

LECTURE 7: SPECIAL PRODUCTION FUNCTIONS, PART I

QUESTIONS AND PROBLEMS

True/False Questions

- _____ A Cobb-Douglas production is the only production function with diminishing *MRTS*
- _____ The Cobb-Douglas production function exhibits increasing *MRTS*
- _____ The production function $f(K,L) = \frac{1}{A} K^\alpha E^\beta$ is a Cobb-Douglas production function.
- _____ If a two-input production function is characterized by increasing returns to scale, then the isoquant for $q=2$ will be closer to the isoquant for $q=1$ than to the isoquant for $q=3$.

Short Questions

1. In general, the MRTS between two inputs depends on the utilization rate of other inputs, i.e., in a production function $f(K,L,E)$, $MRTS_{K,L}$ would in general be a function of E . Show that this is not the case for a Cobb-Douglas production function, i.e., show that for the production function $q = A K^\alpha L^\beta E^\gamma$, $MRTS_{K,L}$ does not depend on E .

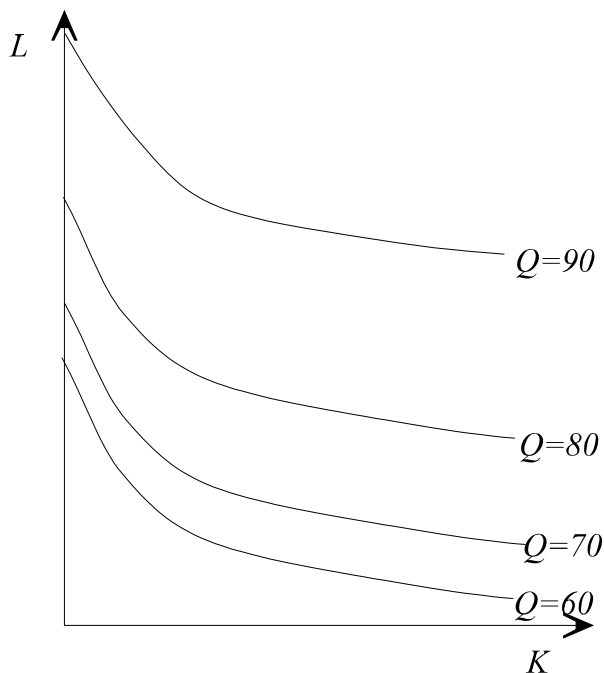
2. Consider the production function

$$f(x, z) = x^2 + 2z^2$$

A. What is the $MRTS_{x,z}$?

B. What is the elasticity of scale for this production function? Show your work, starting from the definition of the elasticity of scale.

3. Consider the production function with isoquants as given in the figure below.



From this figure, does it appear that this production function has increasing or decreasing returns to scale? Explain your answer.

Problems

1. Consider the Cobb-Douglas production function for engineers (E) and technicians (T) given by $Q = 10 E^{0.6} T^{0.3}$.

- a. If the firm employs an equal number of engineers and technicians, how many technicians will it take to do the work of one engineer? In other words, what is the MRTS of engineers for technicians? [Derive your answer from one of the general expressions for MRTS.]
- b. What must be the ratio of engineers to technicians employed in this firm so that one technician will be as productive as one engineer?