On the estimation of ocean engineering design contours

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Motivation

- Environmental design contours required for structural design
- FORM approach available
  - Assumptions might be unrealistic
  - Uncertainty not usually quantified
- Statistical modelling offers an alternative approach to estimate design contours
  - Conditional extremes modelling based on work of Heffernan & Tawn (2004)
- Different forms of design contours are possible
  - Contours of constant probability density
  - Contours of constant exceedence probability

Objectives

- Evaluate different options for design contour estimation
  - By analogy with 1-D, more natural to use constant **exceedence** probabilility $P(X_1 > x_1, X_2 > x_2)$ than constant probability density
  - FORM provides contours of constant probability density
- Evaluate performance of design contour estimation for simulated data
- Estimate design contours for measured and hindcast applications
- Evaluate uncertainty of estimated design contours

Content

- Introduction to FORM
- Introduction to Conditional Extremes modelling
- Illustration of different types of design contours
- Applications to estimation of (H_S, T_P) contours for
  - Measured Northern North Sea
  - Hindcast Northern North Sea
  - Measured Gulf of Mexico
  - Hindcast North-West Shelf of Australia
- Conclusions and recommendations

FORM in outline

- Joint estimation of contours for 2 or more environmental variates
- Independent of structural loading and response
- Used to define **design point** for structural reliability
- Probability integral transform (PIT) used to derive **independent** random variables to derive surface of constant probability **density**
- For example, in the case of 2 variates:
  - $H_S \sim Weibull \overset{PIT}{\Rightarrow} U_1 \sim standard\ Normal$
  - $T_P|H_S \sim log\text{-}Normal \overset{PIT}{\Rightarrow} U_2 \sim standard\ Normal$
  - **Circle** $u_1^2 + u_2^2 = \beta^2$ gives constant probability density
  - $(U_1, U_2) \overset{PIT}{\Rightarrow} (H_S, T_P)$ to get contours on original scale

FORM characteristics

- FORM assumes we can transform to **independent** random variables
- FORM assumes prior knowledge of the distribution of X_1 and $(X_2|X_1)$
  - Usually based on empirical fitting not physics
  - $T_P|H_S \sim$ *log-Normal* commonly used.
    - What about parameters of distribution?
    - Do they vary with location, with time?
  - What about other variates, e.g. *Current speed*$|H_S$?
    - Can we treat current speed and H_S as independent?
    - If not, which functional form for distribution? Parameters?
- Model explains body of distribution, not necessarily tail of distribution
  - $T_P|H_S \sim$ *log-Normal* may be appropriate for body but not tail of distribution
- Difficult in practice to extend beyond 2 variates

Conditional extremes modelling in outline

- Model the conditional distribution of Y_2 given a large value of Y_1
- (X_1, X_2) need to be transformed to (Y_1, Y_2) on the same *standard Gumbel* scale
- **Asymptotic** argument relies on X_1 (and Y_1) being **large**
- In a nut shell:
  - $(X_1, X_2) \overset{PIT}{\Rightarrow} (Y_1, Y_2)$
  - $(Y_2 | Y_1 = y_1) = ay_1 + y_1^b Z$ for large values y_1
  - $(Y_1, Y_2) \overset{PIT}{\Rightarrow} (X_1, X_2)$
  - Simulation to sample joint distribution of (Y_1, Y_2) (and (X_1, X_2))
- Marginal generalised Pareto modelling for (X_1, X_2):
  - $F_{GP}(x_1; \xi, \beta, u) = 1 - (1 + \frac{\xi}{\beta}(x_1 - w_{X1}))_+^{-\frac{1}{\xi}}$

Conditional extremes modelling characteristics

- Value of conditioning variate must be large for conditional extremes model to apply
- No prior knowledge of form of distribution of X_1 and $X_2|X_1$ required
- Models tail of distribution using conditional extremes
- Models body of distribution empirically

Exceedence contours explained

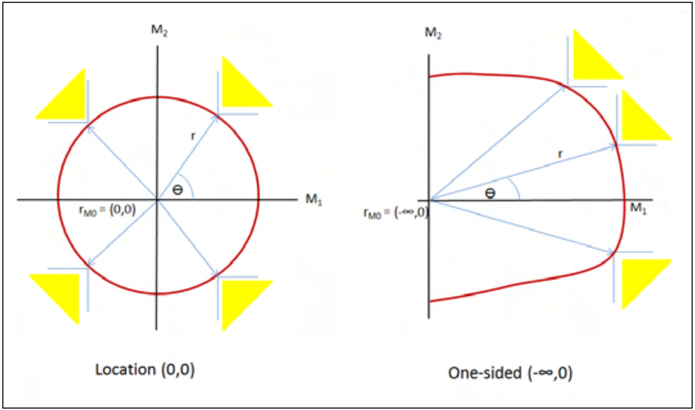


Figure: Contours of constant exceedence probability (C2-C4) radiate outwards from reference location r_{M0}

- $Pr(\bigcap_{j=1}^{2}(r_{Mj}(\theta; r_{M0})M_j > r_{Mj}(\theta; r_{M0})m_j(\theta))) = \alpha$

Constant density contours explained

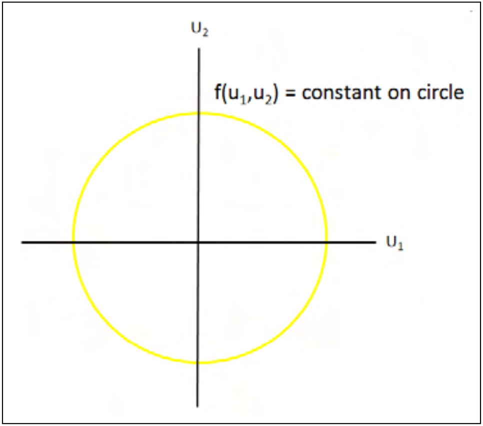


Figure: Contours of constant probability density (C1) are circles in U-space. Density is constant **on the contour only**

- $f(u_1, u_2) = (2\pi)^{-1/2} \exp(-\frac{1}{2}(u_1^2 + u_2^2))$

Contour types C1-C3

- All of C1-C3 use the conditional extremes model to transform: $(H_S, T_P|H_S) \Rightarrow (U_1, U_2) \sim$ *standard Normal* as a first step
- C1: Constant probability **density**, standard **Normal** scale
  - Contours are circles $u_1^2 + u_2^2 = \beta^2$
- C2: Constant **exceedence probability**, standard Normal scale
  - Contour (u_1, u_2) such that: $Pr(\bigcap_{j=1}^{2}(r_{Uj}(\theta; \mathbf{(0,0)})U_j > r_{Uj}(\theta; \mathbf{(0,0)})u_j(\theta))) = \alpha$
- C3: Constant **1-sided** exceedence probability, standard Normal scale
  - Contour (u_1, u_2) such that: $Pr(\bigcap_{j=1}^{2}(r_{Uj}(\theta; \mathbf{(-\infty,0)})U_j > r_{Uj}(\theta; \mathbf{(-\infty,0)})u_j(\theta))) = \alpha$
- All of C1-C3 transform contour in U-space to contours in (H_S, T_P)-space as a final step

Contour type C4

- C4: Constant 1-sided exceedence probability on **original** scale
  - Direct simulation using **full** conditional extremes model
  - Contour (x_1, x_2) such that:
    $Pr(\bigcap_{j=1}^{2}(r_{X_j}(\theta; \mathbf{(\text{-}\infty,0)})X_j > r_{X_j}(\theta; \mathbf{(\text{-}\infty,0)})X_j(\theta))) = \alpha$

Illustration using model of Haver and Nyhus

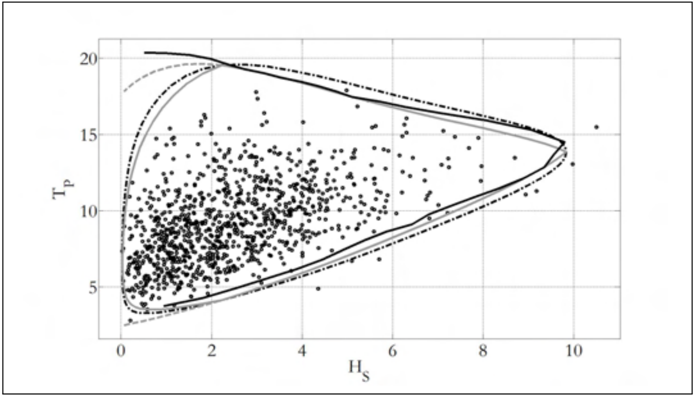
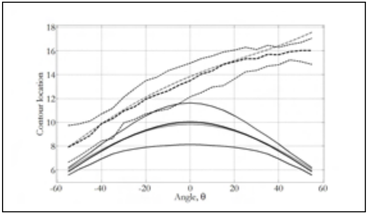


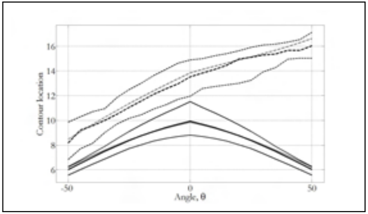
Figure: Contours C1-C4 corresponding to the H_S-T_P model of Haver & Nyhus (equivalent to a 1 in 1000 event of H_S marginally): C1 (dashed black), C2 (solid grey), C3 (dashed grey) and C4 (solid black).

- Haver-Nyhus model: $H_S \sim Weibull$, $T_P | H_S \sim log\text{-}Normal$

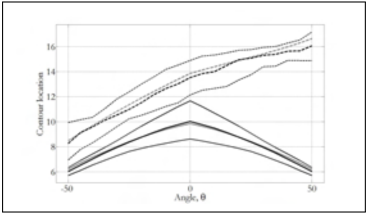
Confidence limits



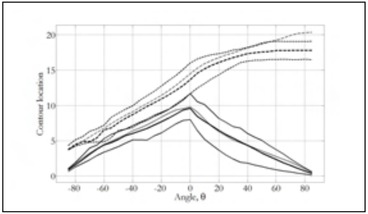
C1: Constant probability **density**, standard normal scale



C3: Constant **1-sided** exceedence probability, standard normal scale



C2: Constant **exceedence probability**, standard normal scale



C4: Constant 1-sided exceedence probability, **original** scale

- Grey: truth. Black: estimated with 95% bands. Dashed: T_P. Solid: H_S.

Data

- Storm peak (H_S, T_P) samples from 4 sources
- Northern North Sea measured
  - Laser: 620 storm peak events (March 1973 - December 2006)
- Northern North Sea hindcast
  - 827 storm peak events (November 1964 - April 1998)
- Gulf of Mexico measured
  - NDBC buoy 42002: 505 storm peak events (January 1980 - December 2007)
- North West Shelf of Australia hindcast
  - 145 storm peak events (February 1970 - April 2006)
- All samples exhibit positive dependence between H_S and T_P

NNS measured

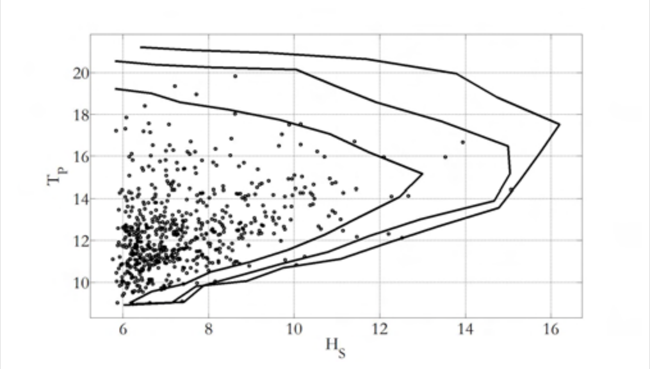


Figure: C4 contours of constant exceedence probability, 1-sided in H_S, (equivalent to 10-, 100- and 1000-year H_S marginally), for measured Northern North Sea data. Also shown is the sample.

- Contour is not closed since it is 1-sided

NNS hindcast

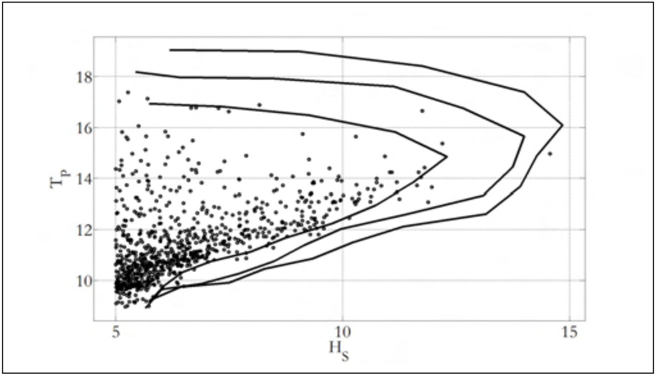


Figure: C4 contours, one-sided in H_S, (equivalent to 10-, 100- and 1000-year H_S marginally), for hindcast Northern North Sea data. Also shown is sample.

- Contours are qualitatively similar to those of measured NNS data

GoM measured

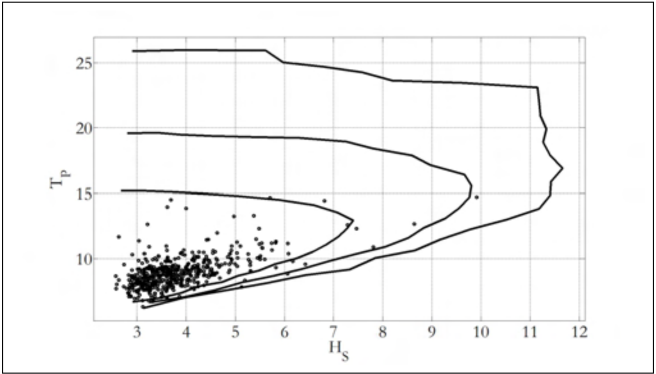


Figure: C4 contours, one-sided in H_S, (equivalent to 10-, 100- and 1000-year H_S marginally), for measured Gulf of Mexico data. Also shown is sample.

- T_P has a longer tail than for NNS

NWS hindcast

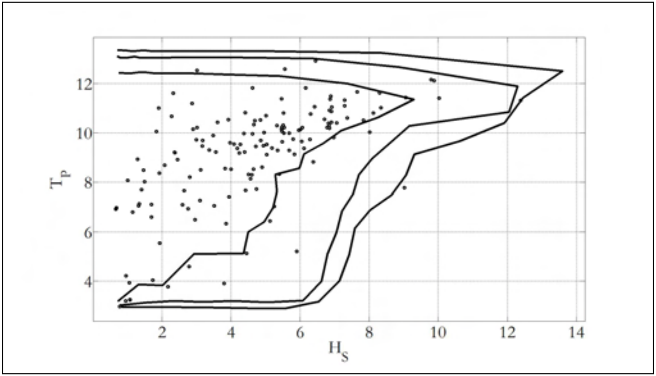
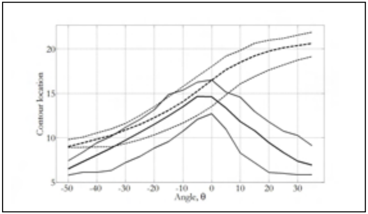


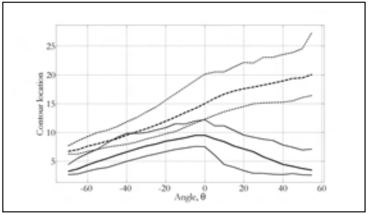
Figure: C4 contours, one-sided in H_S, (equivalent to 10-, 100- and 1000-year H_S marginally), for hindcast North West Shelf data. Also shown is sample.

- T_P not obviously increasing with H_S
- Clear lack of symmetry with respect to T_P

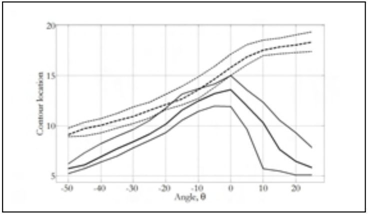
C4 confidence limits



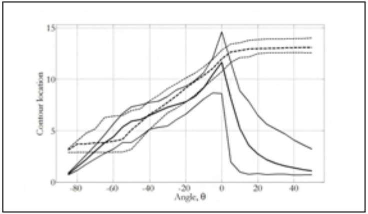
NNS measured



GoM measured



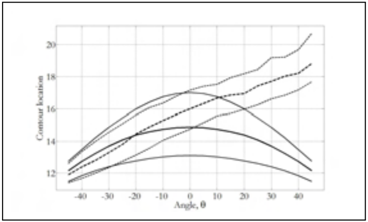
NNS hindcast



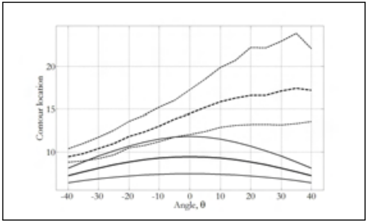
NWS hindcast

- C4: Constant 1-sided exceedence probability, original scale, corresponding to 100-year H_S marginally

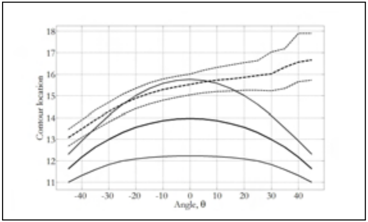
Estimating C1 contours



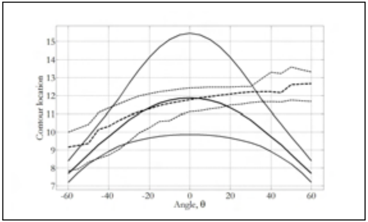
NNS measured



GoM measured



NNS hindcast



NWS hindcast

- C1: Constant probability density, standard normal scale, corresponding to 100-year H_S marginally

Conclusions

- Conditional extremes model:
  - Applicable to all contour estimation provided conditioning variate is large
  - Provides quantification of uncertainty in contour location
- Method has been generalised to p ($p > 2$) dimensions
  - Difficult using FORM
- Failure probability can be estimated directly
  - Rather than using a 'design point' as in FORM
- Further work on incorporating **covariate** effects (e.g. seasonality, directionality) within conditional extremes model is in progress

Thanks

Thanks for listening.

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