Estimation of storm peak and intra-storm directional-seasonal design conditions in the North Sea

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Motivation

- **Rational** design and assessment of marine structures:
  - Reducing **bias** and **uncertainty** in estimation of structural reliability
  - Improved understanding and communication of risk

  - For new (e.g. floating) and existing (e.g. steel and concrete) structures
  - Climate change

  - **Whole-basin** analysis: non-stationary analysis for 1000s of locations with multidimensional covariates

- Other applied fields for extremes in industry:
  - Corrosion and fouling
  - Economics and finance

North Sea

- Model **storm peak significant wave height,** H_S^{sp}
- Incorporate **intra-storm evolution of** H_S
- Estimate **wave height**, **crest elevation**, **tide** and **surge**
- Wave climate is dominated by **extra-tropical storms**
- Fetch (Atlantic, Norwegian Sea, North Sea) and land shadow (Norway, UK)
- **Directional** and **seasonal** variability present in extremes

- Sample of **hindcast** storms for period of $\approx$50 years
- **Marginal** model
- Animation: Link

Storm peak significant wave height H_S^{sp}

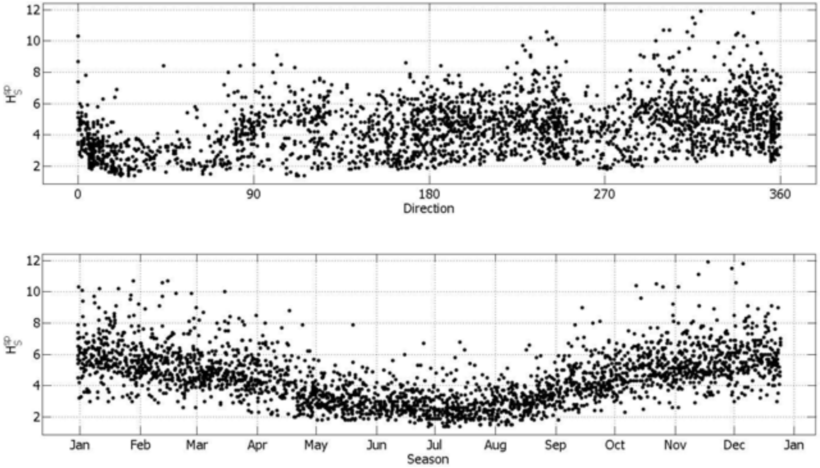


Figure: Storm peak significant wave height H_S^{sp} on storm direction θ^{sp} (upper panel) and storm season ϕ^{sp} (lower panel).

Quantiles of H_S^{sp}

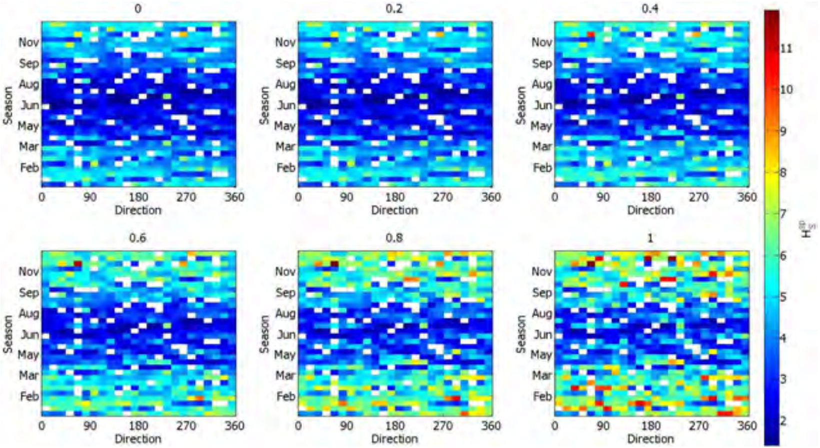


Figure: Empirical quantiles of storm peak significant wave height, H_S^{sp} by storm direction, θ^{sp}, and storm season, θ^{sp}. Empty bins are coloured white.

Storm trajectories of significant wave height, H_S.

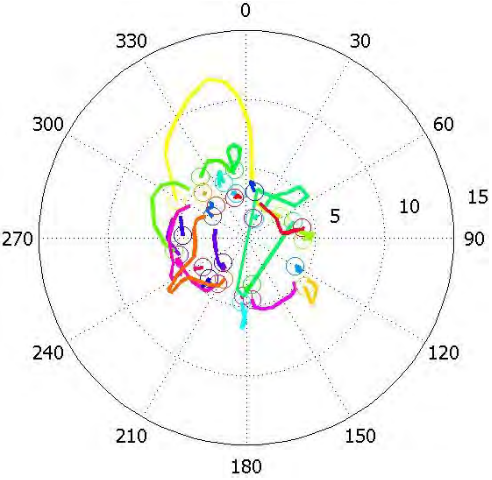


Figure: Storm trajectories of significant wave height, H_S, on wave direction θ for 30 randomly-chosen storm events (in different colours). A circle marks the start of each intra-storm trajectory.

Outline of modelling procedure

Data and model estimation

Storm peak variables

Covariates	θ, ϕ	Isolate from sample & threshold / Poisson model
H_S^{sp}	$H_S^{sp} \mid \theta, \phi$	Isolate from sample & threshold / GP model

Intra-storm variables

Between sea-states	$H_S \mid H_S^{sp}, \theta, \phi$	Isolate trajectories from sample
Within sea-states	$H_{max} \mid H_S$	Known parametric model from literature

Return value inference

Covariates	θ, ϕ	Simulate occurrences of θ, ϕ (corresponding to P years of storm peaks)
H_S^{sp}	$H_S^{sp} \mid \theta, \phi$	Simulate sizes given θ, ϕ
Between sea-state	$H_S \mid H_S^{sp}, \theta, \phi$	Peak-matching (using H_S^{sp}, θ, ϕ) for best trajectory
Within sea-states	$H_{max} \mid H_S$	Sample form known distribution given H_S

Extreme value model components

- Sample $\{\dot{z}_i\}_{i=1}^{\dot{n}}$ of $\dot{n}$ **storm peak** significant wave heights observed with storm peak directions $\{\dot{\theta}_i\}_{i=1}^{\dot{n}}$ and storm peak seasons $\{\dot{\phi}_i\}_{i=1}^{\dot{n}}$

- Model components:
  1. **Threshold** function ψ above which observations $\dot{z}$ are assumed to be extreme estimated using quantile regression
  2. **Rate of occurrence** of threshold exceedances modelled using Poisson model with rate $\rho(\triangleq \rho(\theta, \phi))$
  3. **Size of occurrence** of threshold exceedance using generalised Pareto (GP) model with shape and scale parameters ξ and σ

Extreme value model components

- Rate of occurrence and size of threshold exceedance functionally **independent** (Chavez-Demoulin and Davison 2005)
  - Equivalent to non-homogeneous Poisson point process model (Dixon et al. 1998)
- Smooth functions of covariates estimated using penalised B-splines (Eilers and Marx 2010)
  - Slick linear algebra (c.f. generalised linear array models, Currie et al. 2006)
- Large number of parameters to estimate
  - Computational efficiency essential

Penalised B-splines

- Physical considerations suggest model parameters ψ, ρ, ξ and σ vary smoothly with covariates θ, ϕ
- Values of $(\eta =)\psi, \rho, \xi$ and σ all take the form:

$$\eta = B\beta_\eta$$

  for **B-spline** basis matrix B (defined on index set of covariate values) and some β_η to be estimated
- Multidimensional basis matrix B formulated using Kronecker products of marginal basis matrices:

$$B = B_\theta \otimes B_\phi$$

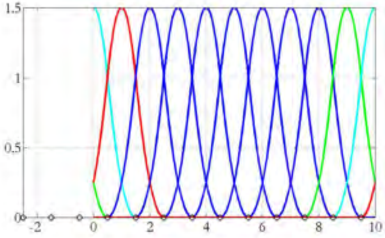
- Roughness R_η defined as:

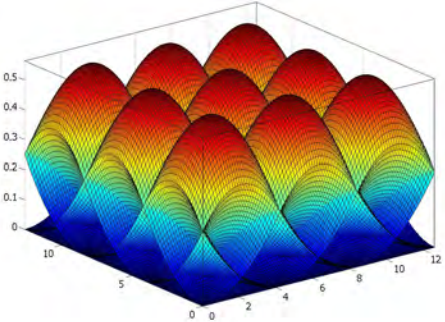
$$R_\eta = \beta_\eta' P \beta_\eta$$

  where effect of P is to difference neighbouring values of β_η

Penalised B-splines

- **Wrapped** bases for periodic covariates (seasonal, direction)
- **Multidimensional** bases easily constructed. **Problem size** sometimes prohibitive
- Parameter **smoothness** controlled by roughness coefficient λ: **cross validation** or similar chooses λ optimally





Quantile regression model for extremal threshold

- Estimate smooth quantile $\psi(\theta, \phi; \tau)$ for non-exceedance probability τ of z (storm peak H_S) using quantile regression by minimising **penalised** criterion ℓ^*_ψ with respect to basis parameters:

$$\ell^*_\psi = \ell_\psi + \lambda_\psi R_\psi$$

$$\ell_\psi = \{\tau \sum_{r_i \geq 0}^{n} |r_i| + (1-\tau) \sum_{r_i < 0}^{n} |r_i|\}$$

  for $r_i = z_i - \psi(\theta_i, \phi_i; \tau)$ for $i = 1, 2, ..., n$, and **roughness** R_ψ controlled by roughness coefficient λ_ψ

- (Non-crossing) quantile regression formulated as linear programme (Bollaerts et al. 2006)

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Directional-seasonal threshold, ψ.

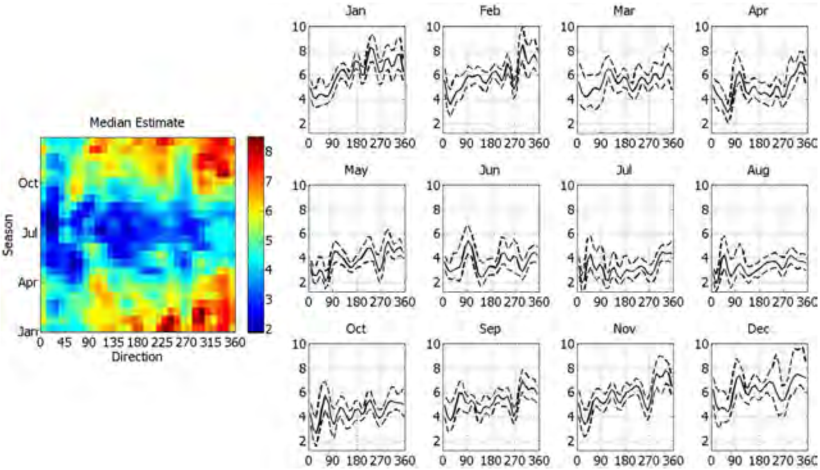

Figure: lhs: bootstrap median. rhs: 12 monthly directional.

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Poisson model for rate of threshold exceedance

- Poisson model for rate of occurrence of threshold exceedance estimated by minimising roughness penalised log likelihood:

$$\ell_\rho^* = \ell_\rho + \lambda_\rho R_\rho$$

- (Negative) penalised Poisson log-likelihood (and approximation):

$$\begin{aligned} \ell_\rho &= -\sum_{i=1}^{n} \log \rho(\theta_i, \phi_i) + \int \rho(\theta, \phi) d\theta dx dy \\ \hat{\ell}_\rho &= -\sum_{j=1}^{m} c_j \log \rho(j\Delta) + \Delta \sum_{j=1}^{m} \rho(j\Delta) \end{aligned}$$

- $\{c_j\}_{j=1}^m$ counts of threshold exceedances on index set of m $(>> 1)$ bins partitioning covariate domain into intervals of volume Δ
- λ_ρ estimated using cross validation or similar (e.g. AIC)

Directional-seasonal exceedance rate, ρ.

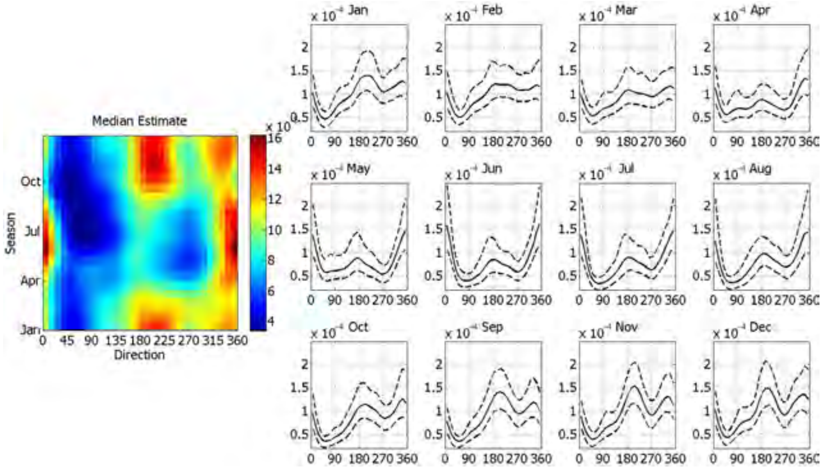

Figure: lhs: bootstrap median. rhs: 12 monthly directional.

GP model for size of threshold exceedance

- Generalise Pareto model for size of threshold exceedance estimated by minimising roughness penalised log-likelihood:

$$\ell^*_{\xi,\sigma} = \ell_{\xi,\sigma} + \lambda_\xi R_\xi + \lambda_\sigma R_\sigma$$

- (Negative) conditional generalised Pareto log-likelihood:

$$\ell_{\xi,\sigma} = \sum_{i=1}^{n} \log \sigma_i + \frac{1}{\xi_i} \log(1 + \frac{\xi_i}{\sigma_i}(z_i - \psi_i))$$

- Parameters: **shape** ξ, **scale** σ
- Threshold ψ set prior to estimation
- λ_ξ and λ_σ estimated using cross validation or similar. In practice set $\lambda_\xi = \kappa\lambda_\sigma$ for fixed κ

Directional-seasonal GP shape, ξ.

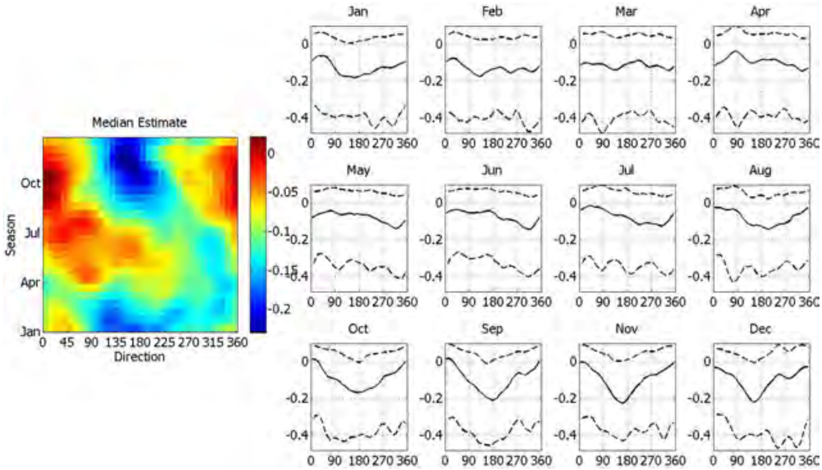

Figure: lhs: bootstrap median. rhs: 12 monthly directional.

Directional-seasonal GP scale, σ.

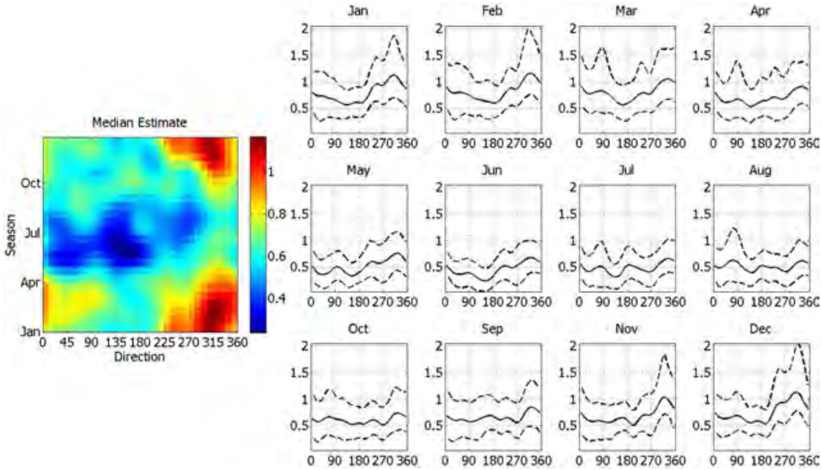

Figure: lhs: bootstrap median. rhs: 12 monthly directional.

Return values

- Estimation of return values by simulation under model
  - Sample number of events in period, directions and seasons of events, sizes of events
- Alternative: closed form function of parameters
  - Return value z_T of storm peak significant wave height corresponding to return period T (years) evaluated from estimates for ψ, ρ, ξ and σ:

$$z_T = \psi - \frac{\sigma}{\xi}(1 + \frac{1}{\rho}(\log(1 - \frac{1}{T}))^{-\xi})$$

  - Interpretation **problematic**
- z_{100} corresponds to 100--year return value, denoted H_{S100}

CDFs for H_{S100}

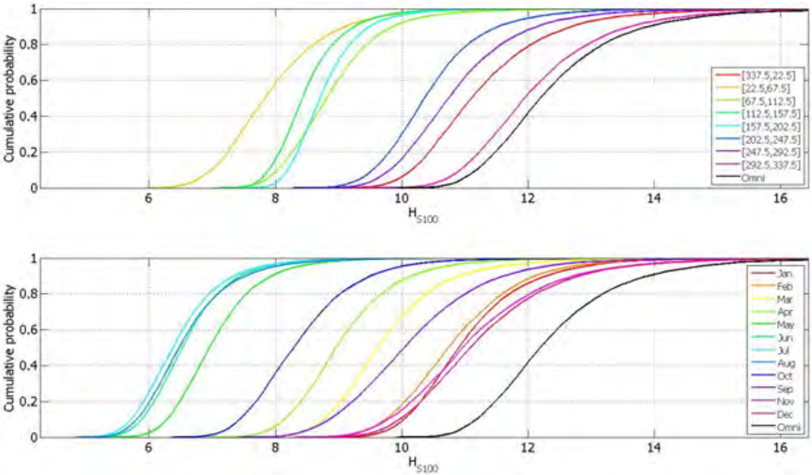

Figure: CDFs incorporating bootstrap uncertainty

Directional-seasonal return value plot for H_{S100}

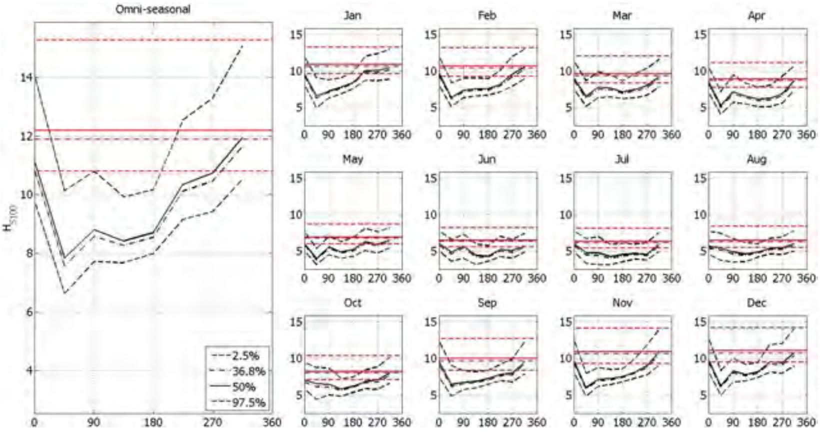

Figure: lhs: Directional omni-seasonal return values. rhs: Directional return values for calendar months.

Directional-seasonal return value plot for H_{S100}

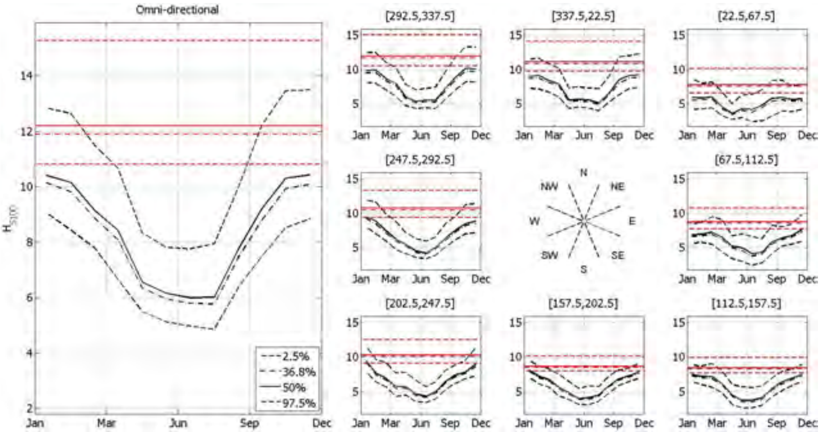

Figure: lhs: Seasonal omni-directional return values. rhs: Seasonal return values for directional octants.

Critical environmental variables

- Peak significant wave height
- Maximum wave height
- Maximum crest elevation
- Peak total water level
- ``Associated'' values of wind speed and direction corresponding to peak significant wave height
- ``Associated'' values of current speed and direction corresponding to peak significant wave height
- Maximum load on structure

Intra-storm variability (e.g. H_S and H_{max})

- Extreme value model allows simulation of H_S^{sp}, θ^{sp} and ϕ^{sp}
- Matching procedure used to estimate storm evolution $(H_S(t), \theta(t), \phi(t))|(H_S^{sp}, \theta^{sp}, \phi^{sp})$ for sea state t
- Empirical literature models for $H(t)|H_S(t)$ and $H_{max}(t)|H_S(t)$

The cumulative distribution function for the maximum wave height H_{max} in a sea-state of n_s waves with significant wave height $H_S = h_s$ is taken (see, for example, Forristall 1978) to be given by:

$$P(H_{max} \leq h_{max}|H_S = h_s, M = n_s) = (1 - \exp(-\frac{1}{\beta}(\frac{h_{max}}{h_s/4})^{\alpha}))^{n_s}$$

with $\alpha = 2.13$ and $\beta = 8.42$. The number of waves n_s in a particular sea state is estimated by dividing the length of the sea-state (in seconds) by its zero-crossing period, T_Z.

Directional-seasonal return value plot for H_{max100}

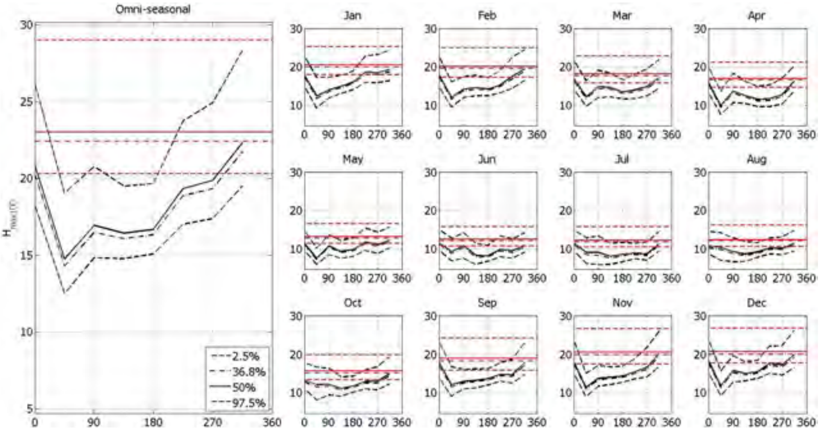

Figure: lhs: Directional omni-seasonal return values. rhs: Directional return values for calendar months.

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Directional-seasonal return value plot for H_{max100}

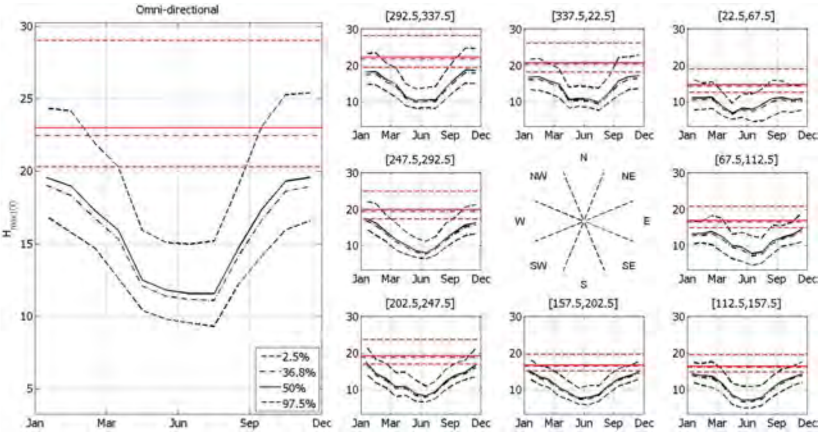

Figure: lhs: Seasonal omni-directional return values on wave season. rhs: Seasonal return values for directional octants.

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Validation of directional-seasonal model for H_S^{sp}

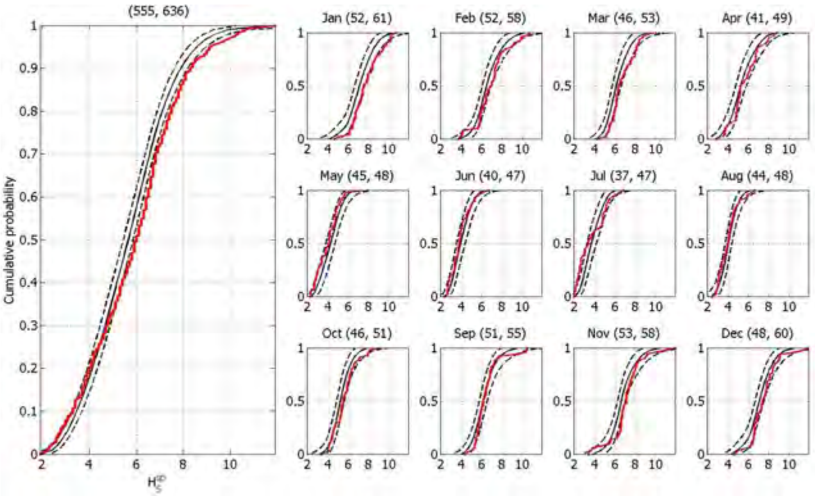

Figure: CDFs for H_S^{sp} for original sample and for 1000 sample realisations under the model corresponding to the same time period as the original sample.

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Validation of directional-seasonal model for H_S

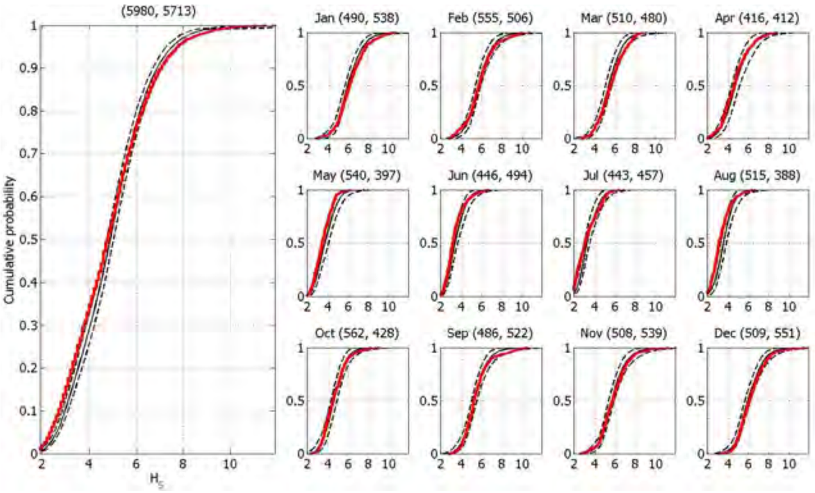

Figure: CDFs for H_S for original sample and for 1000 sample realisations under the model (incorporating ITV) corresponding to the same time period as the original sample.

Summary

- Directional-Seasonal extreme value model for H_S^{sp} for North Sea
- Incorporation of short term effects allowing modelling of associated variables wave height, crest elevation, surge
- Return value distributions vary with direction and season in line with physical intuition
- For operational purposes directional-seasonal model can be re-combined in many ways to quickly get return values without need to do new analysis.
- Generally important to accommodate covariate effects in threshold and rate, sometimes in GP shape and scale

References

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