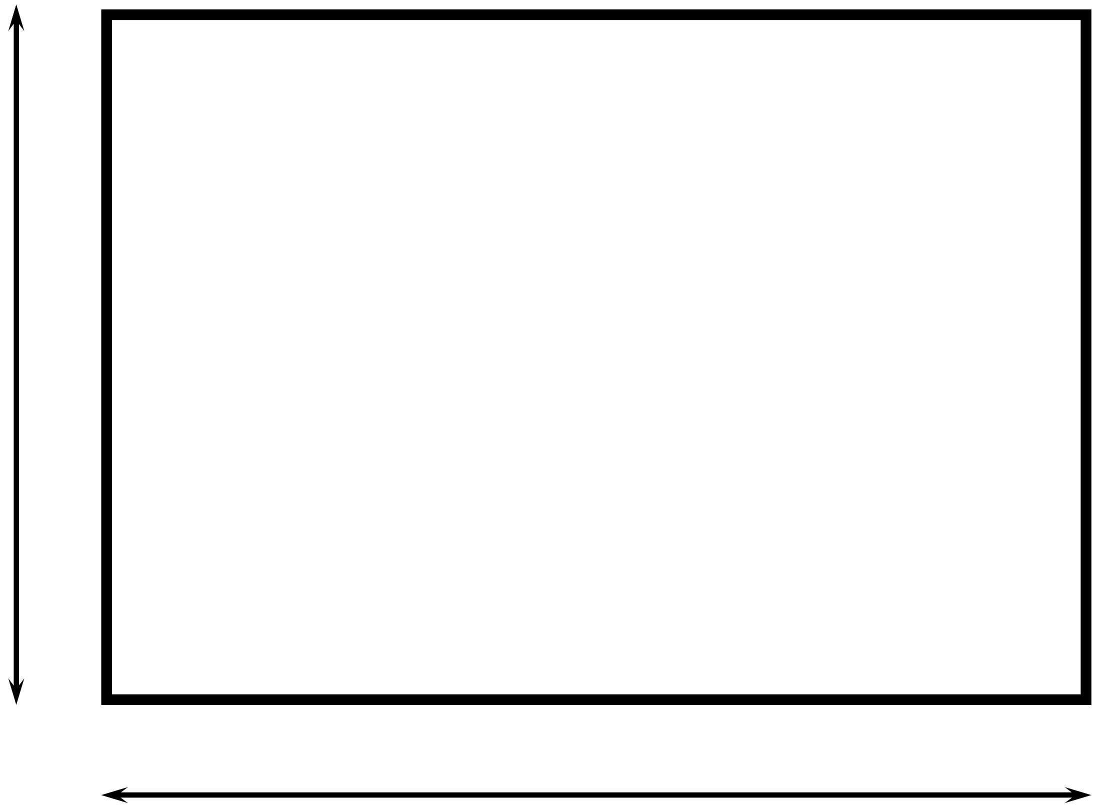
$\omega_2^{A} + \omega_2^{B}$

$= 4 + 2$

$= 6$

Width = $\omega_1^{A} + \omega_1^{B} = 6 + 2 = 8$

The Endowment Allocation



Height =

$\omega_2^A + \omega_2^B$

$= 4 + 2$

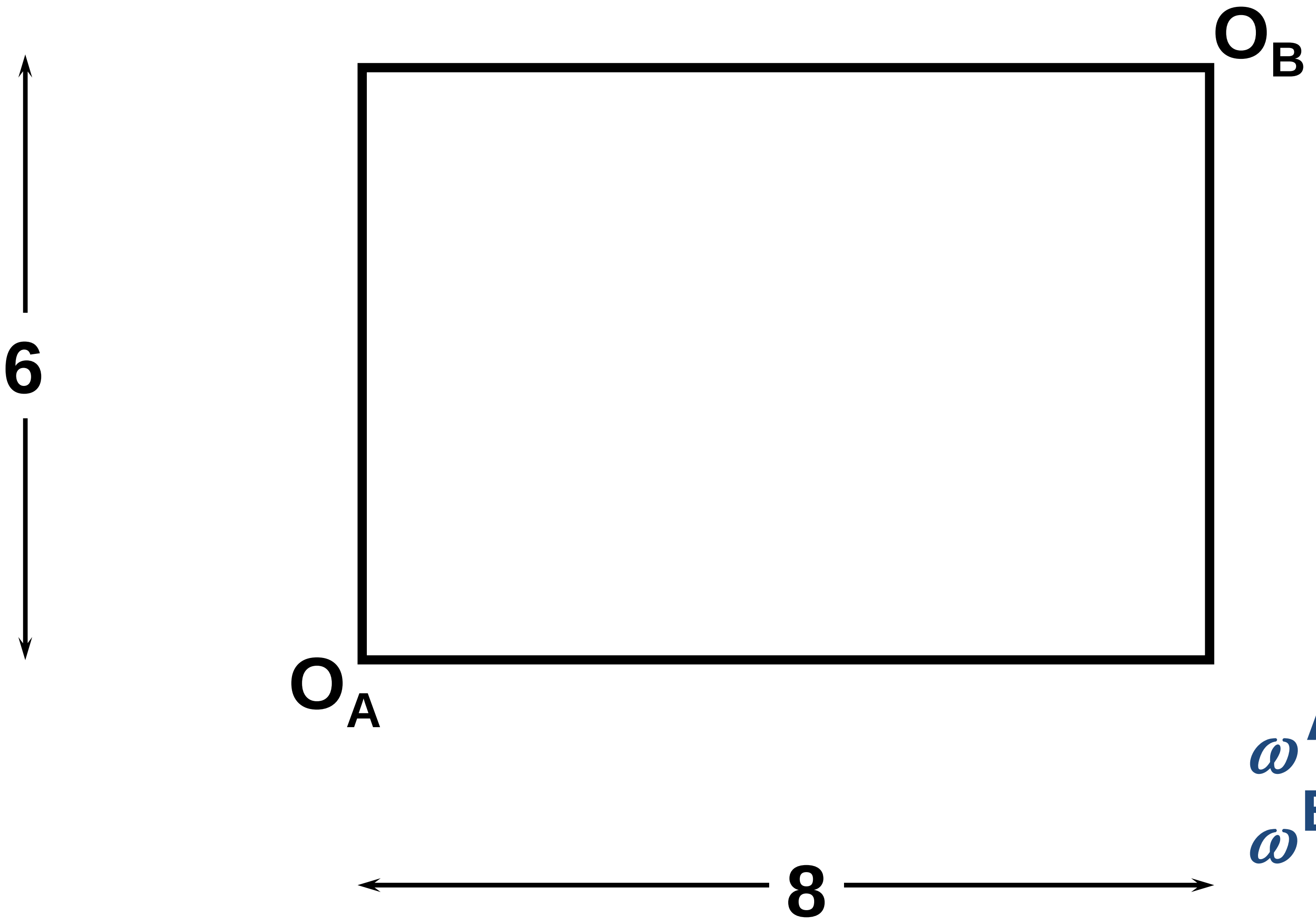
$= 6$

Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

$\omega^A = (6,4)$

$\omega^B = (2,2)$

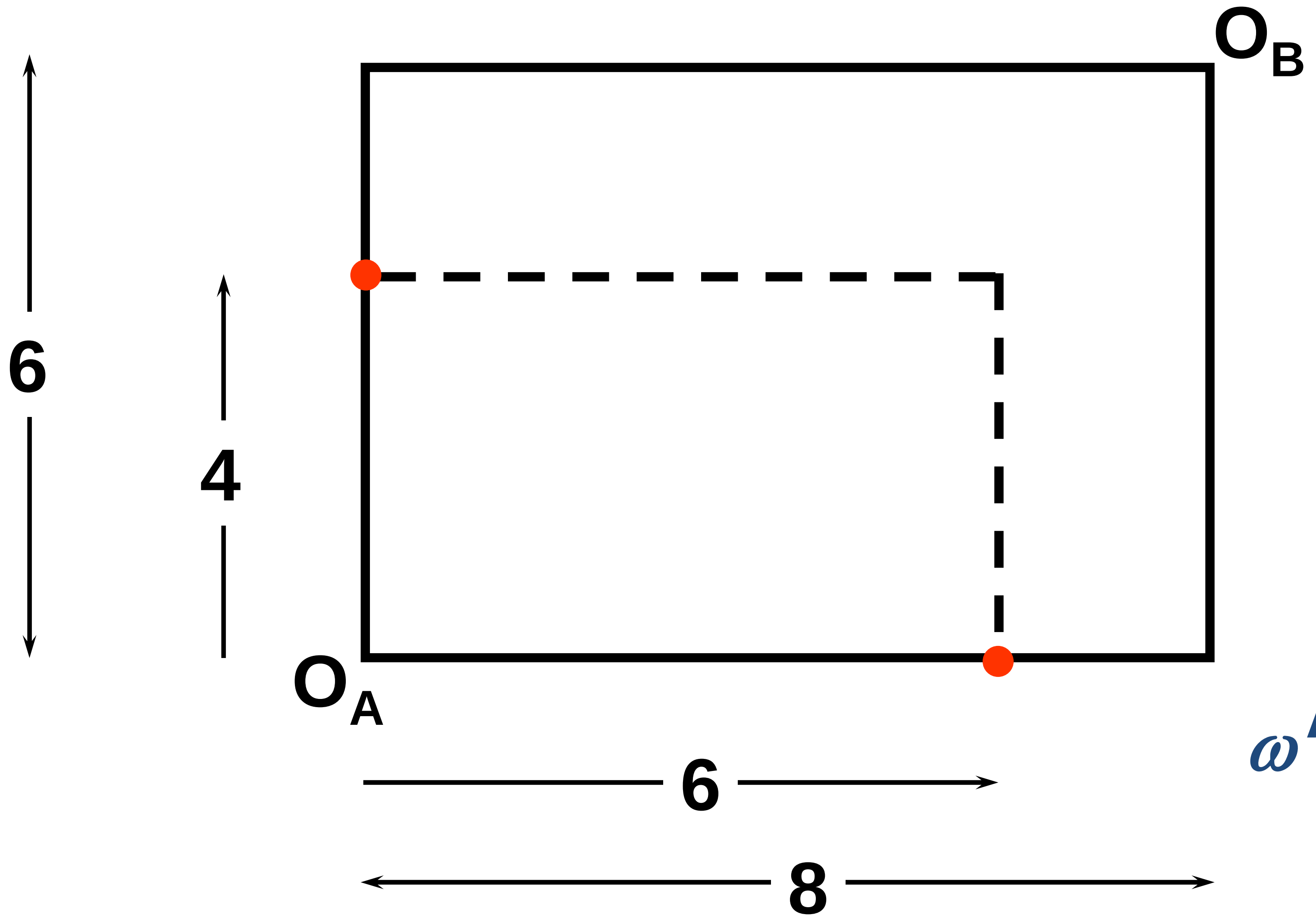
The Endowment Allocation



$\omega^A = (6,4)$

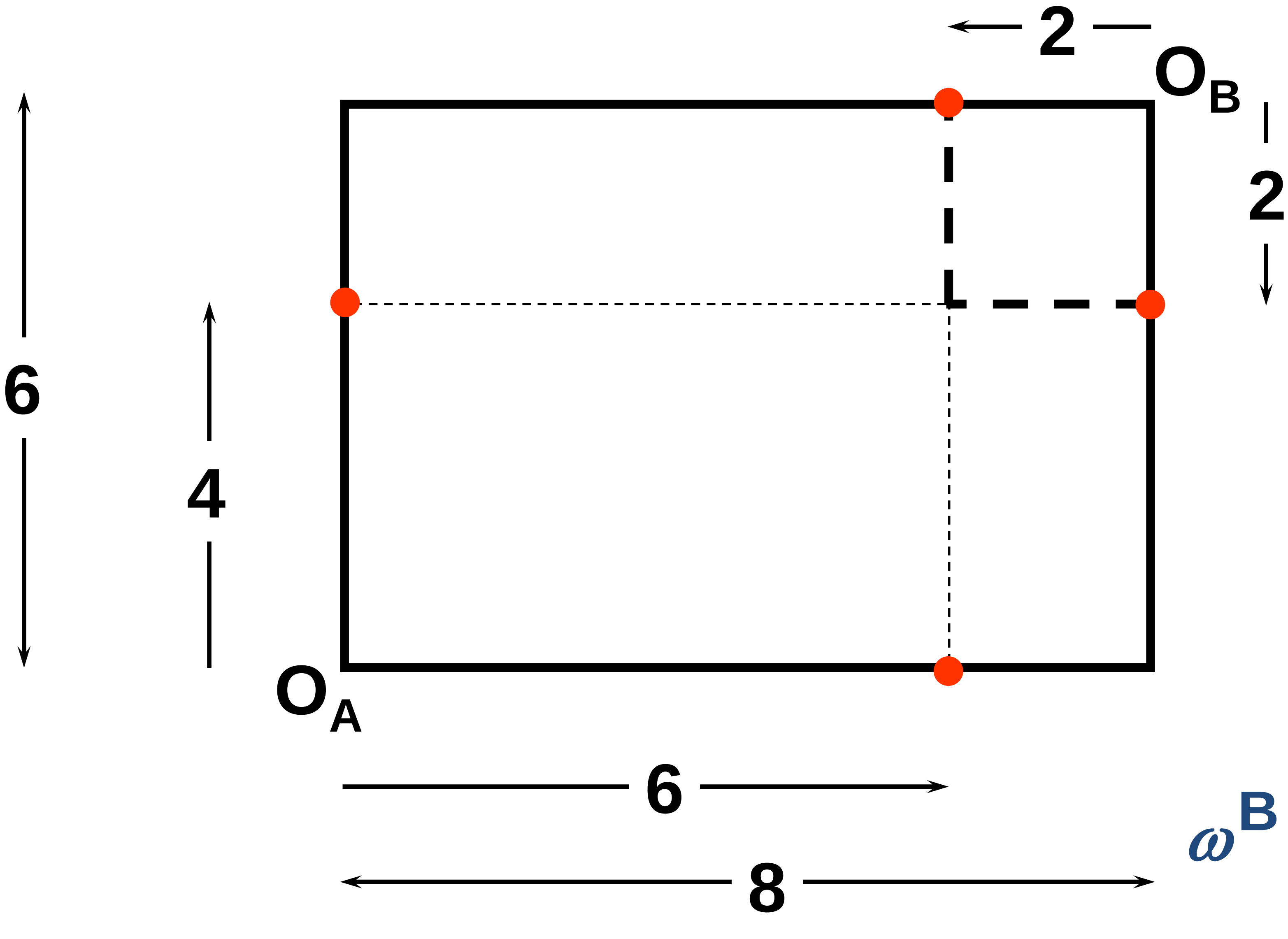
$\omega^B = (2,2)$

The Endowment Allocation



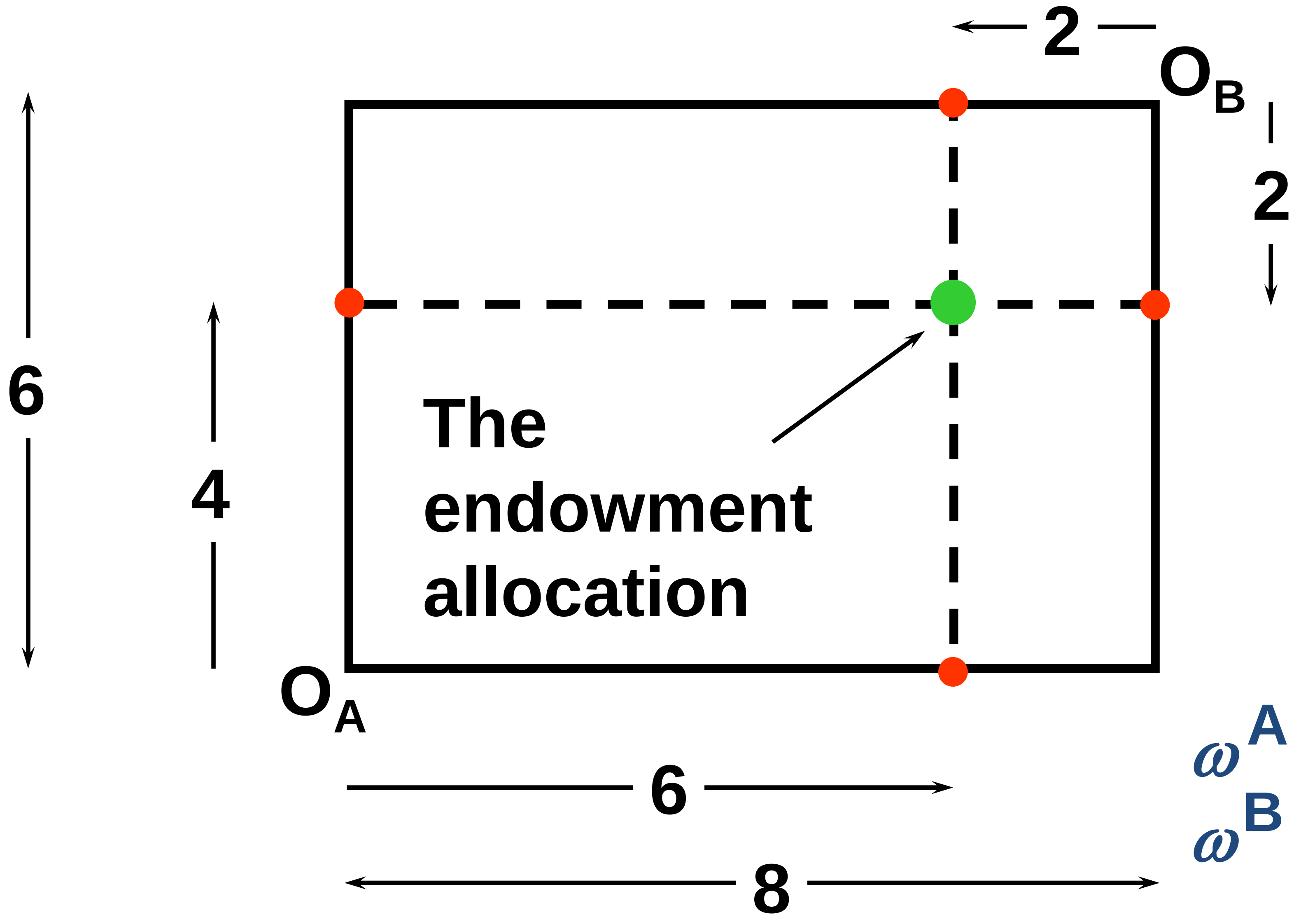
$\omega^{A} = (6,4)$

The Endowment Allocation



$\omega^{\mathbf{B}} = (2,2)$

The Endowment Allocation



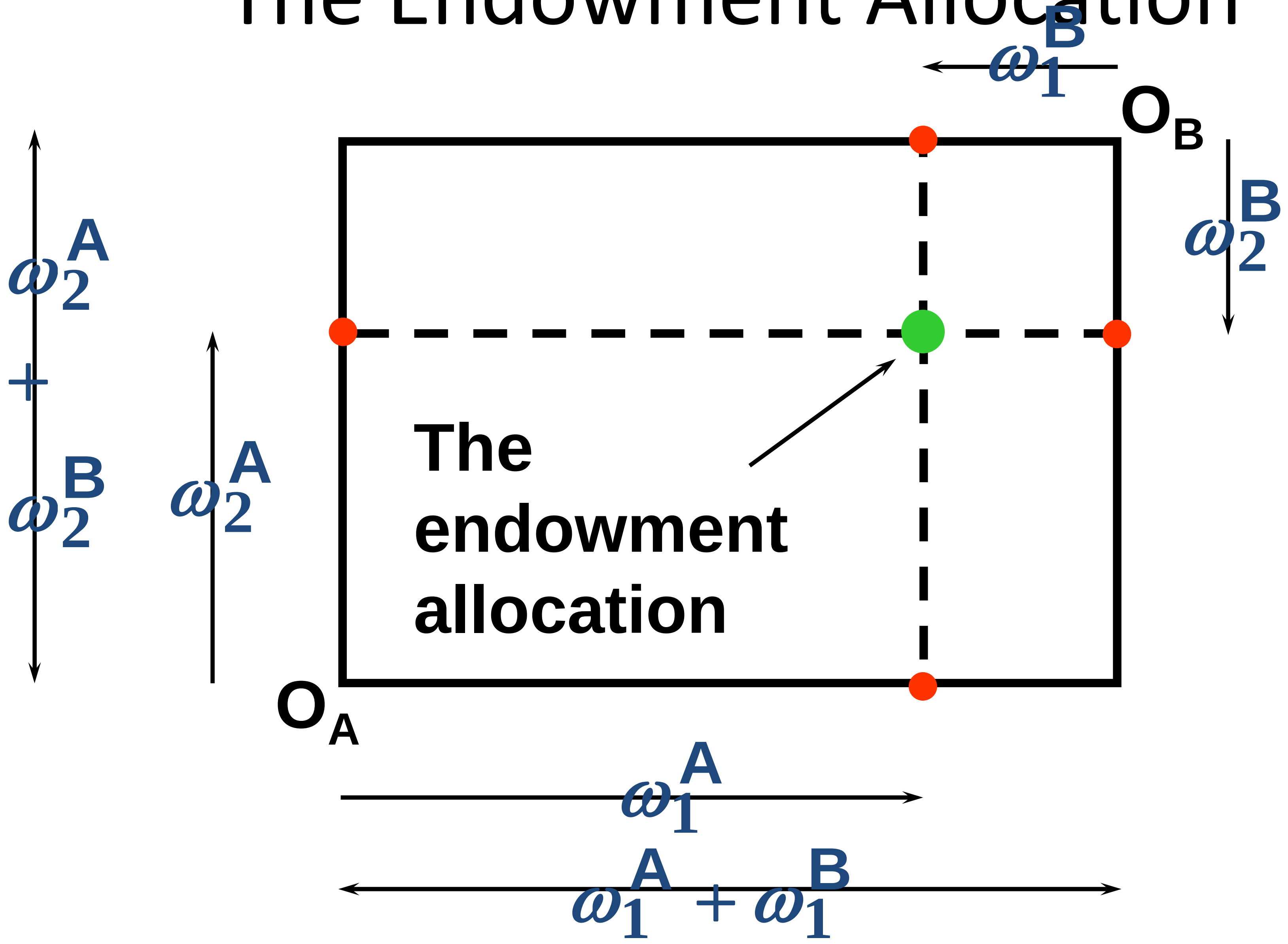
$\omega^A = (6,4)$

$\omega^B = (2,2)$

The Endowment Allocation

More generally, …

The Endowment Allocation



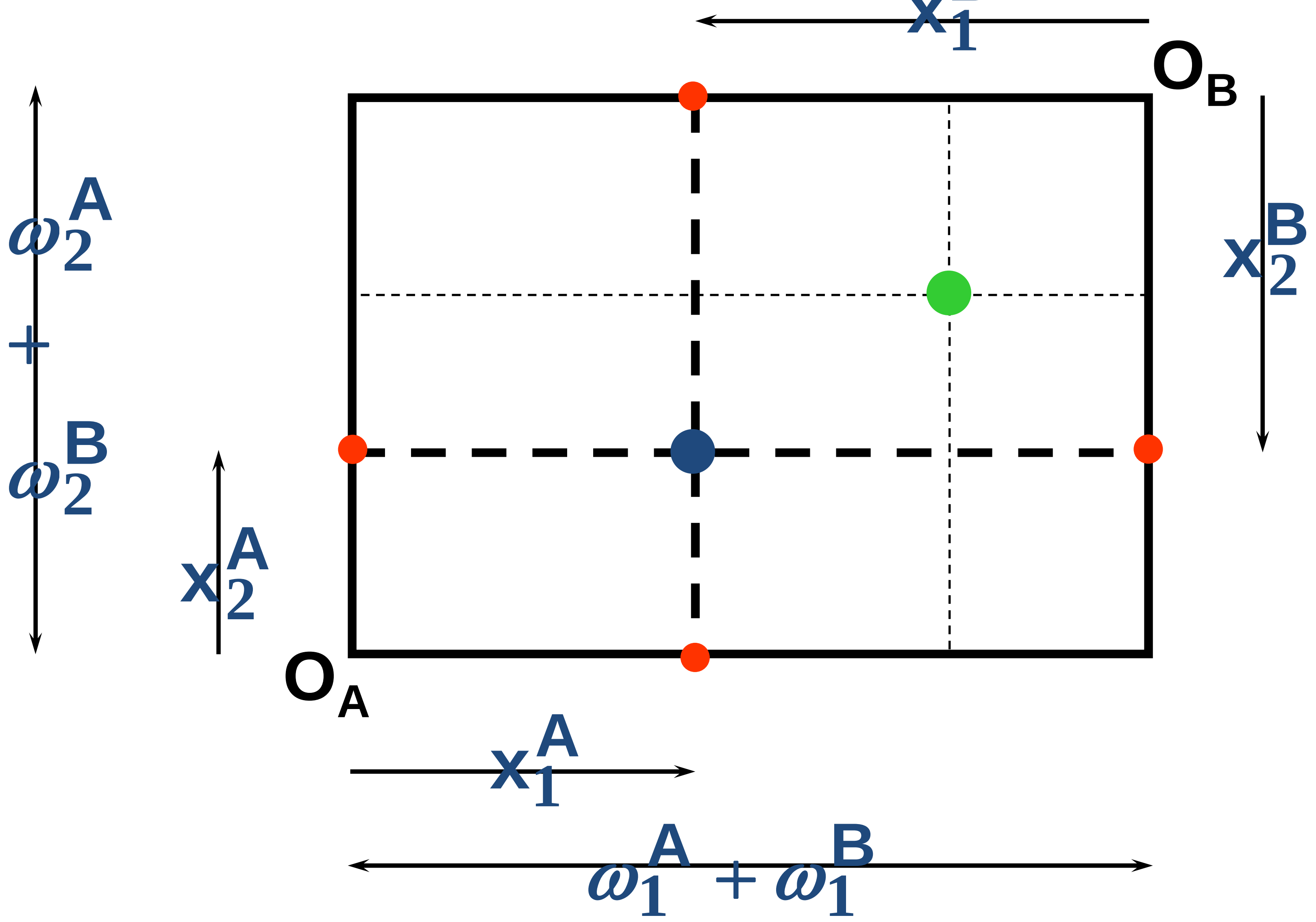
Other Feasible Allocations

- (x_1^A, x_2^A) denotes an allocation to consumer A.
- (x_1^B, x_2^B) denotes an allocation to consumer B.
- An allocation is feasible if and only if

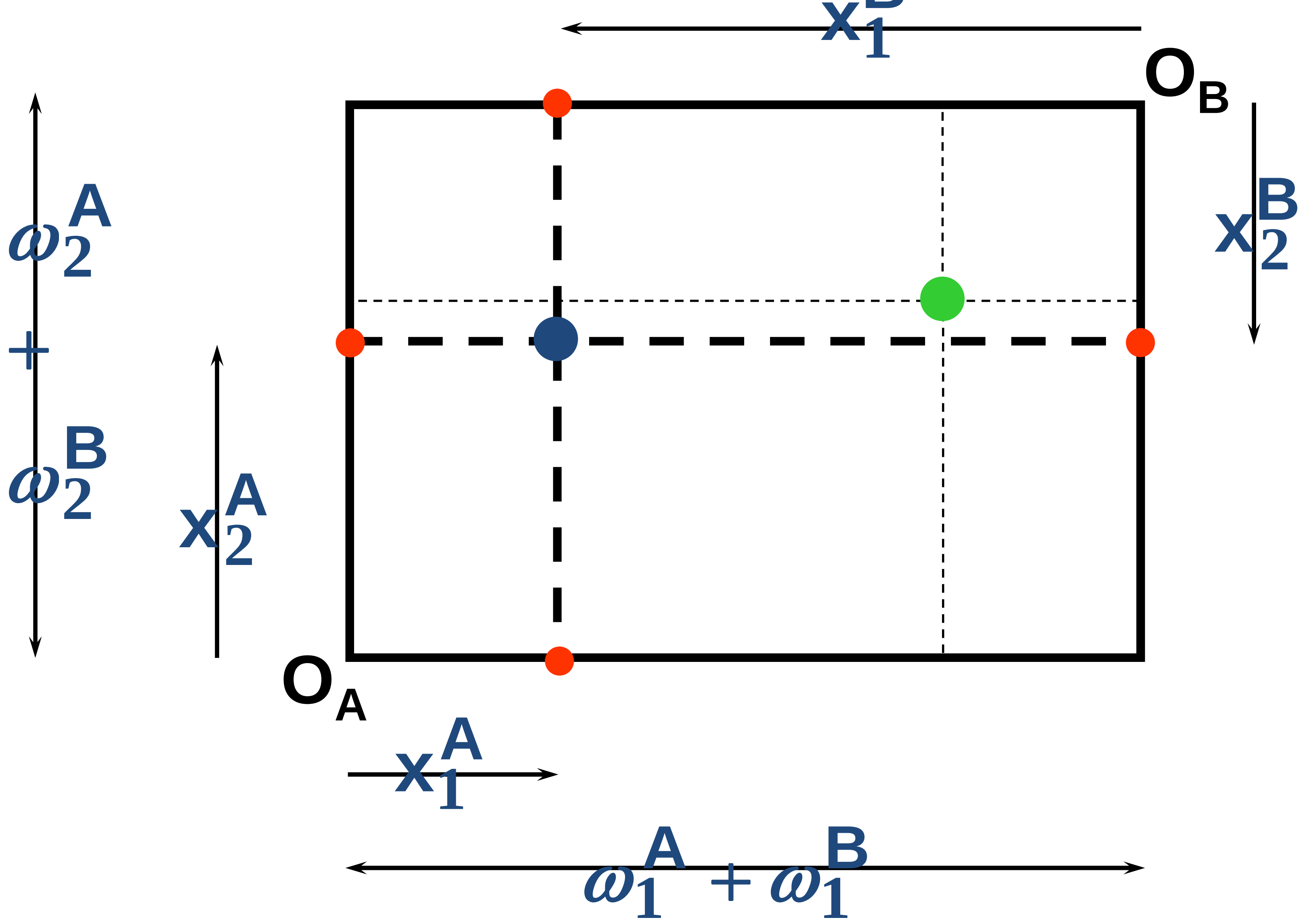
$$x_1^A + x_1^B \leq \omega_1^A + \omega_1^B$$

and $$x_2^A + x_2^B \leq \omega_2^A + \omega_2^B.$$

Feasible Reallocations



Feasible Reallocations



Feasible Reallocations

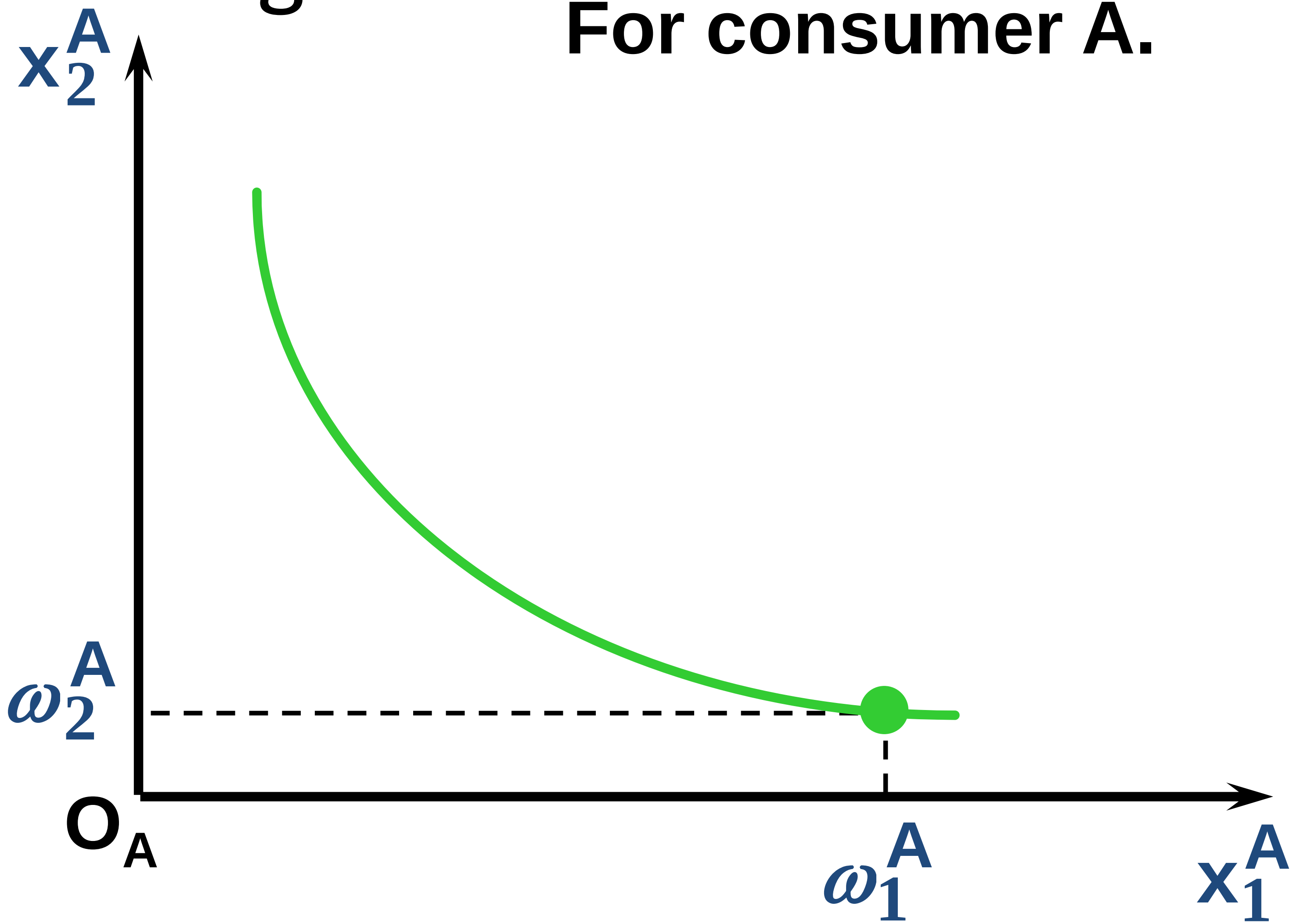
- All points in the box, including the boundary, represent feasible allocations of the combined endowments.

Feasible Reallocations

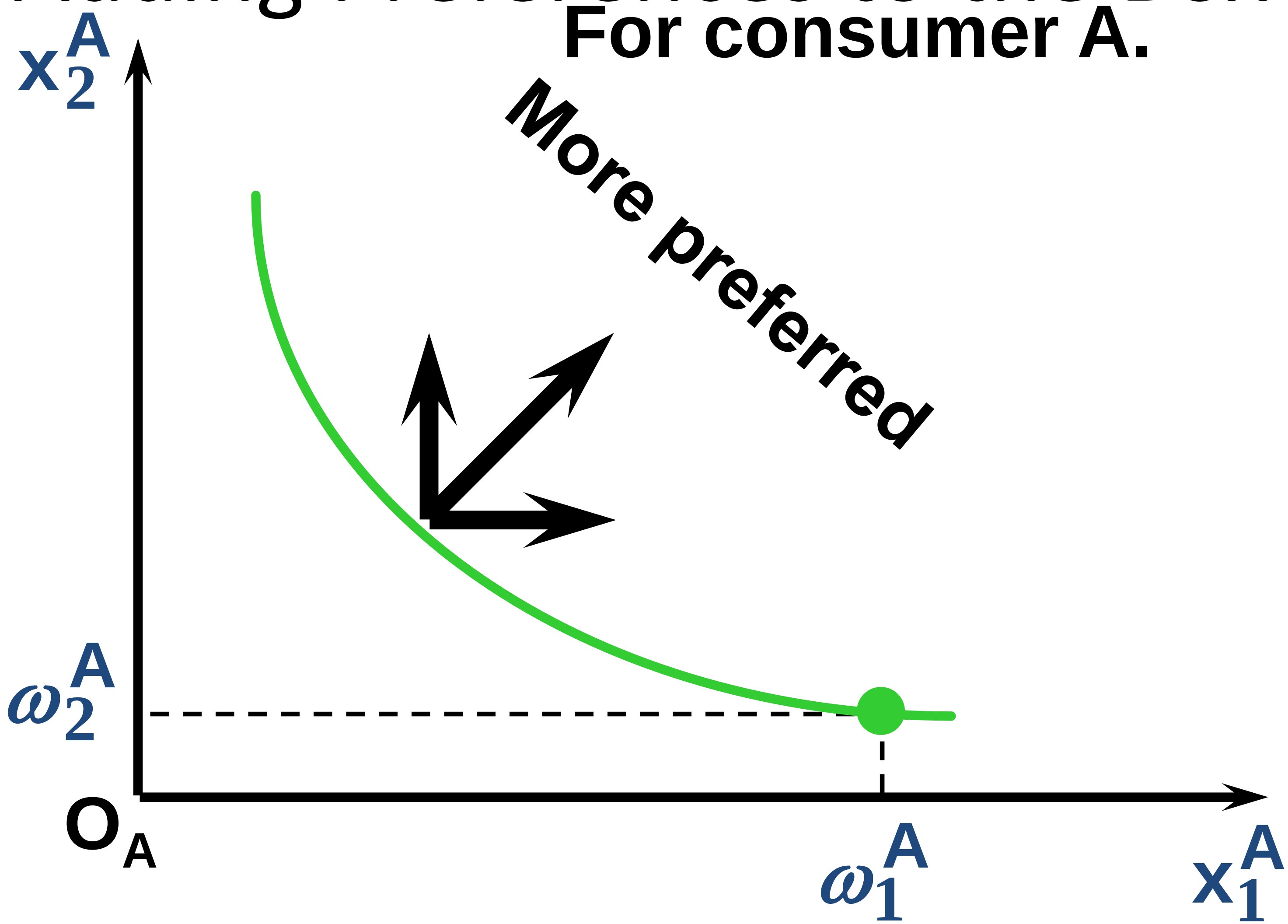
- All points in the box, including the boundary, represent feasible allocations of the combined endowments.
- Which allocations will be blocked by one or both consumers?
- Which allocations make both consumers better off?

Adding Preferences to the Box

For consumer A.

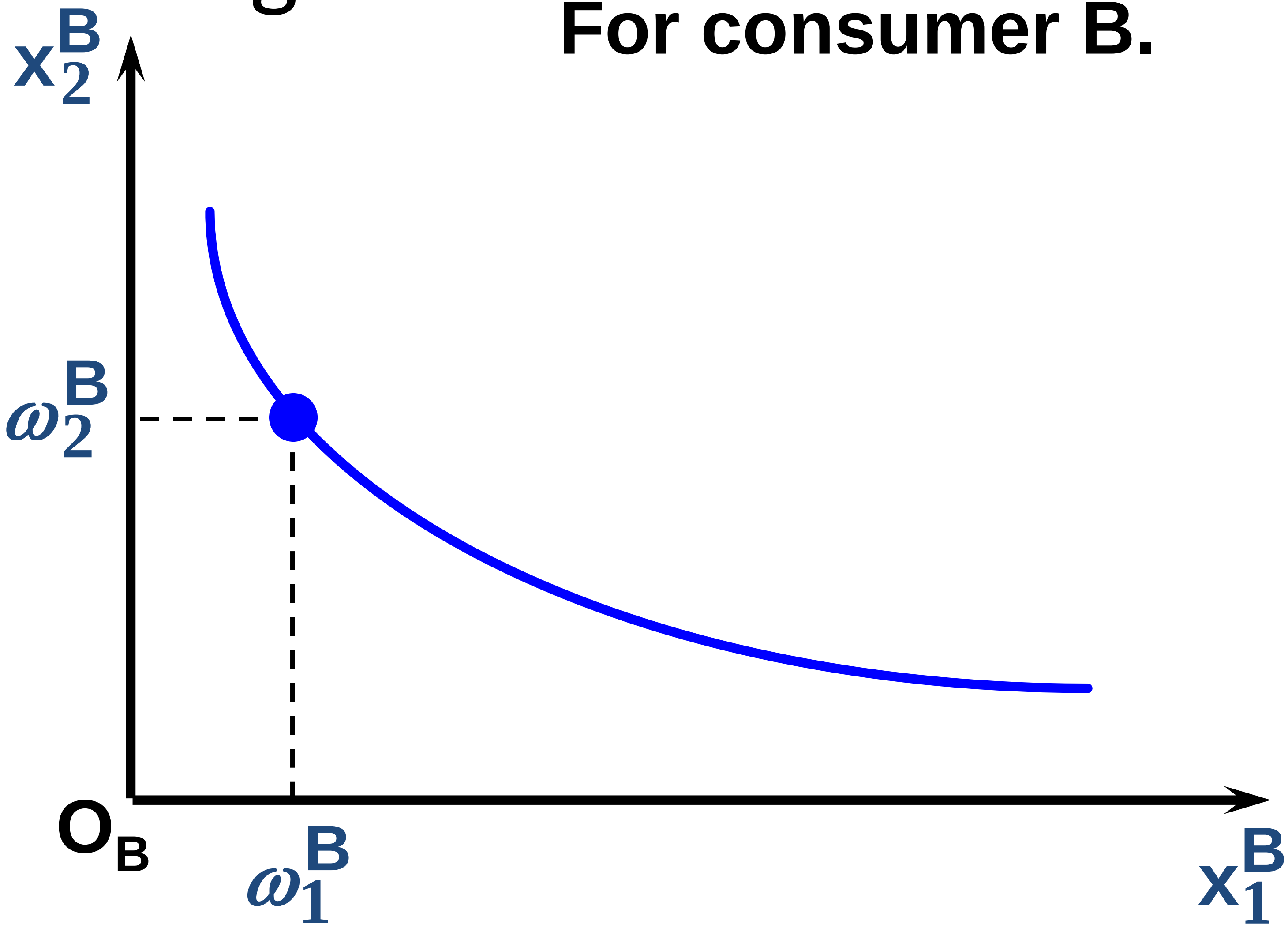


Adding Preferences to the Box

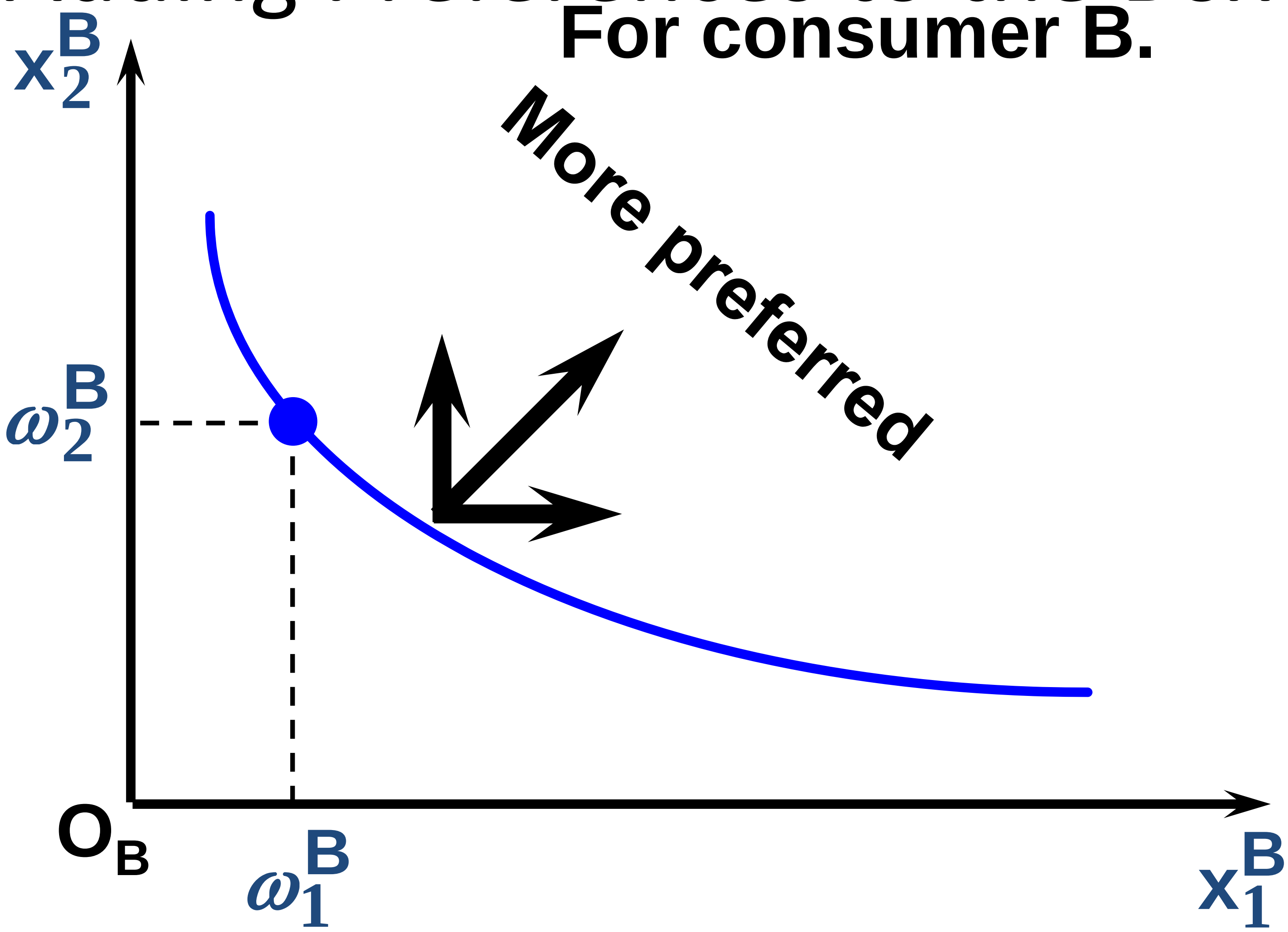


Adding Preferences to the Box

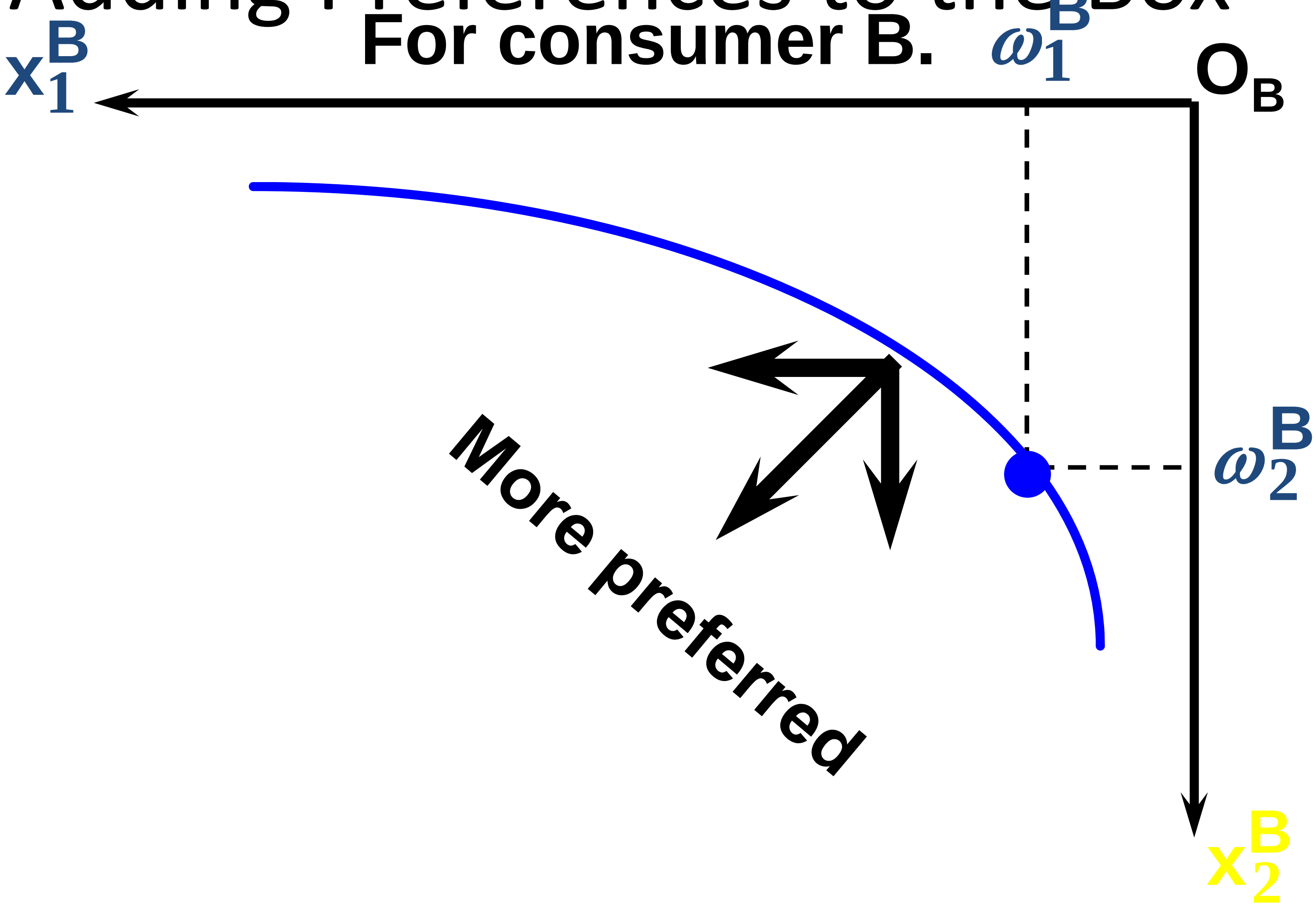
For consumer B.



Adding Preferences to the Box

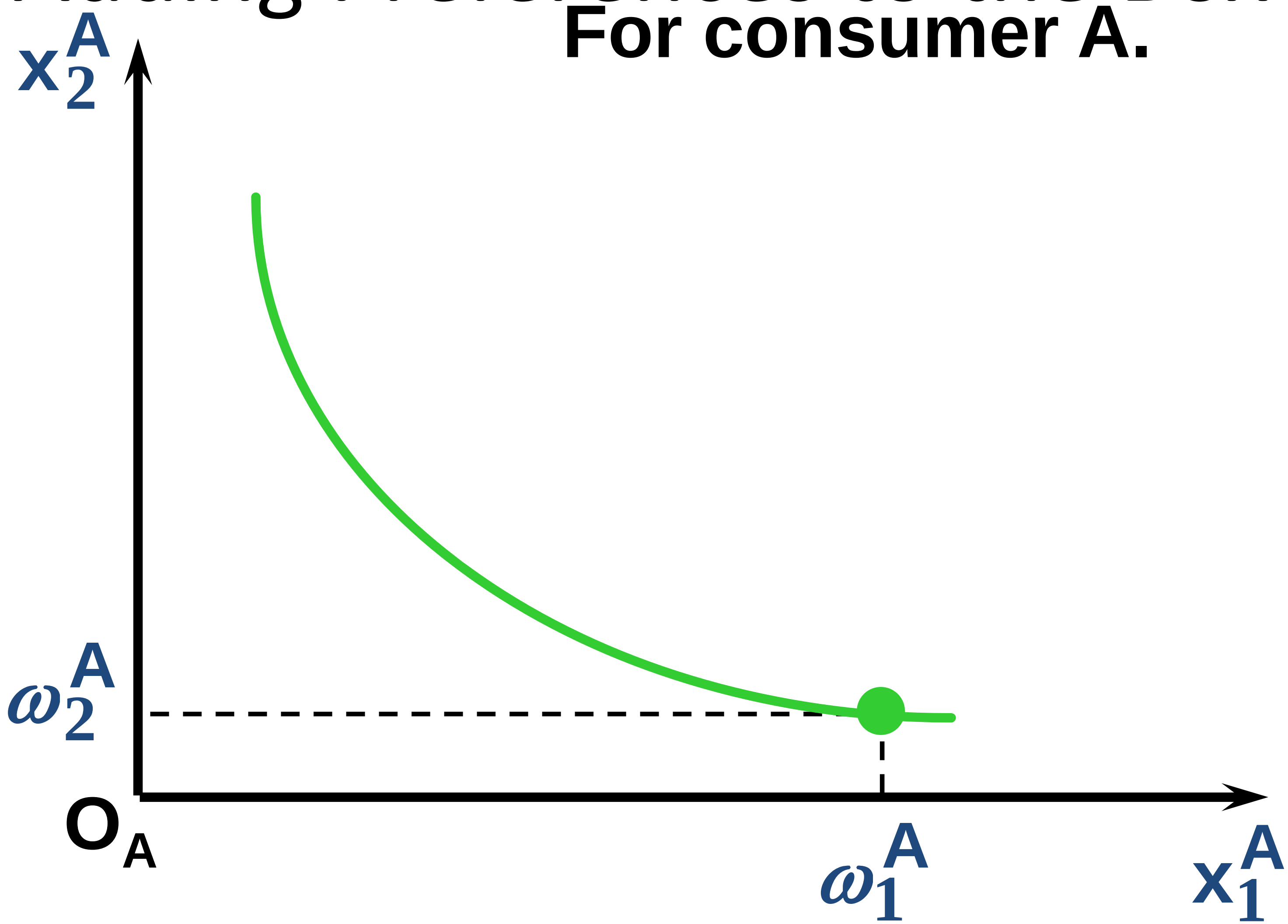


Adding Preferences to the Box

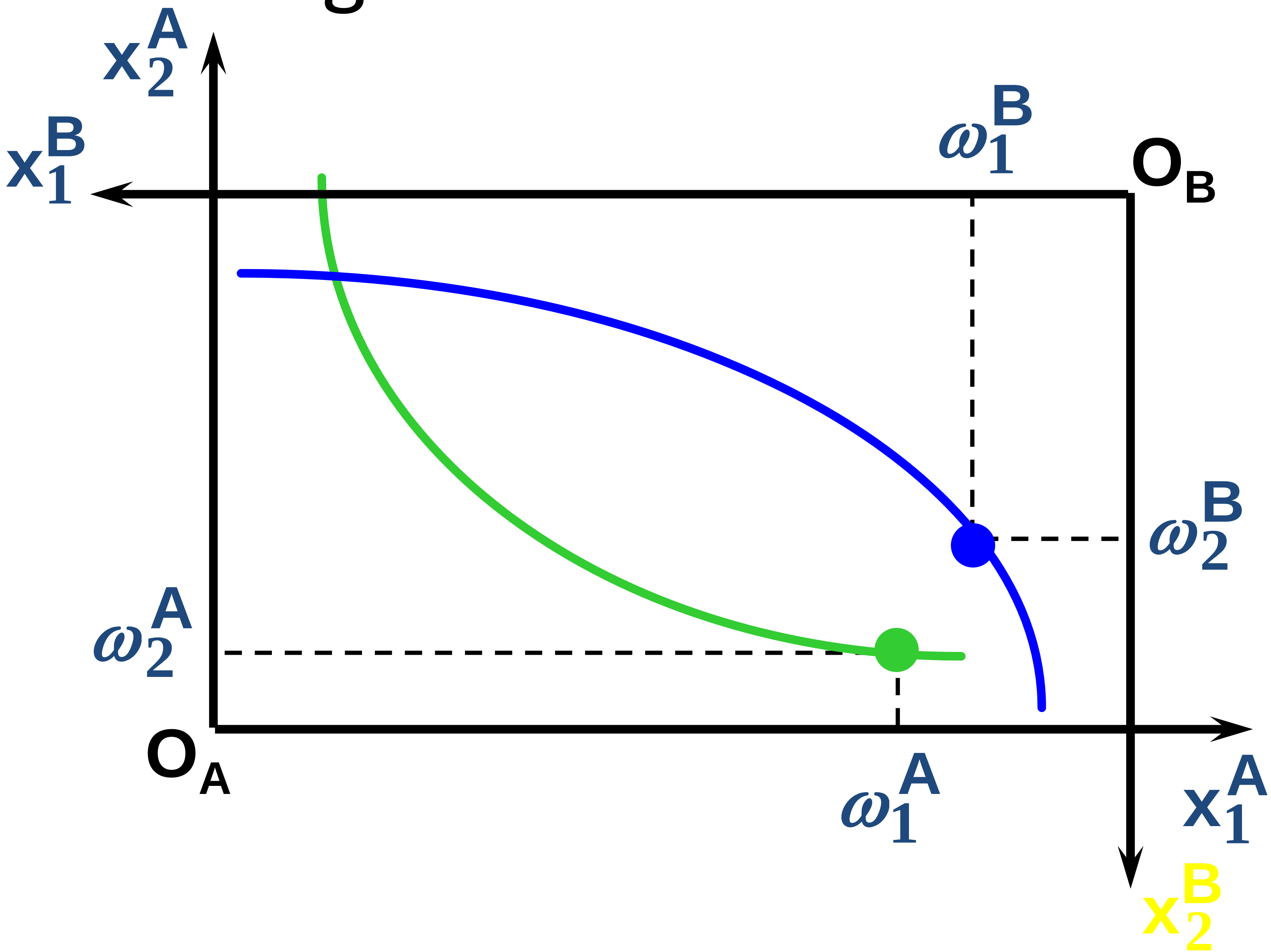


Adding Preferences to the Box

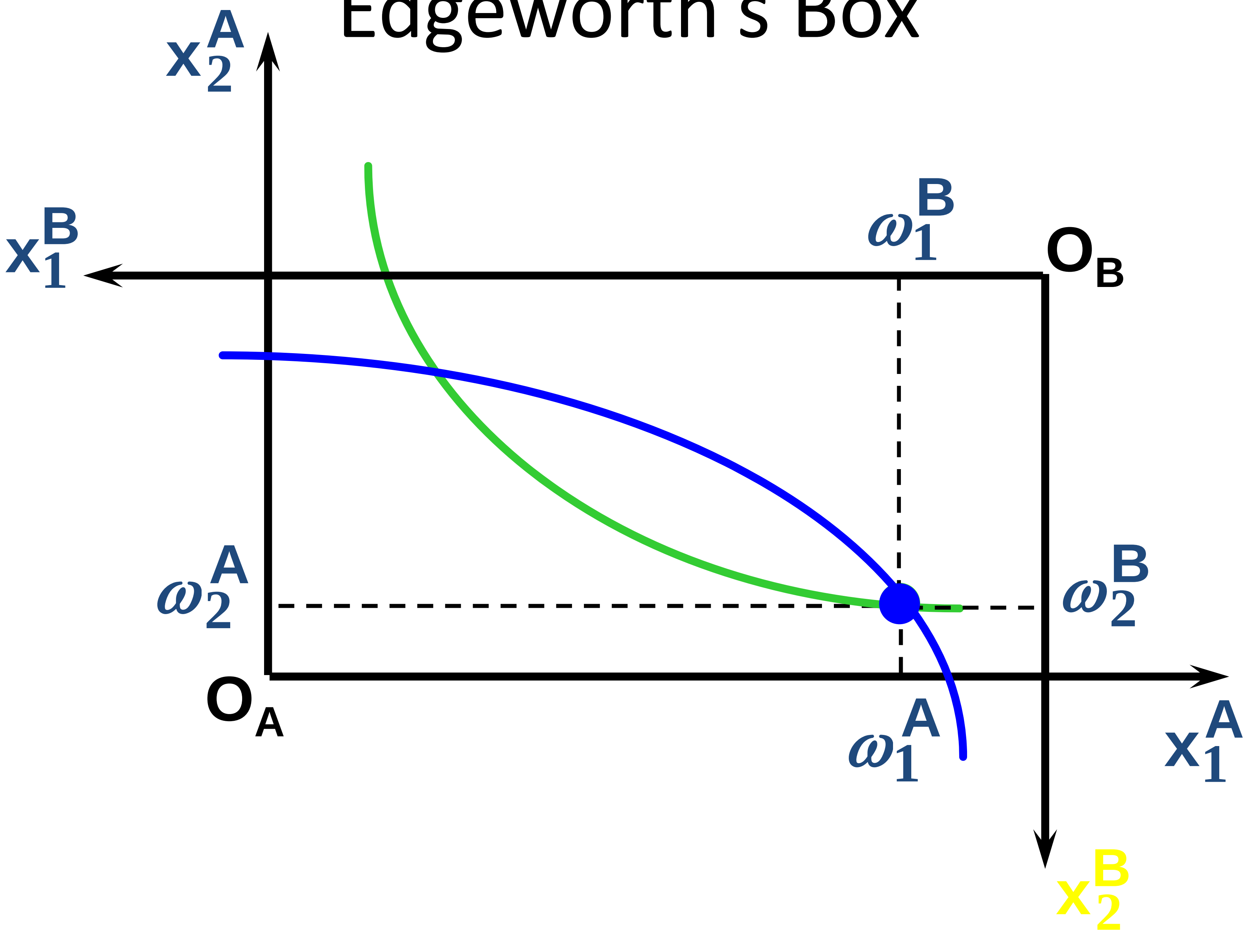
For consumer A.



Adding Preferences to the Box



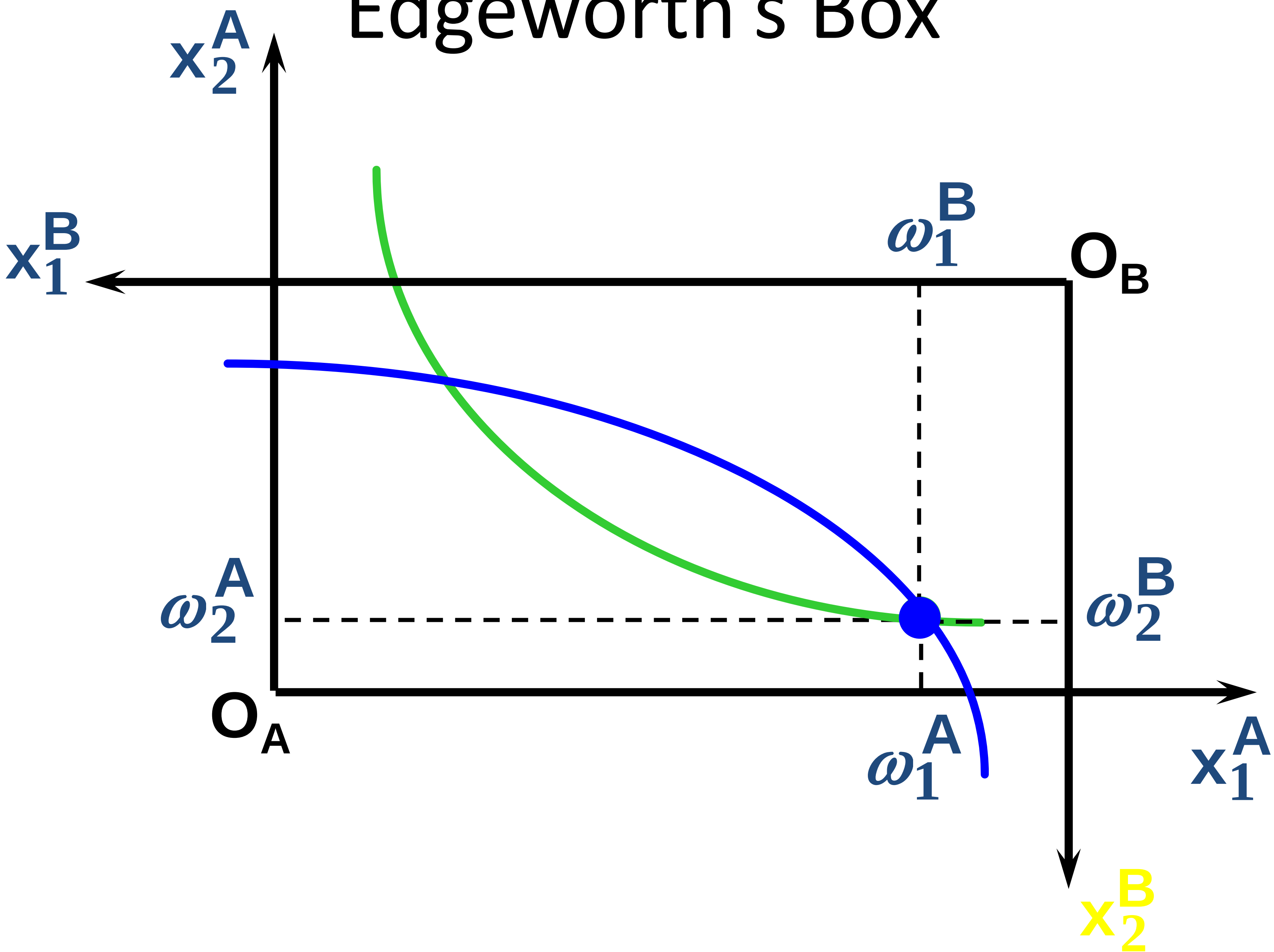
Edgeworth's Box



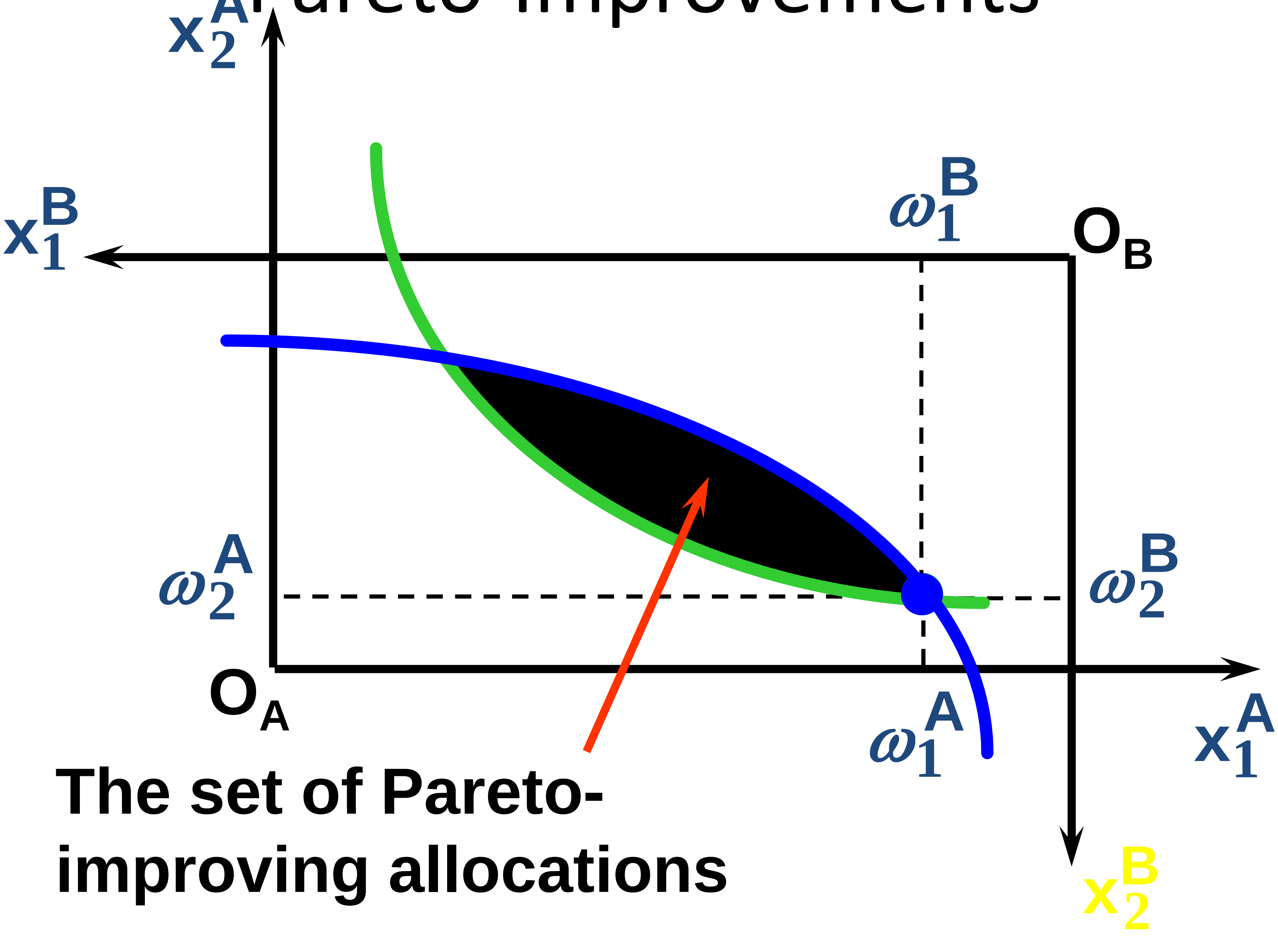
Pareto-Improvement

- An allocation of the endowment that improves the welfare of a consumer without reducing the welfare of another is a Pareto-improving allocation.
- Where are the Pareto-improving allocations?

Edgeworth's Box



Pareto-Improvements

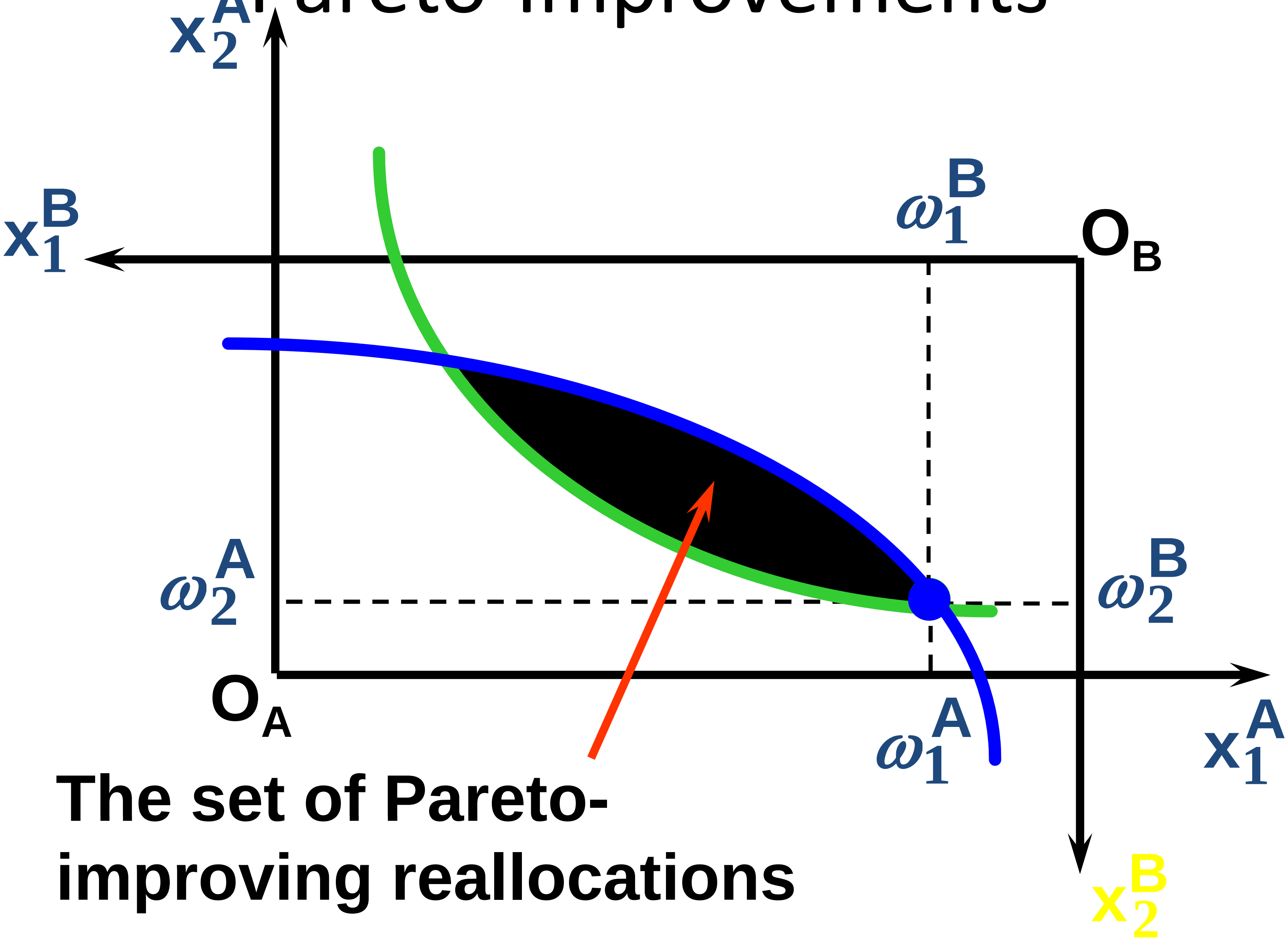


The set of Pareto-improving allocations

Pareto-Improvements

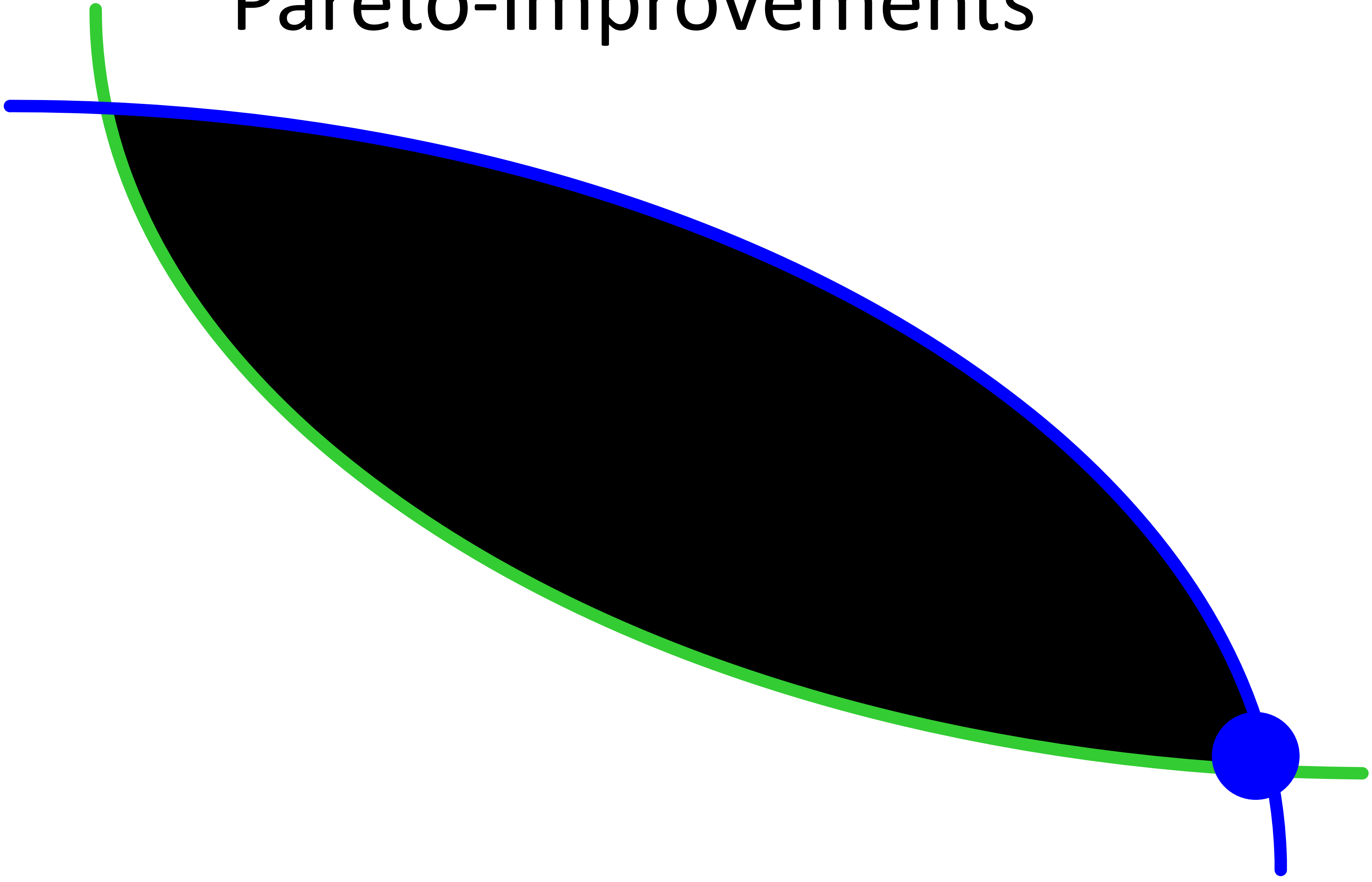
- Since each consumer can refuse to trade, the only possible outcomes from exchange are Pareto-improving allocations.
- But which particular Pareto-improving allocation will be the outcome of trade?

Pareto-Improvements

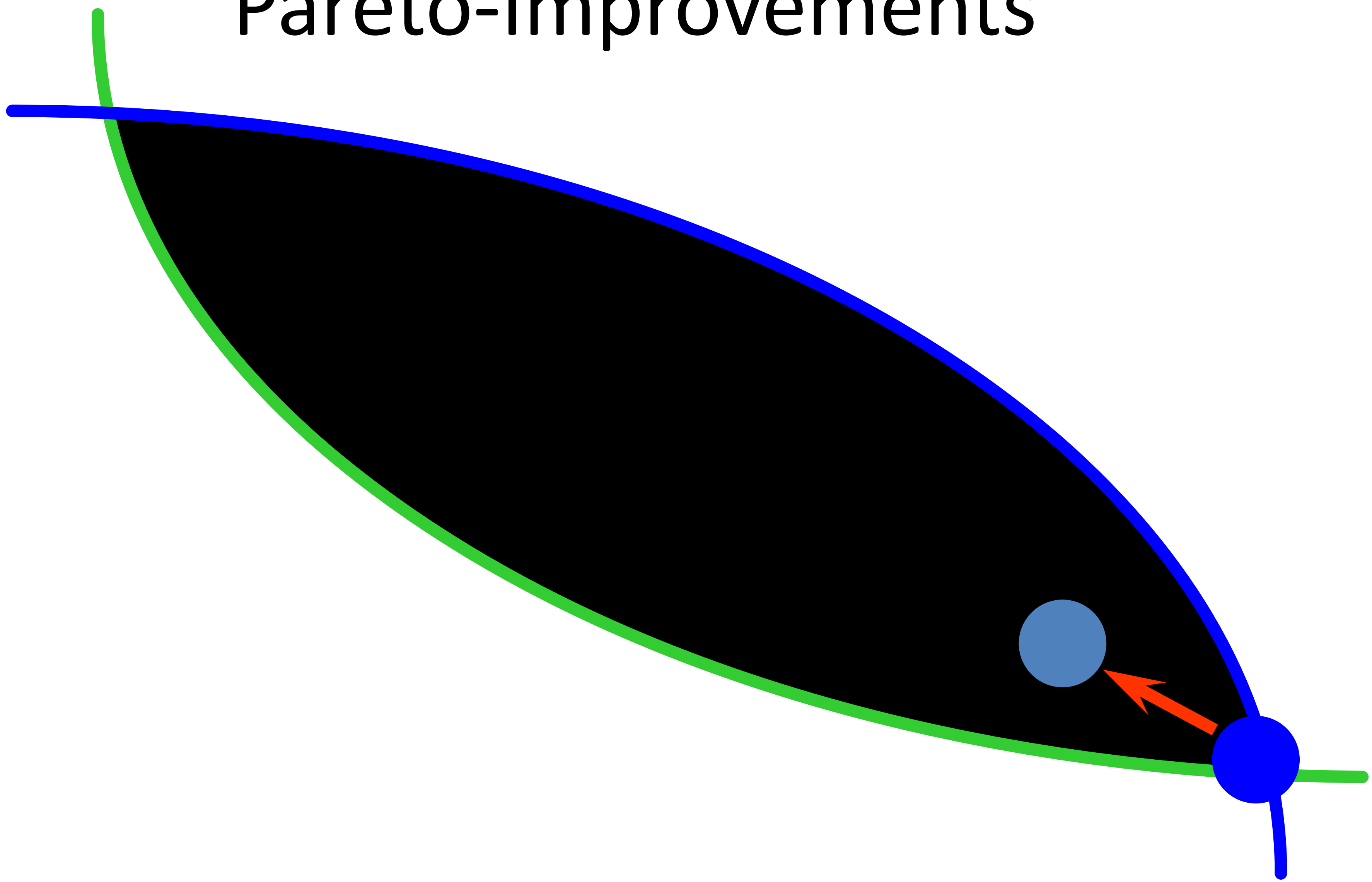


The set of Pareto-improving reallocations

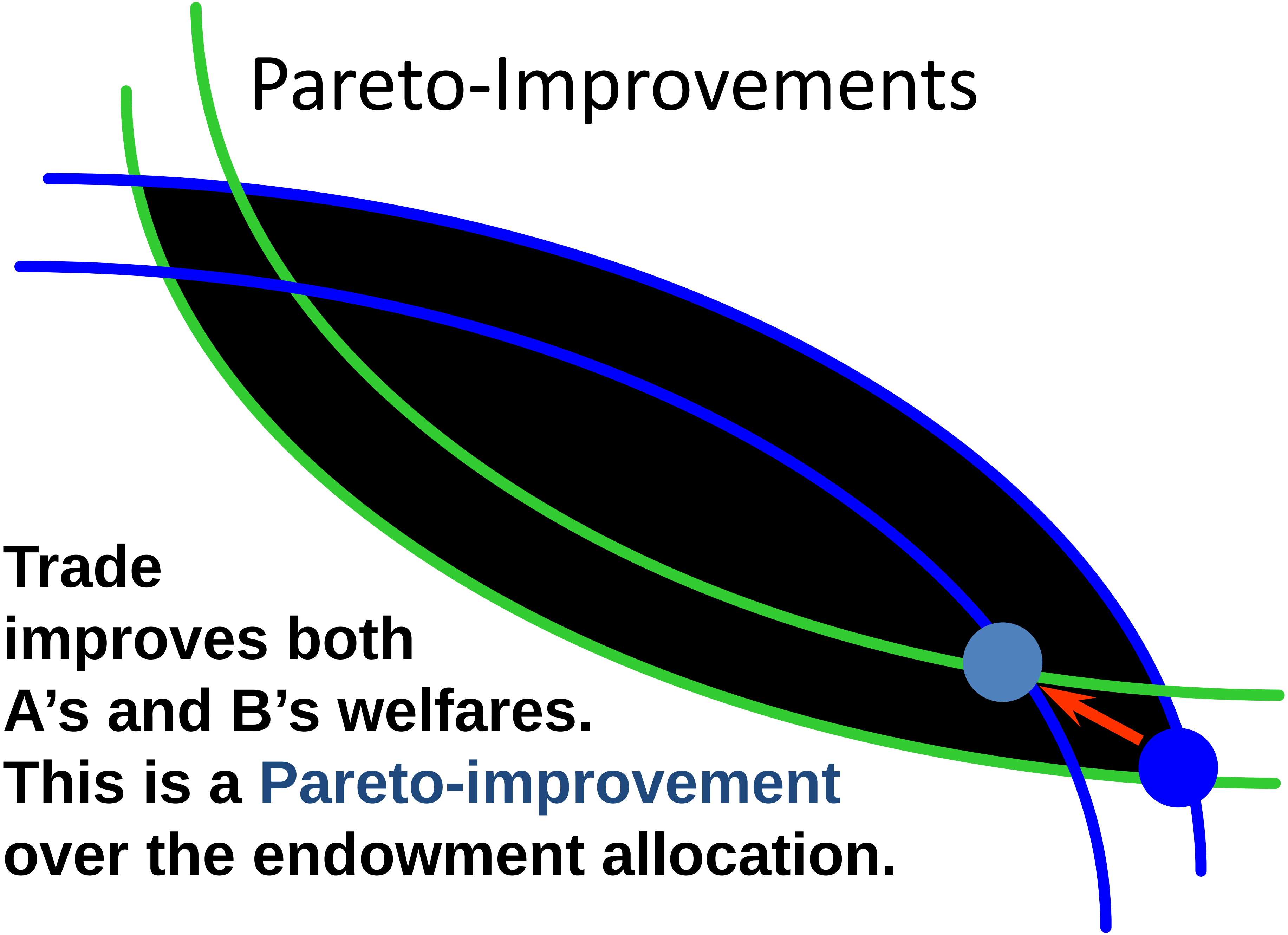
Pareto-Improvements



Pareto-Improvements

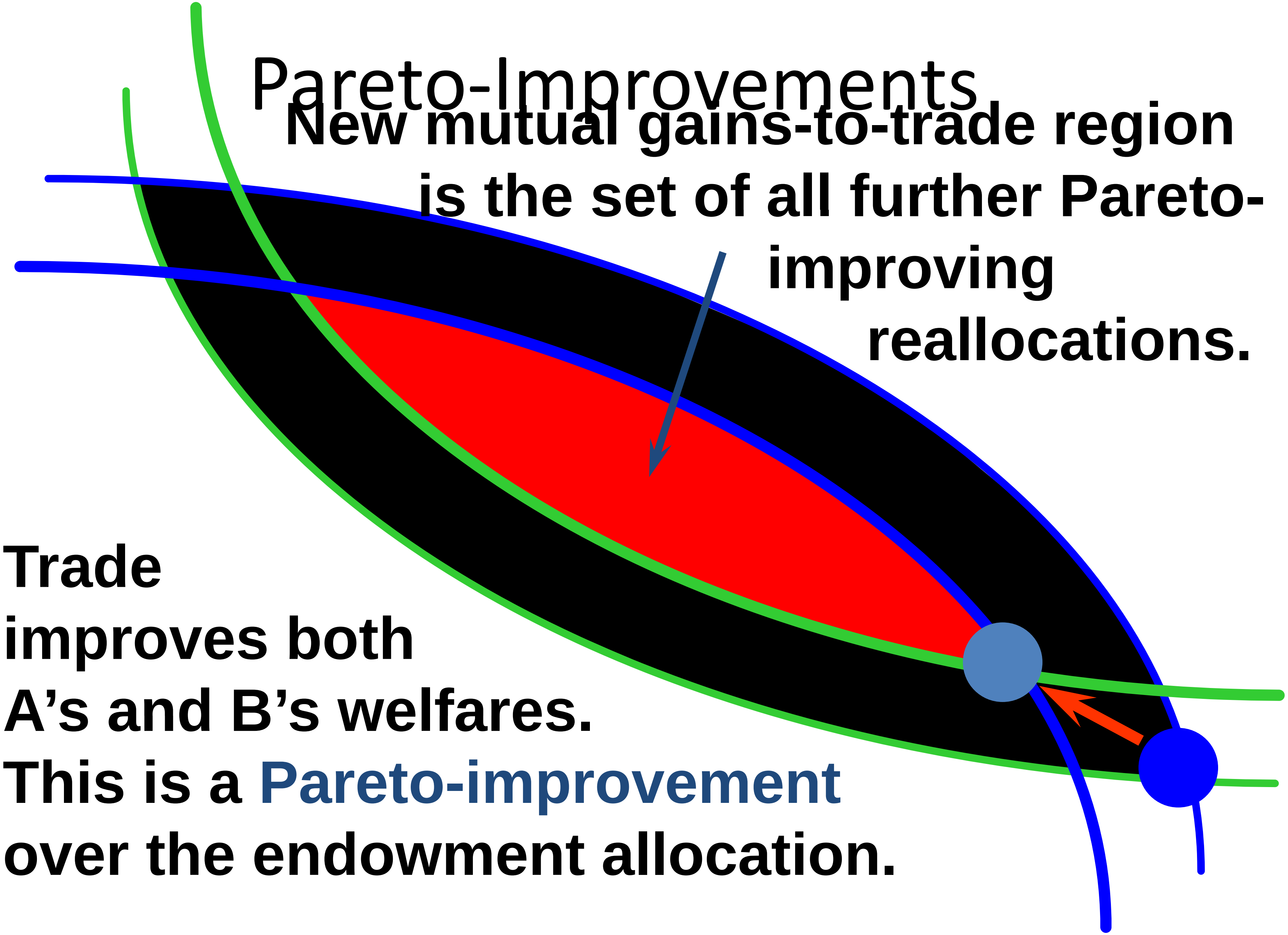


Pareto-Improvements



Trade
improves both
A's and B's welfares.
This is a Pareto-improvement
over the endowment allocation.

Pareto-Improvements

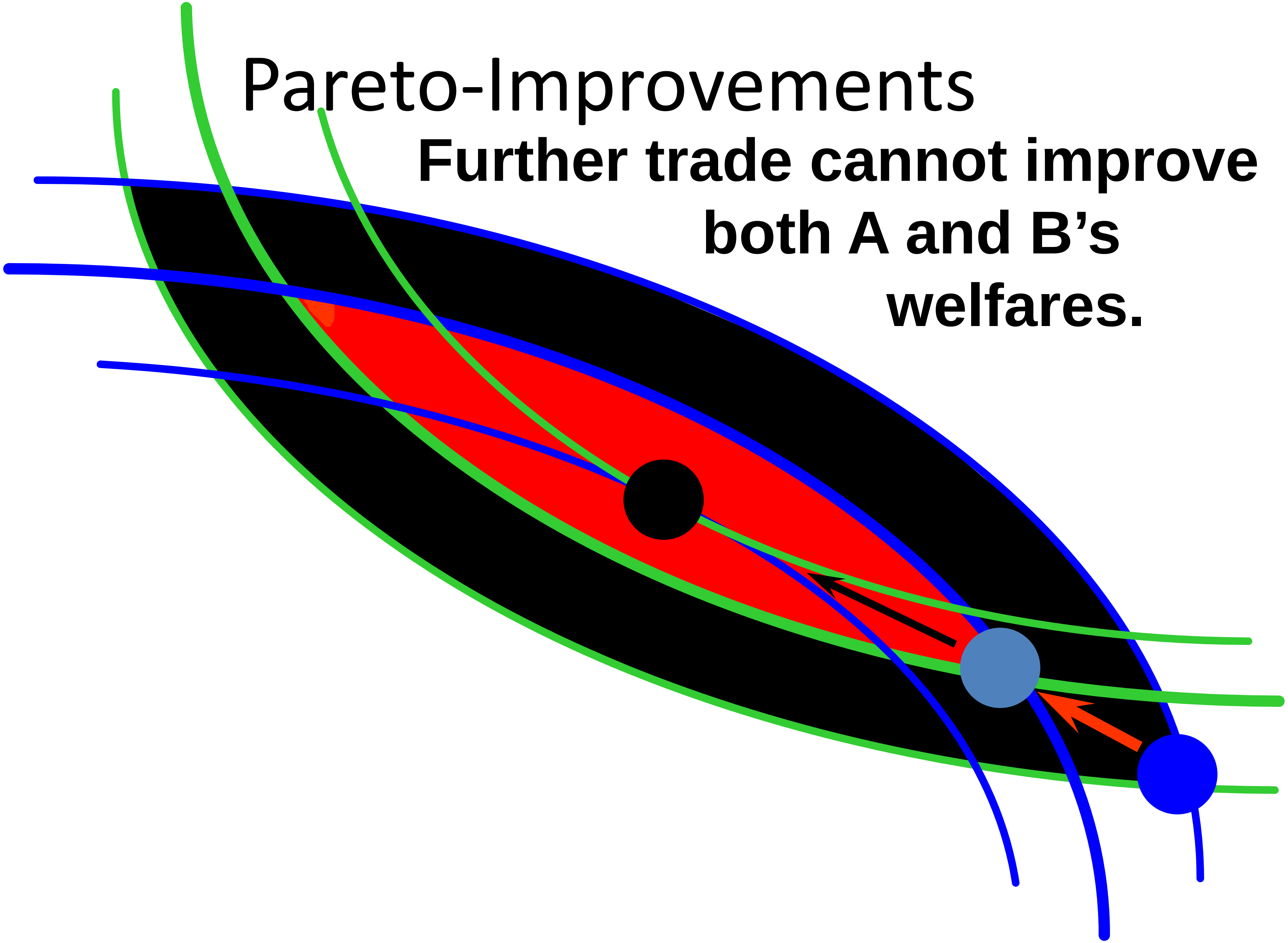


New mutual gains-to-trade region is the set of all further Pareto-improving reallocations.

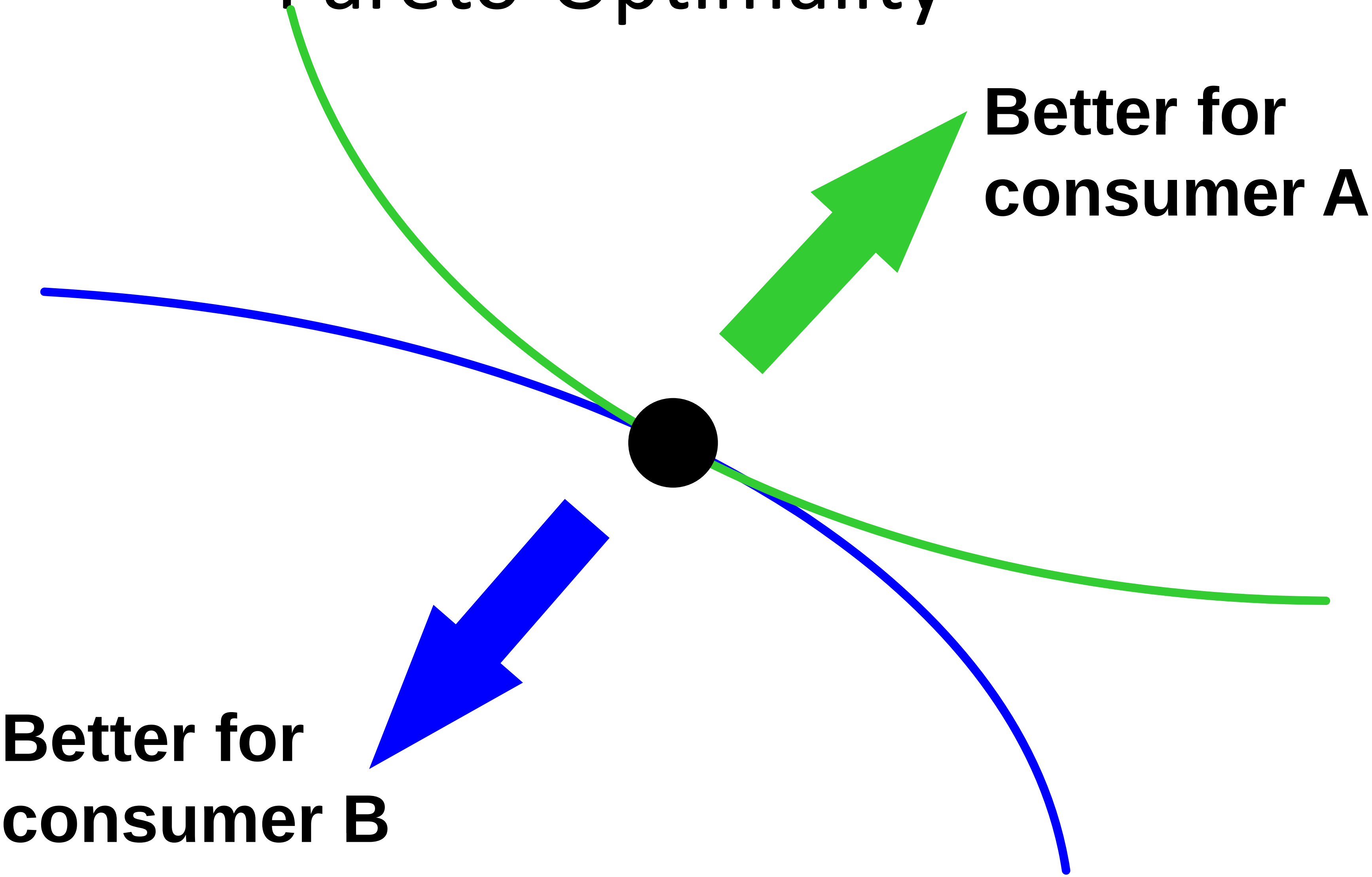
Trade improves both A's and B's welfares. This is a Pareto-improvement over the endowment allocation.

Pareto-Improvements

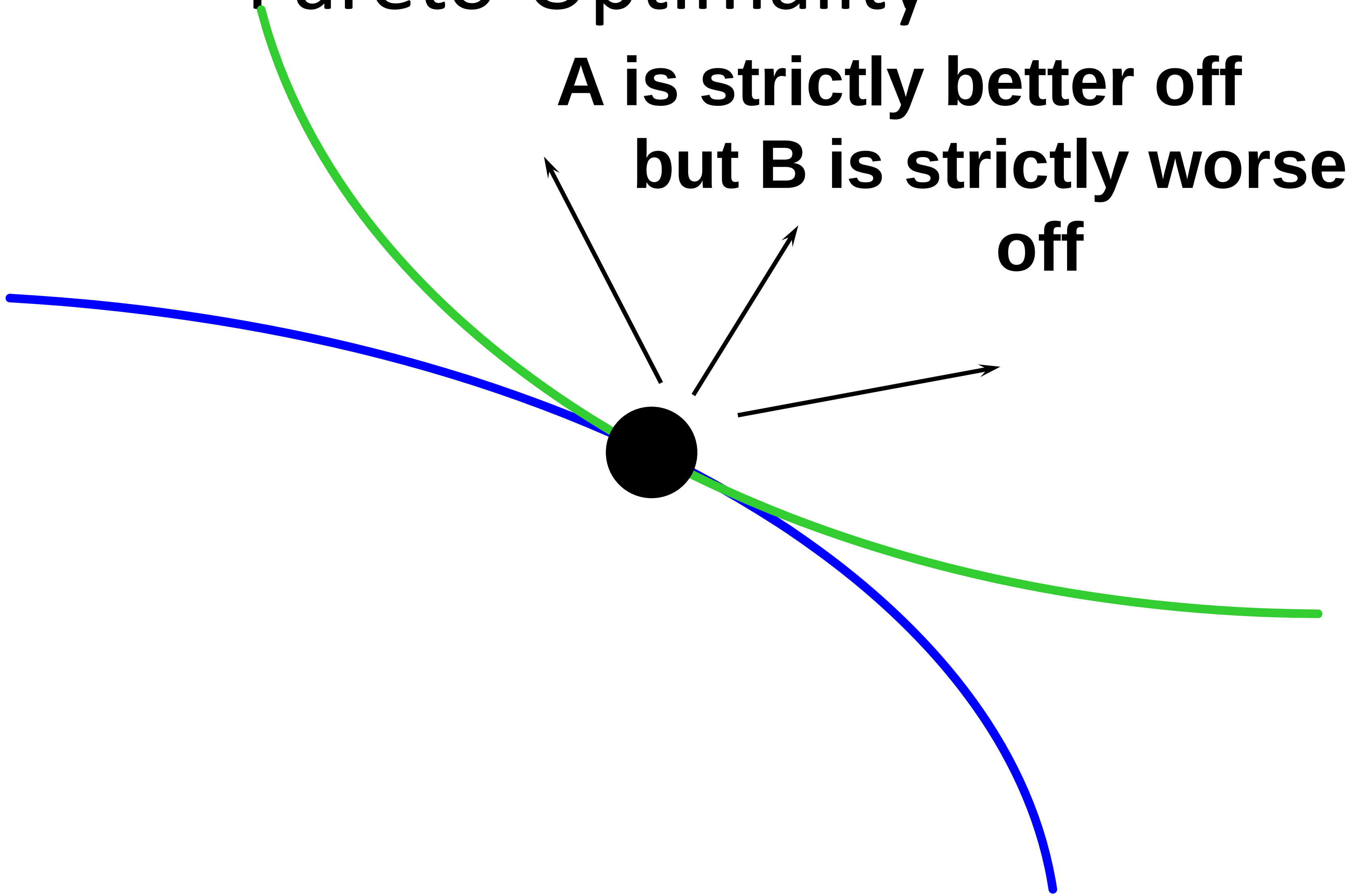
Further trade cannot improve both A and B's welfares.



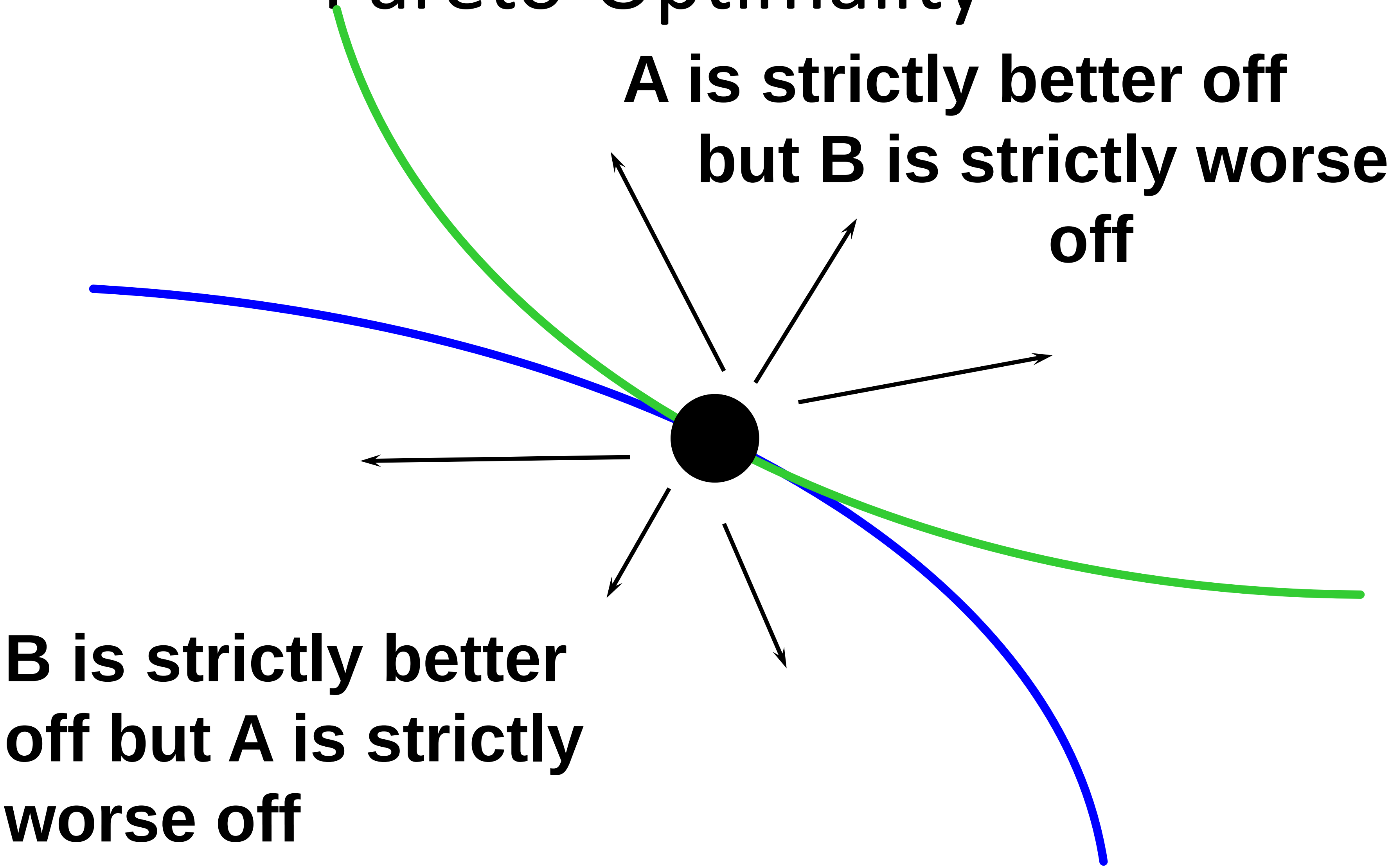
Pareto-Optimality



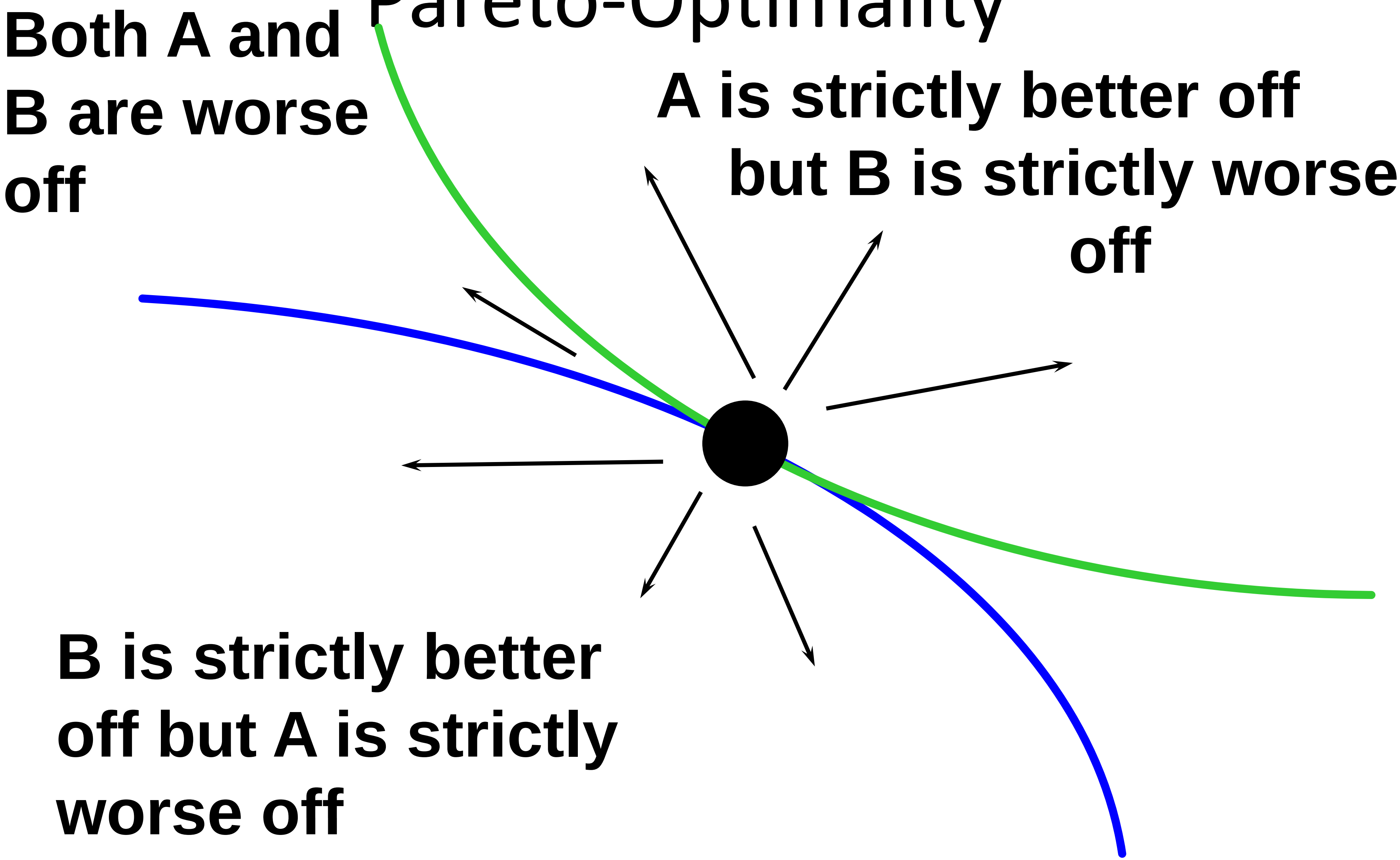
Pareto-Optimality



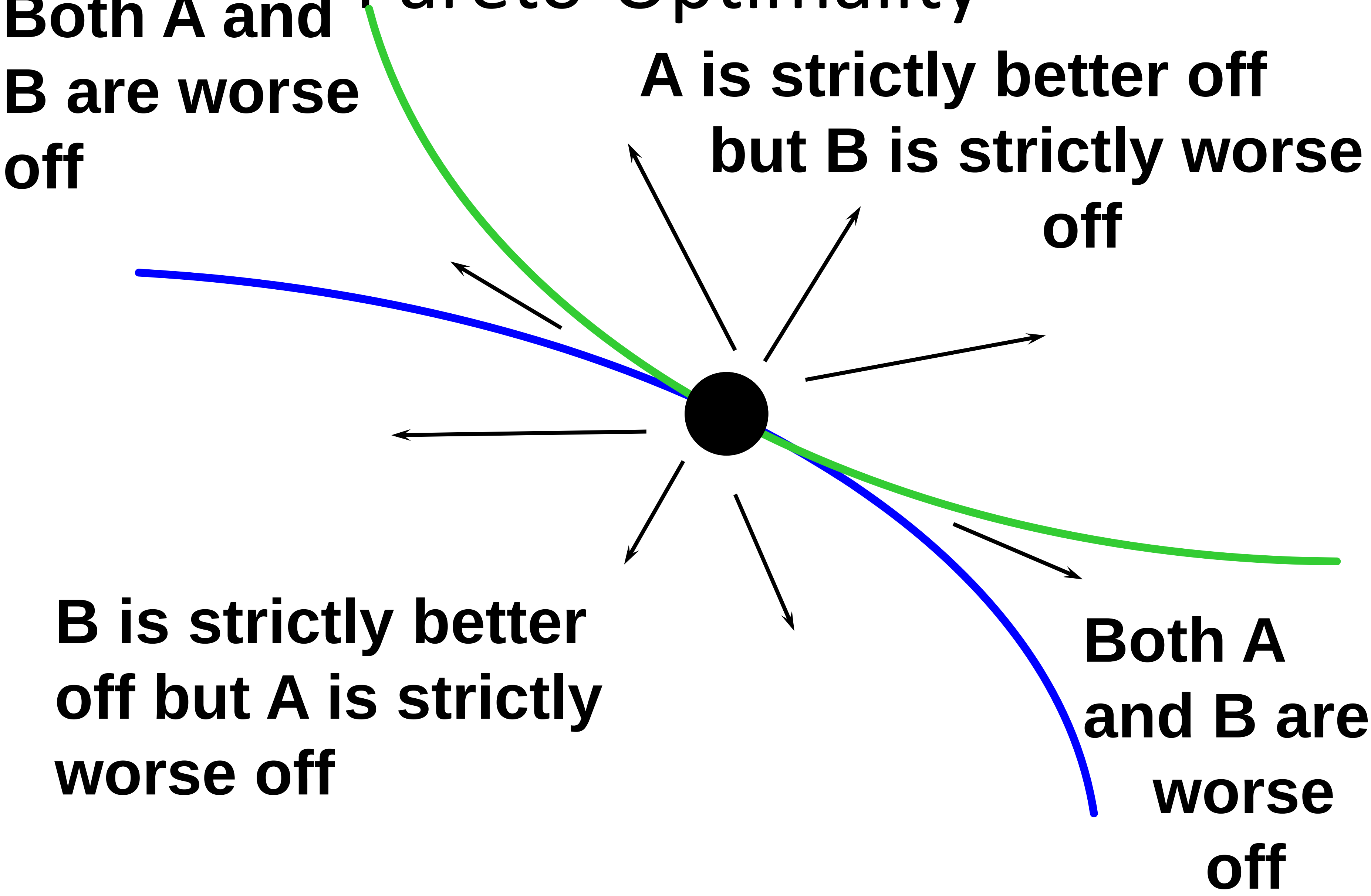
Pareto-Optimality



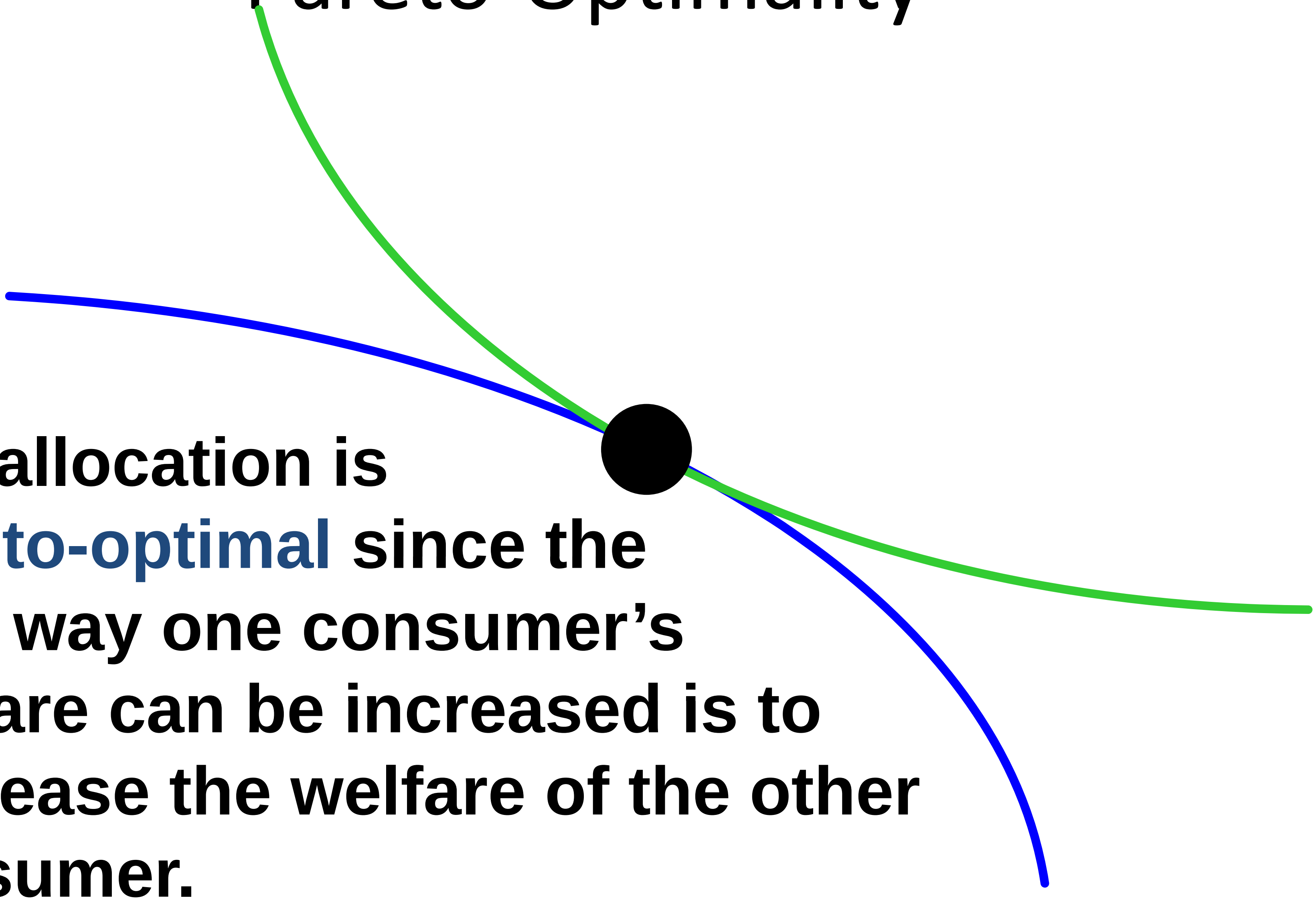
Pareto-Optimality



Pareto-Optimality



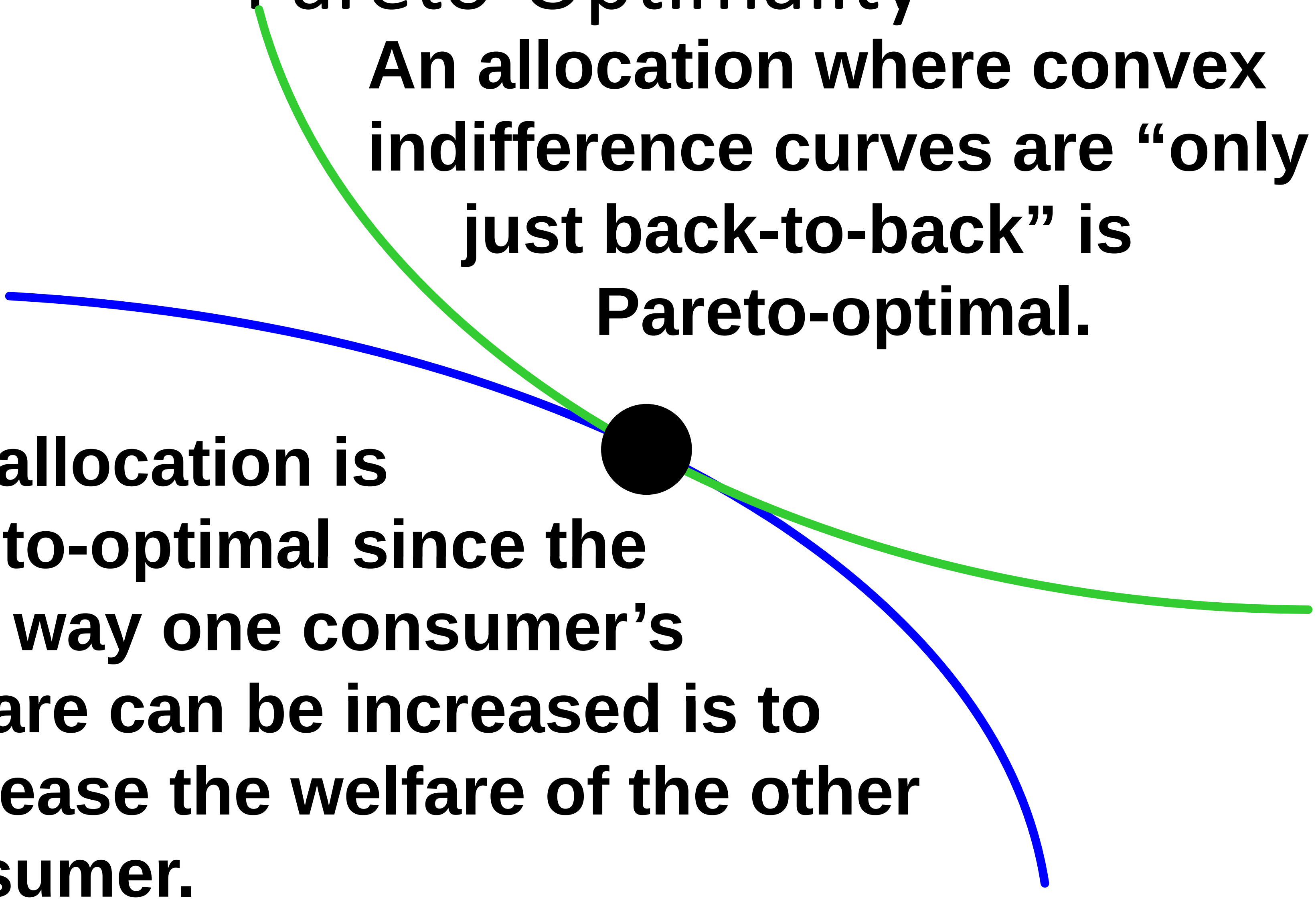
Pareto-Optimality



The allocation is Pareto-optimal since the only way one consumer's welfare can be increased is to decrease the welfare of the other consumer.

Pareto-Optimality

An allocation where convex indifference curves are “only just back-to-back” is Pareto-optimal.

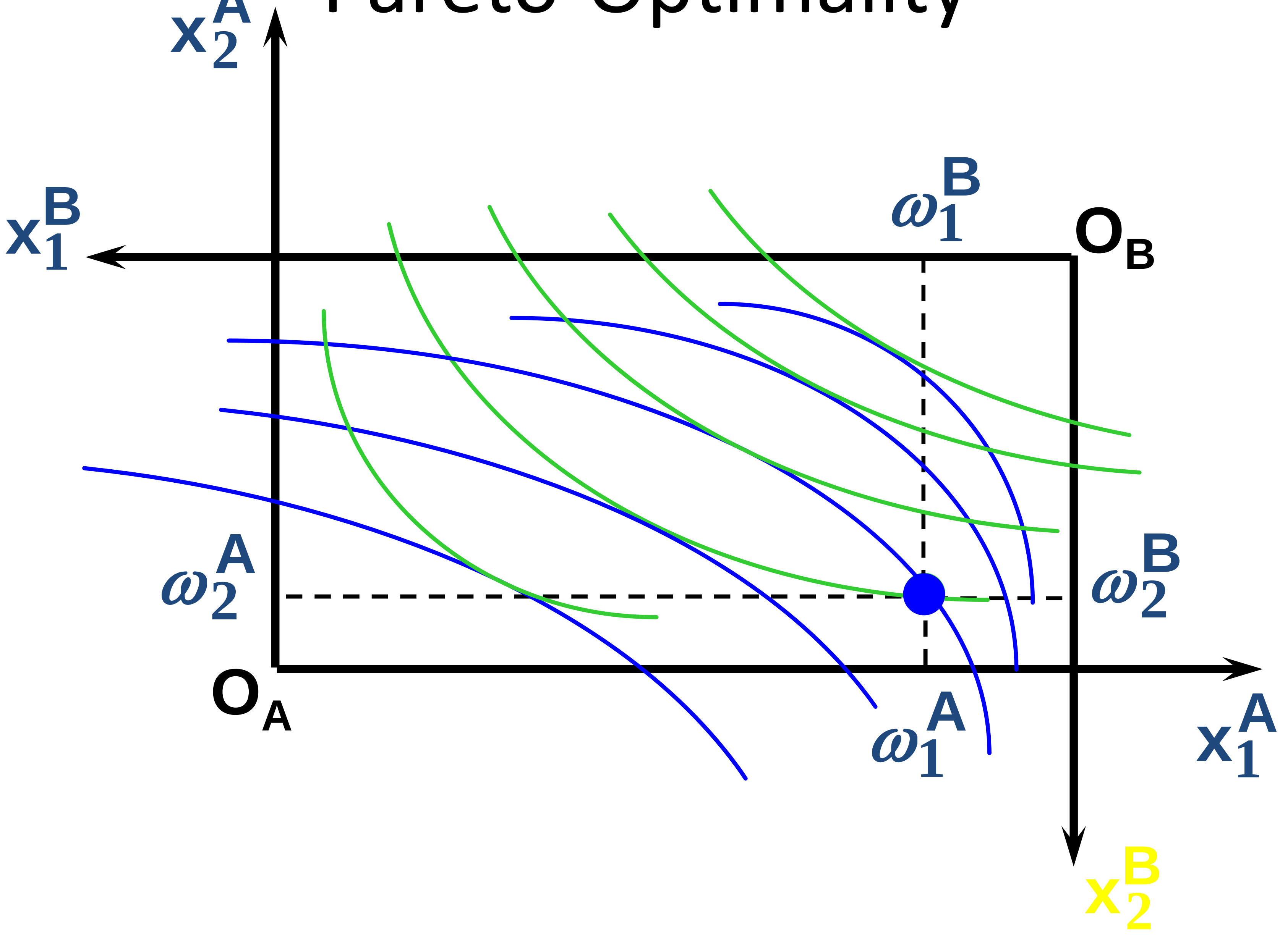


The allocation is Pareto-optimal since the only way one consumer’s welfare can be increased is to decrease the welfare of the other consumer.

Pareto-Optimality

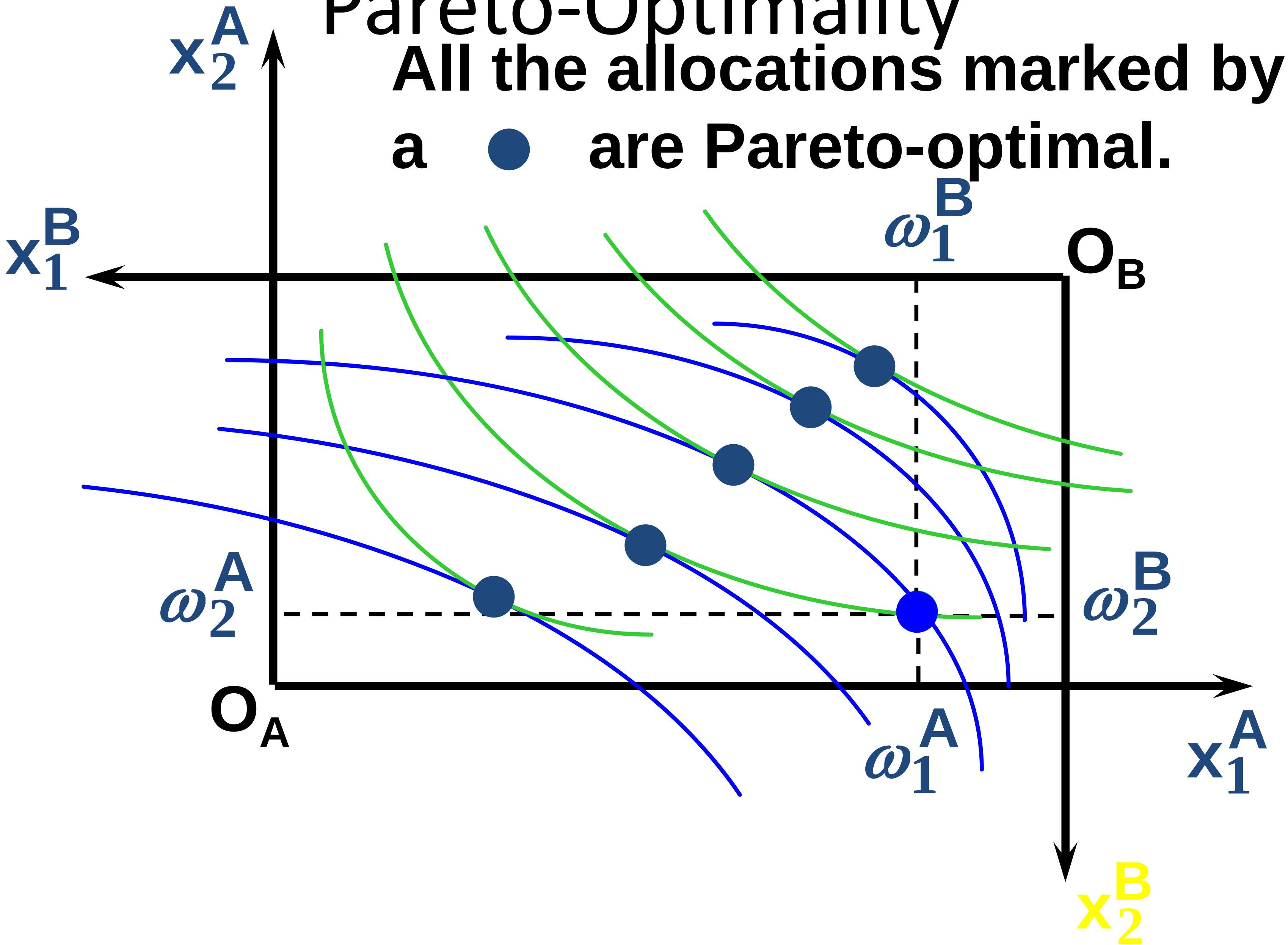
- Where are all of the Pareto-optimal allocations of the endowment?

Pareto-Optimality



Pareto-Optimality

All the allocations marked by a ● are Pareto-optimal.

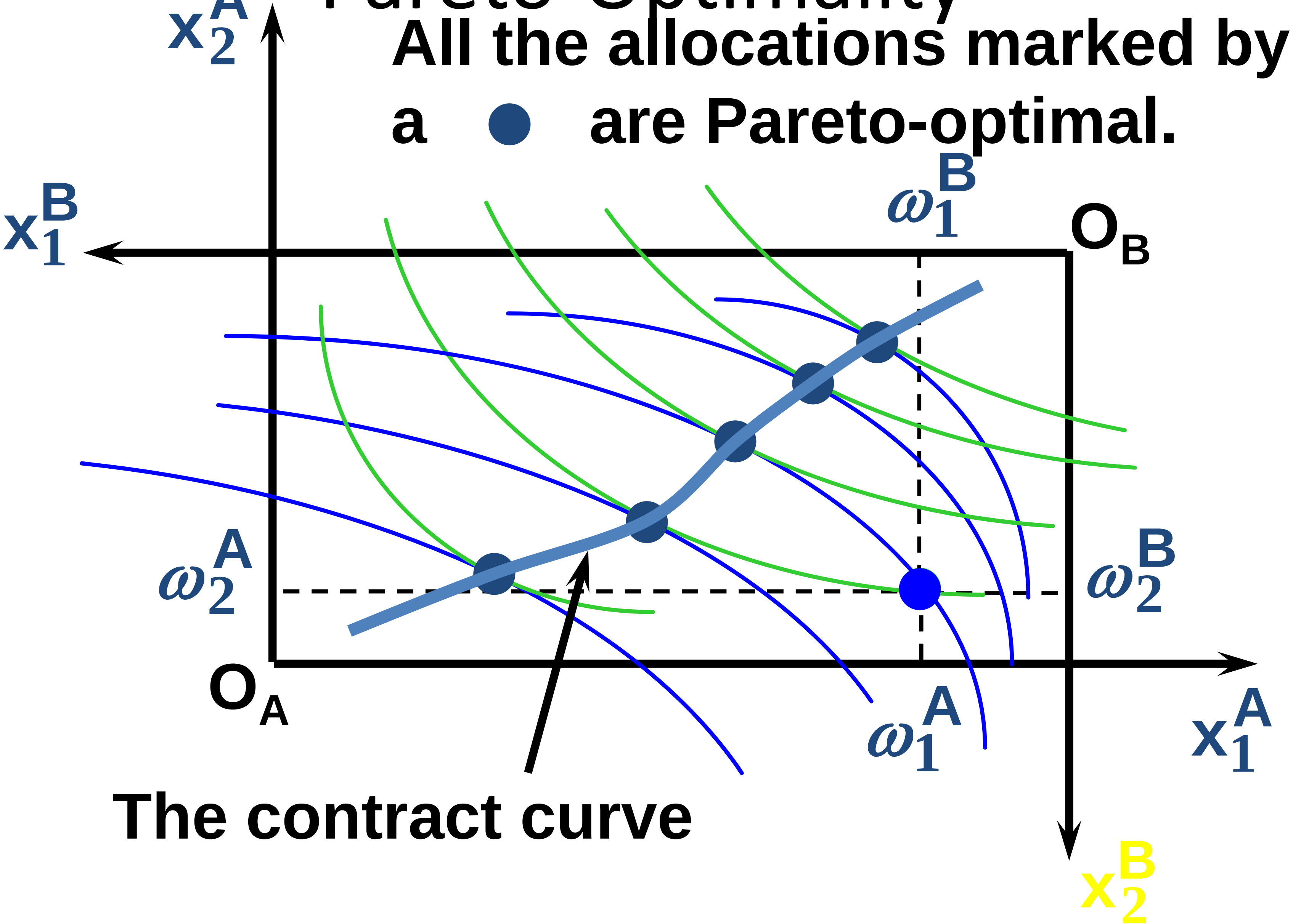


Pareto-Optimality

- The contract curve is the set of all Pareto-optimal allocations.

Pareto-Optimality

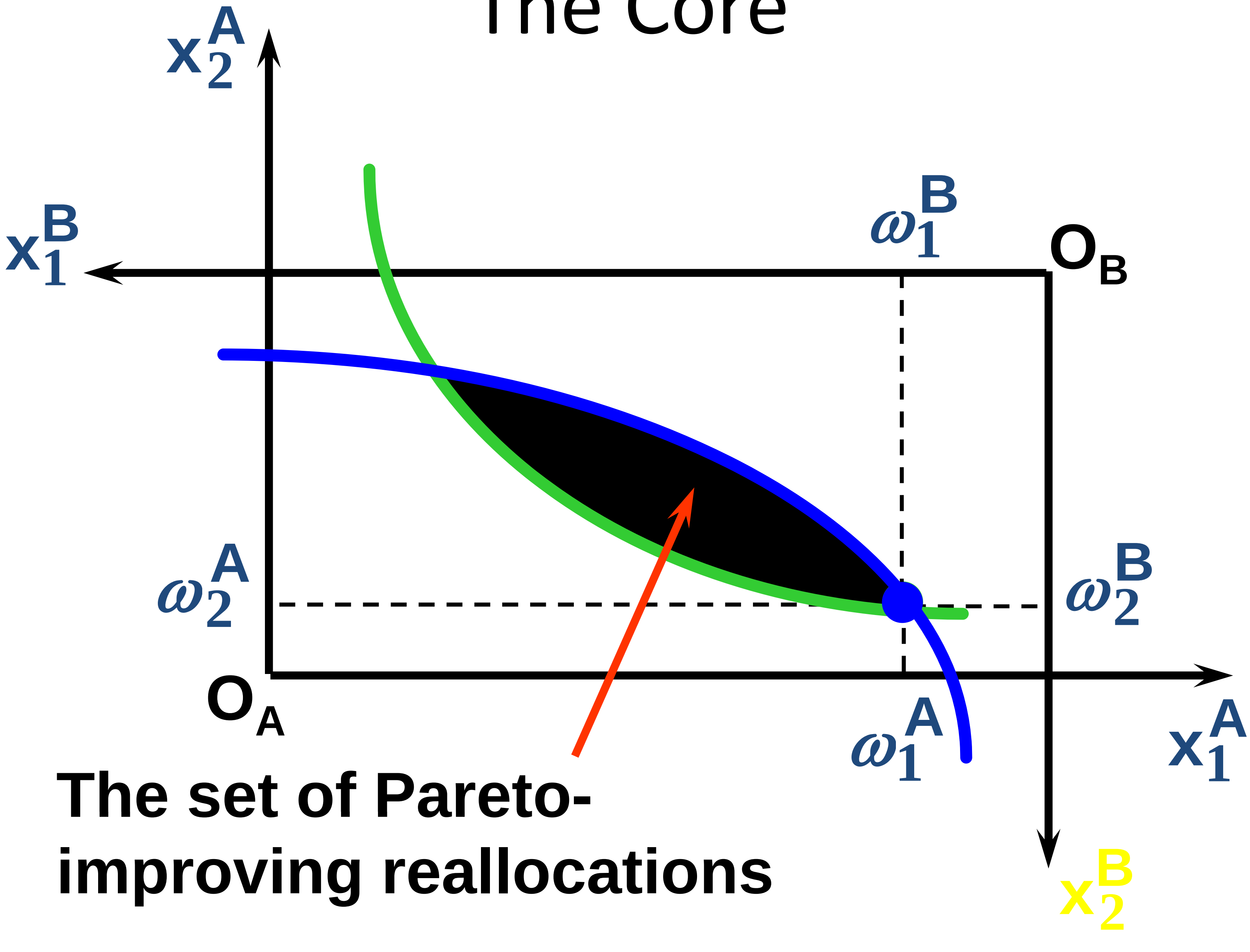
All the allocations marked by a ● are Pareto-optimal.



Pareto-Optimality

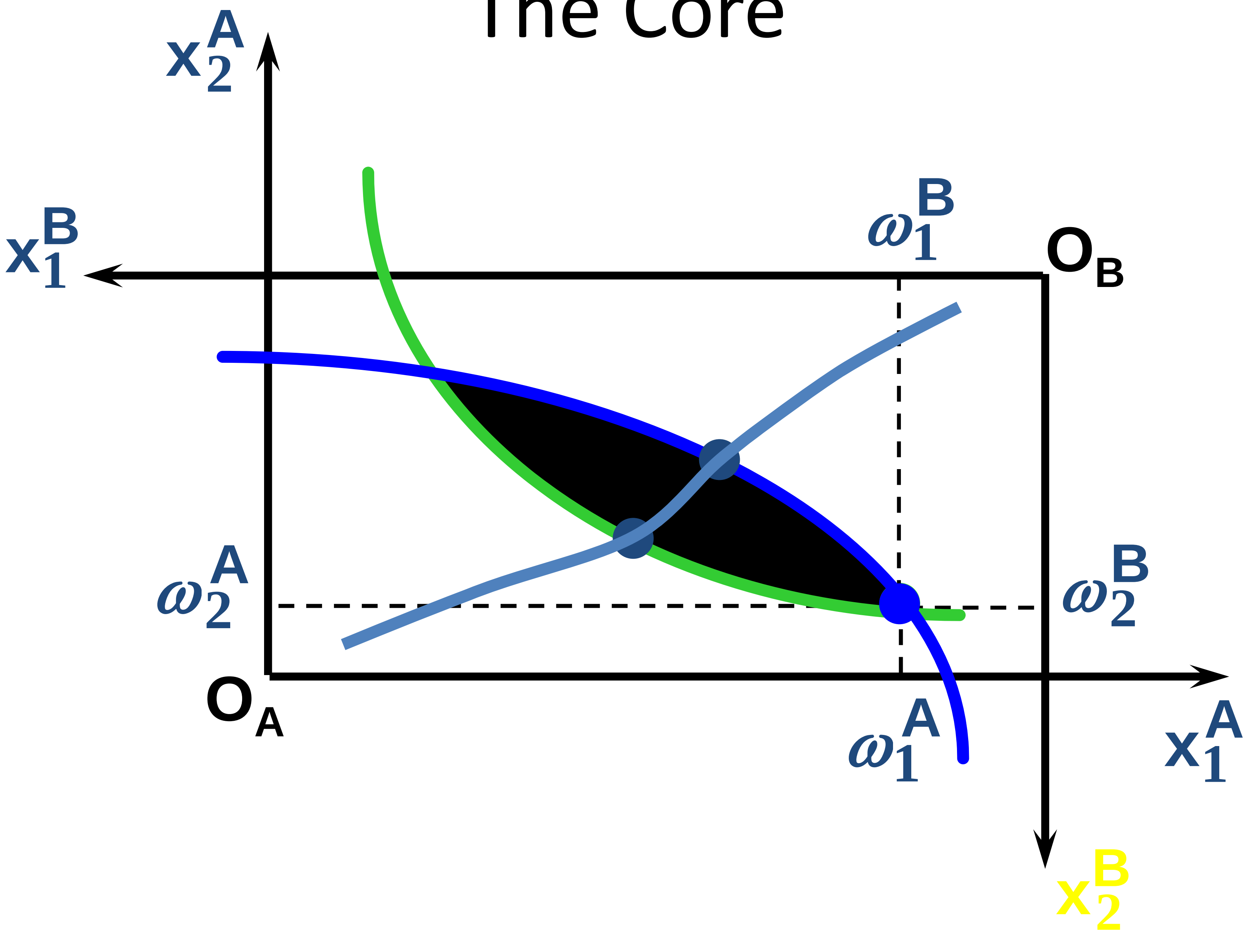
- But to which of the many allocations on the contract curve will consumers trade?
- That depends upon how trade is conducted.
- In perfectly competitive markets? By one-on-one bargaining?

The Core

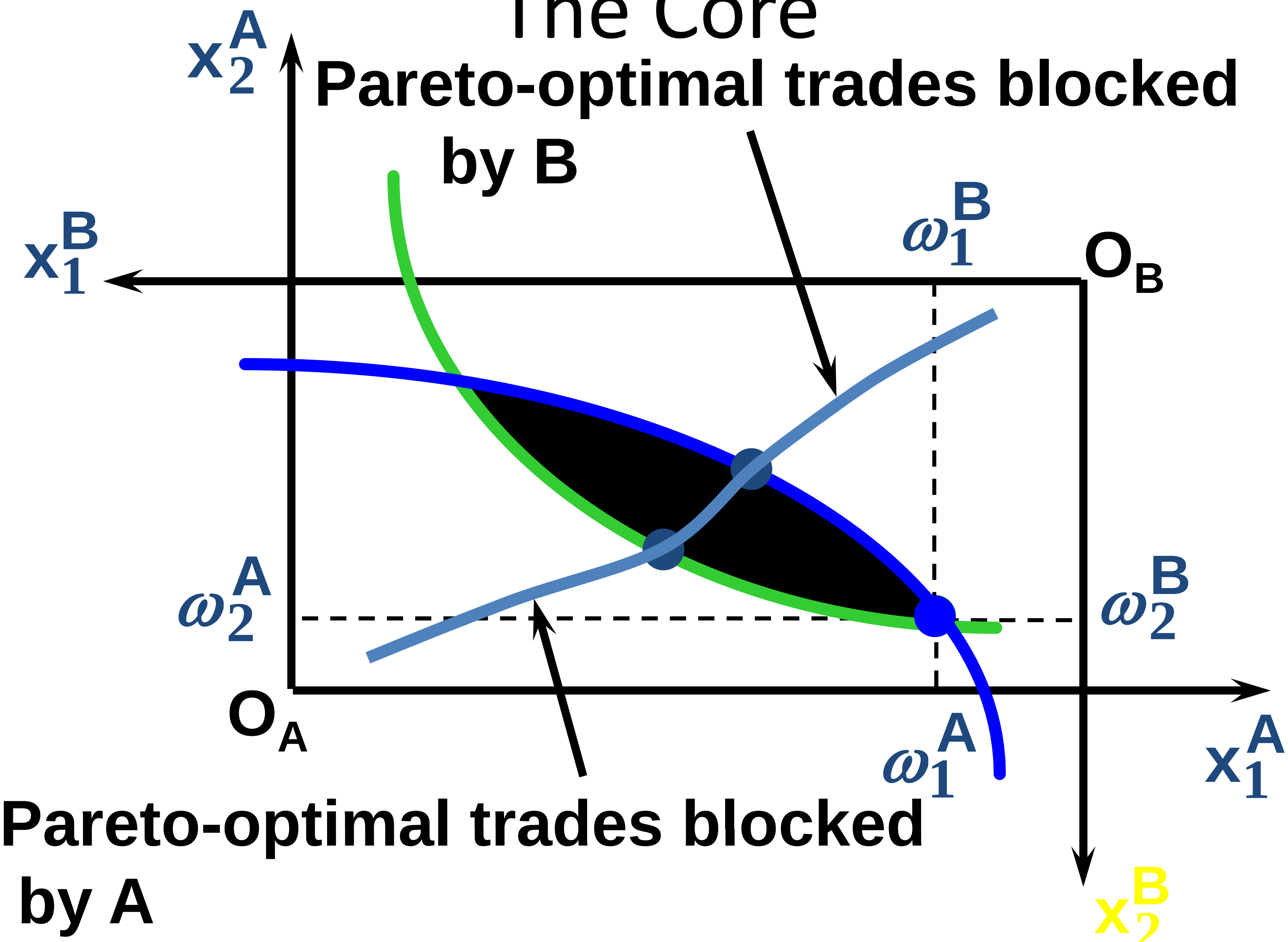


x_2^A
x_1^B
ω_1^B
O_B
ω_2^A
ω_2^B
O_A
ω_1^A
x_1^A
x_2^B
The set of Pareto-improving reallocations

The Core

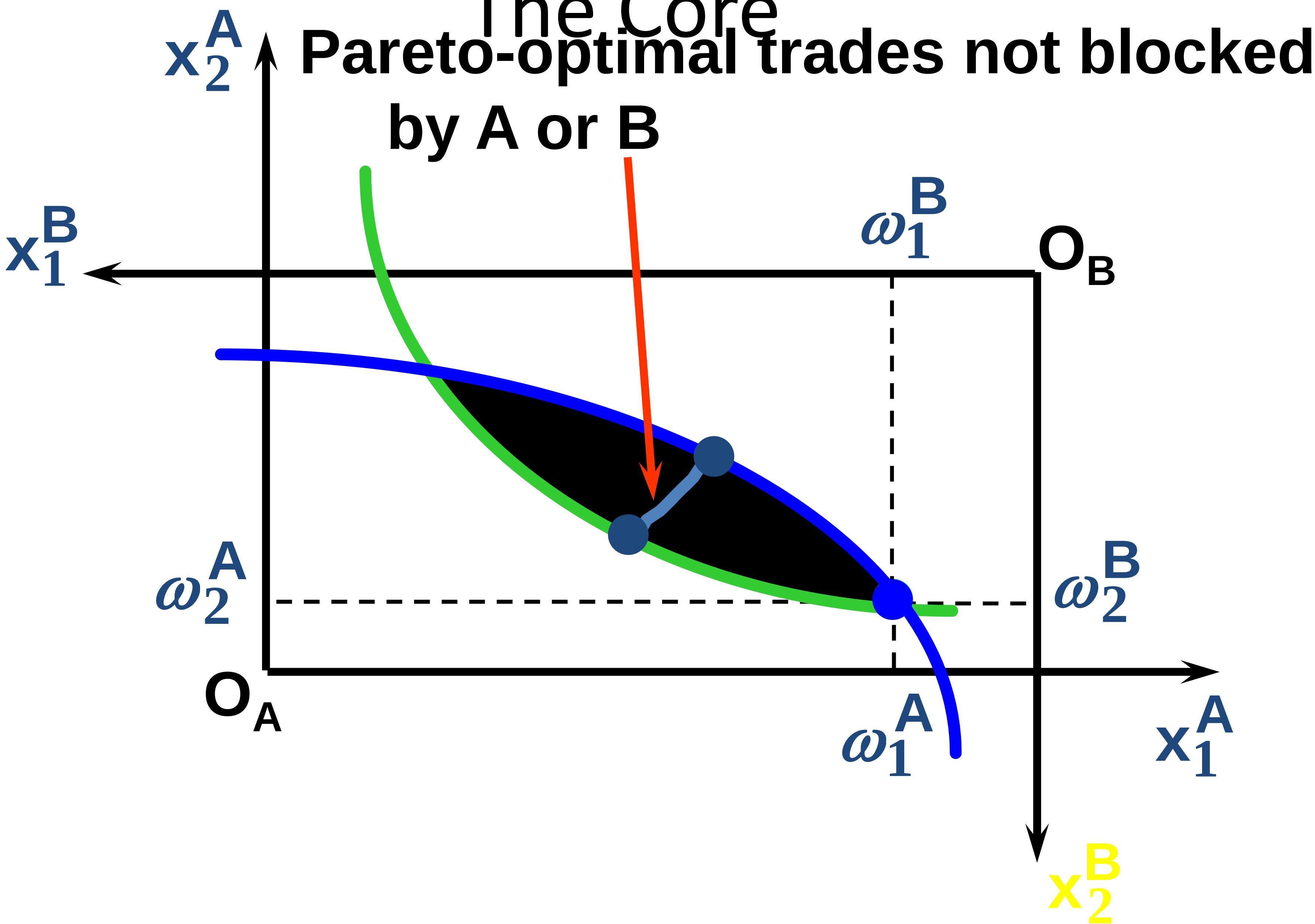


The Core



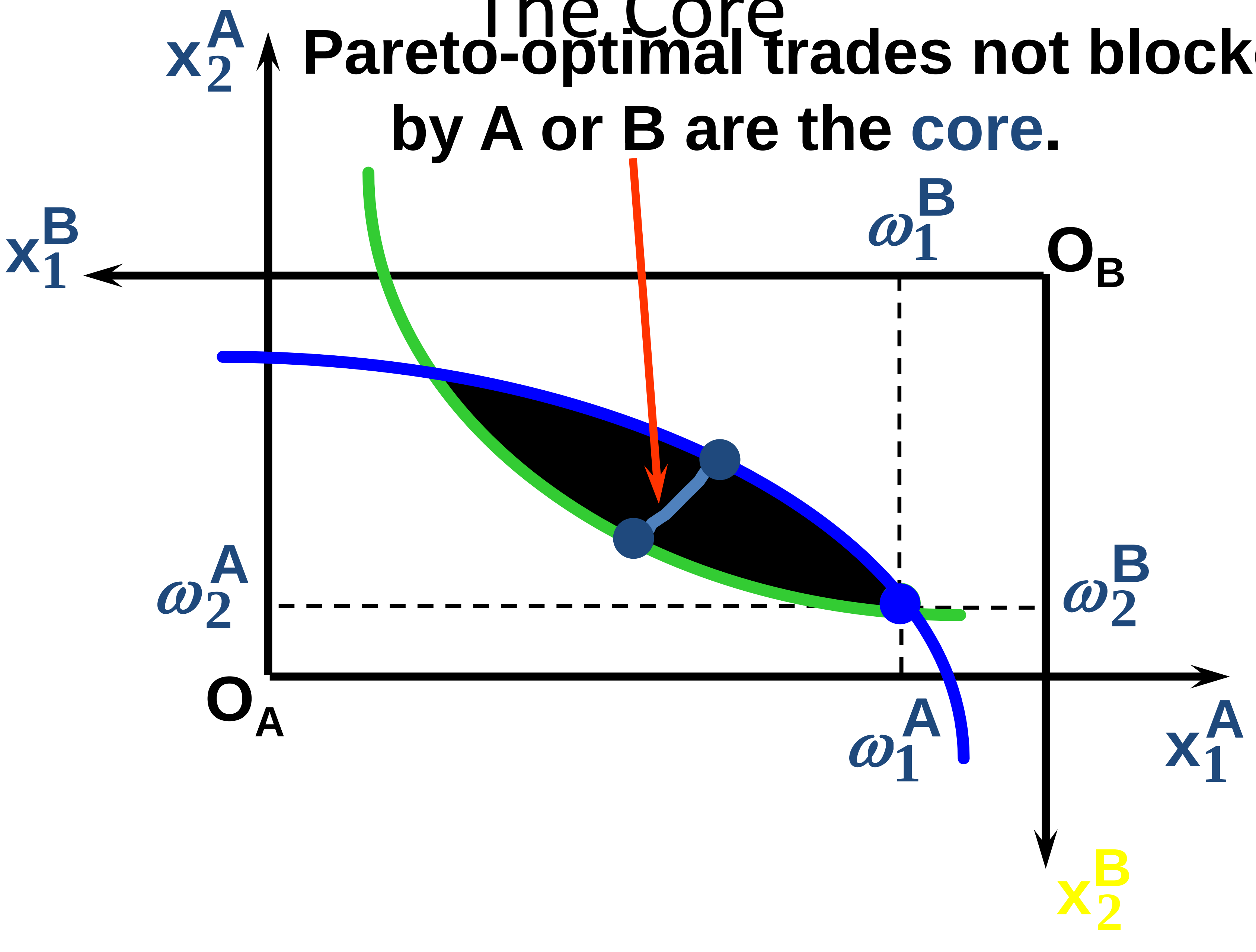
x_2^A
Pareto-optimal trades blocked by B
ω_1^B
x_1^B
O_B
ω_2^A
ω_2^B
O_A
ω_1^A
x_1^A
Pareto-optimal trades blocked by A
x_2^B

The Core


Pareto-optimal trades not blocked by A or B
x_2^A
x_1^B
ω_1^B
O_B
ω_2^A
ω_2^B
O_A
ω_1^A
x_1^A
x_2^B

The Core

Pareto-optimal trades not blocked by A or B are the core.



The Core

- The core is the set of all Pareto-optimal allocations that are welfare-improving for both consumers relative to their own endowments.
- Rational trade should achieve a core allocation.

The Core

- But which core allocation?
- Again, that depends upon the manner in which trade is conducted.

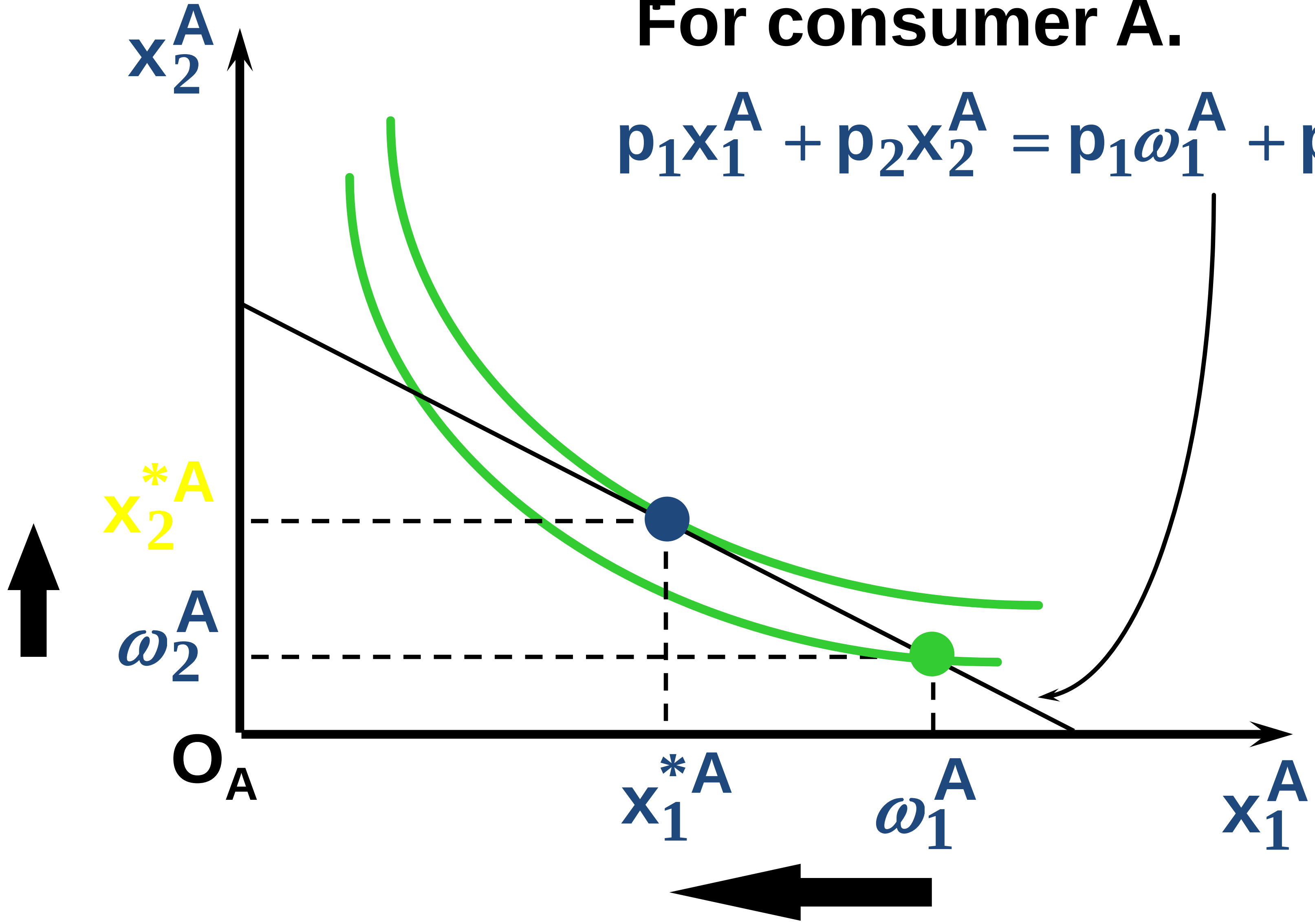
Trade in Competitive Markets

- Consider trade in perfectly competitive markets.
- Each consumer is a price-taker trying to maximize her own utility given p_1, p_2 and her own endowment. That is, ...

Trade in Competitive Markets

For consumer A.

$$p_1x_1^A + p_2x_2^A = p_1\omega_1^A + p_2\omega_2^A$$



Trade in Competitive Markets

- So given p_1 and p_2, consumer A's net demands for commodities 1 and 2 are

$$x_1^{*A} - \omega_1^A \quad \textbf{and} \quad x_2^{*A} - \omega_2^A.$$

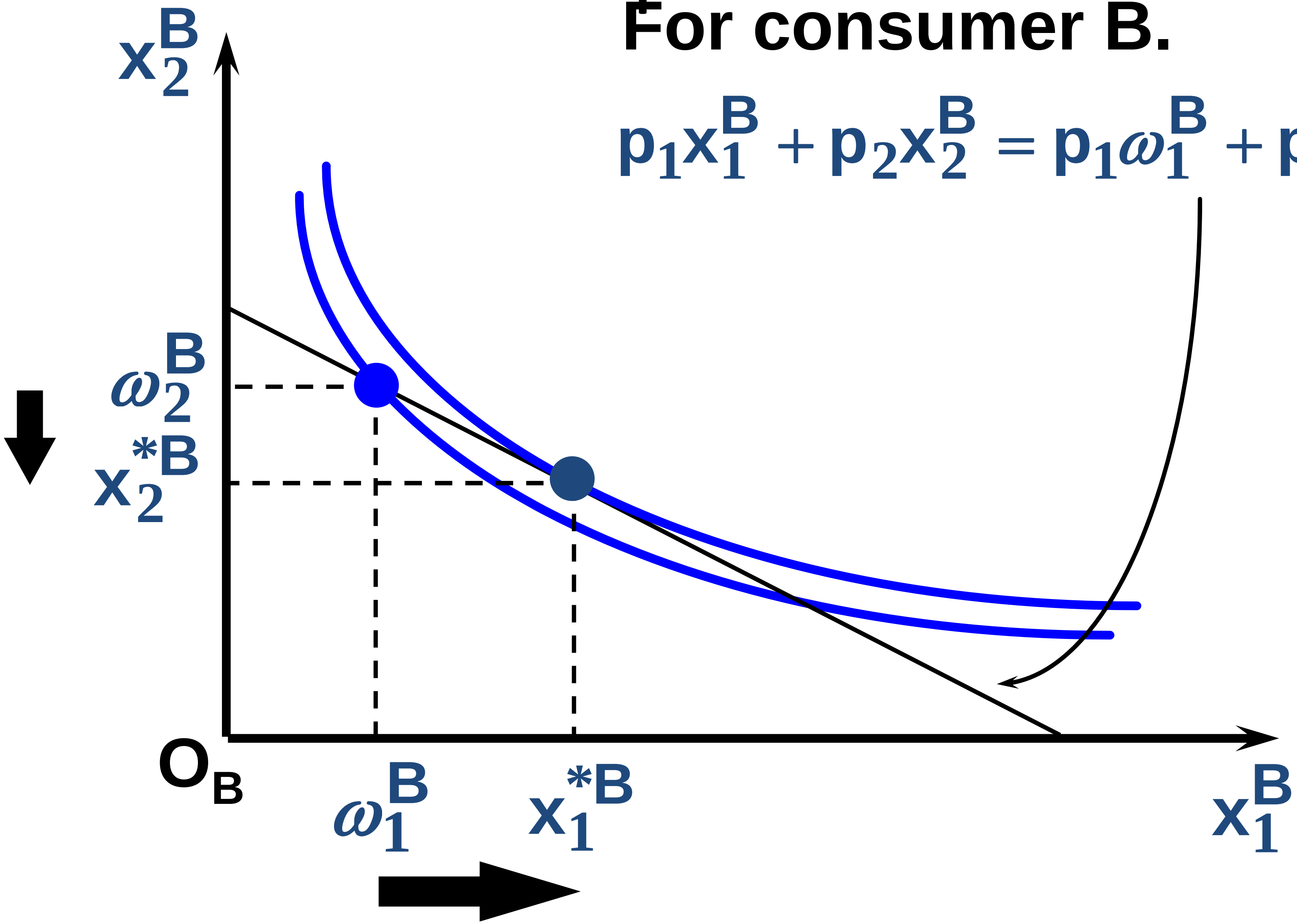
Trade in Competitive Markets

- And, similarly, for consumer B …

Trade in Competitive Markets

For consumer B.

$$p_1x_1^B + p_2x_2^B = p_1\omega_1^B + p_2\omega_2^B$$



Trade in Competitive Markets

- So given p_1 and p_2, consumer B's net demands for commodities 1 and 2 are

$$x_1^{*B} - \omega_1^B \quad \textbf{and} \quad x_2^{*B} - \omega_2^B.$$

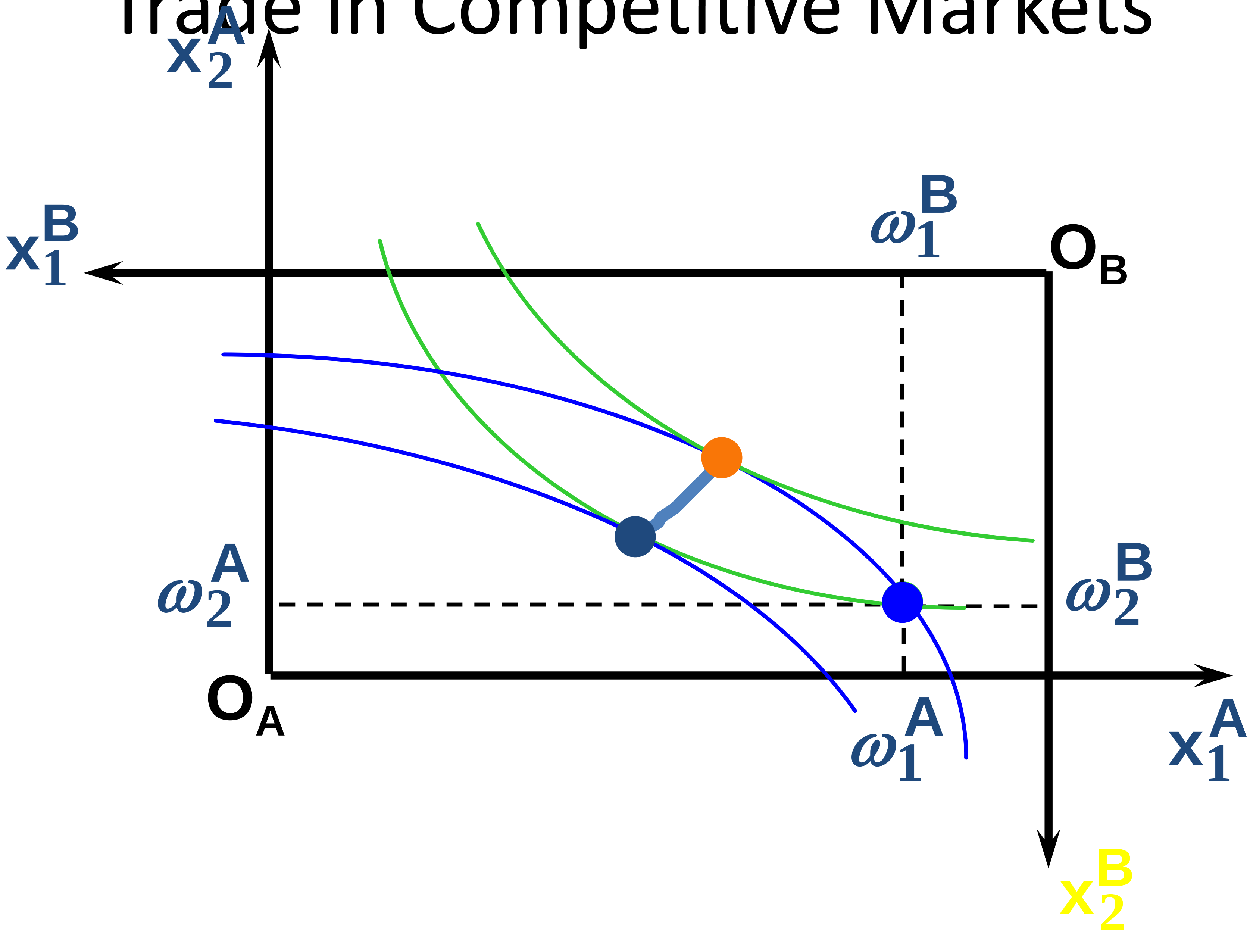
Trade in Competitive Markets

- A general equilibrium occurs when prices p_1 and p_2 cause both the markets for commodities 1 and 2 to clear; i.e.

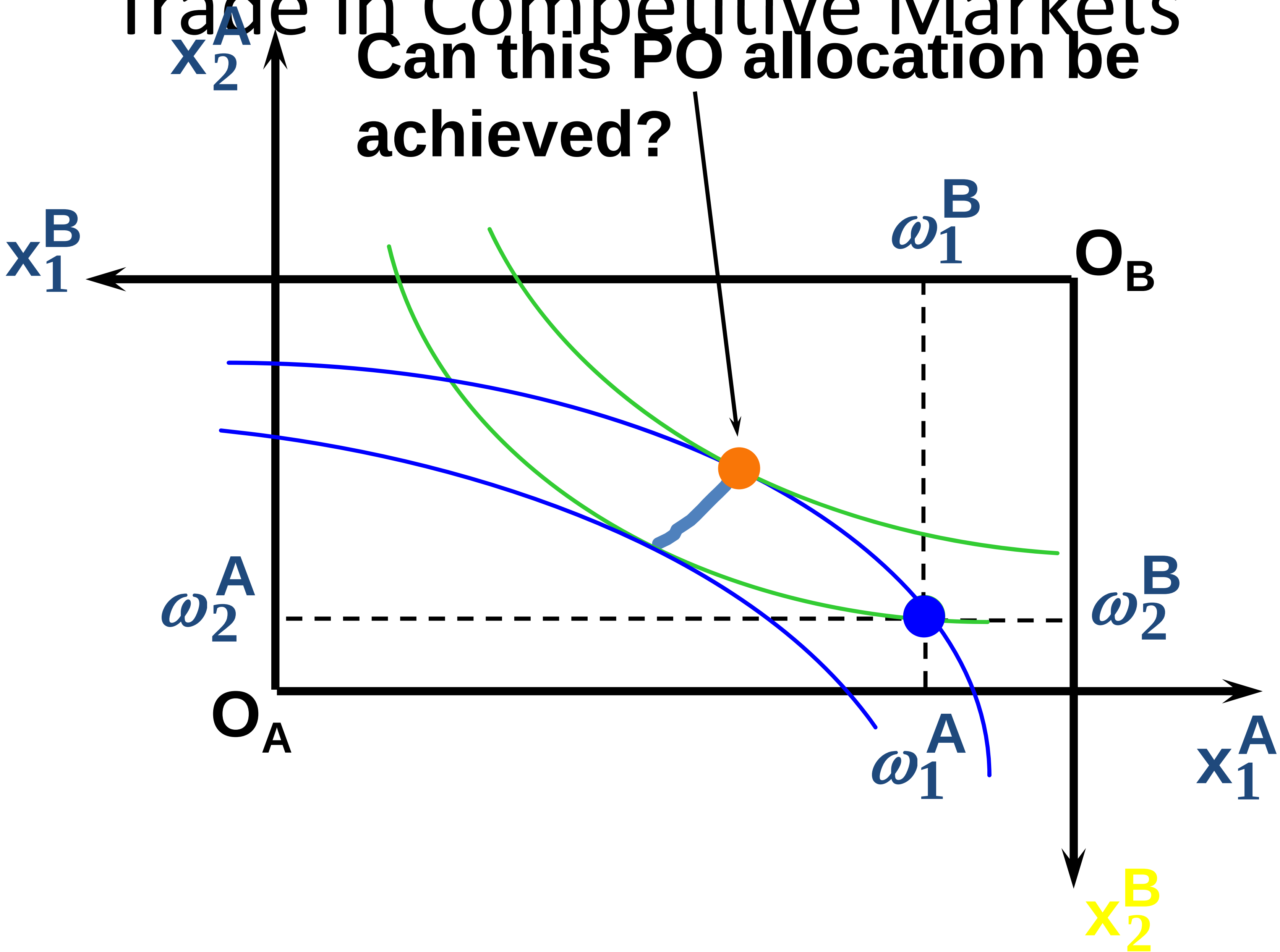
$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$

and $$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B.$$

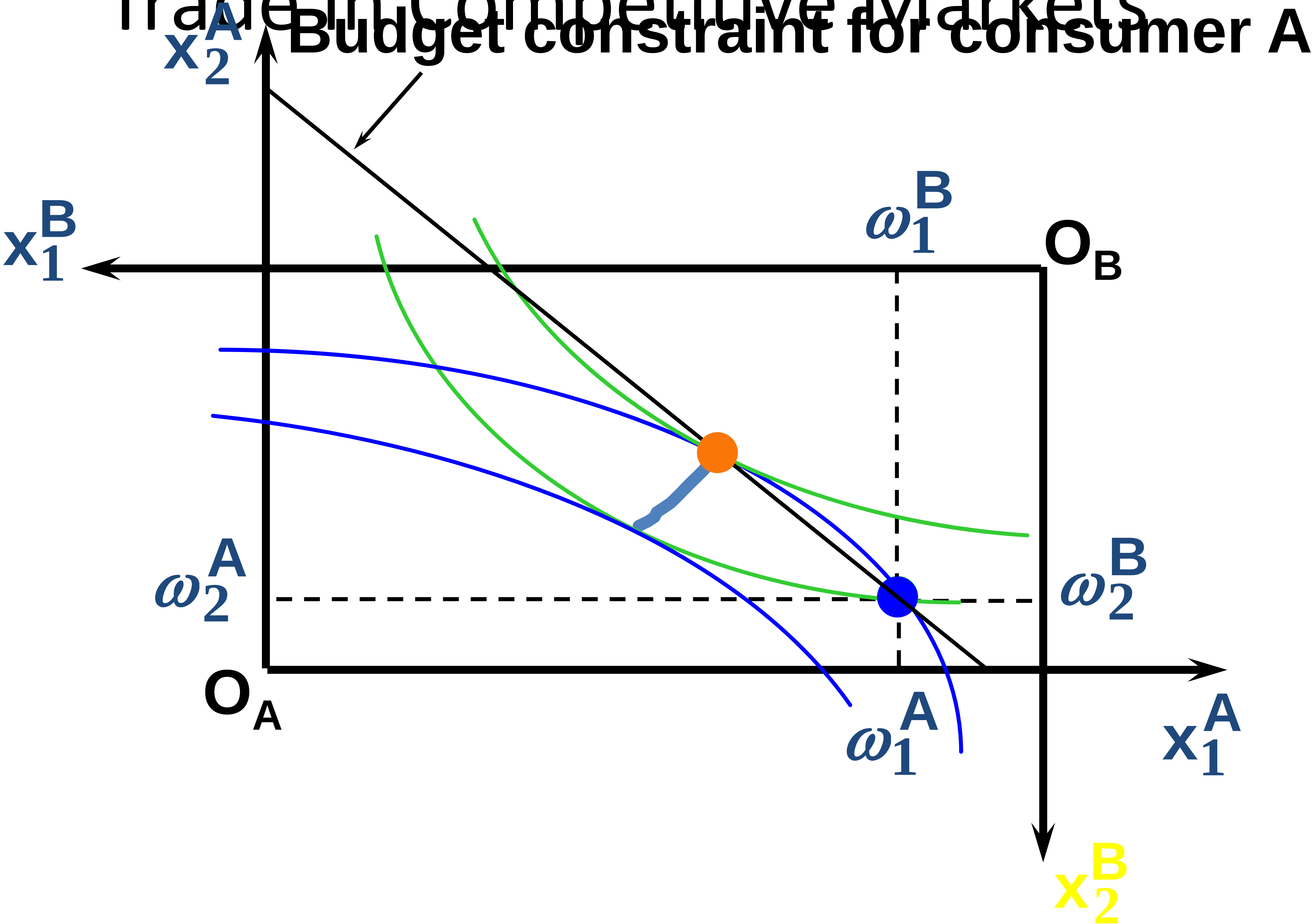
Trade in Competitive Markets



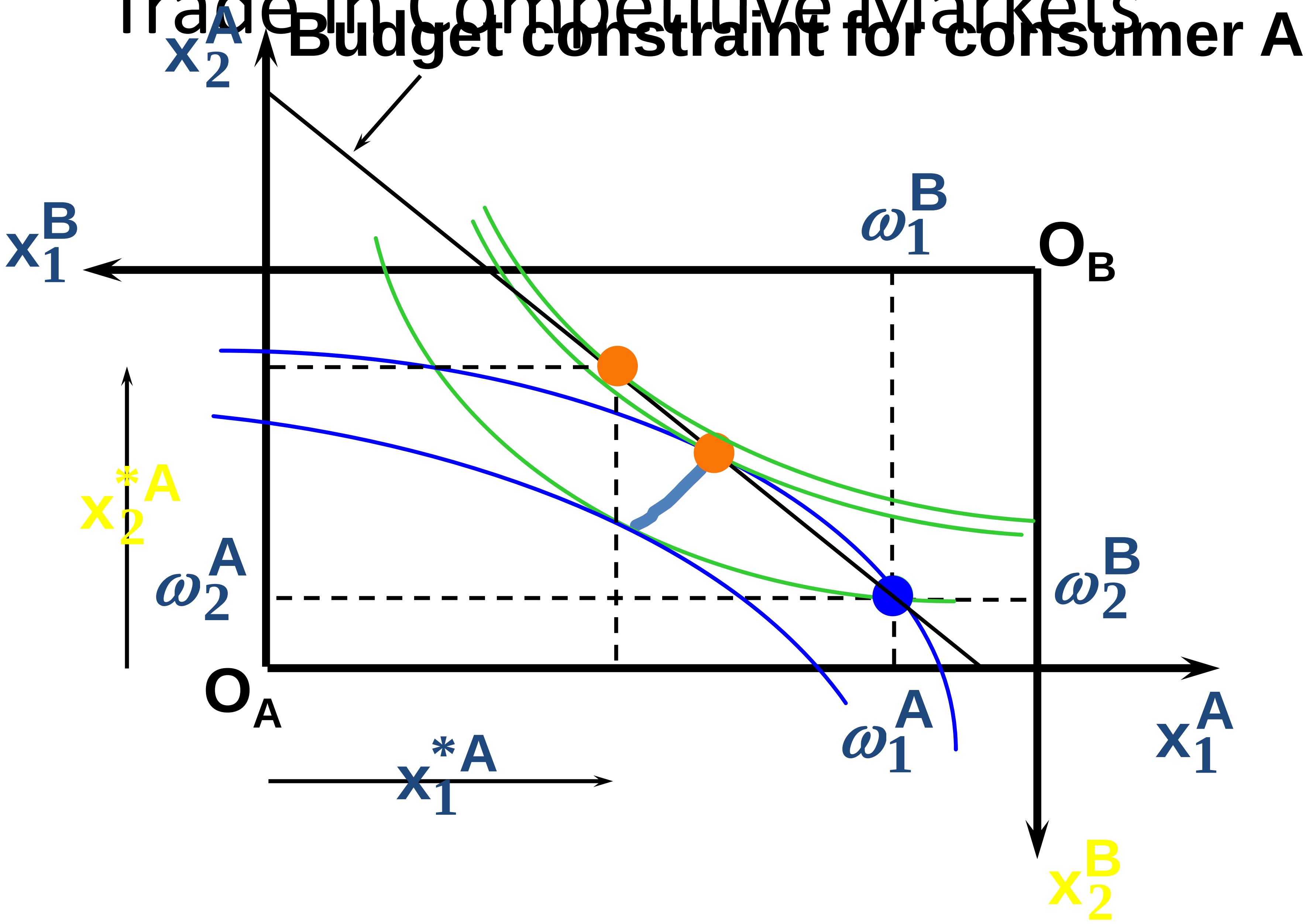
Trade in Competitive Markets



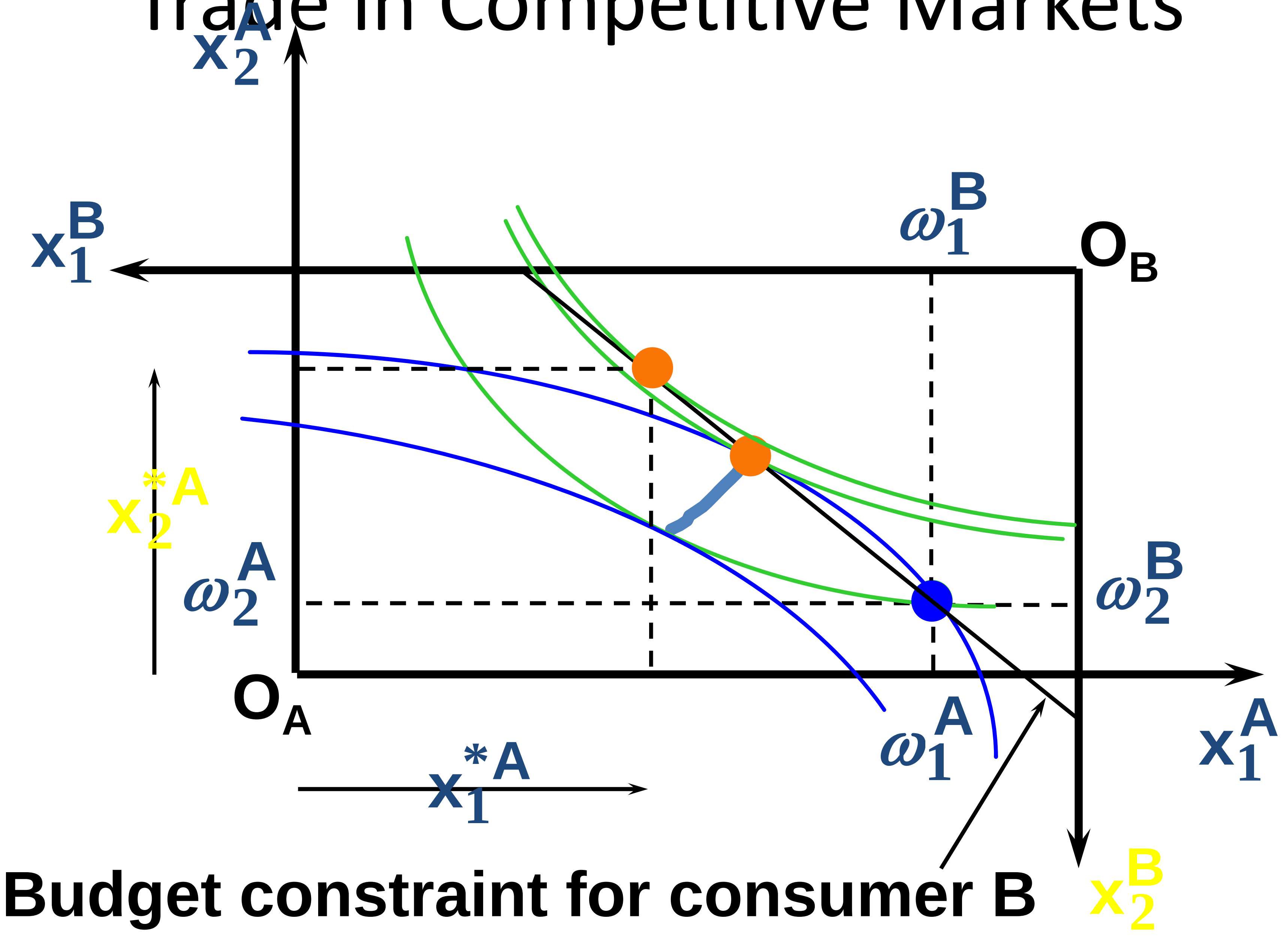
Trade in Competitive Markets



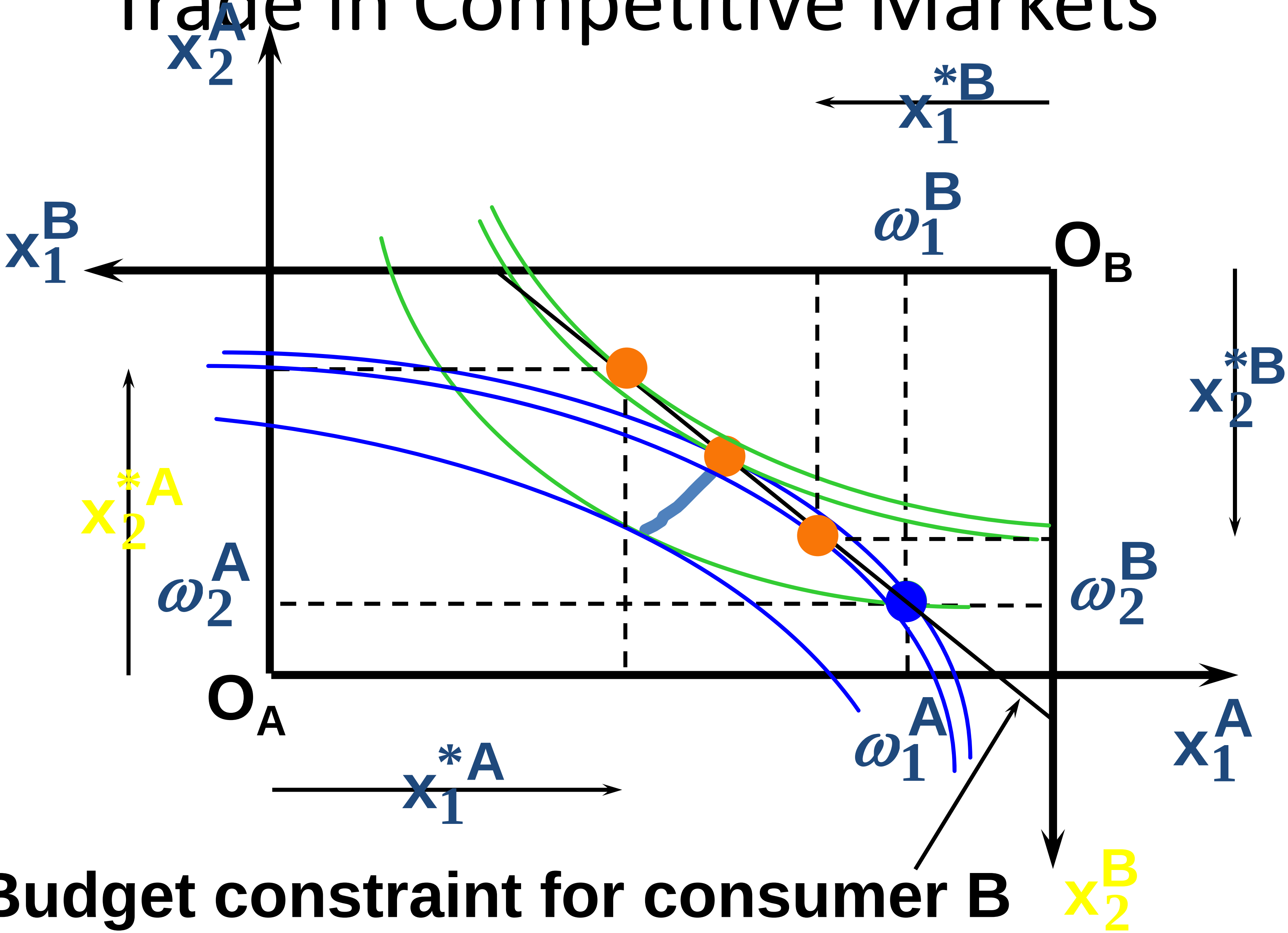
Trade in Competitive Markets



Trade in Competitive Markets

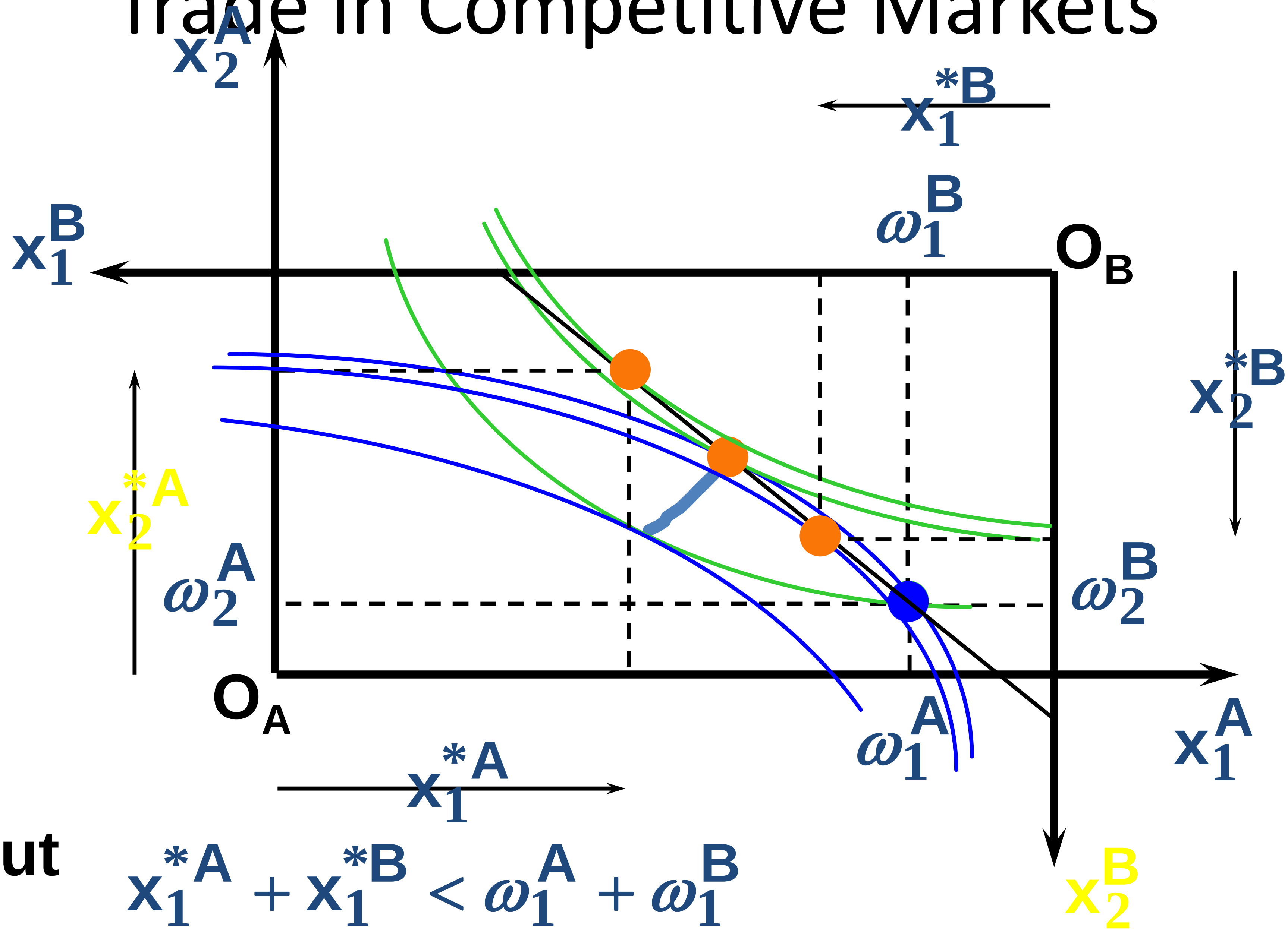


Trade in Competitive Markets



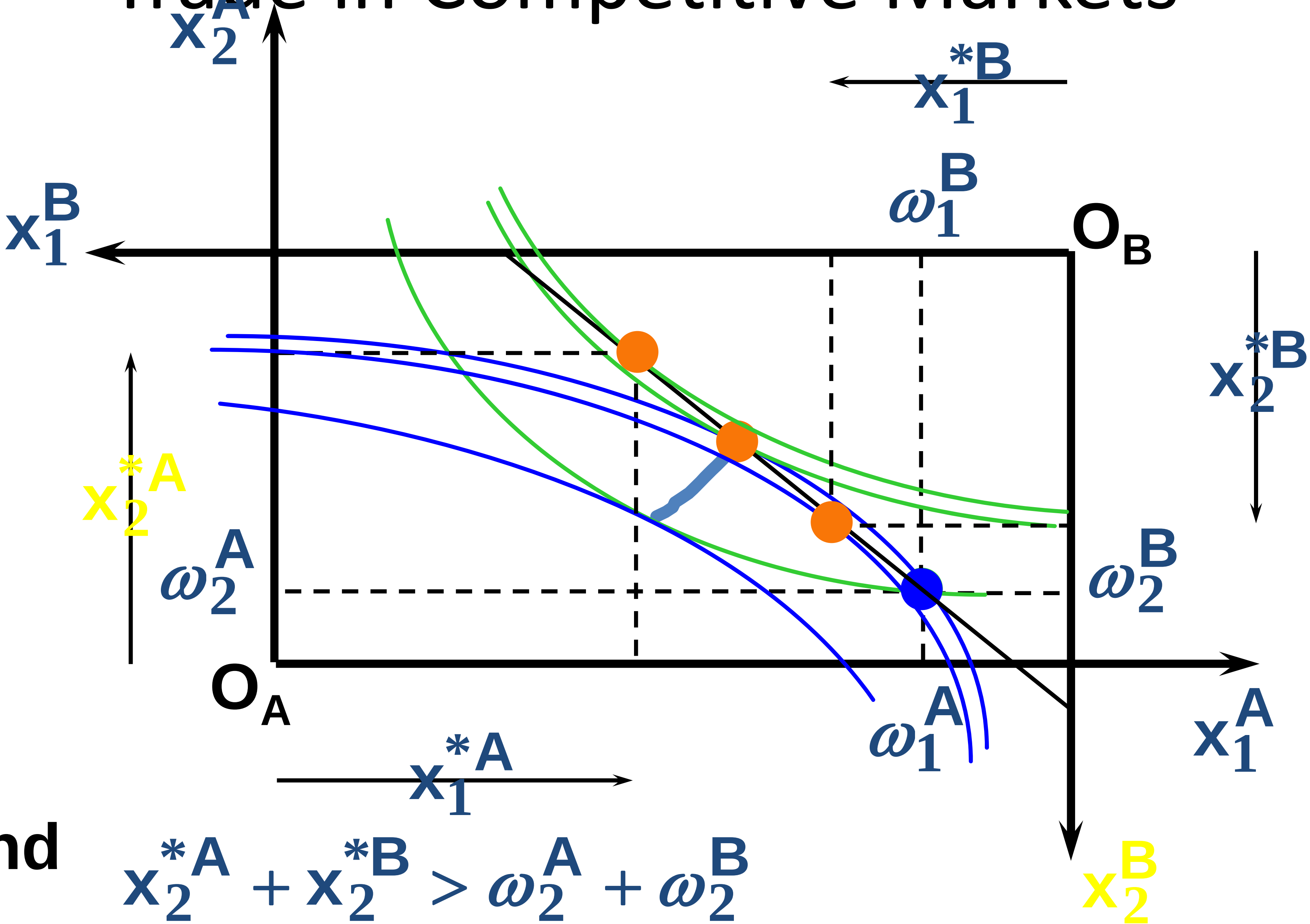
Budget constraint for consumer B

Trade in Competitive Markets



But $x_1^{*A} + x_1^{*B} < \omega_1^A + \omega_1^B$

Trade in Competitive Markets

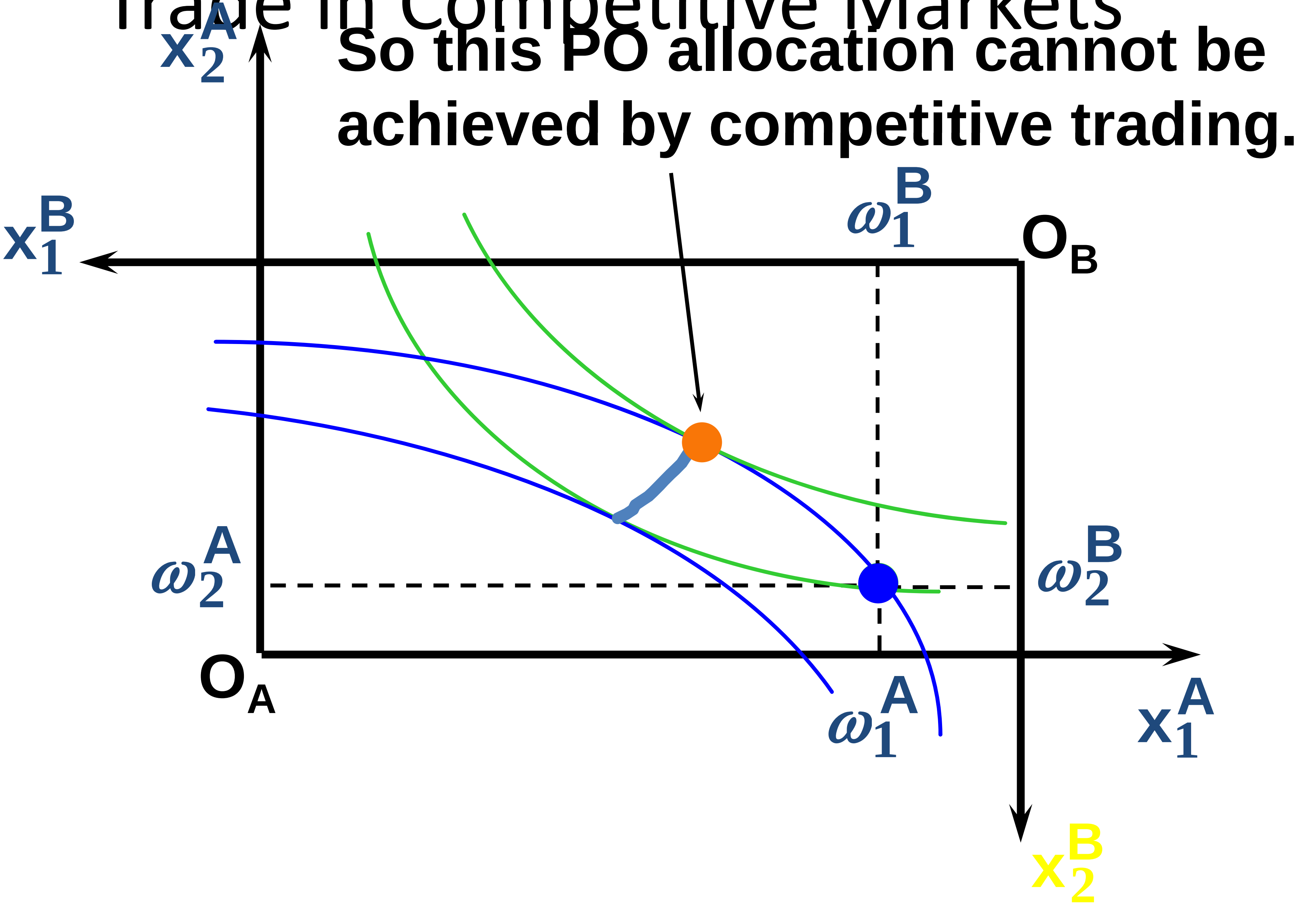


and $x_2^{*A} + x_2^{*B} > \omega_2^A + \omega_2^B$

Trade in Competitive Markets

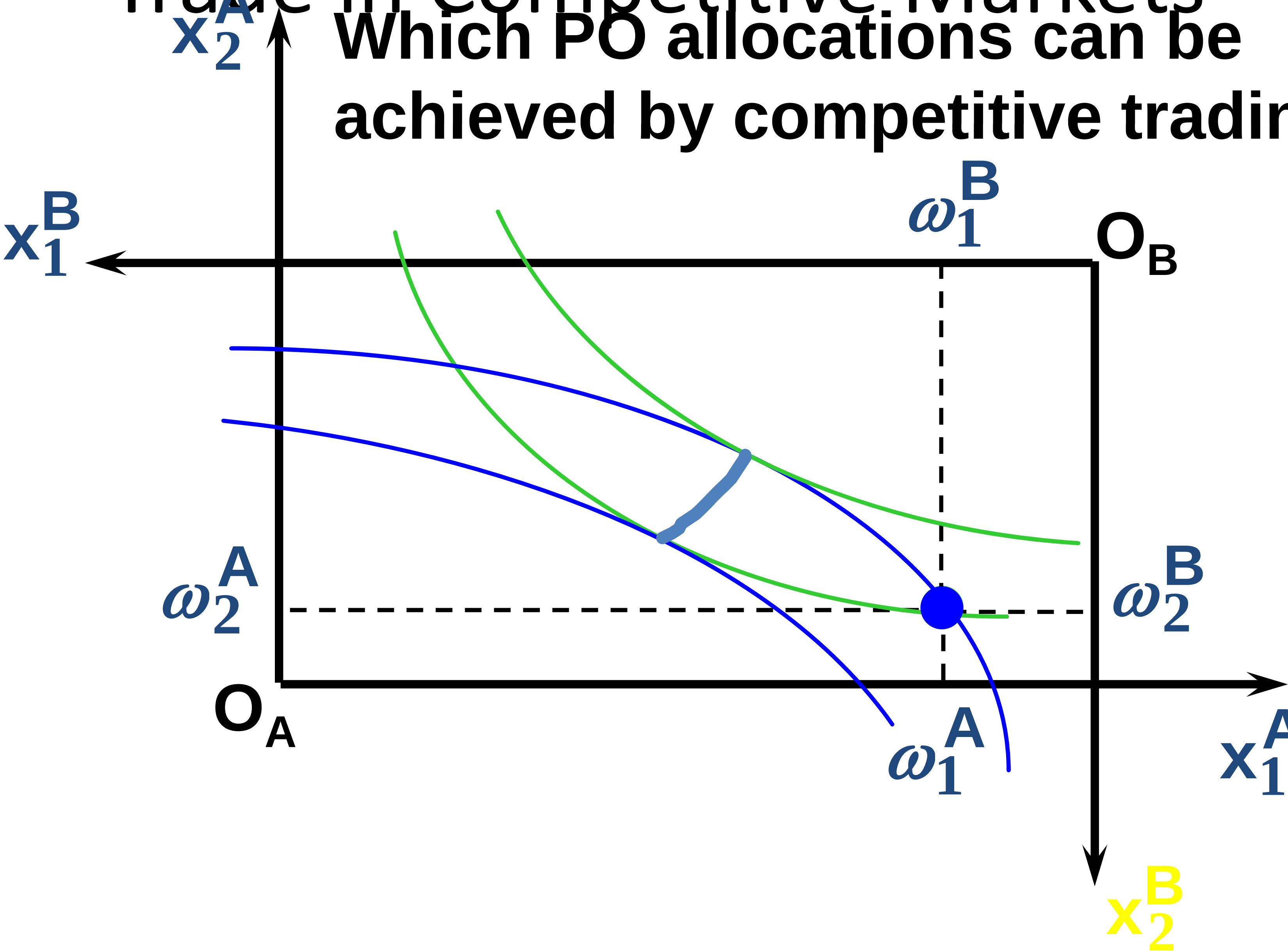
- So at the given prices p_1 and p_2 there is an
  - excess supply of commodity 1
  - excess demand for commodity 2.
- Neither market clears so the prices p_1 and p_2 do not cause a general equilibrium.

Trade in Competitive Markets



Trade in Competitive Markets

Which PO allocations can be achieved by competitive trading?

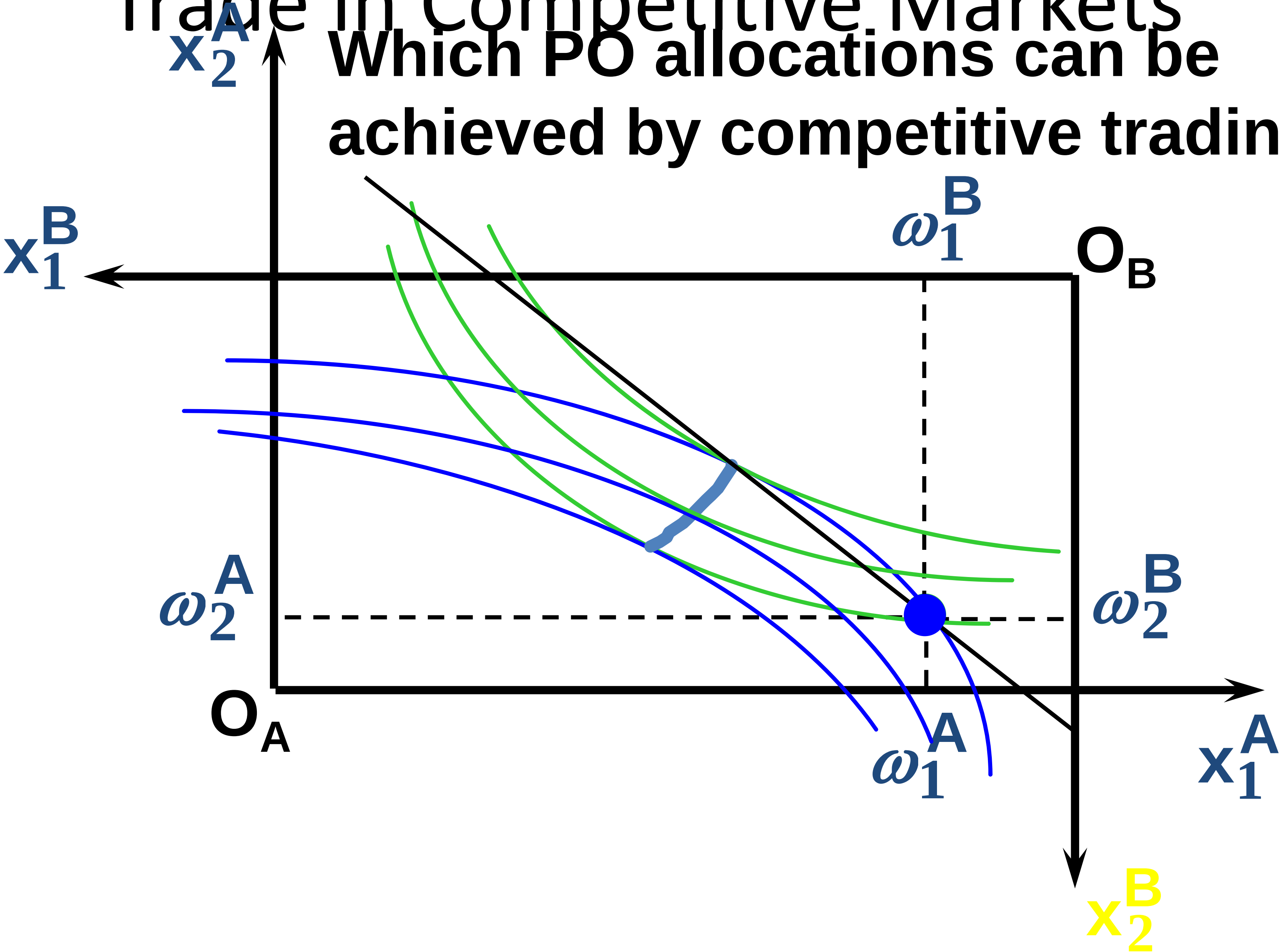


Trade in Competitive Markets

- Since there is an excess demand for commodity 2, p_2 will rise.
- Since there is an excess supply of commodity 1, p_1 will fall.
- The slope of the budget constraints is - p_1/p_2 so the budget constraints will pivot about the endowment point and become less steep.

Trade in Competitive Markets

Which PO allocations can be achieved by competitive trading?



Trade in Competitive Markets

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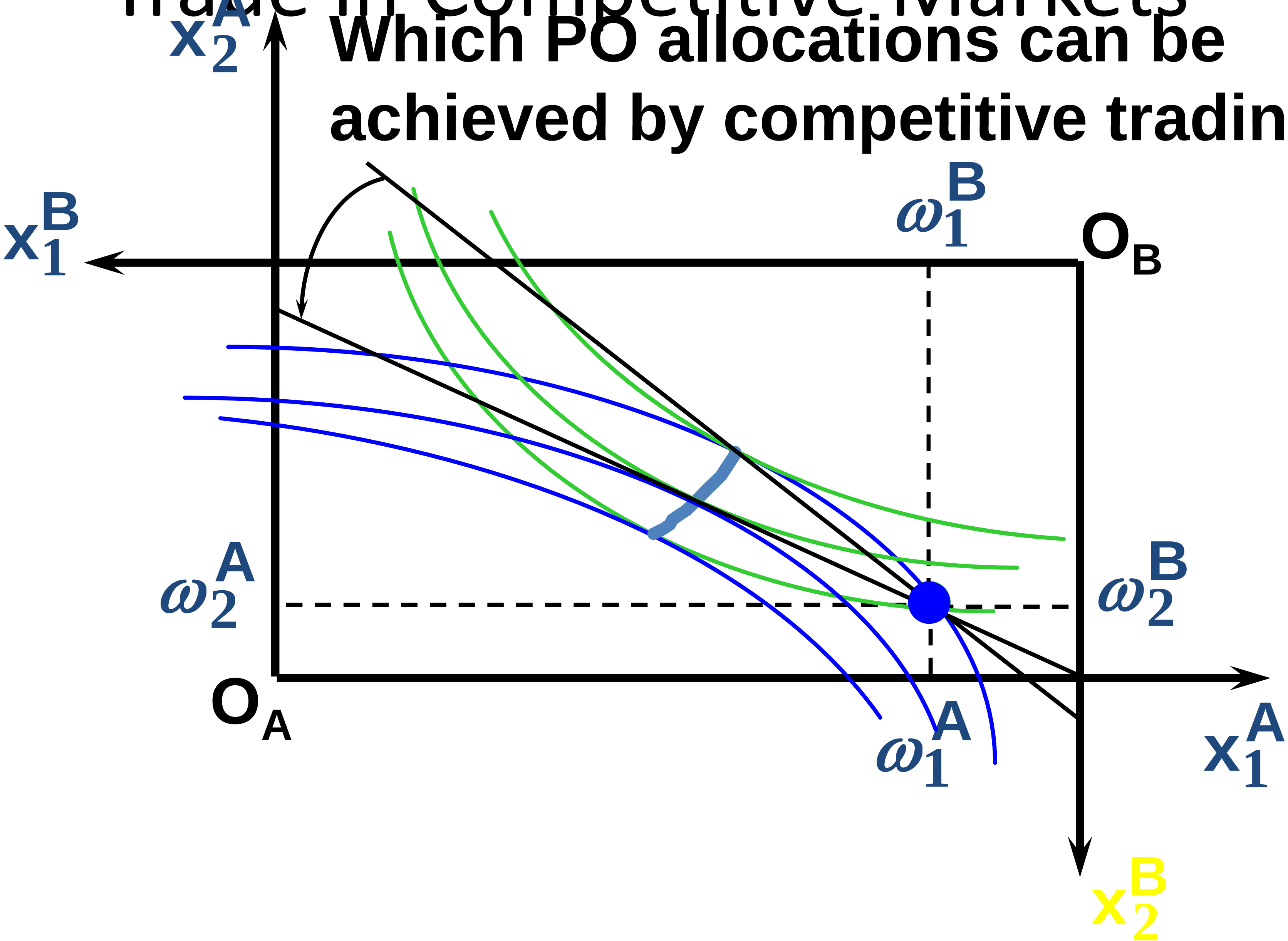

Trade in Competitive Markets

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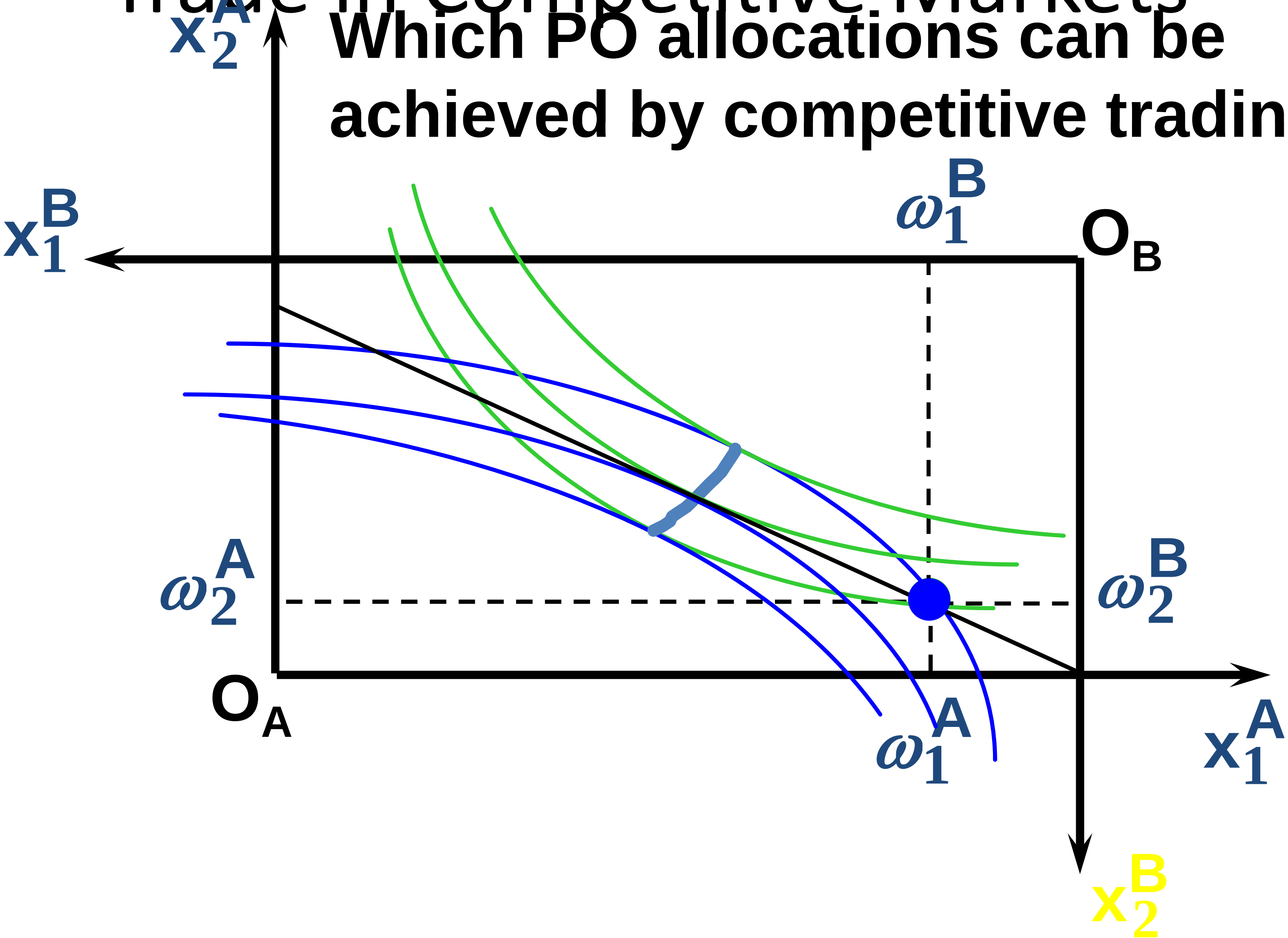
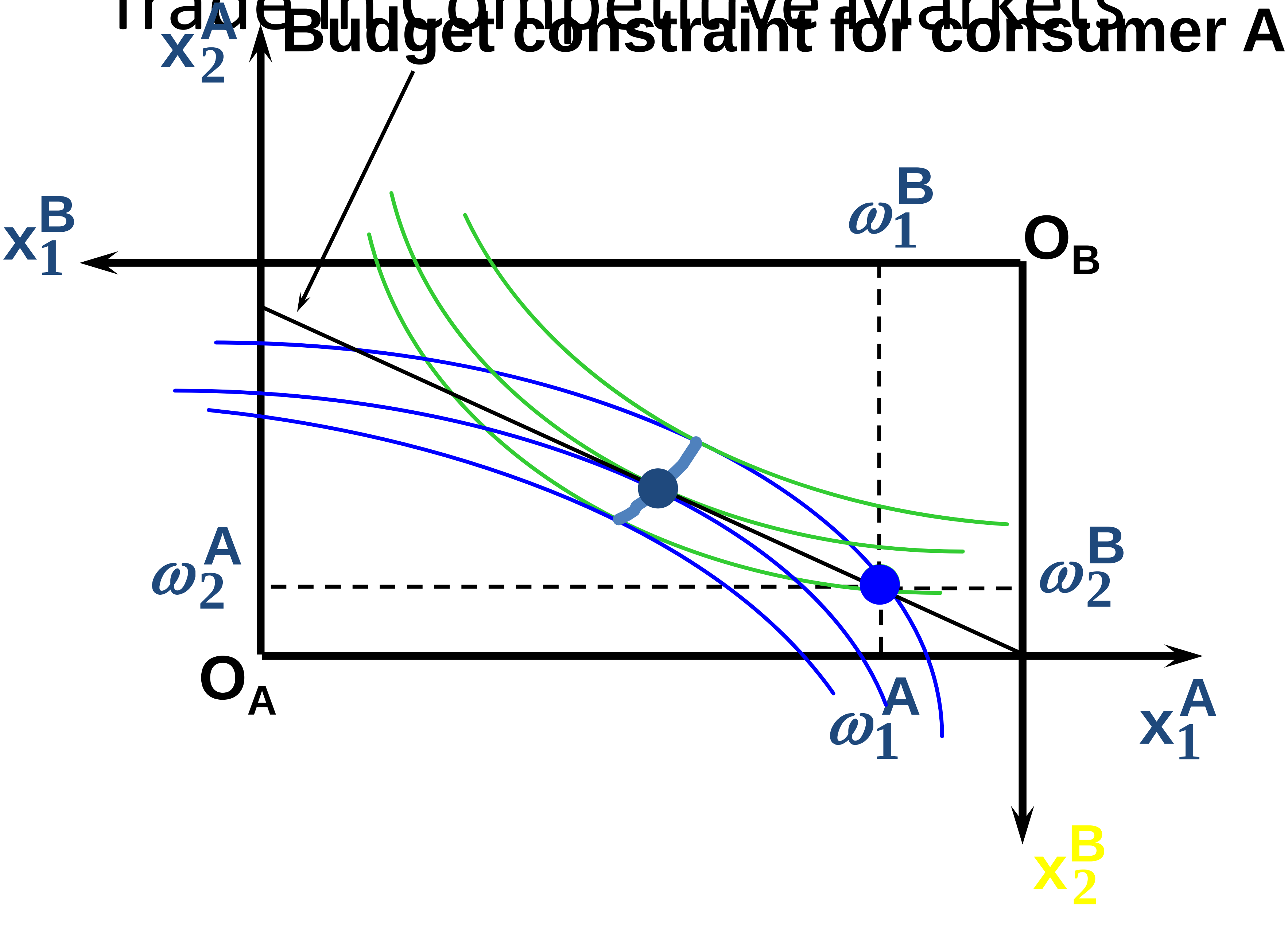
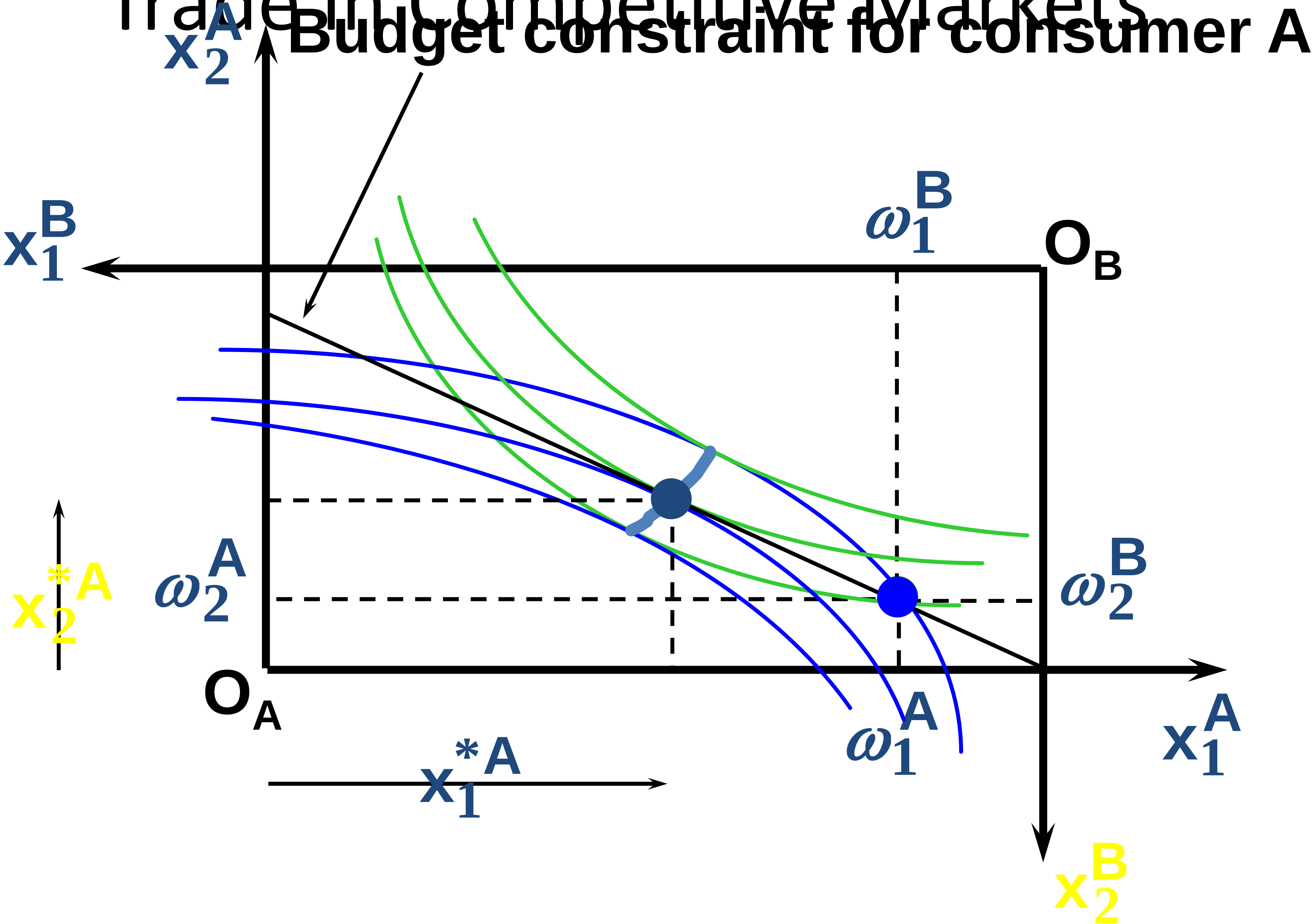


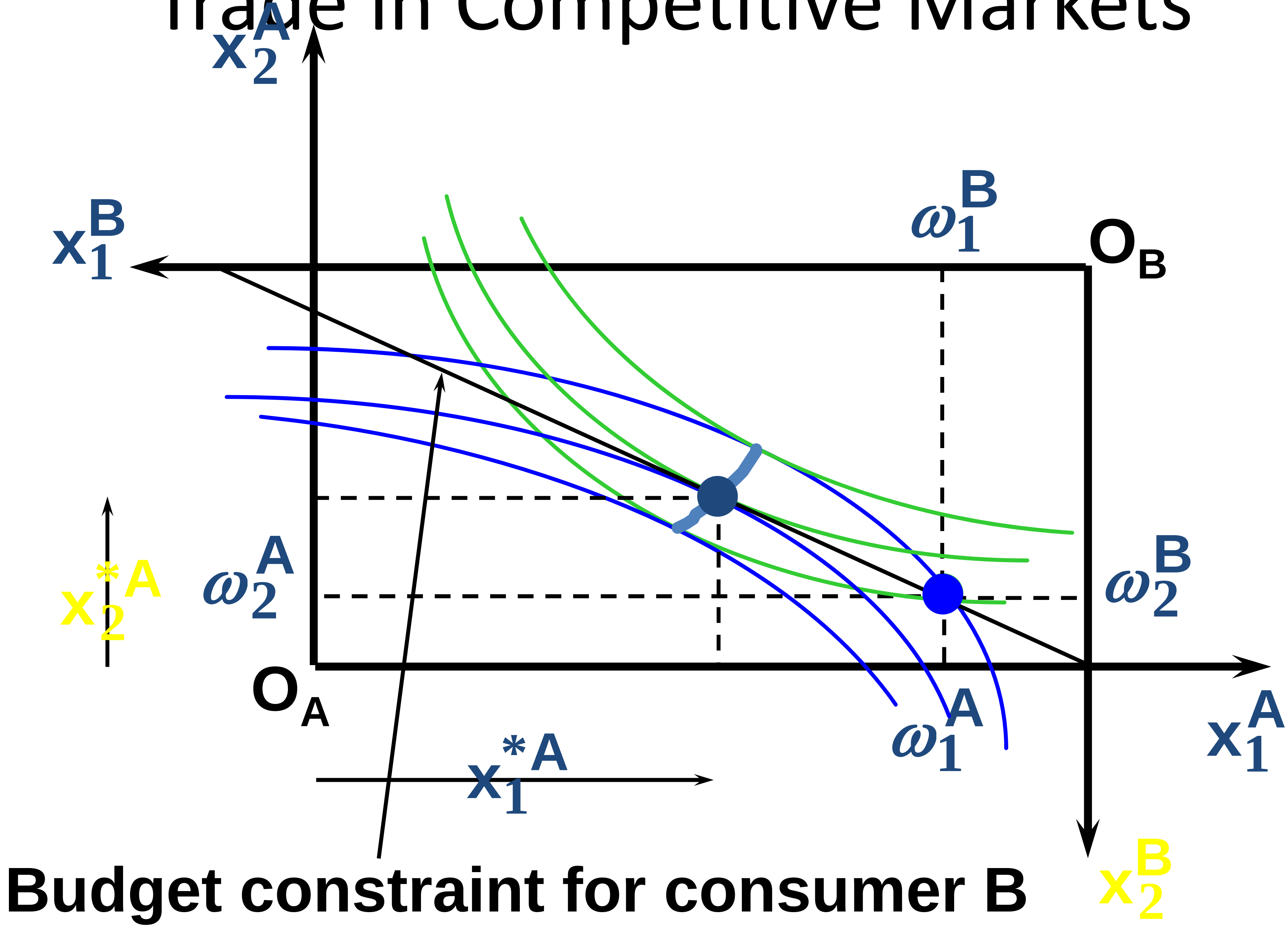
Trade in Competitive Markets



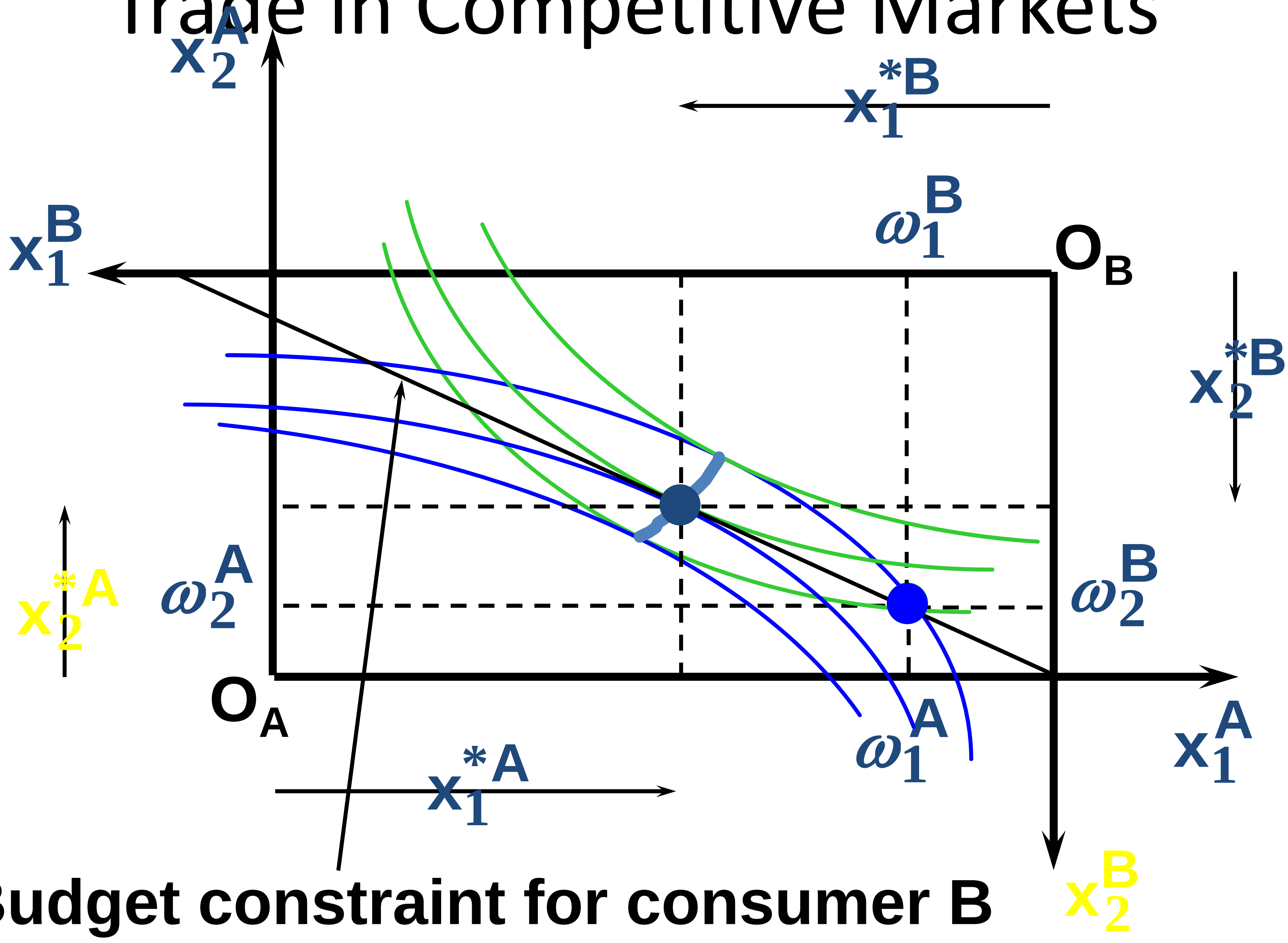
Trade in Competitive Markets



Trade in Competitive Markets

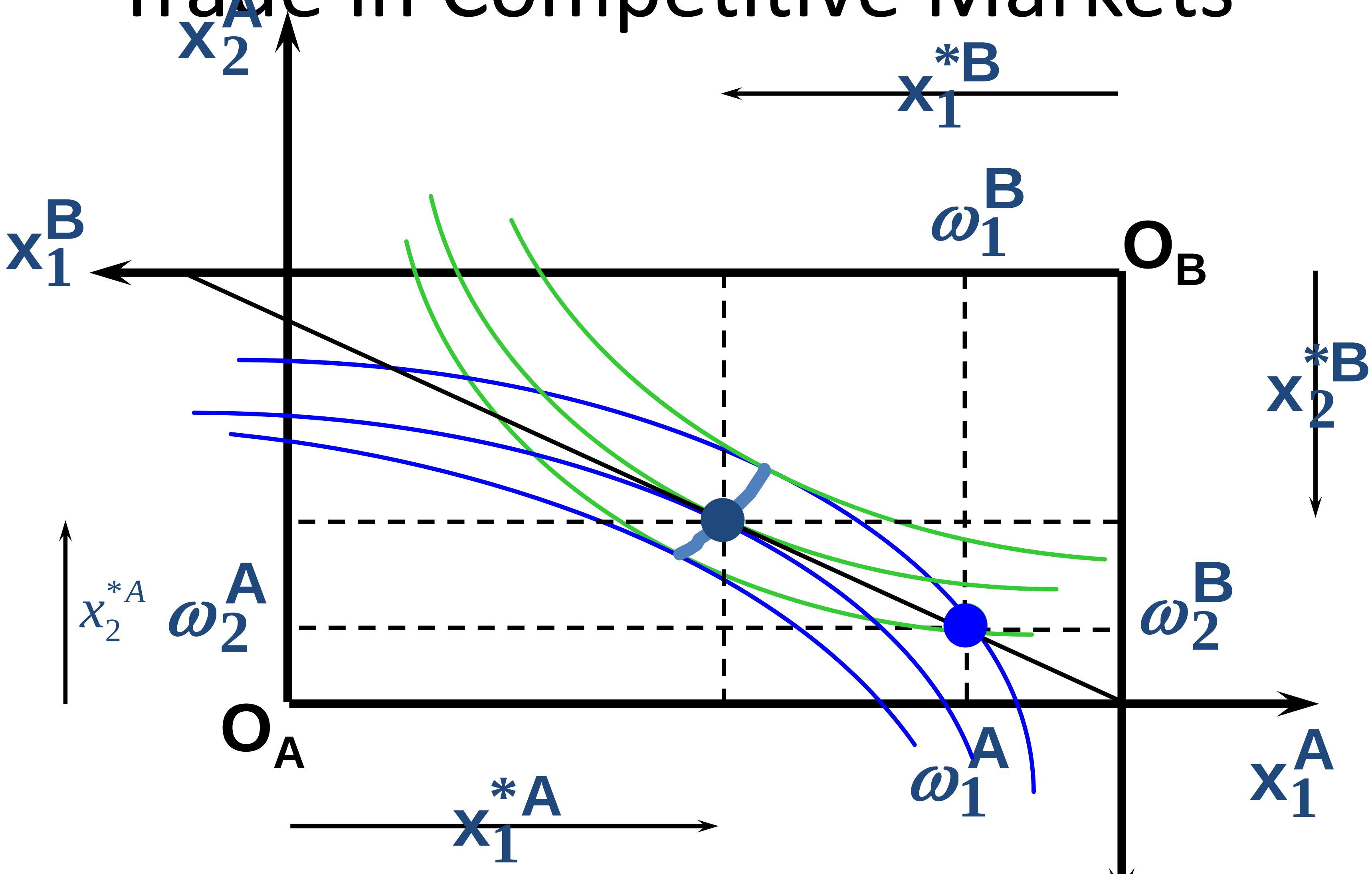


Trade in Competitive Markets



Budget constraint for consumer B

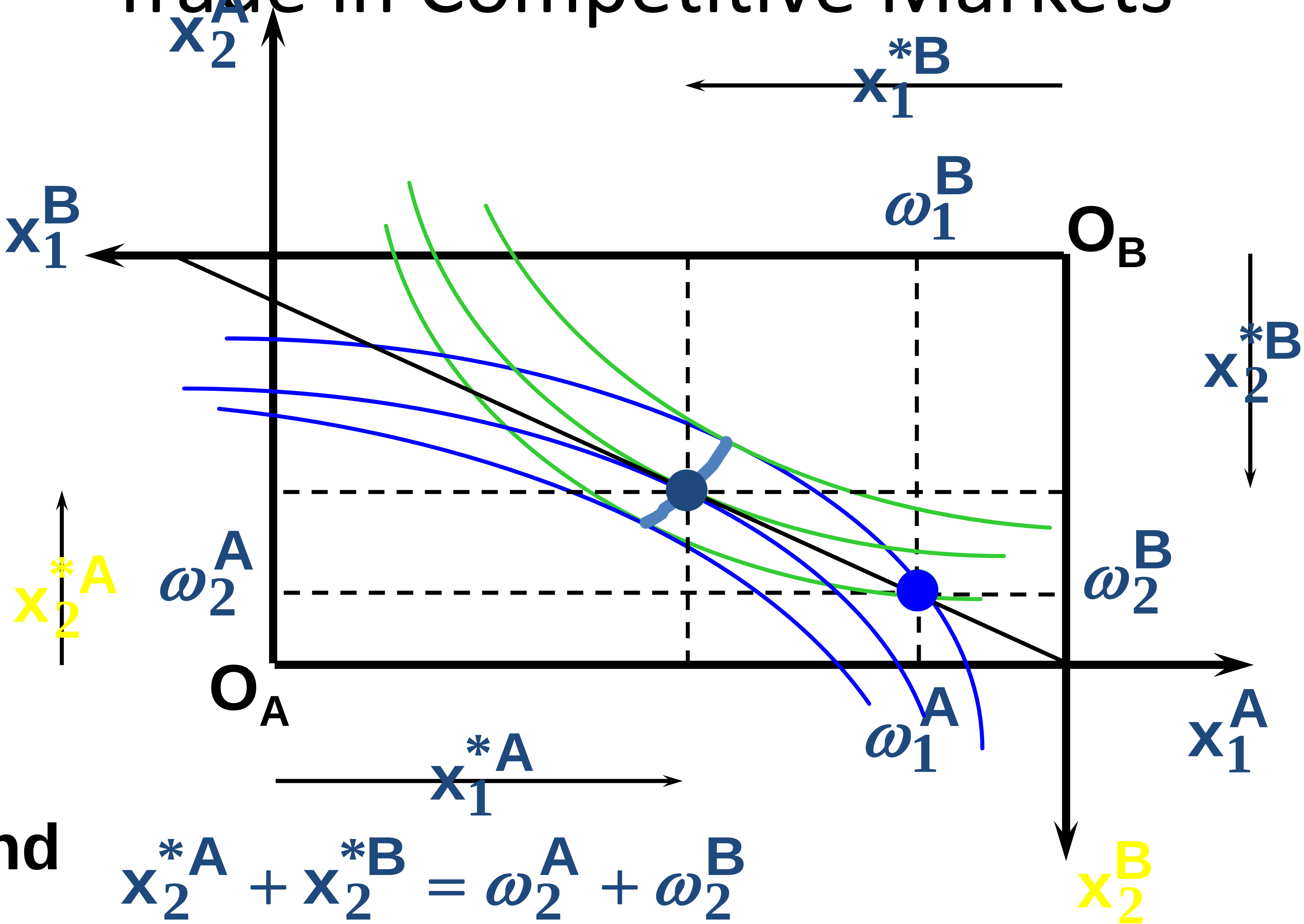
Trade in Competitive Markets



So

$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$

Trade in Competitive Markets



and $x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B$

Trade in Competitive Markets

- At the new prices p_1 and p_2 both markets clear; there is a general equilibrium.
- Trading in competitive markets achieves a particular Pareto-optimal allocation of the endowments.
- This is an example of the First Fundamental Theorem of Welfare Economics.

First Fundamental Theorem of Welfare Economics

- Given that consumers’ preferences are well-behaved, trading in perfectly competitive markets implements a Pareto-optimal allocation of the economy’s endowment.

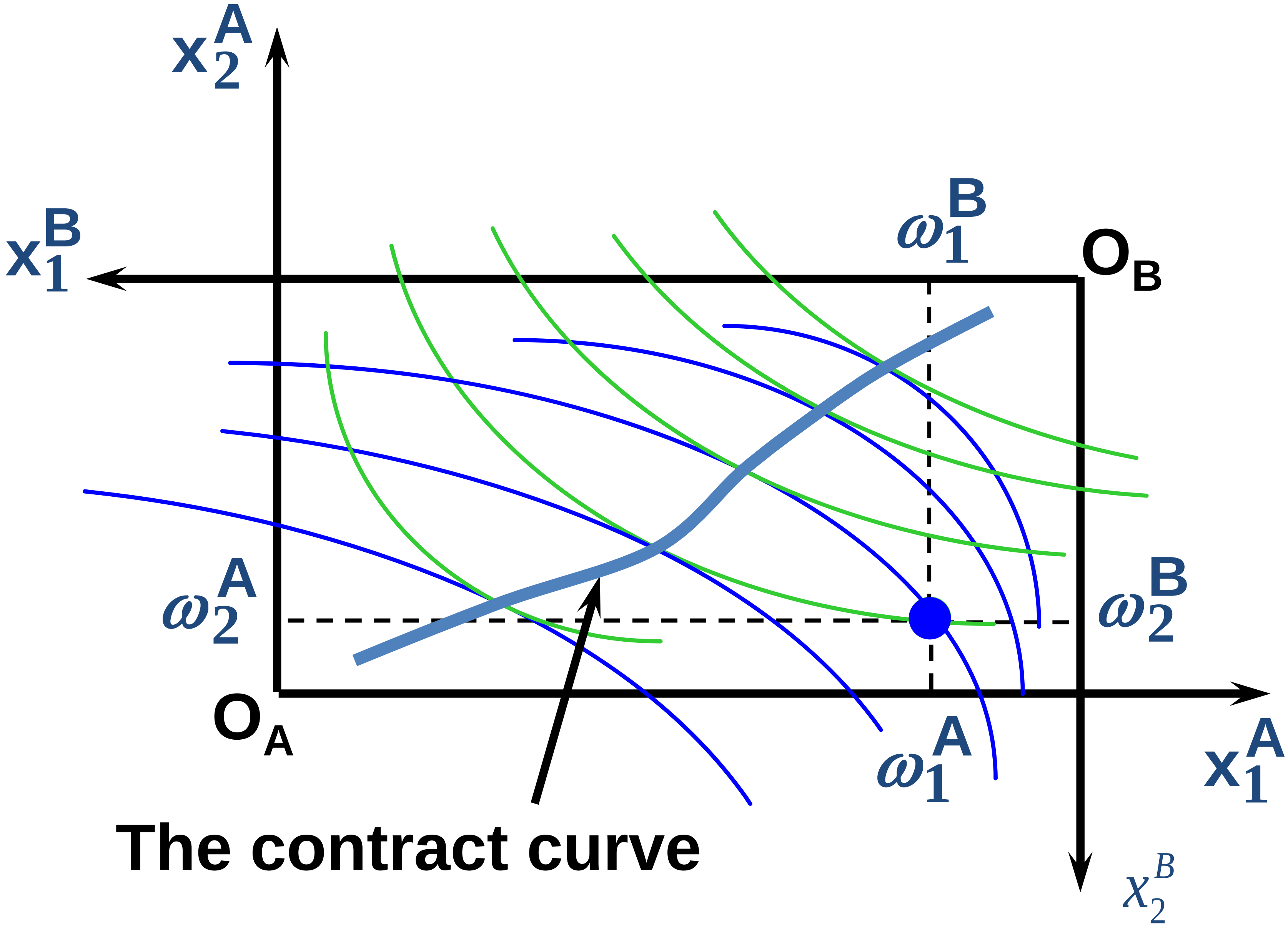
Second Fundamental Theorem of Welfare Economics

- The First Theorem is followed by a second that states that any Pareto-optimal allocation (i.e. any point on the contract curve) can be achieved by trading in competitive markets provided that endowments are first appropriately rearranged amongst the consumers.

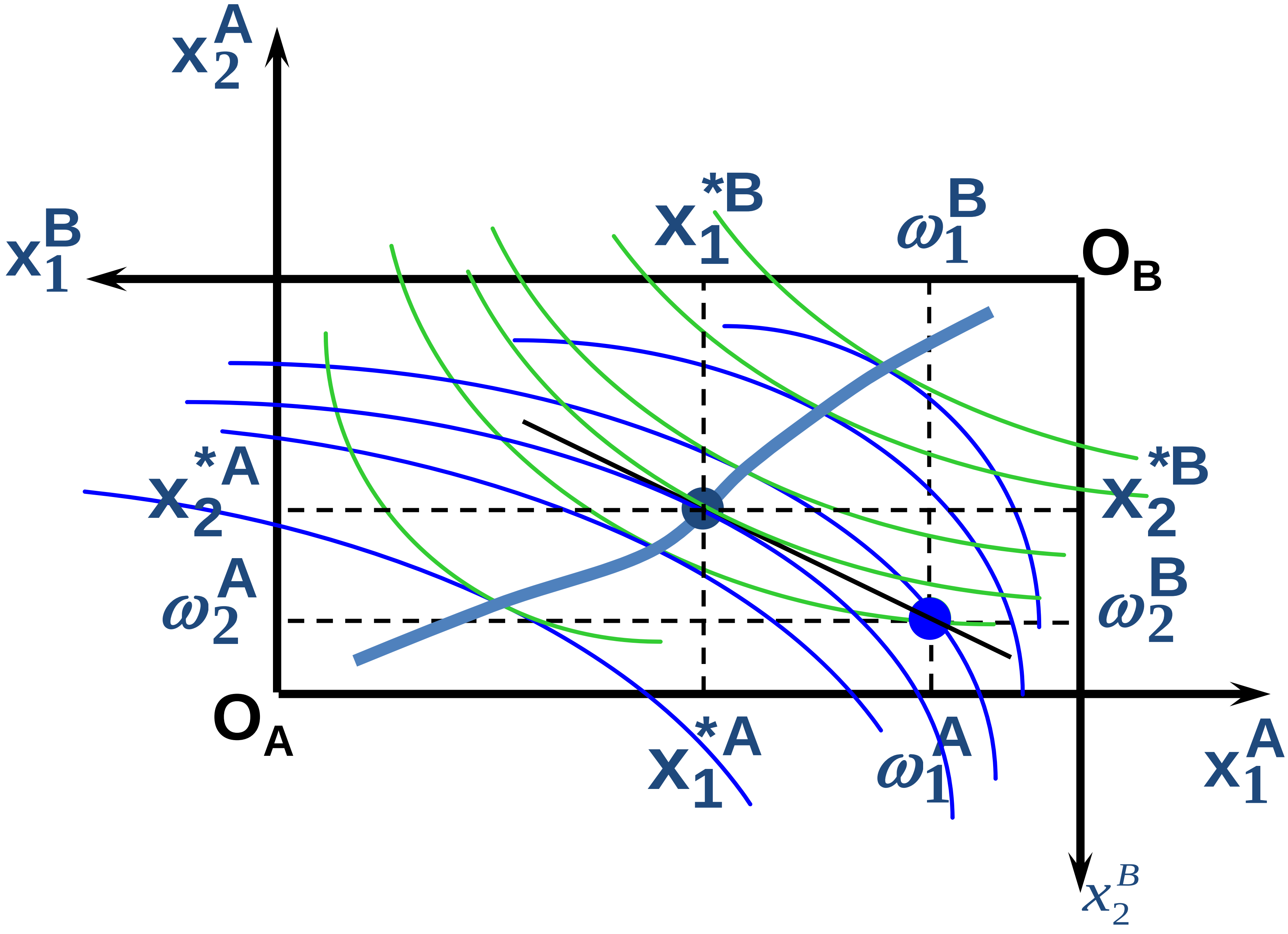
Second Fundamental Theorem of Welfare Economics

- Given that consumers’ preferences are well-behaved, for any Pareto-optimal allocation there are prices and an allocation of the total endowment that makes the Pareto-optimal allocation implementable by trading in competitive markets.

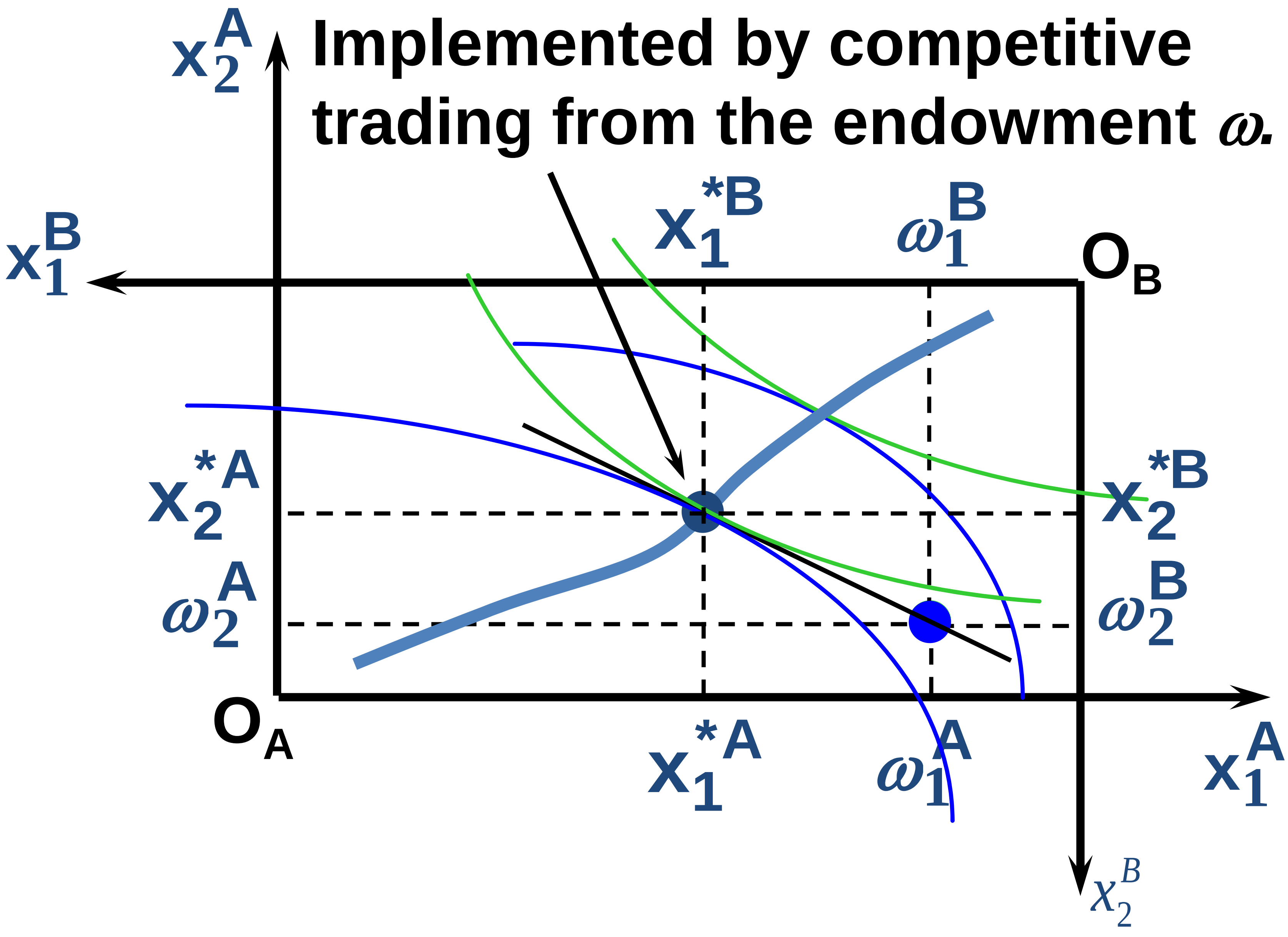
Second Fundamental Theorem



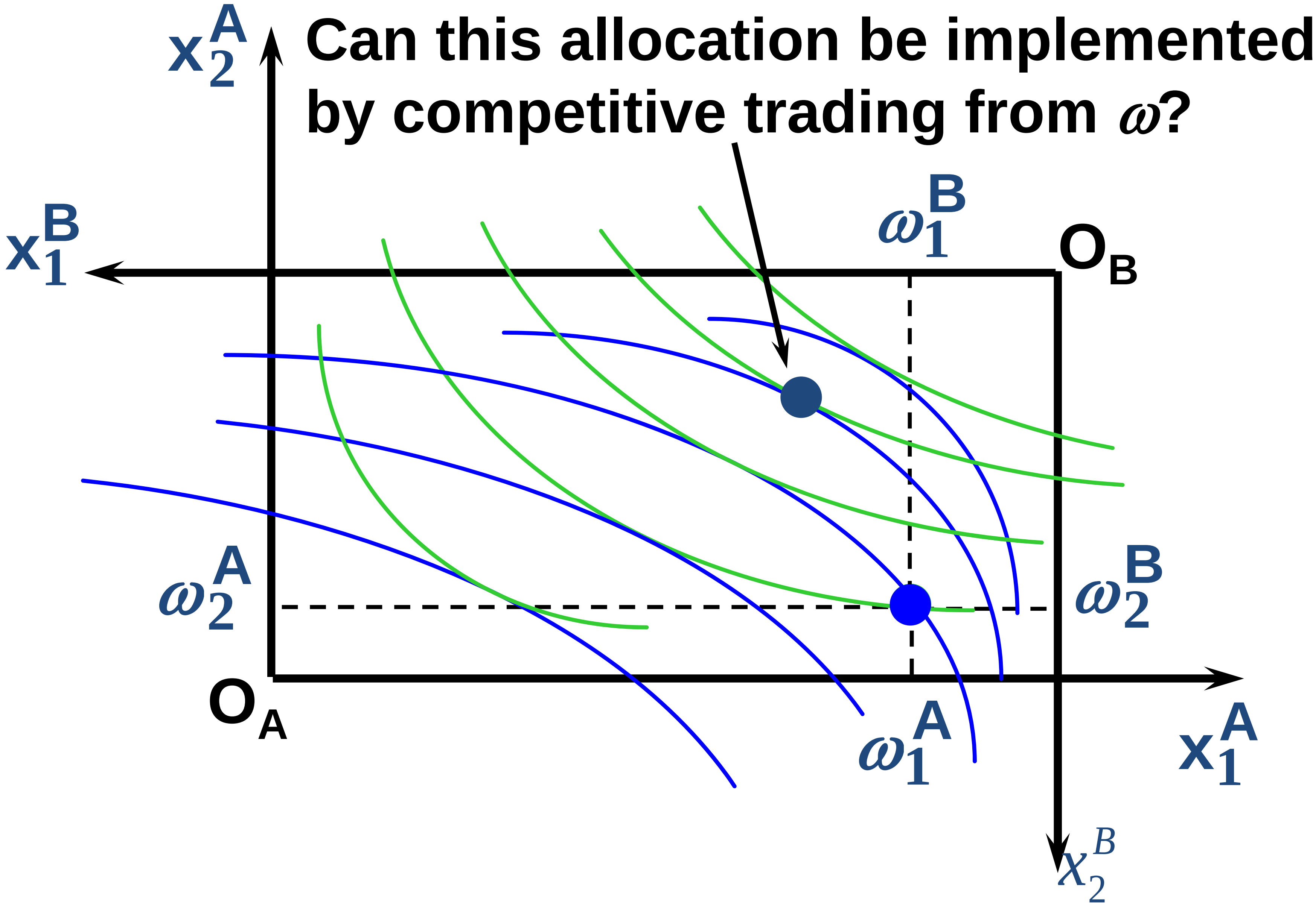
Second Fundamental Theorem



Second Fundamental Theorem

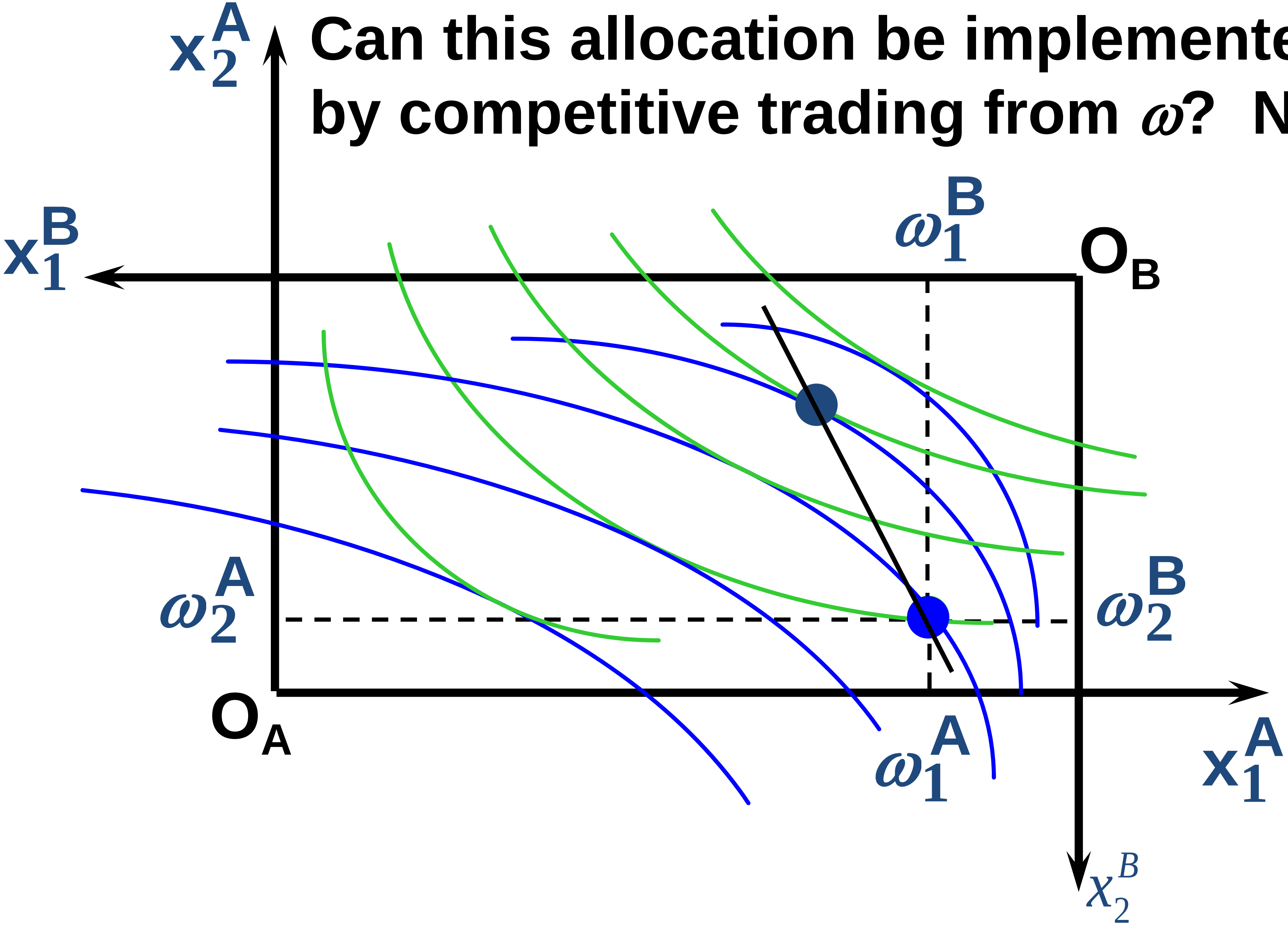


Second Fundamental Theorem



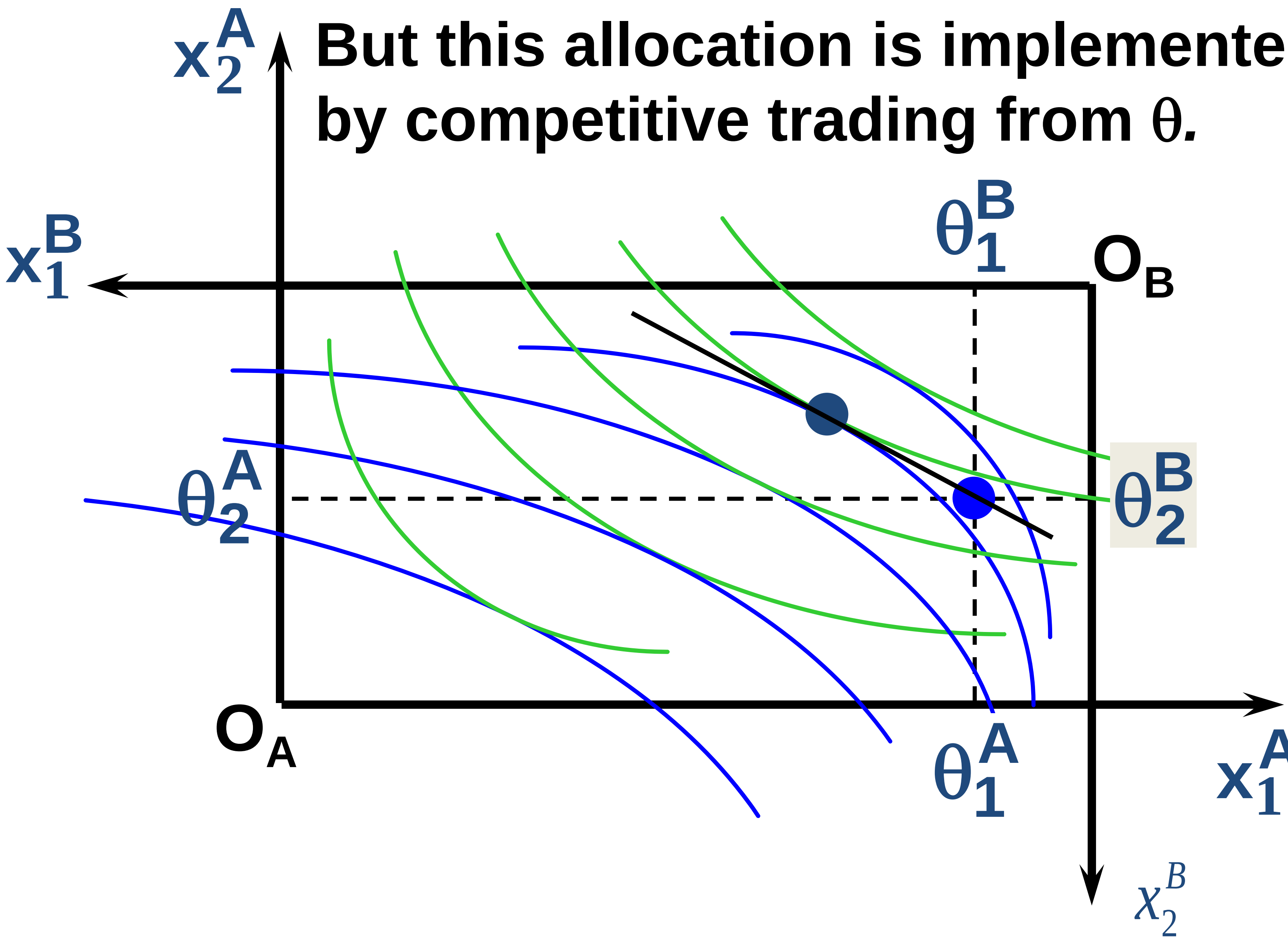
Second Fundamental Theorem

Can this allocation be implemented by competitive trading from ω? No.



Second Fundamental Theorem

But this allocation is implemented by competitive trading from θ.



Walras' Law

- Walras' Law is an identity; i.e. a statement that is true for any positive prices (p_1,p_2), whether these are equilibrium prices or not.

Walras' Law

- Every consumer's preferences are well-behaved so, for any positive prices (p_1,p_2), each consumer spends all of his budget.
- For consumer A:

$$p_1 x_1^{*A} + p_2 x_2^{*A} = p_1 \omega_1^A + p_2 \omega_2^A$$

For consumer B:

$$p_1 x_1^{*B} + p_2 x_2^{*B} = p_1 \omega_1^B + p_2 \omega_2^B$$

Walras' Law

$$p_1 x_1^{*A} + p_2 x_2^{*A} = p_1 \omega_1^A + p_2 \omega_2^A$$

$$p_1 x_1^{*B} + p_2 x_2^{*B} = p_1 \omega_1^B + p_2 \omega_2^B$$

Summing gives

$$p_1(x_1^{*A} + x_1^{*B}) + p_2(x_2^{*A} + x_2^{*B})$$

$$= p_1(\omega_1^A + \omega_1^B) + p_2(\omega_2^B + \omega_2^B).$$

Walras' Law

$$p_1(x_1^{*A} + x_1^{*B}) + p_2(x_2^{*A} + x_2^{*B})$$

$$= p_1(\omega_1^A + \omega_1^B) + p_2(\omega_2^B + \omega_2^B).$$

Rearranged,

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$

$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0.$$

That is, ...

Walras' Law

$$\mathbf{p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +}$$
$$\mathbf{p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B)}$$
$$\mathbf{= 0.}$$

This says that the summed market value of excess demands is zero for any positive prices p_1 and p_2 -- this is Walras' Law.

Implications of Walras' Law

Suppose the market for commodity A is in equilibrium; that is,

$$x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B = 0.$$

Then

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$

$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0$$

implies

$$x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B = 0.$$

Implications of Walras' Law

So one implication of Walras' Law for a two-commodity exchange economy is that if one market is in equilibrium then the other market must also be in equilibrium.

Implications of Walras' Law

What if, for some positive prices p_1 and p_2, there is an excess quantity supplied of commodity 1? That is,

$$\mathbf{x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B < 0.}$$

Then

$$\mathbf{p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +}$$

$$\mathbf{p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0}$$

implies

$$\mathbf{x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B > 0.}$$

Implications of Walras’ Law

So a second implication of Walras’ Law for a two-commodity exchange economy is that an excess supply in one market implies an excess demand in the other market.