NOMA Assisted Wireless Caching: Strategies and Performance Analysis

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Abstract

This paper investigates the coexistence of two important enabling techniques for future wireless networks, non-orthogonal multiple-access (NOMA) and wireless caching, and we show that the use of NOMA ensures that the two caching phases, content pushing and content delivery, can be more effectively carried out, compared to the conventional orthogonal multiple-access (OMA) based case. Two NOMA caching strategies are developed, namely the push-then-delivery strategy and the push-and-delivery strategy. In the push-then-delivery strategy, the NOMA principle is applied in the content pushing and content delivery phases, respectively. The presented analytical framework demonstrates that the push-then-delivery strategy not only significantly improves the cache hit probability, but also considerably reduces the delivery outage probability, compared to the OMA strategy. The push-and-delivery strategy is motivated by the fact that some users’ requests cannot be accommodated locally and the base station has to serve them directly. The key idea of the push-and-delivery strategy is to merge the content pushing and delivery phases, i.e., the base station pushes new content to local servers while simultaneously serving the users. We show that this strategy can be straightforwardly extended to device-to-device caching, and corresponding analytical results are developed to illustrate the superiority of this caching strategy.

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I. INTRODUCTION

Recently non-orthogonal multiple access (NOMA) has received significant attention as a key enabling technique for future wireless networks [1]–[3]. The key idea of NOMA is to encourage spectrum sharing among mobile nodes, which not only improves the spectral efficiency but also ensures that massive connectivity can be effectively supported. Practical concepts for implementing the NOMA principle for a single resource block, such as an orthogonal frequency division multiplexing (OFDM) subcarrier, include power domain NOMA and cognitive radio (CR) inspired NOMA [4]–[6], which provide different tradeoffs between throughput and fairness. When each user is allowed to occupy multiple subcarriers, dynamically grouping the users on different subcarriers is a challenging problem, and various multi-carrier NOMA schemes, such as sparse code multiple access (SCMA) and pattern division multiple access (PDMA) [7], [8], provide practical solutions for achieving different performance-complexity tradeoffs. Unlike single-carrier NOMA, in multi-carrier NOMA, a user’s message is spread over multiple resource blocks, which requires efficient encoding schemes, such as multi-dimensional coding, to be implemented at the transmitter and low-complexity decoding schemes, such as message passing algorithms, to be used at the receivers.

NOMA has been shown to be compatible with many other advanced communication concepts. For example, several features of millimeter-wave (mmWave) communications, such as highly directional transmission, and the mismatch between the users’ channel vectors and the commonly used finite resolution analog beamforming, facilitate the implementation of NOMA in mmWave networks [9], [10]. In addition, NOMA can further improve the spectral efficiency of multiple-input multiple-output (MIMO) systems. For example, MIMO-NOMA can efficiently exploit the spatial degrees of freedom of MIMO channels and, unlike single-input single-output (SISO) NOMA, is beneficial even if the users have similar channel conditions [11]–[13]. Furthermore, conventionally, when the users have a single antenna, cooperative transmission can be used to exploit spatial diversity but suffers from a reduced overall data rate, since relaying consumes extra bandwidth resources [14]. In this context, the application of NOMA can efficiently reduce the number of consumed bandwidth resource blocks, such as subcarriers and time slots, and hence improve the spectral efficiency of cooperative communications [15]–[17]. Furthermore, existing studies have also revealed a strong synergy between NOMA and CR networks, where the use of NOMA can significantly improve the connectivity for the users of the secondary
Wireless caching is another important enabling technique for future communication networks [19], [20], but little is known about the coexistence of NOMA and wireless caching. The key idea of wireless caching is to push the content in off-peak hours during the so-called content pushing phase close to the users before it is requested, and therefore, the users’ requests can be locally served during the so-called content delivery phase. In fact, asking a base station (BS) to serve the users’ requests directly is not preferable, not only because the maximal number of users that a BS can serve concurrently is small, but also because non-caching transmission schemes are severely constrained by the limited backhaul capacity of wireless networks. Most caching schemes can be grouped into one of two classes [20], [23]. The first class assumes the existence of a content caching infrastructure, such as content servers, small cell BSs, etc. [24]–[26]. When caching infrastructure (e.g., content servers) is available, the aim of the content pushing phase is to push the content files to the content servers in a timely and reliable manner, before the users request these files. During the phase of content delivery, an ideal situation is that all the users’ requests can be locally served, without communicating with the central controller of the network, e.g., a BS. The second class, also known as device-to-device (D2D) caching, assumes that there is no dedicated caching infrastructure, and relies on user cooperation [27], [28]. Particularly, during the content pushing phase, all users will proactively cache some content. During the content delivery phase, a user will communicate with its BS only if none of its neighbours can help the user locally, i.e., the user cannot find its requested file in the caches of its surrounding neighbours.

This paper investigates the coexistence of NOMA and wireless caching, which is crucial for their joint implementation in future wireless networks. In particular, we concentrate on the following two questions. The first question is how to realize content pushing in a timely and robust manner. Many existing works on caching assume that content can be pushed to caching infrastructure or D2D caching helpers during off-peak hours through wired connections. However, this assumption might not be realistic due to the dynamic nature of content popularity which implies that some content cached a long time ago may have to be replaced by new content. In addition, caching infrastructure or D2D caching helpers might not have wired connections.

1We note that coded caching, where the number of BS transmissions is reduced by exploiting the structure of the content sent during the content pushing and delivery phases, does not fall into the two considered categories [21], [22].
to the BS, and therefore, content needs to be pushed through the wireless medium which is prone to attenuation and various impairments. Therefore, timely and robust content pushing is critical for efficient wireless caching. The second question is how to cope with the non-ideal situation for wireless caching, when some users’ requests have to be fetched from the BS directly. We note that for wireless caching this non-ideal situation is inevitable and is expected to occur frequently in practice, as the users’ requests cannot be perfectly predicted. When this situation happens, the spectral efficiency of wireless caching is reduced, since the users’ requests cannot be accommodated locally. The two NOMA-assisted caching strategies proposed in this paper address the aforementioned questions. The contributions of the paper are summarized as follows:

- For the case when the content pushing and delivery phases are separated and content servers are available, a NOMA-assisted push-then-delivery strategy is proposed. Particularly, during the content pushing phase, the BS will use the NOMA principle and push multiple files to the content servers simultaneously. A CR inspired NOMA power allocation policy is used to ensure that the most popular file is delivered to the targeted content server with the same outage probability as with orthogonal multiple access (OMA) based transmission. However, by using NOMA, additional files can be pushed to the content servers simultaneously, which significantly improves the cache hit probability. During the content delivery phase, the use of NOMA not only improves the reliability of content delivery, but also ensures that more user requests can be served concurrently by the content server.

- A NOMA assisted push-and-delivery strategy is proposed to efficiently combine the content pushing and delivery phases, in order to effectively cope with the situation when some users’ requests have to be served by the BS directly. Although this situation is not desirable for wireless caching, it is inevitable in practice and constitutes an opportunity for the application of NOMA. Particularly, when a BS serves a user directly, i.e., it delivers a file directly to a user, the NOMA principle enables the BS to perform content delivery and content pushing simultaneously, i.e., it can push new content to the servers while serving users directly. The proposed push-and-delivery strategy is extended to D2D caching without caching infrastructure, and is shown to also effectively improve the spectral efficiency of content pushing and delivery in this paper. We note that the NOMA-multicasting scheme proposed in [29] can be viewed as a D2D special case of the proposed push-and-delivery strategy.
strategy, if the multicasting phase in [29] is viewed as the content delivery phase. However, the impact of integrating content pushing and delivery on the cache hit probability was not investigated in [29].

• Analytical results for the cache hit probability, the transmission outage probability, and the D2D cache miss probability are derived in order to obtain a better understanding to the proposed caching strategies. Particularly, when caching infrastructure is available, the impact of NOMA on the content pushing phase is quantified by adopting the cache hit probability as performance metric and exploiting the joint probability density function (pdf) of the distances between the content servers and the BS. The impact of NOMA on the content delivery phase is investigated by using the transmission outage probability as performance criterion and modelling the locations of the users and the content servers as Poisson cluster processes (PCPs). Furthermore, the impact of NOMA on D2D caching is studied by modelling the effect of content pushing as a thinning Poisson point process and deriving the cache miss probability, i.e., the probability of the event that a user cannot find its requested file in the caches of its neighbours. The provided simulations verify the accuracy of the proposed analysis, and illustrate the effectiveness of the proposed NOMA based wireless caching schemes.

The remainder of the paper is organized as follows. In Section II, the considered system models, including the caching model and the spatial model, are introduced. In Section III, the NOMA-assisted push-then-delivery strategy is presented, and its impact on the content pushing and delivery phases is investigated. In Section IV the proposed push-and-delivery strategy is developed by efficiently merging the two phases, its impact on the cache hit probability is investigated, and its extension to D2D scenarios is discussed. Computer simulations are provided in Section V and the paper is concluded in Section VI. The details of all proofs are collected in the appendix.

II. SYSTEM MODEL

Consider a two-tier heterogeneous communication scenario, in which multiple users request cacheable content with the help of one BS and multiple content servers. The D2D scenario without caching infrastructure, e.g., content servers, will be described in Section IV.C. Assume that each user is associated with a single content server. If the file requested by a user can be found in the cache of its associated content server, this server will serve the user, which means
that multiple content servers can communicate with their respective users concurrently and hence the spectral efficiency is high. However, if the file requested by a user cannot be found locally, the BS will serve the user directly, a situation which is not ideal for caching and should be avoided. The assumption that each user is associated with a single content server facilitates the use of PCP modelling, as discussed in the following subsection.

A. Spatial Clustering Model

Consider that the BS is located in the origin of a two-dimensional Euclidean plane, denoted by \( \mathbb{R}^2 \). As shown in Fig. 1 there are multiple content servers. The locations of the content servers and the users are modelled as PCPs. In particular, assume that the locations of the content servers are denoted by \( x_i \) and are modelled as a homogeneous Poisson point process (HPPP), denoted by \( \Phi_c \), with density \( \lambda_c \), i.e., \( x_i \in \Phi_c \). For notational simplicity, the location of the BS is denoted by \( x_0 \).

Each content server is the parent node of a cluster covering a disk whose radius is denoted by \( \mathcal{R}_c \). Denote the content server in cluster \( i \) by \( \text{CS}_i \). Without loss of generality, assume that there are \( K \) users associated with \( \text{CS}_i \), denoted by \( U_{i,k} \). Note that users associated with the same content server are viewed as offspring nodes [30]. The offspring nodes are uniformly distributed in the disk associated with \( \text{CS}_i \), and their locations are denoted by \( y_{i,k} \). To simplify the notation, the locations of the cluster users are conditioned on the locations of their cluster heads (content servers). As such, the distance from a user to its content server is simply given by \( ||y_{i,k}|| \), and the distance from \( U_{i,k} \) to \( \text{CS}_j \) is denoted by \( ||y_{i,k} + x_i - x_j|| \) [31], [32].

B. Caching Assumptions

Consider that the files to be requested by the users are collected in a finite content library \( \mathcal{F} = \{f_1, \cdots, f_F\} \). The popularity of the requested files is modelled by a Zipf distribution [33]. Particularly, the popularity of file \( f_i \), denoted by \( P(f_i) \), is modelled as follows:

\[
P(f_i) = \frac{\frac{1}{i^\gamma}}{\sum_{p=1}^{F} \frac{1}{p^\gamma}},
\]

where \( \gamma > 0 \) denotes the shape parameter defining the content popularity skewness. We note that \( P(f_i) \) is the probability that a user requests file \( f_i \). Similar to the existing wireless caching literature, [19], [20], [24]–[26], packets belonging to different files are assumed to have the same length. However, unlike the existing literature, we do not assume that the amount of information
contained in the packets of different files is necessarily identical. Particularly, the prefixed data rate of packets of file $f_i$ is denoted by $R_i$.

Assume that the BS has access to all files. In this paper, when content servers exist, we assume that the users have no caching capabilities. On the other hand, for D2D assisted caching, discussed in Section IV.C, it is assumed that each user has a cache as well.

III. PUSH-THEN-DELIVERY STRATEGY

This section considers the case where the two caching phases, content pushing and content delivery, are separated, i.e., content pushing happens during the off-peak hours and content delivery happens during the peak hours. Next, we show that the use of NOMA can improve the reliability of both content pushing and delivery.

A. Content Pushing Phase

This subsection focuses on the phase for content pushing. In order to better illustrate the performance of NOMA assisted content pushing, the conventional OMA based content pushing strategy is introduced first.
OMA - BS pushes $f_1$ to $CS_t$  
NOMA - BS pushes 4 files to CSs

Fig. 2. An illustration of the impact of NOMA on content pushing, where $1 < m < t$, i.e., $CS_m$ is closer to the BS than $CS_t$. CSs denotes content servers in the figure. In OMA, a single file is pushed to $CS_t$, and in NOMA, the BS pushes three additional files by using NOMA, where a content server closer to the BS is likely able to decode more pushed files.

1) OMA Based Content Pushing: Without loss of generality, assume that there is only one time slot for content pushing. If OMA is used, the BS broadcasts the most popular file, $f_1$, to the content servers. Therefore, $CS_m$ is able to decode file $f_1$ with the following achievable data rate:

$$R_{m, OMA}^{CP} = \log \left( 1 + \rho \frac{1}{L(||x_m - x_0||)} \right),$$  

(2)

where $\rho$ denotes the transmit signal-to-noise ratio (SNR), and $\frac{1}{L(||x_m||)}$ is the large scale path loss between $CS_m$ and the BS located at $x_0$. Particularly, the following path loss model is used, $\frac{1}{L(||x_m||)}$, where $L(||x_m||) = ||x_m||^{\alpha}$ and $\alpha$ denotes the path loss exponent. For a large scale network, the probability that $||x_m - x_0|| < 1$ is very small, and therefore, the simplified unbounded path loss model is used in this paper [31], [32], [34]. Nevertheless, the presented analytical results can be extended to other path loss models, e.g., $L(||x_m||) = ||1 + x_m||^{\alpha}$ or $L(||x_m||) = \max\{1, ||x_m||^{\alpha}\}$, in a straightforward manner. We note that small scale multi-path

\[\text{In practice, there will be multiple time slots for content pushing, and the proposed scheme can be straightforwardly extended to the multi-time-slot case by sending different files in different time slots.}\]
fading is not considered for the channel gain associated with CS\textsubscript{m} since the content servers can be deployed such that line-of-sight connections to the BS are ensured, which means that large scale path loss is the dominant factor for signal attenuation. However, small scale fading will be considered for the channel gains associated with the users, since the users may not have light-of-sight connections to their transmitters.

2) NOMA Assisted Content Pushing: By applying the concept of NOMA, more content can be simultaneously delivered from the BS to the content servers, as shown in Fig. 2. Particularly, the BS sends the following mixture, which contains the \( M \text{s} \) most popular files:

\[
s_i = \sum_{i=1}^{M_s} \alpha_i \mathbf{\bar{f}}_i,
\]

where \( \mathbf{\bar{f}}_i \) denotes the signal which represents the information contained in file \( f_i \), \( \alpha_i \) denotes the power allocation coefficient and \( \sum_{i=1}^{M_s} \alpha_i^2 = 1 \). Each content server carries out successive interference cancellation (SIC). The SIC decoding order is determined by the priority of the files, i.e., a more popular file, \( f_i \), will be decoded before a less popular one, \( f_j, i < j \). Suppose that the files \( f_j, j < i \), have been decoded and subtracted correctly by content server CS\textsubscript{m}. In this case, CS\textsubscript{m} can decode the next most popular file, \( f_i \), with the following data rate:

\[
R_{m,i}^{CP} = \log \left( 1 + \frac{\rho \alpha_i^2 \frac{1}{L(||x_m-x_0||)}}{\frac{1}{L(||x_m-x_0||)} \sum_{j=i+1}^{M_s} \alpha_j^2 + 1} \right).
\]

If \( R_{m,i}^{CP} \geq R_i \), then file \( f_i \) can be decoded and subtracted correctly at CS\textsubscript{m}.

To order to compare with OMA, which pushes only one file at a time, a sophisticated power allocation policy is needed for the NOMA scheme. Without loss of generality, we assume that the content servers are ordered as follows:

\[
\frac{1}{L(||x_1-x_0||)} \geq \frac{1}{L(||x_m-x_0||)} \geq \cdots \geq \frac{1}{L(||x_t-x_0||)} \geq \cdots,
\]

for \( 1 \leq m < t \). Furthermore, we make the following quality of service (QoS) assumption, in order to facility the design of the power allocation coefficients:

**QoS Target:** The most popular file, \( f_1 \), needs to reach the \( t \)-th nearest content server.

Both the OMA and NOMA transmission schemes need to ensure this QoS target. Therefore, the CR inspired power allocation policy can be used for NOMA [6], i.e., power allocation coefficient \( \alpha_1 \) is chosen such that \( f_1 \) can be delivered reliably to CS\textsubscript{t}, i.e.,

\[
R_{t,1}^{CP} \geq R_1.
\]
This constraint results in the following choice of $\alpha_1$:

$$\alpha_1^2 = \min \left\{ 1, \frac{\epsilon_1 \left( \frac{1}{\rho L(\|x_t - x_0\|)} + 1 \right)}{\rho (1 + \epsilon_1) \frac{1}{L(\|x_t - x_0\|)}} \right\},$$

(7)

where $\epsilon_l = 2^{R_l} - 1$. As demonstrated in the performance analysis section, the use of the power allocation policy in (7) ensures that the outage probability for the NOMA based pushing strategy for the most popular file, $f_1$, is the same as that for OMA.

Since $\sum_{j=1}^{M_s} \alpha_j^2 = 1$, (7) implies that the sum of the power allocation coefficients, excluding $\alpha_1$, is constrained as follows:

$$\sum_{j=2}^{M_s} \alpha_j^2 = \max \left\{ 0, \frac{\rho L(\|x_t - x_0\|)}{\rho (1 + \epsilon_1) L(\|x_t - x_0\|)} - \epsilon_1 \right\}.$$  (8)

The constraint in (7) is sufficient to guarantee the successful delivery of $f_1$ to the $t$-th nearest content server. How the remaining power shown in (8) is allocated to the other files, $f_i$, $i \neq 1$, does not affect the delivery of $f_1$. Therefore, in this paper, it is assumed that the portion allocated to $f_i$, $i \neq 1$, is fixed, i.e., $\alpha_i^2 = \beta_i P_r$, where $P_r = \max \left\{ 0, \frac{\rho L(\|x_t - x_0\|)}{\rho (1 + \epsilon_1) L(\|x_t - x_0\|)} - \epsilon_1 \right\}$ and the $\beta_i$ are constants, which satisfy the constraint $\sum_{j=2}^{M_s} \beta_i = 1$.

3) Performance Analysis: An effective criterion to evaluate content pushing is the cache hit probability which is the probability that, during the content delivery phase, a user finds its requested file in the cache of its associated content server. Since the request probability for file $l$ is decided by its popularity, the hit probability for a user associated with CS$_m$ can be expressed as follows:

$$P_{m, hit} = \sum_{i=1}^{M_s} P(f_i)(1 - P_{m,i}),$$

(9)

where $P_{m,i}$ denotes the outage probability of CS$_m$ for decoding file $i$. Note that for the OMA case, only file 1 will be sent, and hence the corresponding OMA hit probability is simply given by

$$P_{m, hit, OMA} = P(f_1)(1 - P_{m,1}^{OMA}),$$

(10)

where $P_{m,1}^{OMA}$ denotes the outage probability of CS$_m$ for decoding file 1. The following theorem reveals the benefit of using NOMA for content pushing.
Theorem 1. The cache hit probability achieved by the proposed NOMA assisted push-then-delivery strategy is always larger than or at least equal to that of the conventional OMA based strategy, i.e.,

\[ P_{\text{hit}}^m \geq P_{\text{hit}}^{m,\text{OMA}}, \]  

for \( 1 \leq m \leq t \).

Proof. See Appendix A.

Remark 1: Only the \( t \) nearest content servers are of interest in (11), i.e., \( 1 \leq m \leq t \), which is due to our assumption that the BS aims to push the most popular file, \( f_1 \), to CS\(_t\).

Remark 2: As shown in Appendix A, the key step to prove the theorem is to show \( P_{m,1}^{\text{OMA}} = P_{m,1} \), i.e., the outage performance of NOMA for decoding \( f_1 \) at CS\(_m\) is the same as that of OMA. If \( f_1 \) is viewed as the message to the primary user in a CR NOMA system, this observation about the equivalence between the NOMA and OMA outage performances is consistent with the results in [6] and [12].

While the use of the CR power allocation policy guarantees that CS\(_t\) can decode \( f_1 \), this also implies that the outage performance at CS\(_m\) is impacted by the channel conditions of CS\(_t\). This means that for the calculation of the outage probability, \( P_{m,i} \), the joint distribution of the ordered distances of CS\(_t\) and CS\(_m\) to the BS is needed. The following lemma provides an analytical expression for this joint distribution.

Lemma 1. Denote the distance between the BS and the \( i \)-th nearest content server by \( r_i \). The joint pdf of \( r_m \) and \( r_t \) is given by

\[ f_{r_m,r_t}(x,y) = 4y(\lambda c \pi)^t e^{-\lambda c \pi y^2} x^{2m-1}(y^2 - x^2)^{t-m-1} \frac{(t-m-1)!(m-1)!}{(t-m-1)!(m-1)!}. \]  

Proof. See Appendix B.

Remark 3: It is worth pointing out that the joint pdf obtained in [35] is a special case of Lemma 1 when \( m = 1 \) and \( t = 2 \).

Since the cache hit probability is a function of the outage probability, we provide the outage performance for content pushing in the following lemma.
Lemma 2. Assume \( \epsilon_{M_s} \geq \epsilon_1 \). The outage probability of CS\(_n\), \( 1 \leq n \leq t \), for decoding \( f_1 \) is given by

\[
P_{n,1} = e^{-\lambda_c \pi \left( \frac{1}{\rho_1} \right)^{\frac{1}{2}}} \sum_{k=0}^{n-1} \frac{(\lambda_c \pi)^k}{k!} \left( \frac{\epsilon_1}{\rho_1} \right)^{\frac{2k}{n}} .
\]

The outage probability of CS\(_i\) for decoding \( f_i \), \( 2 \leq i \leq M_s \), is given by

\[
P_{t,i} = e^{-\lambda_c \pi \left( \frac{1}{\rho_i} + \frac{(1+i)}{\rho_0} \right)^{\frac{1}{2}}} \sum_{k=0}^{t-1} \frac{(\lambda_c \pi)^k}{k!} \left( \frac{\epsilon_i}{\rho_i} + \frac{(1+i)}{\rho_0} \right)^{-\frac{2k}{t}},
\]

where \( \phi_i = \min \{ \frac{\xi_2}{\epsilon_2}, \ldots, \frac{\xi_i}{\epsilon_i} \} \), \( \xi_i = \left( \beta_i - \epsilon_i \sum_{j=i+1}^{M_s} \beta_j \right) \) for \( 2 \leq i < M_s \), and \( \xi_{M_s} = \beta_{M_s} \).

The outage probability of CS\(_m\), \( 1 \leq m < t \), for decoding \( f_i \), \( 2 \leq i \leq M_s \), is given by

\[
P_{m,i} \approx P_{t,1} + \frac{4(\lambda_c \pi)^t}{(t-m-1)!} \frac{t-m-1}{(t-m-1)!} \sum_{p=0}^{t-m-1} (-1)^p \binom{t-m-1}{p} \left( \frac{t-m-1}{p} \right)
\]

\[
\times \sum_{l=1}^{N} \frac{\pi \left( \frac{\tau_2 - \tau_1}{2N} \right)}{2N} f_m \left( \frac{\tau_2 - \tau_1}{2} \right) \left( \frac{\tau_2 + \tau_1}{2} \right) \sqrt{1 - w_l^2},
\]

where \( \tau_1 = \left( \frac{\rho_0}{1 + \epsilon_1 + \epsilon_1 \phi_i} \right)^{\frac{1}{n}}, \tau_2 = \left( \frac{\epsilon_i}{\rho_i} \right)^{-\frac{1}{n}}, N \) denotes the parameter for Chebyshev-Gauss quadrature, \( w_l = \cos \left( \frac{2l-1}{2N} \pi \right) \), \( g(y) = \left( \frac{1+i}{\phi_i(\rho - \epsilon_1 y^n)} \right)^{-\frac{1}{n}} \), and

\[
f_m(y) = e^{-\lambda_c \pi y^2} \frac{\lambda_c \pi y^{2(t-m-1)-2p+1}}{2m+2p} \left( y^{2m+2p} - (g(y))^{2m+2p} \right) .
\]

Proof. See Appendix C. \( \square \)

Remark 4: It is worth pointing out that, Lemma 2, it is assumed that the targeted data rates and the power allocation coefficients are chosen to ensure \( \xi_i > 0 \). Otherwise, an outage will always happen for decoding file \( f_i \), \( i \geq 2 \), at the content servers.

Remark 5: In Lemma 2 it is also assumed that \( \epsilon_{M_s} \geq \epsilon_1 \), in order to avoid a trivial case for the integral calculation, as shown in (74). This assumption means that the targeted data rate for file \( M_s \) is larger than that of file 1, which can be justified as follows. If \( R_1 \) is very large, the use of the CR power allocation policy means that most of the transmission power will be consumed in order to ensure that \( f_1 \) is delivered to CS\(_s\), and hence, not much power will be left for pushing additional files. In other words, the case with a large \( R_1 \) is not an ideal situation for applying the proposed NOMA pushing strategy.
B. Content Delivery Phase

In the previous subsection, the cache hit probability for content delivery has been analyzed. However, the event that a user can find its requested file in the cache of its associated content server is not equivalent to the event that this user can receive the file correctly, due to the multi-path fading and path loss attenuation that affect its link to the content server. Hence, in this subsection, the impact of NOMA on the reliability of content delivery is investigated. Similar to the previous subsection, the conventional OMA based content delivery strategy is described first as a benchmark scheme.

1) OMA Based Content Delivery: For the OMA case, during the content delivery phase, each content server randomly schedules a single user whose request is available locally in the cache of the server. We assume that each content server can find a user to serve, and all the content servers transmit simultaneously, which facilitates the used PCP modelling.

Fig. 3. An illustration of the impact of NOMA on content delivery. In OMA, each content server serves a single user. By using NOMA, an additional user can be served.
2) NOMA Assisted Content Delivery: If the NOMA principle is applied in the content delivery phase, each content server can serve two users. Assume that the two users are ordered based on their distances to their associated content servers. As shown in Fig. 3, the weak user, denoted by $U_{m,1}$, is inside a ring with radii $R_s$ and $R_c$, $R_s < R_c$. The strong user, denoted by $U_{m,2}$, is in a disc with radius $R_s$. Without loss of generality, denote the file requested by $U_{m,k}$ by $f_{m,k}$, $f_{m,k} \in \mathcal{F}$. Each content server broadcasts a superposition signal containing two messages, and $U_{m,k}$, which is associated with $CS_m$, receives the following:

$$y_{m,k} = \frac{h_{m,mk}}{\sqrt{L(||y_{m,k}||)}} \sum_{l=1}^{2} \alpha_l \tilde{f}_{m,l}$$

Signals from $CS_m$  

$$+ \sum_{x_j \in \Phi_c \setminus m} \frac{h_{j,mk}}{\sqrt{L(||y_{m,k} + x_m - x_j||)}} \sum_{l=1}^{2} \alpha_l \tilde{f}_{j,l} + n_{m,k},$$

Signals from interfering clusters

where $\tilde{f}_{j,l}$ denotes the signal which represents the information contained in file $f_{j,l}$, $\alpha_l$ denotes the NOMA power allocation coefficient, $n_{m,k}$ is the additive complex Gaussian noise, and $h_{j,mk}$ denotes the Rayleigh fading channel coefficient between $CS_j$ and $U_{m,k}$. In order to obtain tractable analytical results, fixed power allocation is used, instead of CR power allocation, and it is assumed that all content servers use the same fixed power allocation coefficients. In order to keep the notations consistent, the power allocation coefficients are still denoted by $\alpha_i$. We note that the simulation results provided in Section V show that the use of this fixed power allocation can still ensure that NOMA outperforms OMA for both users.

As a result, $U_{m,1}$ will treat its partner’s message as noise and decode its own message $f_{m,1}$ with the following SINR:

$$\text{SINR}_{m,1}^{1} = \frac{\alpha_2^2 |h_{m,m1}|^2}{L(||y_{m,1}||)} + I_{\text{inter}}^{m,1} + \frac{1}{\rho},$$

where

$$I_{\text{inter}}^{m,1} = \sum_{x_j \in \Phi_c \setminus m} \frac{|h_{j,m1}|^2}{L(||y_{m,1} + x_m - x_j||)}.$$ 

We focus on the case with two users since the content delivery phase is analog to the conventional downlink case and two-user NOMA based downlink transmission has been proposed for long term evolution (LTE) Advanced [36]. The analytical results presented in this paper can be extended to the case with more than two users by dividing the disc covered by a content server into multiple rings. In practice, the number of users to be served simultaneously needs to reflect a practical tradeoff between system complexity and throughput.
In practice, the content servers are expected to use less transmission power than the BS, but for notational simplicity, \( \rho \) is still used to denote the ratio between the transmission power of the content servers and the noise power. In Section V, for the presented computer simulation results, different transmission powers are adopted for the BS and the content servers.

The strong user, \( U_{m,2} \), intends to first decode its partner’s message with the data rate 

\[
\log(1 + \text{SINR}_{m,2}^1)
\]

where \( \text{SINR}_{m,2}^1 \) is defined similarly to \( \text{SINR}_{m,1}^1 \), i.e., 

\[
\text{SINR}_{m,2}^1 = \frac{\alpha_2^2 |h_{m,m,2}|^2}{L(\|y_{m,2}\|) \text{inter} + \frac{1}{\rho}}
\]

and the inter-cluster interference, \( I_{\text{inter}}^{m,2} \), is defined similarly to \( I_{\text{inter}}^{m,1} \). If 

\[
\log(1 + \text{SINR}_{m,2}^1) > R_1
\]

i.e., \( U_{m,2} \) can decode its partner’s message successfully, \( U_{m,2} \) will remove \( f_{m,1} \) and decode its own message with the following SINR:

\[
\text{SINR}_{m,2}^2 = \frac{\alpha_2^2 |h_{m,m,2}|^2}{L(\|y_{m,2}\|) \text{inter} + \frac{1}{\rho}}.
\] (18)

The outage probabilities of the two users are defined as follows:

\[
P_{o,m,1} = P(\log(1 + \text{SINR}_{m,1}^1) < R_1),
\] (19)

and

\[
P_{o,m,2} = 1 - P(\log(1 + \text{SINR}_{m,1}^1) > R_1, \log(1 + \text{SINR}_{m,2}^2) > R_2).
\] (20)

The following lemma provides closed-form expressions for these outage probabilities.

**Lemma 3.** The outage probability of \( U_{m,2} \) can be expressed as follows:

\[
P_{o,m,2} \approx 1 - \sum_{n=1}^{N} \bar{w}_n e^{-\frac{c_{n,r}}{\bar{\tau}}} q\left(\frac{c_{n,r}}{\bar{\tau}}\right),
\] (21)

where \( \bar{\tau} = \min\left\{\frac{\alpha_2^2 - \epsilon_1 \alpha_2^2}{\epsilon_1}, \frac{\alpha_2^2}{\epsilon_2}\right\}, \quad q(s) = \exp\left(-2\pi \lambda \frac{\bar{s}}{\alpha} B\left(\frac{2}{\alpha}, \frac{\alpha-2}{\alpha}\right)\right), \quad B(\cdot) \text{ denotes the Beta function, } \bar{w}_n = \frac{\pi}{2N} \sqrt{1 - w_n^2 (w_n + 1)}, \quad w_n \text{ is defined in Lemma 2 and } c_{n,r} = \left(\frac{s}{2} w_n + \frac{r}{2}\right)\alpha.

The outage probability of \( U_{m,1} \) can be expressed as follows:

\[
P_{o,m,1} \approx 1 + \frac{R_s^2}{R_c^2 - R_s^2} \sum_{n=1}^{N} \bar{w}_n e^{-\frac{c_{n,r}}{\alpha_1^2 - \epsilon_1 \alpha_2^2}} q\left(\frac{c_{n,r}}{\alpha_1^2 - \epsilon_1 \alpha_2^2}\right) - \frac{R_c^2}{R_c^2 - R_s^2} \sum_{n=1}^{N} \bar{w}_n e^{-\frac{c_{n,r}}{\alpha_1^2 - \epsilon_1 \alpha_2^2}} q\left(\frac{c_{n,r}}{\alpha_1^2 - \epsilon_1 \alpha_2^2}\right).\] (22)

**Proof.** See Appendix D.
Remark 6: In the previous subsection, the CR power allocation policy is used and this type of power allocation ensures that the NOMA outage performance of the weak user, $U_{m,1}$, is the same as that for OMA. Since fixed power allocation coefficients are used in this subsection for content pushing, the performance of the weak user is no longer guaranteed, but surprisingly, our simulation results indicate that the use of NOMA can still yield an outage performance gain for the weak user, compared to OMA, as shown in Section V.

IV. Push-and-Delivery Strategy

A situation which is undesirable but inevitable for wireless caching is that a user’s request cannot be accommodated by its local content server and hence the BS has to serve the user directly, as shown in Fig. 4(a). Conventionally, when this situation happens, the spectrum efficiency of wireless caching is reduced. The proposed push-and-delivery strategy treats this situation as an opportunity for the application of NOMA to improve the spectrum efficiency of wireless caching. As illustrated in Fig. 4(b), when a user needs to be served directly by its BS, the NOMA principle is applied to achieve two goals simultaneously, namely content pushing and content delivery, which ensures that more content is pushed to the servers for future use, while the BS addresses the current demand of the users directly.

![Diagram](image)

(a) General principle of push-and-delivery  
(b) An illustration of push-and-delivery

Fig. 4. An illustration of the proposed push-and-delivery strategy.

In particular, consider a time slot which is dedicated to user $U_{m,k}$. During this time slot, if OMA is used, only this user can be served by the BS directly. However, the use of the NOMA principle offers the opportunity to also push new content to the servers, i.e., the BS sends a
superposition signal containing the file requested by $U_{m,k}$, denoted by $f_0$, and the $M_s$ most popular files pushed by the BS, denoted by $f_i$, $1 \leq i \leq M_s$. Assume that $f_0$ and $f_i$, $1 \leq i \leq M_s$, belong to different sets of the file library, in order to avoid correlation among these files and to simplify the expression for the cache hit probability. In order to obtain tractable analytical results, it is assumed that $U_{m,k}$ is randomly selected from the offsprings of $CS_m$.

A. Performance Analysis

Following similar steps as in the previous section, the data rate of $U_{m,k}$ for decoding its requested file, $f_0$, which is directly sent by the BS, is given by

$$ R_{m,k} = \log \left( 1 + \frac{\alpha_0^2 |h_{mk}|^2}{\sum_{l=1}^{M_s} \alpha_l^2 |h_{mk}|^2 L(||y_{m,k} + x_m||) + \frac{1}{\rho}} \right), $$

and each content server, $CS_m$, can decode the additionally pushed file $f_i$ with the following data rate:

$$ R_l^m = \log \left( 1 + \frac{\alpha_l^2 L(||x_m||)}{\sum_{l=i+1}^{M_s} \frac{\alpha_l^2 L(||x_m||)}{L(||x_m||) + \frac{1}{\rho}}} \right), $$

if $R_l^m$ is larger than $R_j$, for $0 \leq j \leq i - 1$, where $R_l$ denotes the targeted data rate of $f_l$. Again, small scale multi-path fading is not considered in the channel model of $CS_m$, as we assume that the large scale path loss is dominant in this case, and small scale fading is considered for the users’ channels. Note that the indices of the power allocation coefficients $\alpha_i$ start from 0, due to file $f_0$. Compared to the distance between $CS_m$ and the BS, the corresponding distance between $U_{m,k}$ and the BS has a very complicated pdf, as shown in the following subsection. Therefore, in order to obtain tractable analytical results, fixed power allocation coefficients $\alpha_i$ will be used, instead of the CR based ones. The outage probabilities of the user and the content servers will be studied in the following subsections, respectively.

1) Performance of the user: The main challenge in analyzing the outage performance at the user is the complicated expression for the pdf of the distance $||y_{m,k} + x_m||$. First, we define $\tilde{z}_{m,k} = \frac{|h_{mk}|^2}{L(||y_{m,k} + x_m||)}$. The outage probability at the user can be expressed as follows:

$$ P^1_{m,k} = P(R_{m,k} < R_0) = P \left( \frac{\tilde{z}_{m,k}}{\frac{c_0}{\rho_1}} \right) $$

$$ = \mathcal{E}_L(||y_{m,k} + x_m||) \left\{ 1 - e^{-L(||y_{m,k} + x_m||) \frac{c_0}{\rho_0 \rho}} \right\}, $$

where $\mathcal{E}_L$ is an exponential distribution.
where $\zeta_l = \alpha_l^2 - \epsilon \sum_{j=l+1}^{M_s} \alpha_j^2$ for $0 \leq l < M_s$, and $\zeta_{M_s} = \alpha_{M_s}^2$. Again it is assumed that the power allocation coefficients and the targeted data rates are carefully chosen to ensure that $\zeta_l$ is positive.

In order to derive the pdf of $||y_{m,k} + x_m||$, we first define $r_m = ||x_m||$ and also a function

$$g(r_m, r) = \frac{2r \arccos \frac{r^2 + r^2 - R_c^2}{2r_m r}}{\pi R_c^2}.$$  

Conditioned on $r_m$, the pdf of $||y_{m,k} + x_m||$ is given by [37]

$$f_{||y_{m,k} + x_m||}(r|r_m) = g(r_m, r),$$  \hspace{1cm} (26)

for $r_m - R_c \leq r \leq r_m + R_c$, if $r_m > R_c$. Otherwise, we have

$$f_{||y_{m,k} + x_m||}(r|r_m) = \begin{cases} 2\pi r, & \text{if } r \leq R_c - r_m, \\ 2\pi r - g(r_m, r), & \text{if } R_c - r_m < r \leq \sqrt{R_c^2 - r_m^2}, \\ g(r_m, r), & \text{if } \sqrt{R_c^2 - r_m^2} < r \leq R_c + r_m. \end{cases}$$  \hspace{1cm} (27)

In order to avoid the trivial cases, which lead to $r = 0$, i.e., the user is located at the same place as the BS, we assume that no content server can be located inside the disc, denoted by $B(x_0, \delta R_c)$, i.e., a disc with the BS located at its origin and radius $\delta R_c$ with $\delta > 1$, which means that $r_m \geq \delta R_c$ for all $m \geq 1$. Therefore, only the expression in (26) needs to be used since $r_m$ is strictly larger than $R_c$.

After using the pdf of $||y_{m,k} + x_m||$, the outage probability can be expressed as follows:

$$P_{1m,k} = 1 - \int_{\delta R_c}^{\infty} \int_{z-R_c}^{z+R_c} e^{-\frac{\delta}{R_c} \frac{\pi r^2}{\pi R_c^2}} g(z, r) dr \tilde{f}(z) dz,$$  \hspace{1cm} (28)

where $\tilde{f}(z)$ denotes the pdf of $r_m$. Because of the assumption that no content server can be located inside of $B(x_0, \delta R_c)$, the pdf of $r_m$ is different from that in (50), but the steps of the proof for Theorem 1 in [38] can still be applied to obtain the pdf of $r_m$. Particularly, first denote by $A_r$ the ring with inner radius $\delta R_c$ and outer radius $r$. The cumulative distribution function (CDF) of $r_m$ can be expressed as follows:

$$\tilde{F}_{r_m}(r) = 1 - P(\# \text{ of nodes in the ring } A_r < m)$$  

$$= 1 - \sum_{l=0}^{m-1} \frac{(\lambda_c[\pi r^2 - \pi \delta^2 R_c^2])^l}{l!} e^{-\lambda_c[\pi r^2 - \pi \delta^2 R_c^2]}.$$  \hspace{1cm} (29)
Therefore, the pdf of $r_m$ can be calculated as follows:

$$
\bar{f}_{rm}(r) = -2\pi \lambda_c r e^{-\lambda_c [\pi r^2 - \pi \delta^2 R_c^2]} \left( \sum_{l=1}^{m-1} \frac{\left( \lambda_c [\pi r^2 - \pi \delta^2 R_c^2] \right)^l}{(l-1)!} \right)
$$

$$
- \sum_{l=0}^{m-1} \frac{\left( \lambda [\pi r^2 - \pi \delta^2 R_c^2] \right)^l}{l!} \right)
$$

$$
= 2\pi \lambda_c^m r e^{-\lambda_c [\pi r^2 - \pi \delta^2 R_c^2]} \left[ \pi r^2 - \pi \delta^2 R_c^2 \right]^{m-1} \left( \frac{m-1}{(m-1)!} \right).
$$

Substituting (30) into (28), the outage probability of the user can be obtained.

2) Performance of the content servers: The content servers need to carry out SIC in order to decode the newly pushed files $f_l$. As a result, the outage probability of $CS_m$ for decoding $f_i$ can be expressed as follows:

$$
P_{im} = 1 - P(R_{m}^{i} > R_l, \forall l \in \{0, \ldots, i\})
$$

$$
= P \left( L(||x_m||) > \min \left\{ \frac{\rho \zeta_l}{\epsilon_l}, \forall l \in \{0, \ldots, i\} \right\} \right).
$$

By applying the assumption that $r_m \geq \delta R_c$ and also the pdf in (30), the outage probability of $CS_m$ for decoding $f_i$ can be expressed as follows:

$$
P_{im} = \sum_{l=0}^{m-1} \frac{\left( \lambda_c \left[ \pi \tau_i^l - \pi \delta^2 R_c^2 \right] \right)^l}{l!} e^{-\lambda_c \left[ \pi \tau_i^l - \pi \delta^2 R_c^2 \right]},
$$

where $\tau_i = \left( \frac{1}{\min \left\{ \frac{\rho \zeta_l}{\epsilon_l}, \forall l \in \{0, \ldots, i\} \right\}} \right)^{1/\alpha}$.

Based on the outage probability $P_{im}$, the corresponding cache hit probability for a user associated with $CS_m$ can be expressed as follows:

$$
P_{hit_m} = \sum_{i=1}^{M_s} P(f_i) (1 - P_{im}),
$$

where $f_0$ has been omitted as it is a file currently requested by a user and is assumed to belong to a different library than $f_l$, $1 \leq l \leq M_s$.

B. OMA Benchmarks

A naive OMA based benchmark is that the BS does not push new content while serving a user directly. Compared to this naive OMA scheme, the benefit of the proposed push-and-delivery strategy is obvious since new content is delivered and the cache hit probability will be improved.
A more sophisticated OMA scheme is to divide a single time slot into \((M_s + 1)\) sub-slots. During the first sub-slot, the user is served directly by the BS. From the second until the \((M_s + 1)\)th sub-slots, the BS will individually push the files, \(f_i, i \in \{1, \cdots, M_s\}\), to the content servers. Compared to this more sophisticated OMA scheme, the use of the proposed push-and-delivery strategy can still offer a significant gain in terms of the cache hit probability, as will be shown in Section V.

C. Extension to D2D Caching

The aim of this subsection is to show that the concept of push-and-delivery can also be applied to D2D caching. Again, assume that a time slot is dedicated to a user whose request cannot be found in the caches of its neighbours, and during this time slot, the BS will send the requested file \(f_0\) to the user directly. By applying the push-and-delivery strategy, the BS will also proactively push \(M_s\) new files, \(f_l, 1 \leq l \leq M_s\), to all users for future use. In other words, when the BS addresses the current demand of a user directly, the BS pushes more content files to all users, including the user which requests \(f_0\), for future use.

In the context of D2D caching, content servers are no longer needed. Therefore, the spatial model presented in Section II needs to be revised accordingly. Particularly, it is assumed that the locations of the users are denoted by \(y_k\) and modelled as an HPPP, denoted by \(\Phi_u\), with density \(\lambda_u\).

After implementing the push-and-delivery strategy, following similar steps as in the previous subsection, for a user with distance \(r\) from the BS and Rayleigh fading channel gain \(h\), the outage probability for decoding \(f_i\) is given by

\[
R^i_r = \log \left( 1 + \frac{\alpha^2 |h|^2 r^{-\alpha}}{\sum_{j=i+1}^{M_s} \alpha^2 |h|^2 r^{-\alpha} + \frac{1}{\rho}} \right),
\]

when \(R^l > R_i\), for \(0 \leq l \leq i - 1\). If \(f_j, 0 \leq j \leq M_s - 1\), can be decoded correctly, \(f_{M_s}\) can be decoded by this user with the following data rate:

\[
R^1_{M_s} = \log \left( 1 + \rho \alpha^2 |h|^2 r^{-\alpha} \right).
\]

Consequently, for a user with distance \(r\) from the BS, the probability to successfully decode \(f_i\) can be expressed as follows:

\[
P^i(r) = P \left( R^l_r > R_l, \forall l \in \{0, \cdots, i\} \right)
\]

\[
= e^{-\theta^i_r r^\alpha}.
\]
Following (36), one can draw the conclusion that the locations of the users which can successfully receive $f_i$ no longer follow the original HPPP with $\lambda_u$, but follow an inhomogeneous PPP which is thinned from the original HPPP by $P^i(r)$, i.e., the density of this new PPP is $P^i(r)\lambda_u$. By using this thinning process, the cache miss probability can be characterized as follows.

During the D2D content delivery phase, assume a newly arrived user, whose location is denoted by $y_0$, requests file $f_i$. Denote by $B(y_0,d)$ a disc with radius $d$ and its origin located at $y_0$. This disc is the area in which the user searches for a helpful neighbour which has the requested file in its cache. For this inhomogeneous PPP, the cache hit probability for the user requesting $f_i$ is given by

$$P^{hit}_i = 1 - P(\text{no user in } B(y_0,d) \text{ caches } f_i)$$

$$= 1 - e^{-\Lambda_i(B(y_0,d))},$$

where $\Lambda_i(B(y_0,d))$ denotes the intensity measure of the inhomogeneous PPP for the users which have $f_i$ in their caches. In (37), the hit probability is found by determining the cache miss probability which corresponds to the event that the user cannot find its requested file in the caches of its neighbours located in the disc $B(y_0,d)$. The calculation of the cache hit probability depends on the relationship between $d$ and the distance between the observing user and the BS, denoted by $r_0$, as shown in the following subsections.

Fig. 5. Two possible cases between the radius of the search disc, $B(y_0,d)$, and the distance between the observing user located at $y_0$ and the BS.
1) For the case of \(d < r_0:\) For \(d < r_0,\) define \(\Lambda_i(B(y_0, d)) \triangleq \Lambda_{i \leq r_0}(r_0).\) The assumption, \(d < r_0,\) means that the BS is excluded from \(B(y_0, d).\) Therefore, the intensity measure can be calculated as follows:

\[
\Lambda_{i \leq r_0}(r_0) = \int_{r, \theta \in B(y_0, d)} P_i(r)\lambda u d\theta dr.
\]

As can be observed from Fig. 5, the constraint on \(r\) and \(\theta\) can be expressed as follows:

\[
r^2 + r_0^2 - 2r_0 r \cos \theta \leq d^2.
\]

Therefore, the intensity measure can be expressed as follows:

\[
\Lambda_{i \leq r_0}(r_0) \approx 2\lambda u d \sum_{l=1}^{N} \frac{\pi}{N} g_r(r_0 + dw_l) \sqrt{1 - w_l^2},
\]

where \(g_r(z)\) is given by

\[
g_r(z) = P_i(z) z \arccos \frac{z^2 + r_0^2 - d^2}{2r_0 z}.
\]

2) For the case of \(d \geq r_0:\) For \(d \geq r_0,\) define \(\Lambda_i(B(y_0, d)) \triangleq \Lambda_{d > r_0}.\) The assumption, \(d \geq r_0,\) means that the BS is inside of \(B(y_0, d).\) Following similar steps as in the previous case, the intensity measure can be evaluated as follows:

\[
\Lambda_{d \leq r_0}(r_0) = \int_{r, \theta \in B(y_0, d)} P_i(r)\lambda u d\theta dr
\]

\[
\approx 2\pi \lambda u \gamma \left( \frac{2}{\alpha}, \tilde{r}_i^\alpha(d - r_0)^\alpha \right)
+ 2\lambda u r_0 \sum_{l=1}^{N} \frac{\pi}{N} g_r(d + r_0 w_l) \sqrt{1 - w_l^2},
\]

where \(\gamma(\cdot)\) denotes the incomplete gamma function, and the approximation in the last step follows from the application of Chebyshev-Gauss quadrature.

Finally, the cache hit probability can be obtained by substituting (41) and (43) into (37).
V. NUMERICAL STUDIES AND DISCUSSIONS

In this section, the performances achieved by the proposed push-then-delivery and push-and-delivery strategies are studied by using computer simulations, where the accuracy of the developed analytical results will be also verified.

A. Performance of Push-then-delivery Strategy

In Figs. 6 and 7, the impact of the NOMA assisted push-then-delivery strategy on the cache hit probability is studied. The thermal noise is set as $\sigma_n^2 = -100$ dBm. By applying the NOMA principle to the content pushing phase, more content can be pushed to the content servers simultaneously, and hence, the cache hit probability is improved, compared to the OMA case, as can be observed from Fig. 6. For example, when the transmission power is 40 dBm, $\gamma = 0.5$, and $R_c = 50$ m, the use of OMA yields a hit probability of 0.2, and the use of NOMA improves this value to 0.45, which corresponds to a 100% improvement. At low SNR, NOMA and OMA yield the same performance. This is due to the use of the CR inspired power allocation policy in (7), which implies that at low SNR, all the power is allocated to $f_1$, and hence, there is no difference between the OMA and NOMA schemes. Note that the curves for analysis and simulation match perfectly in Fig. 6, which demonstrates the accuracy of the developed analytical results.

The impact of $\gamma$, the shape parameter defining the content popularity, on the hit probability is significant, as can be observed in Fig. 6. Particularly, increasing the value of $\gamma$ improves the hit probability. This is because a larger value of $\gamma$ means that the first $M_s$ files become more popular, hence ensuring the delivery of these more popular files can significantly improve the hit probability, as indicated by (9). Comparing Fig. 6(a) with Fig. 6(b), one can observe that the impact of $R_c$ on the hit probability is also significant, which is due to the fact that the density of the content servers depends on $R_c$. Particularly, a larger $R_c$ means that the content servers are more sparsely deployed and hence it is more difficult for the BS to push content to these servers, and the cache hit probability decreases.

In Fig. 7, the impact of different choices of $m$ and $t$ on the hit probability is studied. As can be observed from the figure, increasing $t$ will decrease the hit probability. This is again due to the use of the CR power allocation policy. In particular, a larger $t$ means that more transmission power is needed to delivery $f_1$ to CS, and hence, less power is available for other files. An interesting observation in Fig. 7 is that the shape of the hit probability curves is not smooth. This is due to the fact that the hit probability is the summation of popularity probabilities $P(f_i)$
Fig. 6. The cache hit probability for the push-then-delivery strategy. $\alpha = 3$, $\lambda_c = \frac{0.01}{\pi R_c^2}$, $M_s = 3$, $t = 5$, $m = 1$, $M_s = 3$, and $R_l = 1$ bit per channel use (BPCU), for $1 \leq l \leq 3$. The power allocation coefficient for file 1 is based on the CR power allocation policy. The power allocation coefficients for files 2 and 3 are $\beta_2^2 = \frac{3}{4}$ and $\beta_3^2 = \frac{1}{4}$, respectively. $F = \{f_1, f_2, \cdots, f_{10}\}$. 
Fig. 7. The impact of the choices of $m$ and $t$ on the cache hit probabilities for the push-then-delivery strategy. $\alpha = 3$, $R_c = 50$ m, $\lambda_c = \frac{0.01}{\pi R_c^2}$, $M_2 = 3$, $t = 5$, $m = 1$, and $R_l = 1$ BPCU, for $1 \leq l \leq 3$. The power allocation coefficient for file 1 is based on the CR power allocation policy. The power allocation coefficients for files 2 and 3 are $\beta_2^2 = \frac{3}{4}$ and $\beta_3^2 = \frac{1}{4}$, respectively. $\gamma = 0.5$ and $\mathcal{F} = \{f_1, f_2, \cdots, f_{10}\}$. Analytical results are used to generate the figure.

and these popularity probabilities are prefixed and not continuous, as shown in (1). On the other hand, for a fixed $t$, increasing $m$ reduces the cache hit probability, since increasing $m$ means that $\text{CS}_m$ is further away from the BS and hence its reception reliability deteriorates.

In Fig. 8 the impact of using NOMA for content delivery is studied. As can be observed from the figure, the proposed push-then-delivery strategy can improve the reliability of content delivery, particularly for the user with strong channel conditions. For example, when the path loss exponent is set to $\alpha = 3$ and the transmission power of the content servers is 20 dBm, the use of NOMA ensures that the outage probability for the far user is improved from $4.5 \times 10^{-2}$ to $3 \times 10^{-2}$, which is a relatively small performance gain. However, the performance gap between the OMA and NOMA schemes at the near user is much larger, e.g., for the same case as considered before, the outage probability is improved from $5 \times 10^{-1}$ to $1.1 \times 10^{-2}$. Note that the outage probability for content delivery has an error floor, i.e., increasing the transmission power of the content servers cannot reduce the outage probability to zero. This is because multiple content servers transmit simultaneously, and hence, content delivery becomes interference limited at high SNR. We note that the impact of the path loss exponent on the reliability of content delivery is significant, as can be observed by comparing Figs. 8(a) and 8(b). This is due to the fact that a
Fig. 8. The outage probabilities for content delivery for the push-then-delivery strategy. \( R_c = 100 \text{m} \), \( \lambda_c = \frac{0.01}{\pi R_c^2} \), \( R_1 = 1 \) BPCU, and \( R_2 = 6 \) BPCU. The power allocation coefficients are \( \alpha_1 = \frac{3}{4} \) and \( \alpha_2 = \frac{1}{4} \).
smaller value of $\alpha$ results in a lower path loss, which leads to an improved reception reliability.

![Graph]( attachment:graph.png)

**Fig. 9.** The outage probabilities for the proposed push-and-delivery strategy. $R_c = 50m$, $N = 20$, $\alpha = 3$, $\lambda_c = \frac{0.01}{\pi R^2}$, $m = 3$, $\delta = 1.1$. $R_0 = \frac{1}{8}$ BPCU, $R_2 = \frac{3}{4}$ BPCU, $R_3 = \frac{7}{8}$ BPCU, and $R_4 = 1$ BPCU. The power allocation coefficients are $\alpha_0^2 = \frac{3}{8}$, $\alpha_1^2 = \frac{3}{8}$, $\alpha_2^2 = \frac{1}{8}$, and $\gamma = 1.5$. 

B. Performance of Push-and-delivery Strategy

In Fig. 9, the impact of the proposed push-and-deliver strategy on the cache hit probability is studied. As can be observed, the use of the proposed strategy can effectively improve the cache hit probability compared to the OMA case, which is consistent with the conclusions drawn in the previous subsection. In both sub-figures of Fig. 9, the analytical results match perfectly with the simulation results, which verifies the accuracy of the developed analysis.

In Fig. 9, the impact of different choices for the popularity parameters on the cache hit probability is also studied. In particular, the following two cases are considered:

- Case 1: $F_1 = \{f_1, \ldots, f_{10}\}$, and the power allocation coefficient for $f_i$ is $\alpha_l$;
- Case 2: $F_2 = \{f_1, \ldots, f_3\}$, and the power allocation coefficient for $f_i$ is $\alpha_{4-l}$.

The two cases correspond to two different options for mapping files with different popularities to different power levels (or equivalently SIC decoding orders), where in the first case, more popular files are assigned more power, and in the second case, less power is assigned to more popular files.

In Case 1, the performance gap between NOMA and OMA is not significant, as can be observed from Fig. 9(a). For example, when the transmit power is 40 dBm and $m = 5$, the use of OMA results in a hit probability of 0.7 and the use of NOMA yields a hit probability of 0.8, where the gap is 0.1 only. However, for a different set of popularity parameters, i.e., Case 2, the performance gap between OMA and NOMA is significantly increased. For example, for a transmit power of 40 dBm and $m = 5$, the performance gap between OMA and NOMA is enlarged to 0.5. The reason behind this phenomenon is as follows. Recall that the use of NOMA can significantly improve the reception reliability of the files which are decoded at the later stages of the SIC procedure, but the improvement for the files which are decoded during the first few stages of SIC is not significant. In Case 1, the first few files will get larger weights in the sum of the cache hit probability, i.e., file $f_i$, for a small $l$, has more impact on the overall performance. As a result, the gap between OMA and NOMA in Case 1 is small, since the reception reliability for decoding these files in the case of NOMA is not so different from that for OMA. On the other hand, Case 2 means that the most popular file, $f_1$, will be decoded last. As discussed before, the capabilities of OMA and NOMA to decode $f_1$ are quite different, which is the reason for the larger performance gap in Case 2.

Recall that the key idea of the push-and-delivery strategy is to perform content pushing when
asking the BS to serve the users directly. Fig. 9 clearly demonstrates that this strategy can efficiently push new content to the content servers, but it does not demonstrate the impact of this strategy on content delivery, which is studied in Fig. 10. Particularly, as can be observed from the figure, the use of the proposed push-and-delivery strategy does not degrade the reception reliability of content delivery. In fact, the use of NOMA can even improve the outage probability for content delivery.

In Fig. 11, the concept of the proposed push-and-delivery strategy is extended to D2D caching scenarios. Without loss of generality, the newly arrived user is located at $y_0 = (500\text{m}, 500\text{m})$. As expected, the use of the proposed strategy can significantly reduce the miss probability, compared to the case of OMA. For example, for $\lambda_u = 5 \times 10^{-5}$, a transmit power of 40 dBm, and $d = 150$ m, the use of NOMA yields a miss probability of $6 \times 10^{-2}$, whereas the miss probability for OMA is $1.6 \times 10^{-1}$, which is much worse. As can be observed from the figure, increasing the value of $d$ can reduce the miss probability, since the area for searching for a D2D helper is increased. Another important observation is that by increasing the density of the users, the miss probability can be further reduced, since increasing the density means that more users are located in the same area and hence it is more likely to find a D2D helper. We note that, in Fig. 11, computer simulation and analytical results match perfectly, which demonstrates again the accuracy of the developed analysis.

VI. CONCLUSIONS

In this paper, the application of the NOMA principle to wireless caching has been studied. Two NOMA assisted caching strategies have been developed, namely the push-then-delivery strategy and the push-and-delivery strategy. The push-then-delivery strategy is applicable to the case when the content pushing phase and the content delivery phase are separated, and utilizes the NOMA principle independently in both phases. The developed analytical results demonstrate that the proposed NOMA assisted caching scheme can efficiently improve the cache hit probability and reduce the delivery outage probability. The push-and-delivery strategy is motivated by the fact that, in practice, it is inevitable that some user requests cannot be accommodated locally and the BS has to serve the users directly. The key idea of the push-and-delivery strategy is to merge the two phases, i.e., the BS pushes content to the content servers while simultaneously serving users directly. Furthermore, in addition to the caching scenario with caching infrastructure, e.g.,
content servers, we have considered D2D caching, where the use of NOMA has also been shown to yield superior performance compared to OMA.

The results in this paper open several new directions for future research. First, in this paper, the file popularity parameters have been assumed to be given and fixed. As demonstrated in Fig. 9, different choices for these parameters yield different cache hit probabilities, which means that dynamically optimizing the NOMA power allocation (or equivalently the NOMA SIC decoding order) for given content popularity parameters could further improve the performance of NOMA-assisted caching. Second, fixed NOMA power allocation coefficients have been adopted in this paper, except in Section III-A2, where CR inspired power allocation was used. In general, optimizing the power allocation coefficients is expected to further improve the performance of NOMA caching. Third, increasing the density of the users or the search area in D2D caching can increase the cache hit probability, but might also cause stronger interference during the content delivery phase, when the D2D helpers deliver the requested files to their neighbours simultaneously. Therefore, for the content delivery phase, it is important to design low-complexity algorithms for efficiently scheduling of the users’ requests in order to limit co-channel interference.
Fig. 11. The impact of the proposed push-and-delivery strategy on the miss probability in D2D caching scenarios. $\alpha = 3$, $y_0 = (500 \text{ m}, 500 \text{ m})$, $R_0 = 0.5$ BPCU, $M_s = 1$, and $R_1 = 4$ BPCU. The power allocation coefficients are $\alpha_0^2 = \frac{3}{4}$ and $\alpha_1^2 = \frac{1}{4}$.
APPENDIX A

PROOF OF THEOREM

Recall that the NOMA cache hit probability is \( P_{hit}^m = \sum_{i=1}^{M_s} P(f_i)(1 - P_{m,i}) \) and the OMA hit probability is \( P_{hit}^{m,OMA} = P(f_1)(1 - P_{m,1}^{OMA}) \). Since the file popularity probabilities are positive and identical for the NOMA and OMA cases, proving \( P_{m,1}^{OMA} = P_{m,1} \) for all CSs, \( 1 \leq m \leq t \), is sufficient to prove the theorem.

Recall that each content server will carry out SIC, i.e., files \( j, 1 \leq j < i \), are decoded before file \( i \) is decoded. Therefore, the outage probability of CS \( m \) for decoding file \( i \) can be expressed as follows:

\[
P_{m,i} = 1 - P(f_j \text{ is decoded}, \forall j \leq i).
\] (44)

For notational simplicity, first define \( z_m \triangleq \frac{1}{L_t \|x_m - x_0\|} \), and note that these channel gains are ordered as follows: \( z_1 \geq \cdots \geq z_t \). Therefore, the outage probability can be expressed as follows:

\[
P_{m,i} = 1 - P \left( z_m > \frac{\epsilon_l}{\rho \xi_l}, \forall l \leq i \right),
\] (45)

where \( \xi_l = \alpha_l^2 - \epsilon_l \sum_{j=t+1}^{M_s} \alpha_j^2 \).

As discussed previously, showing \( P_{m,1}^{OMA} = P_{m,1} \) is sufficient to prove the theorem. First, we focus on the performance of CS \( t \). According to the definition of the CR NOMA power allocation policy, the outage probability of CS \( t \) for decoding the most popular file, \( f_1 \), is given by

\[
P_{t,1} = P \left( z_t < \frac{\epsilon_1}{\rho \xi_1} \right) \stackrel{(a)}{=} P (\alpha_1 = 1) = P \left( z_t < \frac{\epsilon_1}{\rho} \right) = P_{t,1}^{OMA},
\] (46)

where \( \epsilon_1 \) and \( \xi_1 \) are used since they are corresponding to file \( f_1 \). Step \( (a) \) follows from the fact that an outage occurs at CS \( t \) only if all the power is given to file \( f_1 \), i.e., \( \alpha_1 = 1 \). Therefore, regarding the capability of CS \( t \) to decode \( f_1 \), adopting NOMA does not bring any difference, compared to the OMA case.

Second, the outage probability of CS \( m \), \( 1 \leq m < t \), for decoding \( f_1 \), is given by

\[
P_{m,1} = P \left( z_m < \frac{\epsilon_1}{\rho \xi_1} \right) = P (\alpha_1 = 1, z_m < \frac{\epsilon_1}{\rho \xi_1}) + P (\alpha_1 < 1, z_m < \frac{\epsilon_1}{\rho \xi_1}).
\]
Since the channel conditions of CS_m are better than those of CS_t, the condition that CS_t can decode f_1 correctly, i.e., \( \alpha_1 < 1 \), is sufficient to guarantee successful detection of f_1 at CS_m. Therefore, the outage probability can be simplified as follows:

\[
P_{m,1} = P \left( \alpha_1 = 1, z_m < \frac{\epsilon_1}{\rho \xi_1} \right). \tag{47}
\]

Note that the use of the CR power allocation policy in (7) complicates the expression for the outage probability, since the power allocation coefficients depend on the channel conditions of CS_t. In order to better understand the outage events, we express the event \( \{z_m < \frac{\epsilon_1}{\rho \xi_1}\} \) as follows:

\[
\{z_m < \frac{\epsilon_1}{\rho \xi_1}\} = \left\{z_m < \frac{\epsilon_1}{\rho (1 - P_r - \epsilon_1 P_r)}\right\}
\]

\[
= \left\{z_m < \frac{\epsilon_1}{\rho \left(1 - (1 + \epsilon_1) \max \left\{0, \frac{\rho z_t - \epsilon_1}{\rho (1 + \epsilon_1) z_t}\right\}\right)}\right\}
\]

\[
= \left\{z_m < \frac{\epsilon_1}{\rho \left(1 - \max \left\{0, \frac{\rho z_t - \epsilon_1}{\rho z_t}\right\}\right)}\right\}. \tag{48}
\]

By combining (47) and (48), probability P_{m,1} can be surprisingly simplified as follows:

\[
P_{m,1} = P \left( z_t < \frac{\epsilon_1}{\rho}, z_m < \frac{\epsilon_1}{\rho} \right), \tag{49}
\]

since \( \max \left\{0, \frac{\rho z_t - \epsilon_1}{\rho (1 + \epsilon_1) z_t}\right\} = 0 \) for the case \( z_t < \frac{\epsilon_1}{\rho} \). On the other hand, it is straightforward to show that the outage probability for OMA is given by

\[
P^{OMA}_{m,1} = P \left( z_t > \frac{\epsilon_1}{\rho}, z_m < \frac{\epsilon_1}{\rho} \right) + P \left( z_t < \frac{\epsilon_1}{\rho}, z_m < \frac{\epsilon_1}{\rho} \right)
\]

\[
= P_{m,1}.
\]

Therefore, the NOMA outage performance of CS_m, 1 \( \leq m \leq t \), for decoding f_1 is the same as that of OMA, but the use of NOMA can ensure that more content is delivered to the content servers, which proves the theorem.

**APPENDIX B**

**PROOF OF LEMMA**

Since the content servers follow an HPPP, the pdf for the m-th shortest distance is given by

\[
f_m(x) = \frac{2 \lambda_c^m \pi^m x^{2m-1}}{(m-1)!} e^{-\lambda_c \pi x^2}. \tag{50}
\]
The conditional CDF for the $t$-th shortest distance, given $r_m = x$, can be expressed as follows:

$$F_{r_t|r_m}(y) \triangleq P(r_t \leq y|r_m = x)$$

$$= 1 - P(r_t > y|r_m = x).$$

The event, $(r_t > y|r_m = x)$, corresponds to the case where the $t$-th nearest content server is not located inside the ring between a larger circle with radius $y$ and a smaller one with radius $x$. Or equivalently, the event, $(r_t > y|r_m = x)$, means that at most $(t - m - 1)$ content servers are inside the ring between the two circles. Therefore, the conditional CDF, $F_{r_t|r_m}(y)$, can be explicitly written as follows:

$$F_{r_t|r_m}(y) = 1 - t - m - 1 \sum_{n=0}^{t-m-1} P(#(\text{int}(B(x_0, x), B(x_0, y))) = n),$$

where $#(\mathcal{A})$ denotes the number of points falling into the area $\mathcal{A}$, $B(x_0, x)$ denotes a disc with its origin located at $x_0$ and radius $x$, and $\text{int}(B(x_0, x), B(x_0, y))$ denotes the ring between the boundaries of $B(x_0, x)$ and $B(x_0, y)$.

By applying the HPPP assumption, the conditional CDF can be found as follows:

$$F_{r_t|r_m}(y) = 1 - \sum_{n=0}^{t-m-1} (\lambda_c \pi)^n (y^2 - x^2)^n \frac{e^{-\lambda_c \pi(y^2-x^2)}}{n!}. \quad (53)$$

In order to find the joint pdf between $r_m$ and $r_t$, the conditional pdf is needed first. However, the derivative of the CDF $F_{r_t|r_m}(y)$ shown in the above equation has the following complicated form:

$$f_{r_t|r_m}(y) = \sum_{n=1}^{t-m-1} 2y(\lambda_c \pi)^n (y^2 - x^2)^{n-1} \frac{e^{-\lambda_c \pi(y^2-x^2)}}{n!} \times \left[ \lambda_c \pi (y^2 - x^2) - n \right] + 2\lambda_c \pi y e^{-\lambda_c \pi(y^2-x^2)}. \quad (54)$$

This complicated form makes the calculation of the outage probability very difficult. Instead, the steps provided in [38] can be used to obtain a much simpler form, as shown in the following. First, define $S_n = \left( \frac{\lambda_c \pi (y^2-x^2)}{n!} \right)^n$, and hence the conditional CDF obtained in (53) can be re-written as follows:

$$F_{r_t|r_m}(y) = 1 - \sum_{n=0}^{t-m-1} S_n e^{-\lambda_c \pi(y^2-x^2)}. \quad (55)$$
After taking the derivative of the CDF and exploiting the structure of $S_n$, the conditional pdf can be obtained as follows:

$$f_{r_t|r_m}(y) = 2y\lambda_c\pi e^{-\lambda_c\pi(y^2-x^2)} \left( \sum_{n=0}^{t-m-1} S_n - \sum_{n=1}^{t-m-1} S_{n-1} \right)$$

$$= 2y(\lambda_c\pi)^{t-m} e^{-\lambda_c\pi(y^2-x^2)} \frac{(y^2-x^2)^{t-m-1}}{(t-m-1)!},$$

(56)

which is much simpler than the expression in (54).

By applying Bayes’ rule, the joint pdf between $r_m$ and $r_t$ can be obtained as follows:

$$f_{r_m,r_t}(x,y) = f_{r_m|r_t}(x)f_{r_t}(y)$$

$$= 4y(\lambda_c\pi)^t e^{-\lambda_c\pi y^2} x^{2m-1}(y^2-x^2)^{t-m-1} \frac{(t-m-1)!}{(m-1)!}. $$

(57)

Note that, for the special case of $m = t-1$, the two parameters, $x$ and $y$, are decoupled to yield the following simplified form for the joint pdf:

$$f_{r_m,r_t}(x,y) = \frac{4(\lambda_c\pi)^{m+1}yx^{2m-1}}{(m-1)!} e^{-\lambda_c\pi y^2}. $$

(58)

This completes the proof of the lemma.

**APPENDIX C**

**PROOF OF LEMMA 2**

Following the steps provided in the proof of Theorem 1, the outage probability for CS$_t$ to decode $f_1$ is given by

$$P_{t,1} = P \left( z_t < \frac{\epsilon_1}{\rho} \right).$$

After applying the marginal pdf of the $t$-th shortest distance shown in (50), $P_{t,1}$ can be calculated as follows:

$$P_{t,1} = \frac{2\lambda_c^t\pi^t}{(t-1)!} \int_{\rho \frac{\epsilon_1}{\lambda_c}}^{\infty} y^{2t-1} e^{-\lambda_c\pi y^2} dy$$

$$= e^{-\lambda_c\pi \left( \frac{\epsilon_1}{\rho} \right)^2} \sum_{k=0}^{t-1} \frac{(\lambda_c\pi)^k \left( \frac{\epsilon_1}{\rho} \right)^{2k}}{k!}.$$ 

(59)

According to (49), the outage probability for CS$_m$ to decode $f_1$ is given by

$$P_{m,1} = P \left( z_t < \frac{\epsilon_1}{\rho}, z_m < \frac{\epsilon_1}{\rho} \right).$$
By using the fact that \( r_m \leq r_n \) and again applying the marginal distribution of \( r_m \), the outage probability can be straightforwardly obtained as follows:

\[
P_{m,1} = P \left( z_m < \frac{\epsilon_1}{\rho} \right) = e^{-\lambda_c \pi \left( \frac{\rho}{\epsilon_1} \right) \frac{2h}{\pi}} \sum_{k=0}^{m-1} \left( \frac{\rho}{\epsilon_1} \right)^k \frac{2^k k!}{k!}.
\]  

(60)

Hence, the first part of the lemma is proved.

The outage probability for file \( i, i > 1 \), is more complicated than the case of \( f_1 \). The impact of the channel condition of CS\(_t\) on the outage performance of CS\(_m\) can be made explicit by expressing the individual event \( \left\{ z_m < \frac{\epsilon_i}{\rho\xi_i} \right\}, i > 1 \), as follows:

\[
\left\{ z_m < \frac{\epsilon_i}{\rho\xi_i} \right\} = \left\{ z_m < \frac{\epsilon_i}{\rho} \left( \alpha_i^2 - \frac{\epsilon_i}{\rho} \sum_{j=i+1}^{M_s} \alpha_j^2 \right) \right\}
= \left\{ z_m < \frac{\epsilon_i}{\rho\xi_i \max \left\{ 0, \frac{\rho z_t - \epsilon_1 \rho}{(1+\epsilon_1)z_t} \right\}} \right\},
\]

(61)

where the last step follows from the fact that \( P_t = \max \left\{ 0, \frac{\rho z_t - \epsilon_1 \rho}{(1+\epsilon_1)z_t} \right\} \). Recall that \( \bar{\xi}_i = \left( \beta_i - \frac{\epsilon_i}{\rho} \sum_{j=i+1}^{M_s} \beta_j \right) \) is a constant and not a function of the channel conditions of CS\(_t\). Therefore, the outage probability of CS\(_t\) for decoding \( f_i, i > 1 \), is given by

\[
P_{t,i} = P \left( \alpha_1 = 1, z_t < \max \left\{ \frac{\epsilon_1}{\rho\xi_1}, \cdots, \frac{\epsilon_i}{\rho\xi_i} \right\} \right) \]  

(62)

\[
+ P \left( \alpha_1 < 1, z_t < \max \left\{ \frac{\epsilon_1}{\rho\xi_1}, \cdots, \frac{\epsilon_i}{\rho\xi_i} \right\} \right).
\]

Note that \( \alpha_1 = 1 \) corresponds to the event that all the power is allocated to \( f_1 \). Therefore, \( z_t < \max \left\{ \frac{\epsilon_1}{\rho\xi_1}, \cdots, \frac{\epsilon_i}{\rho\xi_i} \right\} \) is always true if \( \alpha_1 = 1 \), and therefore, the outage probability can be simplified as follows:

\[
P_{t,i} = P(\alpha_1 = 1) \]  

(63)

\[
+ P \left( \alpha_1 < 1, z_t < \max \left\{ \frac{\epsilon_2}{\rho\xi_2}, \cdots, \frac{\epsilon_i}{\rho\xi_i} \right\} \right).
\]

Note that when \( \alpha < 1 \), the expression for the event \( \left\{ z_t < \frac{\epsilon_i}{\rho\xi_i} \right\} \) in (61) can be simplified as follows:

\[
\left\{ z_t < \frac{\epsilon_i}{\rho\xi_i} \right\} = \left\{ z_t < \frac{\epsilon_i}{\rho\xi_i \left( \frac{\rho z_t - \epsilon_1 \rho}{(1+\epsilon_1)z_t} \right)} \right\}.
\]

(64)
Therefore, the outage probability can be rewritten as follows:

\[ P_{t,i} = P(\alpha_1 = 1) \]  
\[ + P\left(\alpha_1 < 1, z_t < \max\left\{\frac{\epsilon_j}{\rho \xi_j \rho^{(1+\epsilon_1)z_t}}, 2 \leq j \leq i\right\}\right) \]  
\[ = P\left(z_t < \frac{\epsilon_1}{\rho}\right) + P\left(z_t > \frac{\epsilon_1}{\rho}, z_t < \frac{\epsilon_1}{\rho} + \frac{(1 + \epsilon_1)}{\rho \phi_i}\right). \]  

By applying the marginal pdf for the \( t \)-th shortest distance, the outage probability for CS\(_t\) to decode \( f_i \) can be obtained as follows:

\[ P_{t,i} = e^{-\lambda_c \pi \left(\frac{\epsilon_1}{\rho} + \frac{(1 + \epsilon_1)}{\rho \phi_i}\right)} - \sum_{k=0}^{t-1} \frac{(\lambda_c \pi)^k \left(\frac{\epsilon_1}{\rho} + \frac{(1 + \epsilon_1)}{\rho \phi_i}\right)^{-\frac{2k}{\rho}}}{k!}. \]  

Hence, the second part of the lemma is proved.

The outage probability for CS\(_m\) to decode \( f_i, i > 1 \), is the most difficult to obtain among the probabilities shown in the lemma. This probability can be first expressed as follows:

\[ P_{m,i} = P\left(\alpha_1 = 1, z_m < \max\left\{\frac{\epsilon_1}{\rho \xi_1}, \ldots, \frac{\epsilon_i}{\rho \xi_i}\right\}\right) \]  
\[ + P\left(\alpha_1 < 1, z_m < \max\left\{\frac{\epsilon_1}{\rho \xi_1}, \ldots, \frac{\epsilon_i}{\rho \xi_i}\right\}\right). \]  

Note that \( \alpha_1 = 1 \) results in the situation that no power is allocated to \( f_j, j > 1 \), which means that the event \( z_m < \max\left\{\frac{\epsilon_1}{\rho \xi_1}, \ldots, \frac{\epsilon_i}{\rho \xi_i}\right\} \) always happens, if \( \alpha_1 = 1 \). In addition, by using the fact that \( r_m \leq r_t \), the outage probability can be simplified as follows:

\[ P_{m,i} = P\left(z_t < \frac{\epsilon_1}{\rho}\right) \]  
\[ + P\left(z_t > \frac{\epsilon_1}{\rho}, z_m < \max\left\{\frac{\epsilon_2}{\rho \xi_2}, \ldots, \frac{\epsilon_i}{\rho \xi_i}\right\}\right). \]  

Note that \( z_t > \frac{\epsilon_1}{\rho} \) guarantees \( z_m > \frac{\epsilon_1}{\rho \xi_1} \), as \( z_t \leq z_m \) and \( z_t > \frac{\epsilon_1}{\rho \xi_1} \) is equivalent to \( z_t > \frac{\epsilon_1}{\rho} \). However, \( z_t > \frac{\epsilon_1}{\rho} \) does not guarantee \( z_m > \frac{\epsilon_j}{\rho \xi_j}, j > 1 \). Recall that conditioned on \( z_t > \frac{\epsilon_1}{\rho} \), the term \( \frac{\epsilon_j}{\rho \xi_j}, j > 1 \), can be simplified as follows:

\[ \frac{\epsilon_j}{\rho \xi_j} = \frac{\epsilon_j}{\xi_j \rho^{(1+\epsilon_1)z_t}}. \]
Therefore, the term $Q_1$ can be calculated as follows:

$$Q_1 = P \left( z_t > \frac{\epsilon_1}{\rho}, z_m < \max \left\{ \frac{\epsilon_2}{\rho \xi_2}, \ldots, \frac{\epsilon_i}{\rho \xi_i} \right\} \right)$$  \tag{70}$$

$$= P \left( z_t > \frac{\epsilon_1}{\rho}, z_m < \frac{(1 + \epsilon_1)}{\phi_i (\rho - \epsilon_1)} \right).$$

After applying the path loss model, $z_t$ ($z_m$) can be replaced by the distance between the BS and CS$_t$ (CS$_m$), and the outage probability can be expressed as follows:

$$Q_1 = P \left( y < \left( \frac{\epsilon_1}{\rho} \right)^{-\frac{1}{\alpha}}, x > \left( \frac{(1 + \epsilon_1)}{\phi_i (\rho - \epsilon_1 y^\alpha)} \right)^{-\frac{1}{\alpha}} \right),$$  \tag{71}$$

where $x$ denotes the distance between the BS and CS$_t$ and $y$ denotes the distance between the BS and CS$_m$. However, there is an extra constraint on $y$ as follows:

$$\left( \frac{\epsilon_1}{\rho} \right)^{-\frac{1}{\alpha}} > \left( \frac{(1 + \epsilon_1)}{\phi_i (\rho - \epsilon_1 y^\alpha)} \right)^{-\frac{1}{\alpha}},$$  \tag{72}$$

which leads to the following constraint on $y$:

$$y^\alpha > \frac{\rho}{\epsilon_1} \left[ 1 - \frac{1 + \epsilon_1}{\epsilon_1 \phi_i} \right].$$  \tag{73}$$

To better understand this constraint, the term $\frac{1 + \epsilon_1}{\epsilon_1 \phi_i}$ is rewritten as follows:

$$\frac{1 + \epsilon_1}{\epsilon_1 \phi_i} = \frac{1 + \epsilon_1}{\epsilon_1 \min \left\{ \xi_2, \ldots, \xi_M \right\}} \geq \frac{1 + \epsilon_1}{\epsilon_1 \xi_{M_s}} = \frac{\epsilon_{M_s} 2^{R_1}}{\epsilon_1 \xi_{M_s}},$$  \tag{74}$$

where $\xi_{M_s} \leq 1$ and $2^{R_1} \geq 1$ hold. The only uncertainty for the comparison between the term $\frac{1 + \epsilon_1}{\epsilon_1 \phi_i}$ and 1 is caused by the relationship between $\epsilon_1$ and $\epsilon_{M_s}$. In the lemma, it is assumed that $\epsilon_1 \leq \epsilon_{M_s}$. As a result, the constraint in (73) is always satisfied since $\frac{1 + \epsilon_1}{\epsilon_1 \phi_i} \geq 1$. However, the probability in (71) also implies the following constraint:

$$y > \left( \frac{(1 + \epsilon_1)}{\phi_i (\rho - \epsilon_1 y^\alpha)} \right)^{-\frac{1}{\alpha}}.$$  \tag{75}$$

This leads to the following constraint on $y$:

$$y > \left( \frac{\rho \phi_i}{1 + \epsilon_1 + \epsilon_1 \phi_i} \right)^{\frac{1}{\alpha}} \triangleq \tau_1.$$  \tag{76}$$

After understanding the ranges of $x$ and $y$, we can now apply the joint pdf to calculate the outage probability, which yields the following:

$$Q_1 = \int_{\tau_1}^{\tau_2} \int_{\tau_1}^{y} f_{r_m, r_t}(x, y) \, dx \, dy,$$  \tag{77}$$
where $\tau_2$ is defined in the lemma. To facilitate the calculation of this integral, the joint pdf is rewritten as follows:

$$f_{r_m, r_1}(x, y) = \frac{4(\lambda_c \pi)^t}{(t - m - 1)! (m - 1)!} e^{-\lambda_c \pi y^2} \sum_{p=0}^{t-m-1} (-1)^p$$

$$\times \left( \frac{t - m - 1}{p} \right) y^{2(t-m-1)-2p+1} x^{2m+2p-1}. \tag{78}$$

Now, we can apply the joint pdf which yields the following:

$$Q_1 = \frac{4(\lambda_c \pi)^t}{(t - m - 1)! (m - 1)!} \sum_{p=0}^{t-m-1} (-1)^p \left( \frac{t - m - 1}{p} \right)$$

$$\times \int_{\tau_1}^{\tau_2} f_m(y) dy,$$

where $f_m(\cdot)$ is defined in the lemma. One can apply Chebyshev-Gauss quadrature to obtain the following expression for $Q_1$:

$$Q_1 \approx \frac{4(\lambda_c \pi)^t}{(t - m - 1)! (m - 1)!} \sum_{p=0}^{t-m-1} (-1)^p \left( \frac{t - m - 1}{p} \right)$$

$$\times \sum_{l=1}^{N} \pi \left( \frac{\tau_2 - \tau_1}{2N} \right) f_m \left( \frac{\tau_2 - \tau_1}{2} w_l + \frac{\tau_2 + \tau_1}{2} \right) \sqrt{1 - w_l^2}. \tag{80}$$

Substituting (80) and (60) into (68), the third part of the lemma is proved.

**APPENDIX D**

**PROOF OF LEMMA 5**

Because the two users associated with the same content server are located in different regions inside the disc with radius $R_c$, the density functions for their channel gains are different, and therefore, the two users’ outage probabilities will be calculated separately in the following subsections.

1) The outage performance at $U_{m, 2}$: First define the composite channel gain as $z_{m,k} \triangleq \frac{|h_{m, mk}|^2}{L(||y_{m,k}||)}$, for $k \in \{1, 2\}$. Recall that, for a user which is uniformly distributed in a disc with radius $r$, the CDF of its composite channel gain which includes the effects of small scale Rayleigh fading and path loss can be expressed as follows [5]:

$$F_r(z) \approx \sum_{n=1}^{N} \bar{w}_n \left( 1 - e^{-cn_r z} \right), \tag{81}$$

and the corresponding pdf of the channel gain is $f_r(z) \approx \sum_{n=1}^{N} \bar{w}_n c_{n_r} e^{-cn_r x}$. Recall that $U_{m, 2}$ is uniformly distributed in a disc with radius $R_s$, and therefore, the CDF and pdf of the channel
gain of $U_{m,2}$ are simply $F_{R_s}(z)$ and $f_{R_s}(z)$ by replacing $r$ with $R_s$. The reason for using the approximated form in (81) is that both the approximated CDF and pdf are in forms of exponential functions. In the following, we will show that these exponential functions will significantly simplify the application of the probability generating functional (PGFL).

With the definition of $z_{m,k} \triangleq \frac{|h_{m,mk}|^2}{L(||y_{m,k}||)}$, the SINR of $U_{m,2}$ for decoding the first message, $f_{m,1}$, is given by

$$\text{SINR}^1_{m,2} = \frac{\alpha_1^2 z_{m,2}}{\alpha_2^2 z_{m,2} + \Gamma_{\text{inter}}^m + \frac{1}{\rho}}.$$  \hspace{1cm} (82)

Similarly, the SINR of $U_{m,2}$ for decoding its own message, $f_{m,2}$, can be rewritten as follows:

$$\text{SINR}^2_{m,2} = \frac{\alpha_2^2 z_{m,2}}{\Gamma_{\text{inter}}^m + \frac{1}{\rho}}.$$  \hspace{1cm} (83)

Therefore, the outage probability of $U_{m,2}$ for decoding its own message can be expressed as follows:

$$P_{o,m,2} = 1 - P\left(\log(1 + \text{SINR}^l_{m,2}) > R_l, l \in \{1, 2\}\right)$$

$$= \mathcal{E}_{\text{inter}}^m \left\{ \mathbb{P}\left(z_{m,2} < \max \left\{ \frac{\epsilon_1 \Gamma_{\text{inter}}^m + \frac{1}{\rho}}{\alpha_1^2 - \epsilon_1 \alpha_2^2}, \frac{\epsilon_2 \Gamma_{\text{inter}}^m + \frac{1}{\rho}}{\alpha_2^2} \right\} \right\},$$

where $\mathcal{E}_x \{\cdot\}$ denotes the expectation operation with respect to $x$. In order to facilitate the application of the PGFL, the outage probability is first rewritten as follows:

$$P_{o,m,2} = \mathcal{E}_{\text{inter}}^m \left\{ \mathbb{P}\left(z_{m,2} < \min \left\{ \frac{\Gamma_{\text{inter}}^m + \frac{1}{\rho}}{\alpha_1^2 - \epsilon_1 \alpha_2^2}, \frac{\Gamma_{\text{inter}}^m + \frac{1}{\rho}}{\alpha_2^2} \right\} \right\} \right\}$$

(85)

$$= \mathcal{E}_{\text{inter}}^m \left\{ \mathbb{P}\left(z_{m,2} < \frac{\Gamma_{\text{inter}}^m + \frac{1}{\rho}}{\min \left\{ \alpha_1^2 - \epsilon_1 \alpha_2^2, \alpha_2^2 \right\}} \right\}.\right.$$
Denote the Laplace transform of \( I_{m,2}^{\inter} \) by \( \mathcal{L}_{I_{m,2}^{\inter}}(s) \). Then, the outage probability can be rewritten as follows:

\[
P_o^m \approx 1 - \sum_{n=1}^{N} w_n e^{-\frac{c_n R_s R_s \min \left\{ \frac{\alpha_1 - \epsilon_1 \alpha_2}{\epsilon_1^2}, \frac{\alpha_2}{\epsilon_2^2} \right\}}{}} \mathcal{L}_{I_{m,2}^{\inter}} \left( \frac{c_n R_s}{\min \left\{ \frac{\alpha_1 - \epsilon_1 \alpha_2}{\epsilon_1^2}, \frac{\alpha_2}{\epsilon_2^2} \right\}} \right).
\]

Therefore, the outage probability can be calculated if the Laplace transform of \( I_{m,2}^{\inter} \) is known. Particularly, the Laplace transform of \( I_{m,2}^{\inter}, \mathcal{L}_{I_{m,2}^{\inter}}(s) \), can be first expressed as follows:

\[
\mathcal{L}_{I_{m,2}^{\inter}}(s) = \mathcal{E} \left\{ \prod_{x_j \in \Phi_c \setminus x_m} \exp \left( -s \frac{|h_{j,m,2}|^2}{L(\|y_{m,2} + x_m - x_j\|)} \right) \right\}.
\]

By using the assumption that \( h_{j,m,2} \) is Rayleigh distributed, the small scale fading gain can be averaged out in the expression, and the Laplace transform can be expressed as follows:

\[
\mathcal{L}_{I_{m,2}^{\inter}}(s) = \mathcal{E} \left\{ \prod_{x_j \in \Phi_c \setminus x_m} \frac{1}{s + \frac{L(\|y_{m,2} + x_m - x_j\|)}{}} + 1 \right\}.
\]

By applying the Campell theorem and the PFGL \([30, 31, 39]\), the Laplace transform can be simplified as follows:

\[
\mathcal{L}_{I_{m,2}^{\inter}}(s) = \exp \left( -\lambda_c \int_{\mathbb{R}^2} \left( 1 - \mathcal{E}_{y_{m,2}} \left\{ \frac{1}{l(\|y_{m,2} + x_m - x\|)} + 1 \right\} \right) dx \right).
\]

Denote the pdf of \( y_{m,2}, y_{m,2} \in \mathcal{B}(x_m, R_s) \), by \( f_{y_{m,2}}(y) \), where we recall that \( \mathcal{B}(x_m, R_s) \) denotes the disc with radius \( R_s \) and with the origin located at \( x_m \). Therefore, the Laplace transform can be expressed as follows:

\[
\mathcal{L}_{I_{m,2}^{\inter}}(s) = \exp \left( -\lambda_c \int_{\mathcal{B}(x_m, R_s)} f_{y_{m,2}}(y) \int_{\mathbb{R}^2} \left( 1 - \frac{1}{L(\|y + x_m - x\|)} + 1 \right) dxdy \right).
\]

Following similar steps as in \([30, 31, 39]\), the substitution of \( y + x_m - x \rightarrow x' \) can be used to simplify the expression of the Laplace transform as follows:

\[
\mathcal{L}_{I_{m,2}^{\inter}}(s) = \exp \left( -\lambda_c \int_{\mathcal{B}(x_m, R_s)} f_{y_{m,2}}(y) \int_{\mathbb{R}^2} \left( 1 - \frac{1}{L(\|x'\|)} + 1 \right) dx'dy \right).
\]

After applying the Beta function \([40]\), the Laplace transform can be obtained as follows:

\[
\mathcal{L}_{I_{m,2}^{\inter}}(s) = \exp \left( -2\pi \lambda_c \frac{s^2}{\alpha} \frac{B \left( \frac{2}{\alpha}, \frac{\alpha - 2}{\alpha} \right)}{B \left( \frac{2}{\alpha}, \frac{\alpha - 2}{\alpha} \right)} dy \right).
\]
where the last equality follows from the fact that the integral with respect to \( y \) is not a function of \( x \). Substituting (92) into (87), the first part of the lemma is proved.

2) The outage performance at \( U_{m,1} \): Recall that \( U_{m,1} \) is located inside a ring with \( R_s \) as the inner radius and \( R_c \) as the outer radius. Therefore, the CDF of this user’s channel gain needs to be calculated differently compared to that of \( U_{m,2} \). First, by using the assumptions that the user is uniformly distributed inside the ring and the fading gain is Rayleigh distributed, the CDF of \( z_{m,1} \) can be expressed as follows [41]:

\[
F_{z_{m,1}}(z) = \frac{2}{R_c^2 - R_s^2} \int_{R_s}^{R_c} \left( 1 - e^{-r^\alpha z} \right) r \, dr
\]  

(93)

Comparing this with [5, Eq. (3)], one can find that the approximated form shown in (81) can be applied to each term in the above expression, and hence the CDF can be approximated as follows:

\[
F_{z_{m,1}}(z) = \frac{1}{R_c^2 - R_s^2} \left[ R_c^2 F_{R_c}(z) - R_s^2 F_{R_s}(z) \right].
\]  

(94)

Following similar steps as in the previous subsection, the outage probability of \( U_{m,1} \) for decoding \( f_{m,1} \) can be obtained as follows

\[
P_{m,1}^o = \mathcal{E}_{m,1}^{inter} \left\{ \mathbb{P} \left( z_{m,1} < \frac{\epsilon_{1m,1}^{inter} + \epsilon_{1}^1}{\alpha_1^1 - \epsilon_1^{2\alpha_2}} \right) \right\}. 
\]  

(95)

After using the approximated expression for the pdf of \( z_{m,1} \), the outage probability can be approximated as follows:

\[
P_{m,1}^o \approx \frac{R_c^2}{R_c^2 - R_s^2} \mathcal{E}_{m,1}^{inter} \left\{ \sum_{n=1}^{N} \bar{w}_n \left( 1 - e^{-c_{n,R_c} \frac{\epsilon_{1m,1}^{inter} + \epsilon_{1}^1}{\alpha_1^1 - \epsilon_1^{2\alpha_2}}} \right) \right\}
\] 

\[
- \frac{R_s^2}{R_c^2 - R_s^2} \mathcal{E}_{m,1}^{inter} \left\{ \sum_{n=1}^{N} \bar{w}_n \left( 1 - e^{-c_{n,R_s} \frac{\epsilon_{1m,1}^{inter} + \epsilon_{1}^1}{\alpha_1^1 - \epsilon_1^{2\alpha_2}}} \right) \right\}
\]

\[
\approx 1 + \frac{R_s^2}{R_c^2 - R_s^2} \sum_{n=1}^{N} \bar{w}_n e^{-c_{n,R_c} \frac{\epsilon_{1}^1}{\alpha_1^1 - \epsilon_1^{2\alpha_2}}} \mathcal{E}_{m,1}^{inter} \left\{ \frac{e^{-c_{n,R_s} \frac{\epsilon_{1m,1}^{inter}}{\alpha_1^1 - \epsilon_1^{2\alpha_2}}}}{e^{-c_{n,R_s} \frac{\epsilon_{1m,1}^{inter}}{\alpha_1^1 - \epsilon_1^{2\alpha_2}}}} \right\}
\] 

(96)

It is straightforward to show that the Laplace transform of \( \mathcal{E}_{m,1}^{inter} \) is the same as that of \( \mathcal{E}_{m,2}^{inter} \). Therefore, substituting (92) with (96), the second part of the lemma is proved.
REFERENCES


