LECTURE 5: PRINCIPLES OF PRODUCTION FUNCTIONS - SINGLE INPUT FUNCTIONS

QUESTIONS AND PROBLEMS

True/False Questions

____ If a production function has an elasticity of scale of 3, this means that increasing all inputs by 1% will increase output by 3%.

____ The marginal product of an input x is higher than the average product of x.

____ Consider a single-input production function. If the marginal product exceeds the average product for all levels of output and there are no fixed inputs, then this production function is characterized by decreasing returns to scale.

____ Consider a single-input production function. If the marginal product is increasing at some level of output q, then the production function is characterized by increasing returns to scale at that level of output.

____ If a production function has an elasticity of scale of 2, this means that increasing all inputs by 2% will increase output by 1%.

____ The average product of input x is higher than the marginal product of input x.

____ If the average product of a single input production function is increasing for all levels of output, then this production function is characterized by increasing returns to scale.

____ If the marginal product of a single input production function exceeds the average product for all levels of output and there are no fixed inputs, then this production function is characterized by increasing returns to scale.

____ Consider a single-input production function. If the average product is increasing at some level of output q, then the production function is characterized by increasing returns to scale at that level of output.

____ The average product of a single input production function is given by $q^{0.5}$ where q is the output level. This production function is characterized by increasing returns to scale.

Short Questions

1. Explain what is meant by “decreasing returns to scale”.

Problems

1. The production function of an oil-field is given by

$$Q = \log(1 + \beta D)$$

where Q is the quantity of oil produced, D reflects the amount of drilling in the field (measured in thousands of feet of extraction holes drilled into the oil-field), and β is a positive number that indexes the effectiveness of drilling.

a. What is the marginal product of additional drilling? [Assume for simplicity that D is a real number, i.e, that the firm can drill increments of thousands of feet.] Are there decreasing returns to increasing drilling in that field?

b. Suppose that an other oil-field yields oil on the basis of the production function

$$Q = 2 \log(1 + \beta D)$$

That is, this oil-field is more productive than the first one. Suppose that the firm that owns the oil-fields has drilled 50 thousand feet in the first field and 120 thousand feet in the second field. For what values of β would the firm choose to expand drilling in the first (relatively unproductive) field rather than the second?

c. How does the effectiveness of drilling technology affect that decision? That is, will the firm choose to expand drilling in the first field of the second when drilling technology is effective or when it is ineffective?

2. The number of berries harvested in a field depends on the number of workers employed in that field. Let the quantity harvested be given by

$$q = 100 \sqrt{L}$$

where L the number of labor-hours used.

a. Graph the relationship between q and L, with q on the vertical axis and L on the horizontal axis. Does this production function exhibit decreasing, constant, or increasing returns to scale ?

b. What is the average productivity of labor in this farm? Graph this relationship and show that AP_L diminishes as labor input increases.

c. Derive the marginal productivity of labor for this farm. Show that $MP_L < AP_L$ for all values of L.

3. Consider the production function

$$f(x) = \begin{cases} \alpha \sqrt{x} - \beta & \text{if } \alpha \sqrt{x} - \beta \geq 0 \\ 0 & \text{if } \alpha \sqrt{x} - \beta < 0 \end{cases}$$

The parameter β captures the notion of the set-up cost: you need to use a critical level of input before you get any positive output.

a. Graph this production function for $\alpha = 1$ and $\beta = 2$. Label any intercepts. Show your work.

b. Calculate the marginal product and the average product of x. Treat α and β as unknown parameters.

c. Does this production function have increasing or decreasing returns to scale? Base your answer and explanation on your graph in part (a). [Using math is fine, but it is the hard way to show this, and you don't need to do it.]

4. Consider the production function

$$f(x) = \begin{cases} \alpha x^2 - \beta & \text{if } \alpha x^2 - \beta \geq 0 \\ 0 & \text{if } \alpha x^2 - \beta < 0 \end{cases}$$

The parameter β captures the notion of the set-up cost: you need to use a critical level of input before you get any positive output.

a. Graph this production function for $\alpha = 1$ and $\beta = 2$.

b. Calculate the marginal product and the average product of x. Treat α and β as unknown parameters.

c. Does this production function have increasing or decreasing returns to scale? Base your answer and explanation on your graph in part (a). [Using math is fine, but it is the hard way to show this, and you don't need to do it.]

d. Suppose you did not know the values of parameters α and β but you only knew that they were positive constants. Would you still have been able to reach the same the conclusion as that obtained in part (c) above?