Secure Communication in Cooperative Network with Wireless Information and Power Transfer

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Abstract

This paper considers the secure communication issue in an amplify-and-forward (AF) relaying cooperative network, in which the energy harvesting relay is powered by radio-frequency (RF) signals from the source. Based on the two different energy harvesting strategies, i.e. power splitting-based relaying (PSR) protocols and time switching-based relaying (TSR) protocols, we formulate the problem for maximizing the secrecy rate subject to the transmitted power constraint by taking into account the power splitting coefficient and time switching coefficient. We solve these problems by applying the semidefinite programming relaxation approach and 1-D optimization over the coefficients of energy harvesting protocols. The simulation results are presented to verify the effectiveness of these solutions proposed in this paper.

I. INTRODUCTION

Wireless energy harvesting is an effective approach to prolong the lifetime of wireless communication networks [1]–[3]. The traditional power supplies of wireless devices, such as batteries, are difficult to replace or recharge in many practical environments [1]. For instance, it is challenging to replace the batteries for the implanted medical devices used to record data from patient body. These difficulties have motivated energy harvesting technologies which are to recharge the batteries of wireless nodes by collecting energy from the surrounding environments. Particularly some previous works proposed that sources for energy harvesting can be solar, wind and thermoelectric power, [4]–[7]. Recently, using radio-frequency (RF) signals to replace traditional energy sources for energy harvesting has received a lot of attention [8]. However, having a single receiver circuit to receive information and to harvest energy at the same time is challenging due to the practical circuit limitations. Therefore, the idea for receivers to perform energy harvesting and to information detection separately was proposed in [3], where an energy harvesting node can be operated in a time switching protocol or a power splitting protocol. Many existing works based on energy harvesting protocols considered point-to-point communication models [1], [8]–[10]. Recently the application of energy harvesting protocols to wireless cooperative networks has received a lot of attention [11]–[14]. Particularly the authors in [11] investigated the outage probability and the throughput in an energy harvesting cooperative network with an energy harvesting relay operating at the amplify-to-forward (AF) mode. In [15], a cooperative network with multiple pairs of sources and destinations and one energy harvesting relay was considered and the various strategies for power allocation were investigated.

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As any other communication networks, the issue of security for wireless information and power transfer is paramountly important, where passive eavesdroppers can potentially intercept the source transmission due to the openness of wireless channels. There were many existing works related to general physical (PHY) layer security \[16\]–\[18\]. In \[16\], the overhearing channel can be considered as a degraded one to receive the information and multiple antennas were used in secure communication systems, where the secrecy capacity is optimized by carefully designing beamforming. Based on the study for secrecy rates in single-input-single-output (SISO) fading channels \[19\], researchers optimized the secure problem in multi-input-single-output (MISO) and multi-input-multi-output (MIMO) systems \[20\], \[21\]. Since the wireless relaying network can enhance the spectrum efficiency and the physical layer security, secure optimization in cooperative networks has been studied \[22\]. The cooperative beamforming optimization problem was studied in \[23\]–\[25\], in which relays use amplify-and-forward (AF) and decode-and-forward (DF) strategies to transmit the confidential messages to legitimate receivers.

In this paper, we study secrecy rate maximization in amplify-and-forward (AF) wireless cooperative network with an energy harvesting relay. The authors in \[26\]–\[28\] studied the security issue with simultaneous wireless information and power transfer for MISO broadcast channels. The secure beamforming (SRB) at the relay study in \[29\] proposed an iterative algorithm to solve the SRB problem. Motivated by these aforementioned works, we consider the secure issue in a wireless cooperative network in which the energy harvesting relay harvests energy from the RF signals transmitted by the source and then assist the source to deliver its information to the destination. We formulate the problem for maximizing the secrecy rate subject to the transmitted power constraint based on the two different energy harvesting strategies, i.e. power splitting-based relaying (PSR) protocols and time switching-based relaying (TSR) protocols. We solve these problems by using the semidefinite programming (SDP) relaxation approach and 1-D optimization over the coefficients of energy harvesting protocols.

The rest of this paper is organized as follows. Section II presents a general system model of an energy harvesting wireless cooperative network. Section III presents the details for the transmission model based on PSR protocols, formulates the corresponding secrecy rate maximization problem and proposes an optimal solution. Section IV investigates the transmission model based on TSR protocols, the associated problem and its solution. The numerical results are provided in V and section VI concludes this paper.

Notations: The bold uppercase and lowercase letters are denoted as matrices and vectors, respectively. \(E(\cdot)\) is the expectation over the random variables within the bracket. \((\cdot)^*\), \((\cdot)^T\) and \((\cdot)^H\) are denoted as conjugate, transpose and conjugate transpose, respectively. \(|x|^+\) denotes \(\max\{0, x\}\). \(I_N\) denotes \(N \times N\) identity matrix. \(Tr(\cdot)\) is the trace of a matrix. vec is the matrix vectorization. \(\otimes\) represents the Kronecker operator. \(|x|\) denotes the norm of a complex number \(x\). \(|x|^2\) is the squared Euclidean norm of a complex \(x\). \(\|X\|_F^2\) is the Frobenius norm of a complex matrix \(X\). \(\mathbb{C}^{M \times N}\) denotes the space of \(M \times N\) matrices with complex entries.

II. System Model

Consider a wireless cooperative network consisting of one source node, one relay node, one destination node and one eavesdropper node. Except that the relay node which has \(N\) antennas, all other nodes are equipped with
a single antenna. We assume that there is no direct link between the source and the eavesdropper/destination, i.e. the $N$-antenna energy harvesting relay transmits the source data to the destination node while preventing the eavesdropper from intercepting the source messages. The perfect channel state information is assumed available at the relay node. This may be feasible for the scenario that this eavesdropper is also a user in the network but is malicious to overhear the messages from other users.

The energy harvesting relay can harvest energy from the RF signals transmitted by the source node and utilize the energy as the transmission power to forward the source information. In this paper, we assume the relay node has an initial power and further collects power from the source node to relay the information, therefore the energy harvesting technique can further improve the performance [30]. The battery capacity of the energy harvesting relay is assumed as infinite. The amplify-and-forward (AF) scheme is considered in the cooperative network. The two relaying protocols for energy harvesting are considered in this paper. Detailed analysis based on the time switching-based relaying (TSR) protocol and the power splitting-based relaying (PSR) protocol are given in the following two sections, respectively. It is assumed that the power consumed in the transmit circuit at the relay is negligible, when compared to the transmission power used for sending the source data to the destination.

III. SECRECY RATE MAXIMIZATION BASED ON PSR PROTOCOL

In this section, we study secrecy rate maximization based on the power splitting-based relaying protocol. The transmission scheme with PSR protocols and the corresponding optimization problem are given as follows.

A. Power Splitting-based Relay Protocol

In a wireless cooperative network with power splitting relays, one transmission block is split into two phases. At the end of the first phase, the power splitting relay receives the observations from the source with the transmitted power $P_s$ and then splits them into two streams. One part of the observations is used for energy harvesting, from which energy is harvested for the next phase transmission. The remaining is used for information decoding at the relay.

B. System Transmission Model and Problem Statement

The energy harvesting relay harvests the energy and detects the transmitted signal from the source in the first phase. The received signal at the relay can be express as

$$y_r = \sqrt{P_s} h_r s + n_r,$$

where $P_s$ is the transmitted power at the source node, $h_r \in \mathbb{C}^{N \times 1}$ denotes the channel vector the between the source node and the relay node, $s$ is the transmitted data from the source node with $\mathbb{E}(|s|^2) = 1$, $n_r$ is the additive complex Gaussian noise vector at the relay node following $CN(0, \sigma_r^2 I_N)$.

Because the power splitting protocol is applied at the relay, the observation is split into two streams: one is for the energy harvesting and the other is for information decoding respectively. It results in the following received signal

$$y_r^{ID} = \sqrt{\rho P_s} h_r s + \rho n_r + n_c,$$
where $n_c$ is the additive complex Gaussian noise vector due to the RF to baseband conversion following $\mathcal{CN}(0, \sigma^2_c \mathbf{I}_N)$. At the same time, the energy harvesting relay harvests energy from the source, the signal for energy harvesting is given by

$$y_{EH}^r = \sqrt{(1-\rho)P_s} \mathbf{h}_r s + (1-\rho) \mathbf{n}_r.$$  

Therefore the amount of the harvested energy is

$$E = \eta (1-\rho) (P_s ||\mathbf{h}_r||^2 + \sigma^2_r),$$

where $\eta$ is the energy conversion efficiency coefficient at the relay. The relay node has an initial power $P_r$ and further harvests energy. Therefore, after the relay is powered by the source, the total available transmit power at the relay node is expressed as

$$P_{r_{\text{max}}} = \eta (1-\rho) (P_s ||\mathbf{h}_r||^2 + \sigma^2_r) + P_r.$$  

Since the energy harvesting relay is operated with the AF protocol, upon receiving the signal, the relay processes the signal by relay precoding. The signal transmitted by the relay node is given by

$$x_r = \sqrt{\rho P_s} \mathbf{W} s + \rho \mathbf{W} \mathbf{n}_r + \mathbf{W} n_c,$$

where $\mathbf{W} \in \mathbb{C}^{N \times N}$ is the precoding matrix. The power of $x_r$ at the relay node is

$$P_r = \rho P_s ||\mathbf{W}||^2 + \rho \sigma^2_r ||\mathbf{W}||_F^2 + \sigma^2_c ||\mathbf{W}||_F^2.$$  

Then the received signal at the destination node in the second phase is given by

$$y_d = \sqrt{\rho P_s} \mathbf{h}_d^T \mathbf{W} s + \rho \mathbf{h}_d^T \mathbf{W} \mathbf{n}_r + \mathbf{h}_d^T \mathbf{W} n_c + n_d,$$

where $\mathbf{h}_d \in \mathbb{C}^{N \times 1}$ denotes the channel vector between the relay node and the destination node and $n_d$ is the additive Gaussian noise at the destination following $\mathcal{CN}(0, \sigma^2_d)$. Similarly, the received signal at the eavesdropper can be written as

$$y_e = \sqrt{\rho P_s} \mathbf{h}_e^T \mathbf{W} s + \rho \mathbf{h}_e^T \mathbf{W} \mathbf{n}_r + \mathbf{h}_e^T \mathbf{W} n_c + n_e,$$

where $\mathbf{h}_e \in \mathbb{C}^{N \times 1}$ denotes the channel vector between the relay node and the destination node, and $n_e$ is the additive Gaussian noise at the destination following $\mathcal{CN}(0, \sigma^2_e)$.

According to the aforementioned signal model, the receive signal-to-noise ratios (SNRs) at the destination node and the eavesdropper node are

$$SNR_d = \frac{\rho P_s ||\mathbf{h}_d^T \mathbf{W}||^2}{\rho \sigma^2_r ||\mathbf{h}_d^T \mathbf{W}||_F^2 + \sigma^2_c ||\mathbf{h}_d^T \mathbf{W}||_F^2 + \sigma^2_d},$$  

and

$$SNR_e = \frac{\rho P_s ||\mathbf{h}_e^T \mathbf{W}||^2}{\rho \sigma^2_r ||\mathbf{h}_e^T \mathbf{W}||_F^2 + \sigma^2_c ||\mathbf{h}_e^T \mathbf{W}||_F^2 + \sigma^2_e},$$

respectively.
The achievable secrecy rate can be presented as

$$R_s = \left| \frac{1}{2} \log_2 (1 + SNR_d) - \frac{1}{2} \log_2 (1 + SNR_e) \right|^+. \quad (12)$$

In this paper, we formulate the problem for maximizing the secrecy rate subject to the transmitted power constraint at the relay node, via jointly optimizing relay precoding and the power splitting coefficient. Particularly, the addressed optimization problem is formulated as follows:

$$\max_{W, \rho} R_s$$

$$s.t. \quad C1 : P_r \leq P_r^{max},$$

$$C2 : 0 \leq \rho \leq 1. \quad (13)$$

It is obvious that the problem stated above is not a standard convex problem because the objective function is non-concave and the power splitting coefficient results in the problem being not convex. In the next subsection, we find the solution to this problem.

C. The Optimal Solution

In this subsection, we proposed an optimal solution based on SDR. Firstly we introduce a new variable $\gamma_e$ to represent the predefined threshold for the eavesdropper’s SNR and reformulate the original problem as follows:

$$\max_{W, \rho} SNR_d$$

$$s.t. \quad C1 : SNR_e \leq \gamma_e,$$

$$C2 : P_r \leq P_r^{max},$$

$$C3 : 0 \leq \rho \leq 1. \quad (14)$$

The new obtained problem is still not a convex problem. We further transform the objective function as follows:

$$SNR_d = \frac{\rho P_s |h_d^T W h_r|^2}{\rho \sigma_r^2 ||h_d^T W||^2 + \sigma_e^2 ||h_d^T W||^2 + \sigma_d^2},$$

$$= \frac{w^H Q_{a1} w}{w^H Q_{a2} w + \sigma_d^2}, \quad (15)$$

where $w = vec(W)$ and

$$Q_{a1} = \rho P_s (h_r h_r^H) \otimes (h_d^* h_d^T),$$

$$Q_{a2} = \rho \sigma_r^2 I_N + \sigma_e^2 I_N \otimes (h_d^* h_d^T). \quad (16)$$

The above transform is obtained by using the rule as follows [31]

$$Tr(ABCD) = (vec(D^T))^T (C^T \otimes A) vec(B).$$

Then we transform the SNR of the eavesdropper constraint as

$$SNR_e = \frac{\rho P_s |h_e^T W h_r|^2}{\rho \sigma_r^2 ||h_e^T W||^2 + \sigma_e^2 ||h_e^T W||^2 + \sigma_e^2},$$

$$= \frac{w^H Q_{b1} w}{w^H Q_{b2} w + \sigma_e^2}, \quad (19)$$
where

\[ Q_{b1} = \rho P_s (h_r h_r^H) \otimes (h_e^* h_e^T), \]

\[ Q_{b2} = (\rho \sigma^2 e + \sigma^2 c I_N) \otimes (h_e^* h_e^T). \]

(20)

The transmitted power constraint can be rewritten as

\[ P_r = \rho P_s ||W h_r||_2^2 + \rho \sigma^2 ||W||_F^2 + \sigma^2 ||W||_F^2, \]

\[ = w^H Q_c w, \]

(22)

where

\[ Q_c = (\rho P_s h_r^H + \rho \sigma^2 e I_N + \sigma^2 c I_N)^T \otimes I_N. \]

(23)

Define a new variable \( X = w w^H \), the optimization problem can be reformulated as

\[
\max_{X, \rho} \frac{Tr(Q_{a1} X)}{Tr(Q_{a2} X) + \sigma_d^2}
\]

s.t. \( C_1 : \frac{Tr(Q_{b1} X)}{Tr(Q_{b2} X) + \sigma_e^2} \leq \gamma_e \),

\( C_2 : Tr(Q_c X) \leq P_r^\text{max} \),

\( C_3 : 0 \leq \rho \leq 1 \),

\( C_4 : \text{Rank}(X) = 1. \)

(24)

It can be seen that the rank-one constraint makes the problem still difficult to solve. Therefore we drop the rank-one constraint and a semi-definite programming (SDP) problem can be obtained as

\[
\max_{X, \rho} \frac{Tr(Q_{a1} X)}{Tr(Q_{a2} X) + \sigma_d^2}
\]

s.t. \( C_1 : Tr(Q_{b12} X) \leq \sigma_e^2 \),

\( C_2 : Tr(Q_c X) \leq P_r^\text{max} \),

\( C_3 : 0 \leq \rho \leq 1 \).

(25)

where \( Q_{b12} = \frac{1}{\sigma_e^2} Q_{b1} - Q_{b2} \). It is worthy to point out that it is still challenging to solve this problem directly, mainly due to the power splitting coefficient in the problem, which renders the joint optimization problem unsolvable. In order to tackle this difficulty, we perform 1-D optimization over the power splitting coefficient. Then we can select the best performance among those possible choices of the power splitting coefficient. If the value of \( \rho \) is set, the problem can be treated as a quasi-convex SDP problem. Instead of employing the bisection search approach, we use the Charnes-Cooper transformation [32] to solve it. Specifically, we define a new variable \( t = \frac{1}{Tr(Q_{a2} X) + \sigma_d^2} \)
and let $\tilde{X} = tX$. The problem can be recast as follows:

$$\max_{\tilde{X}, t} \text{Tr}(Qa_1\tilde{X})$$

subject to

$$C1: \text{Tr}(Qa_2\tilde{X}) + t\sigma^2 = 1,$$

$$C2: \text{Tr}(Q_{b12}\tilde{X}) \leq t\sigma^2 e,$$

$$C3: \text{Tr}(Q_c\tilde{X}) \leq tP_{\text{max}}.$$ (26)

Then the problem is a standard SDP problem and one can efficiently find its global optimal solution via available solvers [33]. The optimal solution is supposed as $\tilde{X}^\star, t^\star$. Then the optimal solution of $X$ denoted by $X^\star$ is obtained by $X^\star = \frac{\tilde{X}^\star}{t^\star}$. If the rank of $\tilde{X}^\star$ is one, $w$ is exactly computed via eigenvalue decomposition. Otherwise we can employ the following theorem which is obtained from the result in [34] in order to obtain the rank-one solution.

**Theorem 1.** For a matrix $\tilde{X}^\star$ which has a higher rank, the rank-one solution can be acquired via the following procedure.

1. Decompose $\tilde{X}^\star$ as $\tilde{X}^\star = VV^H$ with $V \in \mathbb{C}^{N \times r}$, where $r$ is the rank of $\tilde{X}^\star$.
2. Find a nonzero $r \times r$ Hermitian matrix $M$ to satisfy the equations as follows $\text{Tr}(V^HQ_{a2}VM) = 0$, $\text{Tr}(V^HQ_{b12}VM) = 0$ and $\text{Tr}(V^HQ_cVM) = 0$.
3. Evaluate all eigenvalues $\lambda_1, \lambda_2 \cdots \lambda_r$ for matrix $M$ and define $|\lambda| = \max\{|\lambda_1|, |\lambda_2| \cdots |\lambda_r|\}$, where $|\lambda|$ represents the modulus of $\lambda$.
4. Update matrix $\tilde{X}^\star = V(I_r - \frac{1}{|\lambda|}M)V^H$. If $\tilde{X}^\star$ still has the higher rank, we repeat the step 1-3 until the rank of $\tilde{X}^\star$ is one.

**Proof.** The proof steps are similar to the ones in [34], [35]. Since there are three linear equations in Step 2, we can always find a nonzero solution $M$ if $r^2 \geq 3$. It can be found that the rank of $\tilde{X}^\star$ is reduced at least one by performing one iteration. And the updated $\tilde{X}^\star$ in step 4 satisfies equations in step 2, which is also a solution for the SDP problem, i.e. the updated $\tilde{X}^\star$ can achieve the same value of objective function for our transformed problem but with the lower rank. We can finally acquire a rank-one solution by repeating the procedure. The proof is completed.

**Remark 1.** In step 2 of Theorem 1, we need to solve an undetermined system of linear equations to find $M$. There are 3 equations and $r^2$ real-valued unknowns. If $r^2 \geq 3$, then it can be found a nonzero solution for linear equations. We can rewrite the system of linear equations into the form $Ax = 0$, where $A \in \mathbb{R}^{3 \times r^2}$ and $x \in \mathbb{R}^{r^2}$. The matrix $A$ is the combination of the matrix $V^HQ_{a2}V$, $V^HQ_{b12}V$ and $V^HQ_cV$. Then finding a nonzero $x$ is equal to finding the null space of $A$. Recall that $M \in \mathbb{R}^{r \times r}$, after reshaping $x$, $M$ is obtained.
Remark 2. The rank of $\tilde{X}^*$ will be reduced at least one by performing Theorem 1 one time. The number of iterations depends on the initial rank of $\tilde{X}^*$. For example, if the initial rank of $\tilde{X}^*$ is 9, at worst we need to perform Theorem 1 eight times.

IV. SECRECY RATE MAXIMIZATION BASED ON TSR PROTOCOL

In this section, we study the secrecy rate maximization problem on the basis of time switching-based relaying protocol. The transmission model with the TSR protocol and the corresponding optimization problem are given as follows.

A. Time Switching-based Relay Protocol

Different from PSR protocols, the information transmission process based on TSR ones is split into three phases. In the first phase, the source transmits RF signals to power the energy harvesting relay for $\alpha T$, where $T$ denotes the duration of one block and $0 \leq \alpha \leq 1$. During the rest of the block time $(1-\alpha)T$, the information is transmitted from the source to the relay and then the relay will use the harvested energy to deliver the source information to the destination.

B. System Transmission Model and Problem Statement

The received signal at the relay can be express as follows

$$y_r = \sqrt{P_s} h_r s + n_r + n_c.$$ \hspace{1cm} (27)

where $P_s$, $h_r$, $s$, $n_r$ and $n_c$ have the same definition as previously.

The relay harvests energy from the RF signals sent by the source for $\alpha$ time, and the amount of the harvested energy is given by

$$E = \eta \alpha (P_s ||h_r||^2 + \sigma_r^2).$$ \hspace{1cm} (28)

The total available transmit power at the relay is then expressed as follows:

$$P_r^{\text{max}} = \frac{2 \eta \alpha (P_s ||h_r||^2 + \sigma_r^2)}{1 - \alpha} + P_r.$$ \hspace{1cm} (29)

Since the energy harvesting relay is operated with the AF protocol, the signal transmitted by the relay node can be written as follows:

$$x_r = \sqrt{P_r} W h_r s + W n_r + W n_c.$$ \hspace{1cm} (30)

The relay transmission power is given by

$$P_r = P_s ||Wh_r||^2 + \sigma_r^2 ||W||^2_F + \sigma_c^2 ||W||^2_F.$$ \hspace{1cm} (31)

Then the received signal at the destination node is given by

$$y_d = \sqrt{P_s} h_d^T W h_r s + h_d^T W n_r + h_d^T W n_c + n_d.$$ \hspace{1cm} (32)
Similarly the received signal at the eavesdropper can be written as follows:

\[ y_e = \sqrt{P_s} h_e^T W_{r} s + h_e^T W_{n} + h_e^T W_{c} + n_e. \]  

(33)

According to the described signal model, the received signal-to-noise ratios (SNRs) at the destination node and the eavesdropper node are

\[ SNR_d = \frac{P_s |h_d^T W_{r}|^2}{\sigma_r^2 \|h_d^T W\|^2_2 + \sigma_e^2 \|h_d^T W\|^2_2 + \sigma_d^2}, \]  

(34)

and

\[ SNR_e = \frac{P_s |h_e^T W_{r}|^2}{\sigma_r^2 \|h_e^T W\|^2_2 + \sigma_e^2 \|h_e^T W\|^2_2 + \sigma_d^2}, \]  

(35)

respectively. The achievable secrecy rate can be expressed as

\[ R_s = \left| \frac{1}{2} \log_2 (1 + SNR_d) - \frac{1}{2} \log_2 (1 + SNR_e) \right|^+. \]

We formulate the problem of maximizing the secrecy rate subject to the transmitted power constraint at the relay node, via jointly optimizing relay precoding and the time switching coefficient. The optimization problem is formulated as follows:

\[
\begin{align*}
\max_{W, \alpha} R_s \\
\text{s.t.} \quad & C1: P_r \leq P_r^{\max}, \\
& C2: 0 \leq \alpha \leq 1.
\end{align*}
\]  

(36)

This problem is in a form similar to the one proposed in the previous section, and also not a standard convex problem. We can perform the similar transform steps as follows.

C. The Optimal Solution

Firstly we introduce a new variable \( \gamma_e \) to represent the predefined threshold for the eavesdropper’s SNR and reformulate the original problem as follows:

\[
\begin{align*}
\max_{W, \alpha} SNR_d \\
\text{s.t.} \quad & C1: SNR_e \leq \gamma_e, \\
& C2: P_r \leq P_r^{\max}, \\
& C3: 0 \leq \alpha \leq 1.
\end{align*}
\]  

(37)

Then we further transform the objective function as follows:

\[
SNR_d = \frac{P_s |h_d^T W_{r}|^2}{\sigma_r^2 \|h_d^T W\|^2_2 + \sigma_e^2 \|h_d^T W\|^2_2 + \sigma_d^2} = \frac{w^H Q_{a1} w}{w^H Q_{a2} w + \sigma_d^2},
\]  

(38)
where
\[ Q_{a1} = P_s (h_r h_r^H)^T \otimes (h_d^* h_d^T), \]  
\[ Q_{a2} = (\sigma_d^2 I_N + \sigma_c^2 I_N) \otimes (h_d^* h_d^T). \]  

The SNR of the eavesdropper constraint is given by
\[ \text{SNR}_e = \frac{P_s |h_e^T W h_r|^2}{\sigma_r^2 ||h_e^T W||^2_F + \sigma_c^2 ||h_e^T W||^2_F + \sigma_e^2}, \]
\[ = \frac{w^H Q_{b1} w}{w^H Q_{b2} w + \sigma_e^2}, \]  

where
\[ Q_{b1} = P_s (h_r h_r^H)^T \otimes (h_e^* h_e^T), \]  
\[ Q_{b2} = (\sigma_r^2 I_N + \sigma_c^2 I_N) \otimes (h_e^* h_e^T). \]  

The transmitted power constraint can be rewritten as follows:
\[ P_r = P_s ||W h_r||^2_F + \sigma_r^2 ||W||^2_F + \sigma_c^2 ||W||^2_F, \]
\[ = w^H Q_c w, \]  

where
\[ Q_c = (P_s h_r h_r^H + \sigma_r^2 I_N + \sigma_c^2 I_N)^T \otimes I_N. \]

Define \( X = w w^H \), and the optimization problem can be reformulate as follows:
\[ \max_{X, \alpha} \frac{\text{Tr}(Q_{a1} X)}{\text{Tr}(Q_{a2} X) + \sigma_d^2} \]
\[ \quad \text{s.t.} \quad C1 : \frac{\text{Tr}(Q_{b1} X)}{\text{Tr}(Q_{b2} X) + \sigma_e^2} \leq \gamma_e, \]
\[ C2 : \text{Tr}(Q_c X) \leq P_r^{max}, \]
\[ C3 : 0 \leq \alpha \leq 1, \]
\[ C4 : \text{Rank}(X) = 1. \]

We drop the rank-one constraint and a SDP problem can be obtained as follows:
\[ \max_{X, \alpha} \frac{\text{Tr}(Q_{a1} X)}{\text{Tr}(Q_{a2} X) + \sigma_d^2} \]
\[ \quad \text{s.t.} \quad C1 : \text{Tr}(Q_{b12} X) \leq \sigma_e^2, \]
\[ C2 : \text{Tr}(Q_c X) \leq P_r^{max}, \]
\[ C3 : 0 \leq \alpha \leq 1, \]
where \( Q_{b12} = \frac{1}{\sigma_c^2} Q_{b1} - Q_{b2} \). The time switching coefficient is still a challenge to be optimized. We again perform 1-D optimization over the time switching coefficient. If the value of \( \alpha \) is set, the problem can be treated as a quasi-convex SDP problem. We also use the Charnes-Cooper transformation. Define a new variable \( t = \frac{1}{Tr(Q_{a2}\tilde{X}) + \sigma_d^2} \) and let \( \tilde{X} = tX \). The problem can be recast as follows:

\[
\begin{align*}
\max_{\tilde{X}, t} & \quad Tr(Q_{a1}\tilde{X}) \\
\text{s.t.} & \quad C1 : Tr(Q_{a2}\tilde{X}) + t\sigma_d^2 = 1, \\
& \quad C2 : Tr(Q_{b12}\tilde{X}) \leq t\sigma_c^2, \\
& \quad C3 : Tr(Q_c\tilde{X}) \leq tP_{r, max}.
\end{align*}
\]

(48)

Then the problem is solvable like the problem with PSR protocols. The optimal solution is denoted by \( \tilde{X}^*, t^* \). Then the solution of \( X \) denoted by \( X^* \) is obtained by \( X^* = \frac{\tilde{X}^*}{t^*} \). If the rank of \( \tilde{X}^* \) is not one, we can employ Theorem 1 to obtain the rank-one solution.

V. Numerical Results

In this section, simulation results are provided to evaluate the performance of the proposed secrecy rate optimization solution. The number of antenna is 3. We use the TGn path loss model with the directional transceiver antenna gain of 10dBi. The distance between each node is set as 10m. The carrier frequency is 470 MHz which is accorded with the IEEE 802.11af Wi-Fi parameters [36]. The noise variance is assumed as \( \sigma_r^2 = \sigma_c^2 = \sigma_d^2 = \sigma_e^2 = \sigma_f^2 = -25 \) dBm. The initial power at the relay node is 10dBm. The energy conversion efficient is 0.8. The maximum SNR tolerance for the eavesdropper is assumed as -10dB. Simulation results were averaged over 1000 independent trials.

In Fig. 1, the secrecy rates achieved by different secure transmission schemes versus the transmitted power of the source are plotted. The results for PSR and TSR schemes are obtained with its optimal energy harvesting coefficient. It can be seen that curves of secrecy rates for the two proposed energy harvesting schemes are monotonically non-decreasing functions of the transmitted power because the higher transmitted power results in more available power at the relay for the relay-destination transmission. The proposed scheme with PSR protocol outperforms that with TSR protocol. For comparison, Fig. 1 also presents the performance for non-energy harvesting cooperative networks. In the non-energy harvesting scheme, the relay node only use its initial power to relay the information. It can be observed that performances of two energy harvesting schemes and the non-energy scheme are close in the low source power region. This is because that if the transmitted power at the source node is low, the energy harvesting relay can not collect much power to improve the system performance. However, if the source power is large enough, the energy harvesting schemes can obviously outperform the non-energy harvesting scheme. The secrecy capacity in the non-energy harvesting scheme is limited by the relay power and can not further been improved with the increased source power. From the figure, we can see that the energy harvesting relaying schemes have better performance than the non-energy harvesting scheme, which demonstrates energy harvesting relay schemes can improve the secrecy rate in a resource-limited relay network.
In Fig. 2, we present the secrecy rate versus the transmitted power for the proposed scheme with PSR and TSR protocols by using different parameters. It can be seen from the figure that the secrecy rates achieved by these two protocols improve with the increased number of the antenna and are degraded with the increase of the distance between each node. This is attributed to the fact that the relay could exploit the array gain to achieve better performance with more antennas and the longer distance makes channel attenuation larger in turns resulting in the worse performance of relay networks.

VI. CONCLUSIONS

In this paper, we formulated the secrecy rate maximization in an AF relay network with PSR and TSR energy harvesting protocols, while taking into account relay precoding and the energy harvesting coefficient. The optimal solution was founded by applying the SDP relaxation approach and 1-D optimization. Simulation results illustrated the energy harvesting relaying schemes could enhance the secrecy rate in a resource-limited relay network and yield an improved trade-off between the secrecy rate and system parameters.
REFERENCES


