LECTURE 8: SPECIAL PRODUCTION FUNCTIONS, PART II

QUESTIONS AND PROBLEMS

True/False Questions

____ The elasticity of scale of a fixed proportions production function is not defined because the fixed proportions production function is not differentiable.

____ The *MRTS* between two inputs for a fixed proportions production function is either zero or infinity or not defined depending on the input mix. No other values are possible.

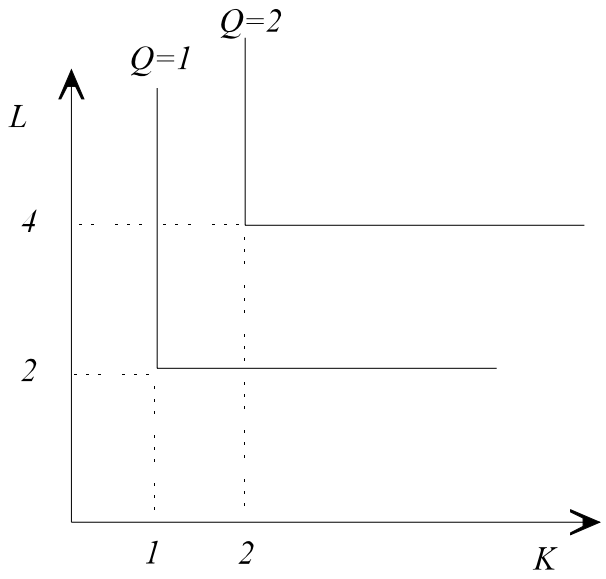
____ If a firm's production function is linear, then the marginal product of each input is constant and independent of the level of the other inputs.

____ The $MRTS_{x,y}$ of a linear production function that has two inputs, *x* and *y*, is constant if the production function is of constant returns to scale, increasing if the production function is of increasing returns to scale, and decreasing if the production function is of decreasing returns to scale.

____ The $MRTS_{x,y}$ of a linear production function that has two inputs, *x* and *y*, is constant if the production function is of constant returns to scale, decreasing if the production function is of increasing returns to scale, and increasing if the production function is of decreasing returns to scale.

Short Questions

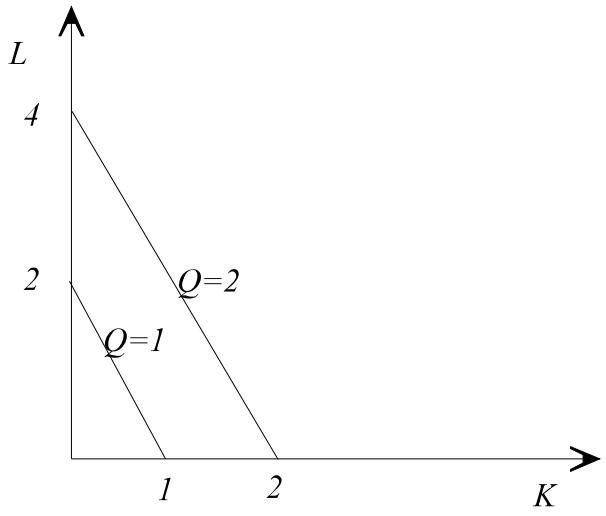
1. Consider a firm whose production function is characterized by the following isoquants.



A. Is the production function of this firm Cobb-Douglas, Leontieff, or Linear?

B. Write the mathematical expression for this production function.

2. Consider a firm whose production function is characterized by the following isoquants:



A. Is the production function of this firm Cobb-Douglas, Leontieff, or Linear?

B. Write the mathematical expression for this production function.

3. Consider the production function

$$q = K^{\gamma} E^{0.2} L^{\frac{1}{\theta}}$$

Answer the following questions. No need to show any calculations. Just provide the answer, using what we have learned from the lectures.

a. Is this production function Cobb-Douglas, Leontieff, Linear, or of some other type?

b. Does this production exhibit decreasing or increasing *MRTS*?

c. What is the elasticity of scale of this production function?

d. Which of the three inputs to this production function are essential for production?

e. Sketch an isoquant of this production function with K on the vertical axis and L on the horizontal axis, for E=1. No need to label any points; just make sure the overall shape is consistent with the above production function.

4. Consider the production function

$$q = \min\{2K, \frac{1}{3}L\}^{1/\theta}$$

where θ is an unknown positive parameter. Answer the following questions using what we learned from the lectures (no need to provide explanations).

a. Is this production function Cobb-Douglas, Leontieff, Linear, or of another type?

b. What is the elasticity of scale of this production function?

c. Are any of the two inputs essential for producing output? If so, which one(s)?

d. Draw below the isoquant for this production function that corresponds to $q = 1$, by putting labor (L) in the vertical axis and capital (K) on the horizontal axis.

5. Consider the production function

$$f(K, L) = (K^2 + \beta L^2)^{\epsilon}$$

where K is a measure of capital inputs, L is a measure of labor inputs, the parameter β reflects productivity specific to labor and the parameter ϵ is related to the returns to scale.

a. What is the $MRTS_{K,L}$?

b. What is the elasticity of scale for this productions function?

Problems

1. A power plant can produce electricity using natural gas or fuel oil, or a combination of the two. In particular, its production function for electricity is given by

$$E = (\alpha G + \beta F)^{\epsilon}$$

where is E is the output of electricity, G in the input of natural gas, F is the input of fuel oil, and α, β, and ϵ are parameters.

a. Show that the elasticity of scale is equal to ϵ.

b. Draw the isoquant for $E = 9$, if $\alpha = 2$, $\beta = 3$ and $\epsilon = 2$.

2. Fixing a bug in the computer code of Windows requires 100 hours of an experienced programmer or 300 hours of an inexperienced programmer, or a linear combination of experienced and inexperienced programmers.

a. How many Windows computer code bugs could Microsoft fix if it had at its disposal 1000 hours worth of experienced programers and 1,500 hours worth of inexperienced programmers?

b. Write down the expression for the production function that relates the number of available hours of experienced programmers and the number of available hours of inexperienced programmers to the number of bugs of Windows computer code that can be fixed.

3. Producing sweetening each unit of diet *FineSoda* requires either θ units of NutraSweet or λ units of Splenda, or a linear combination of the two. The sweetening process is constant returns to scale, so increasing the output by any given percentage would require increasing the inputs by that same percentage.

a. Write down the production function for sweetening *FineSoda*.

b. Graph the isoquant that corresponds to sweetening one unit of *FineSoda*. In the same figure, graph the isoquant that corresponds to sweetening two units of *FineSoda*. Make sure to draw the two isoquants so as to reflect the fact that this is a constant returns to scale production function.

4. The number of heart transplants that can be done at Humana hospital depends on the number of surgeon-hours and nurse-hours available. There is no substitutability between surgeons and nurses: For each surgery one needs 4 surgeon-hours and 60 nurse-hours.

a. Write down the production function for heart transplants at Human hospital.(assume for simplicity that one can have fractional transplants)

b. Graph the isoquant that corresponds to 2 transplants. In the same figure, graph the isoquant that corresponds to 3 transplants.