The Extremal Dependence of Storm Severity, Wind Speed and Surface Level Pressure in the Northern North Sea

Ross Towe, Emma Eastoe, Jonathan Tawn, Philip Jonathan and Yanyun Wu

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Return levels and Pooling

- Return levels are used in the design criteria of an offshore structure to ensure the structure withstands a particularly extreme event.
- Can calculate return levels for Hs for a given site in the North sea.
Return levels and Pooling

- Return levels are used in the design criteria of an offshore structure to ensure the structure withstands a particularly extreme event.
- Can calculate return levels for Hs for a given site in the North sea.
- A model is fitted to estimate the return levels of $H_s$, as there is not enough data to estimate the return levels empirically.
- To increase our confidence in the estimates of the return levels, we can pool data across sites.
Pooling

What happens if there is dependence between sites?

<table>
<thead>
<tr>
<th>Data set</th>
<th>SE(50yr)</th>
<th>SE(100yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>One site</td>
<td>0.65</td>
<td>1.00</td>
</tr>
<tr>
<td>Pooled (perfectly dependent) sites</td>
<td>0.62</td>
<td>0.94</td>
</tr>
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<td>Pooled (independent) sites</td>
<td>0.22</td>
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Standard errors (uncertainty) of the 50 and 100 year return levels
Pooling

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Standard errors (uncertainty) of the 50 and 100 year return levels

- Pooling dependent data produces false estimates of the standard errors
- The effective sample size is smaller for pooled perfectly dependent data compared to pooled independent data
Extremal dependence

- We can also consider the extremal dependence **Wind Speed** (WS) and **Significant Wave Height** ($H_s$) at a given site.
- A storm is characterised by a number of different factors, such as wind and waves.
- A severe storm can be a result of high waves but also a combination of extreme wind and waves.
We can also consider the extremal dependence **Wind Speed** (\(WS\)) and **Significant Wave Height** (\(H_s\)) at a given site.

A storm is characterised by a number of different factors, such as wind and waves.

A severe storm can be a result of high waves but also a combination of extreme wind and waves.

**Should we model** \(WS\) and \(H_s\) **jointly or independently?**

**We are interested in the joint behaviour** \(WS\) and \(H_s\) **during a storm**.
Extremal dependence

Bivariate Normal distribution

The dependence in the body of the data is different to that of the tail

Consider the bivariate Normal distribution with $\rho = 0.8$

Calculate,

$$P(X > z \mid Y > z)$$

for $z \to \infty$,

<table>
<thead>
<tr>
<th>$z$</th>
<th>$P(X &gt; z \mid Y &gt; z)$</th>
</tr>
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<tbody>
<tr>
<td>1.00</td>
<td>0.60</td>
</tr>
<tr>
<td>1.50</td>
<td>0.49</td>
</tr>
<tr>
<td>2.00</td>
<td>0.33</td>
</tr>
<tr>
<td>2.50</td>
<td>0.00</td>
</tr>
<tr>
<td>3.00</td>
<td>0.00</td>
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Extremal dependence

Bivariate extreme value distribution

The dependence in the body of the data is different to that of the tail

Consider the bivariate extreme value distribution with $\rho = 0.8$.

Calculate

$$P(X > z \mid Y > z) \text{ for } z \to \infty,$$

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<tr>
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<tr>
<td>2.00</td>
<td>0.72</td>
</tr>
<tr>
<td>4.00</td>
<td>0.79</td>
</tr>
<tr>
<td>6.00</td>
<td>1.00</td>
</tr>
<tr>
<td>8.00</td>
<td>1.00</td>
</tr>
<tr>
<td>10.00</td>
<td>1.00</td>
</tr>
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</table>
Data

- Storm peak events from a 50 year data set:
  - Significant wave height [metres]
  - Wind speed [miles per hour]
  - Surface level pressure [hectopascal]
  - Storm direction [degrees]

- Sites have different local characteristics
Original margins

(NSLP vs $H_s$)  (WS vs $H_s$)

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Fréchet margins

Consider the data on Fréchet margins, \( F(z) = \exp(-1/z) \)

\((\text{NSLP vs } H_s)\) \hspace{1cm} \((\text{WS vs } H_s)\)
Definition

Consider a pair of random variables $(X, Y)$ with identical margins,

**Asymptotic independence:**

$$P(X > z, Y > z) = 0, \text{ as } z \to \infty$$

*Two extreme events are unlikely to occur simultaneously*

Consider the measure: $\bar{\chi}$

**Asymptotic dependence:**

$$P(X > z, Y > z) > 0, \text{ as } z \to \infty$$

*Two extreme events are likely to occur simultaneously*

Consider the measure: $\chi$
Extremal dependence measures

**Asymptotic independence:**
- \( \bar{\chi} \) determines the strength of asymptotic independence
- \( \bar{\chi} \in (-1, 1) \).
- If \( \bar{\chi} = 1 \) \( \Rightarrow \) asymptotic dependence
Extremal dependence measures

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**Asymptotic dependence:**
- \( \chi \) determines the strength of asymptotic dependence
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  \chi = P(X > z \mid Y > z)
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  \[ \chi = P(X > z \mid Y > z) \]
- $\chi \in (0, 1)$.
- When $\chi = 0 \Rightarrow$ asymptotic independence

$\bar{\chi}$ and $\chi$ can be estimated empirically or by using a model.
Asymptotic independence case

\((\text{NSLP } v \ H_s)\)

<table>
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<tr>
<th>Site</th>
<th>(\hat{\chi}) (95% CI)</th>
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<tr>
<td>A</td>
<td>0.61 (0.50, 0.72)</td>
</tr>
<tr>
<td>B</td>
<td>0.61 (0.50, 0.72)</td>
</tr>
<tr>
<td>C</td>
<td>0.59 (0.47, 0.70)</td>
</tr>
<tr>
<td>D</td>
<td>0.65 (0.53, 0.77)</td>
</tr>
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Estimates of \(\hat{\chi}\) for the 4 sites with the threshold set at the respective 80\% quantile estimation threshold
Asymptotic dependence case

\((WS \text{ v } H_s)\)

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<tr>
<td>A</td>
<td>0.95(0.81,1.00)</td>
<td>0.73(0.69,0.79)</td>
</tr>
<tr>
<td>B</td>
<td>0.86(0.73,0.98)</td>
<td>0.74(0.69,0.78)</td>
</tr>
<tr>
<td>C</td>
<td>0.85(0.71,0.99)</td>
<td>0.73(0.68,0.78)</td>
</tr>
<tr>
<td>D</td>
<td>0.84(0.71,0.97)</td>
<td>0.72(0.67,0.77)</td>
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Estimates of ($\hat{\chi}$ and $\chi$) for the four sites with the threshold set at the respective 80% quantile estimation threshold.
Possible covariates

- The marginal behaviour of the **height of waves** and the **speed of winds** is influenced by:
  - Land shadow
  - Storm direction
- Can consider storm direction to be a covariate
- Is the dependence structure, therefore influenced by the same covariates?
Site locations
Asymptotic independence case

Introduce storm direction \( (\theta) \) as a covariate by using a Fourier series:

\[
\bar{\chi}(\theta) = 2 \min\{\eta_+(\theta), 1\} - 1, \quad \text{where}
\]

\[
\log[\eta_+(\theta)] = \alpha_0 + \sum_{i=1}^{d} \alpha_{1i} \cos(\alpha_{2i} + i\theta), \quad \theta \in [0, 360^\circ).
\]
Motivation
Extremal dependence
Covariate modelling
Concluding Remarks

\( (\text{NSLP vs } H_s) : \bar{\chi}(\theta) \)

\( \bar{\chi}(\theta) \) for Site A

\( \bar{\chi}(\theta) \) for Site B

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$(NSLP \text{ vs } H_s) : \tilde{\chi}(\theta)$

$\tilde{\chi}(\theta)$ for Site C

$\tilde{\chi}(\theta)$ for Site D

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Asymptotic dependence case

Introduce storm direction ($\theta$) as a covariate by using a Fourier series:

$$\logit[\chi(\theta)] = \beta_0 + \sum_{i=1}^{d} \beta_i \cos[\beta_{2i} + i\theta], \text{ for } \theta \in [0, 360^\circ)$$
(WS vs $H_s$) : $\chi(\theta)$

$\chi(\theta)$ for Site A

$\chi(\theta)$ for Site B

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\( (WS \ vs \ H_s) : \chi(\theta) \)

\( \chi(\theta) \) for Site C

\( \chi(\theta) \) for Site D

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Concluding Remarks

- Need to think carefully about pooling of dependent samples, and dependence between extreme values. Intuition can be misleading.
- Wind speed and wave height are asymptotically dependent.
- NSLP and wave height are asymptotically independent.
- Extremal dependence varies with storm direction are vital for reliable estimation of joint design values for winds, waves and currents.
- Empirical modelling of $\chi$ and $\bar{\chi}$ may guide the choice of covariates for joint extreme value modelling.