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## DRAFT: ESTIMATING EXTREME WAVES IN THE GULF OF MEXICO USING A SIMPLE SPATIAL EXTREMES MODEL

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### ABSTRACT

We seek to characterize the behavior of extreme waves in the Gulf of Mexico, using a 109 year-long wave hindcast (GOMOS). The largest waves in this region are driven by strong winds from hurricanes. Design of offshore production systems requires the estimation of extreme metocean conditions corresponding to return periods from 1 year to 10,000 years and beyond. For extrapolation to long return periods, estimation using data for around 100 years from a single location will incur large uncertainties. Approaches such as spatial pooling, cyclone track-shifting and explicit track modeling have been proposed to alleviate this problem. The underlying problem in spatial pooling is the aggregation of dependent data and hence underestimation of uncertainty using naïve analysis; techniques such as block-bootstrapping can be used to inflate uncertainties to more realistic levels. The usefulness of cyclone track-shifting or explicit track modeling is dependent on the appropriateness of the physical assumptions underpinning such a model.

In this paper, we utilize a simple spatial statistical model for extreme value estimation of significant wave height under tropical cyclones, known as STM-E, proposed in Wada et al. (2018). The STM-E model was developed to characterize extreme waves offshore Japan, also dominated by tropical cyclones. The method relies on the estimation of two distributions from a sample of data, namely the distribution of spatio-temporal maximum (STM) and the exposure (E). In the current work, we apply STM-E to extreme wave analysis in Gulf of Mexico. The STM-E estimate provides a parsimonious spatially-smooth distribution of extreme waves, with smaller uncertainties per location compared to estimates using data from a single location. We also discuss the estimated characteristics of extreme wave environments in this region.

### 1. INTRODUCTION

Design of offshore production system requires estimation of extreme metocean conditions that account for very long return periods, e.g. 200 years or 10,000 years. Estimation of such low frequency events requires extrapolation far beyond the period of observation. In this paper, we discuss the extreme wave behavior in the Gulf of Mexico based on data from the GOMOS<sup>[1]</sup> wave hindcast. Large waves in this region are known to be dominated by strong winds from hurricanes, i.e. tropical cyclones. For such extrapolation, estimation from 100-year long simulation using a conventional “per location” approach will still have large uncertainty in the extreme estimation result.

Numerous approaches have been proposed to address the issue of limited sample data, such as spatial pooling<sup>[2]</sup>, cyclone track-shifting<sup>[3]</sup> and explicit track modeling<sup>[4]</sup>. The underlying problem of spatial pooling is the dependency of extreme values from neighboring locations and the requirement of techniques such as block-bootstrapping for estimation of uncertainties. The usefulness of cyclone track-shifting and explicit track modeling depends on the validity of the physical assumptions underlying such model.

In this paper, we utilize a simple statistical model for extreme value estimation of significant wave height under tropical cyclones proposed in Wada et al. (2018)<sup>[5]</sup>. The method extracts two distributions from the data, namely the distribution of spatio-temporal maximum (STM) and distribution of exposure (E). The STM-E model was developed to analyze extreme waves offshore Japan, also dominated by tropical cyclones.

The aim of this paper is to apply STM-E to Gulf of Mexico and discuss the results in comparison with competing approaches.

## 2. METHODOLOGY

### (1) Description of STM-E

Here, we describe the methodology of STM-E following Wada et al. (2018)<sup>[5]</sup>. Briefly, the modelling procedure is as follows.

- (a) A certain ocean region is selected for analysis;
- (b) For each tropical cyclone event occurring in the region, the largest value of significant wave height observed anywhere in the region for the period of the cyclone is retained: this is the STM for the cyclone;
- (c) Per location in the region, the largest value of significant wave height observed during the period of the cyclone, expressed as a fraction of STM, is retained: this is exposure E;
- (d) An extreme value model is estimated using the sample of STM;
- (e) Simulation of random occurrences of STM from (d) each combined with a randomly-sampled exposure E, permits estimation of the spatial distribution of significant wave height correspond to return periods of arbitrary length;
- (f) Diagnostic tools are used to confirm the consistency of simulations (e) under the model with historical cyclone characteristics.

### (2) GOMOS08 data set

GOMOS is a comprehensive metocean study for the Gulf of Mexico made by Oceanweather<sup>[1]</sup>. The wave data is based on 3rd generation wave model with 1/16th degree grid (7km) resolution with analysis of all significant storm events. "GOMOS08" includes wave data from 1900 to 2009, holding 348 tropical events (hurricanes) for the period.

## 3. VALIDATION OF ASSUMPTIONS

Here, we validate the applicability of STM-E model in Gulf of Mexico by testing the underlying assumptions of STM-E. Further details of the validation tests is given in Wada et al. (2018)<sup>[5]</sup>. Summarizing findings, STM-E assumptions appear reasonable.

### (1) Absence of linear time trend

Fig. 1 shows the time series of extracted STM values, having set a threshold of 8 [m] for extracting the STM. A permutation test was conducted by randomly permuting the times of occurrence of extreme values, then fitting a linear regression to STM value in time. The corresponding P-value for the original time trend was small, suggesting there is no temporal trend in STM (Fig. 2).

### (2) Absence of linear spatial trend

Fig. 3 shows the location of extracted STM occurrence in GOMOS. A permutation test was conducted by randomly permuting locations of occurrences and estimating the size of the greatest linear spatial trend over all directions. The P-value of the original linear trend was small, suggesting there is no spatial trend in the occurrence of STM (Fig. 4).

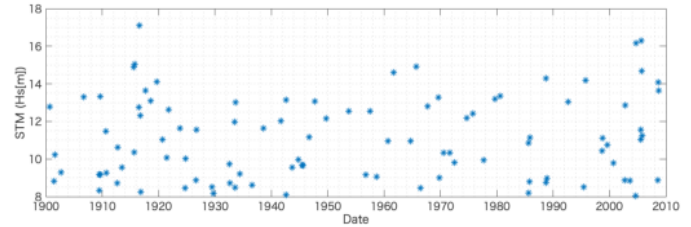


FIG 1. Time series of STM over 8m (1900-2008)

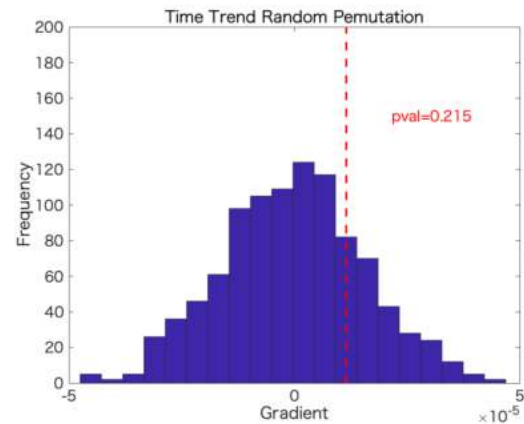


FIG 2. Result of random permutation test for temporal trend. Red dashed line indicates value from original data.

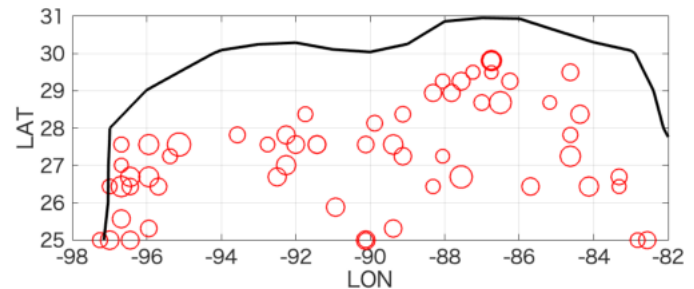


FIG 3. Map of location of STM occurrence. The marker size corresponds to the magnitude of STM.

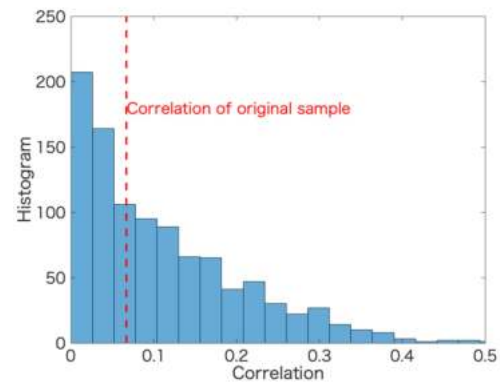


FIG 4. Random permutation test for spatial trend. Red dashed line indicates value from original data

#### 4. RESULTS

Here, we apply the STM-E model to GOMOS data and the result is compared with a traditional location-by-location extreme value analysis.

##### (1) STM model

In the extraction of STM, we consider three threshold values, 8, 10 and 12 [m]. For each threshold, a total of 44, 26 and 12 extreme values were extracted. We apply extreme value analysis for the STM using general Pareto fitting<sup>[6]</sup> and the LWM<sup>[7]</sup> method. This provides posterior predictive estimates for the distribution of shape parameter  $\xi$ , scale parameter  $\sigma$  and return values using Bayesian inference.

Fig. 5 shows the LWM estimation results for threshold 8[m]. The estimated posterior distribution for  $\xi$  has a mode around -0.5 but extends beyond -0.2. The trend is similar for thresholds 10 and 12 [m]. A most probable value of  $\xi$  around -0.5 is considerably smaller than typically found in previous location-by-location analysis in the Gulf of Mexico<sup>[8]</sup>.

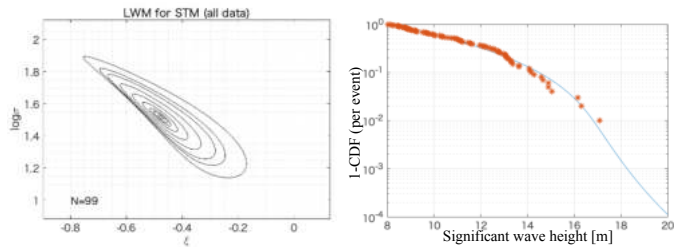
##### (2) Exposure model

In Fig. 6, the exposure calculated from 60 TC with the largest and the smallest STM values are presented. We can see that exposure from the large STM has TC like tracks, as can be seen in Wada et al. (2018)<sup>[5]</sup>. However, the exposure from small STM seems to be scattered, not following the spatial pattern of a tropical cyclone. Therefore, we apply threshold of 8[m] for the exposure model as well.

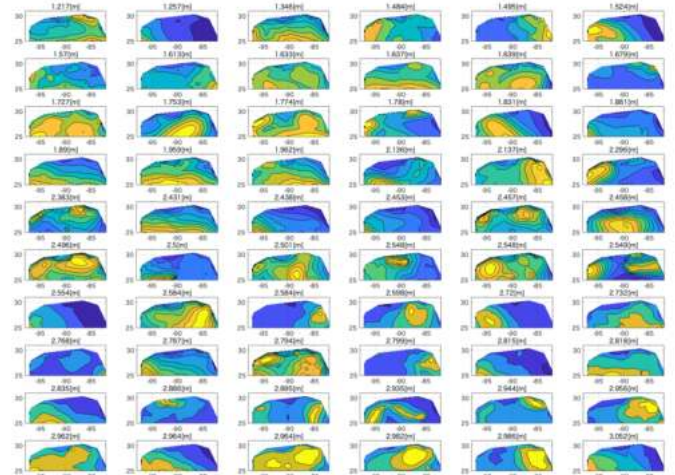
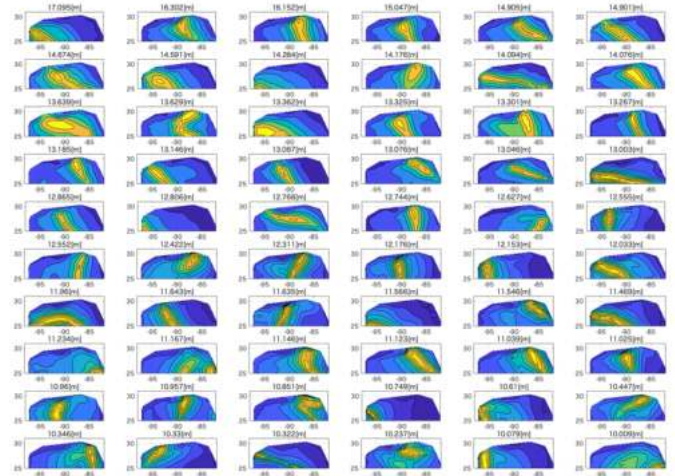
##### (3) STM-E

Finally, the STM-E model is obtained by combining the STM and Exposure models. In Fig. 7, estimates of the 100-year return period value are shown, for thresholds of 8 [m] and 10 [m].

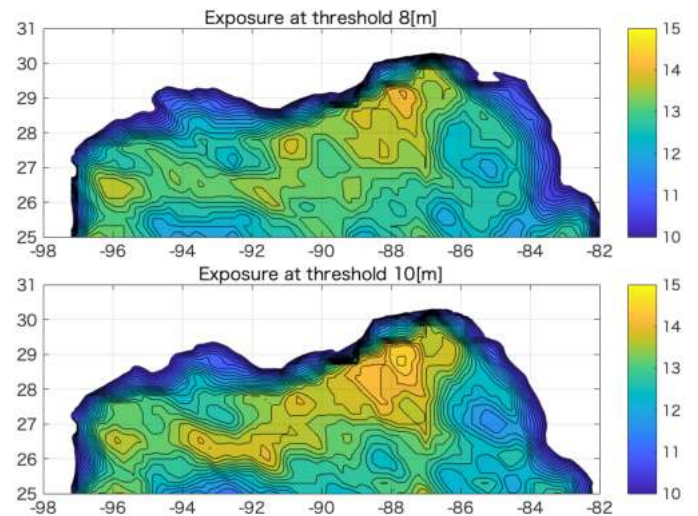
As can be seen in the figure, the two estimates share similar spatial characteristics. For example, large 100-year values occur at the northeast area of the GOM. Estimates differ, but typically by around 1 [m] only. The 100-year values are calculated from the posterior predictive distributions of parameters, allowing uncertainty bands to be calculated easily.



**FIG 5. STM analysis (Threshold=8 [m]). The left-hand plot shows contours of the joint posterior distribution of model parameters. The right-hand plot shows the observed and posterior predictive tails for STM**



**FIG 6. Map of Exposure distribution from TCs with Above) largest 60 STM, Below) smallest 60 STM.**



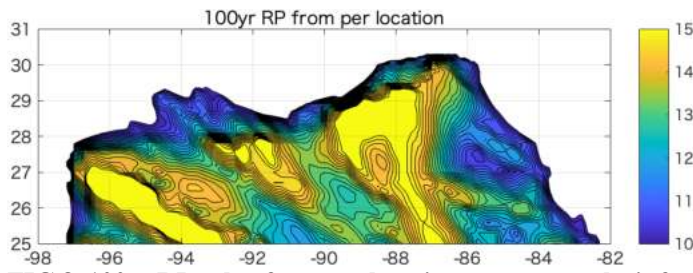
**FIG 7. 100-year return values from STM-E for thresholds of 8 and 10 [m]**



## 5. DISCUSSION

(1) Comparison with “per location” return value estimates  
 100-year return value were also estimated using location-by-location analysis of the same GOMOS data set. The analysis follows the same procedure as described above for extreme value analysis for STM. Posterior predictive distributions of extreme value parameters per location are obtained, leading to estimates of estimates of posterior predictive distributions for the 100-year return value. Return value estimates are shown in Fig. 8.

The figure shows track-like spatial patterns which coincide with observations of exposure from the few largest STMs depicted in Fig. 6. Although color-saturated in the figure, the largest values of return value are larger than 19 [m]. As is well-known, per location estimates are strongly affected by the few very strong tropical cyclones approaching the location by chance.

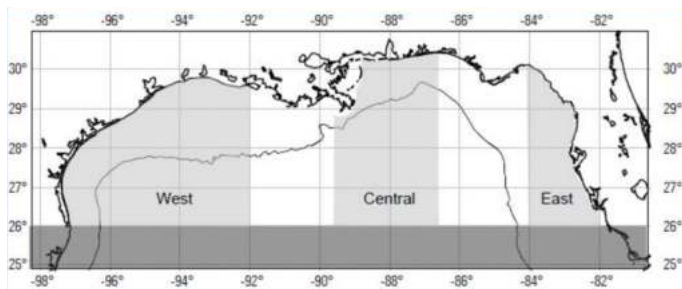


**FIG 8. 100yr RP value from per location extreme analysis for thresholds of 8 [m]**

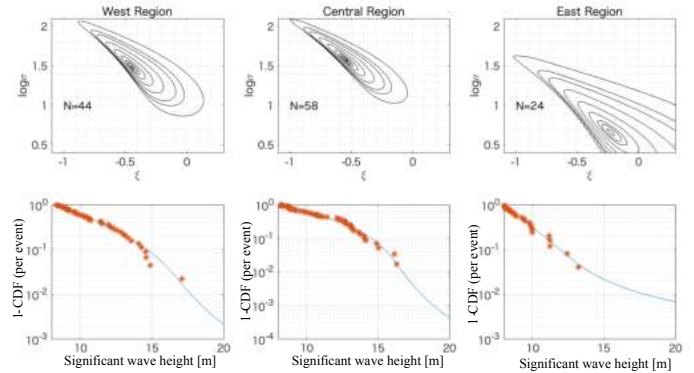
### (2) Spatial characteristics

Guidance for extreme wave estimation in the Gulf of Mexico is provided in API RP 2MET<sup>[9]</sup>. Characteristics of extreme waves are discussed for three regions: West, Central and East (Fig. 9). Here, we discuss how the results of STM-E compare for the API regions. Fig. 10 shows the LWM result for STM in the three regions. The East region has a lower number of occurrences of STM over 8m. Uncertainties are large and the extreme model is not well fitted. Results in the West and Central regions are similar to each other.

Revisiting Fig. 7, there is evidence of change in variance in the latitude. For example, Central regions shows higher values near 29N.



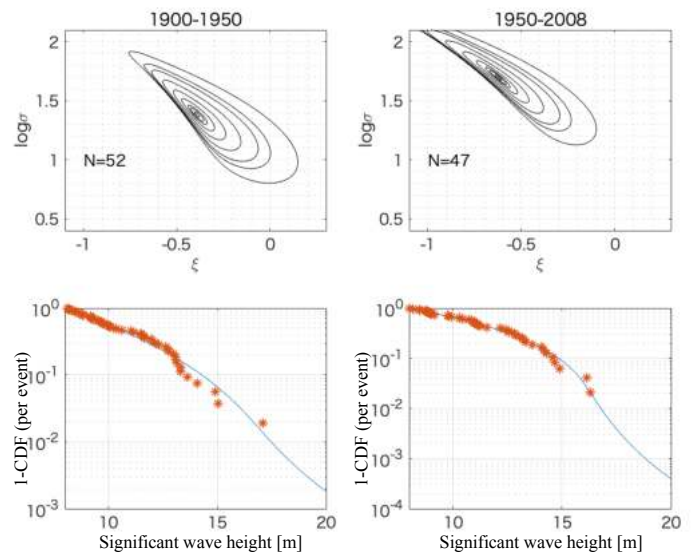
**FIG 9. Regions indicated in API RP 2MET<sup>[9]</sup>**



**FIG 10. LWM from regions indicated in API RP 2MET. See Fig. 5 for further description**

### (3) Temporal characteristics

There is some speculation concerning the relative quality of observations of hurricane characteristics before 1950, and hence of the quality of hindcasts based on those observations. Here, we partition the data into 1900-1950 and 1951-2009 periods and examine whether extreme characteristics change between these periods. As seen in Fig. 11, LWM analysis for the two periods suggests similar behavior, especially in the posterior predictive extreme value distribution. The estimated value of  $\xi$  in the later period is particularly low.



**FIG 11. LWM from 1900-1950 and 1951-2009. See Fig. 5 for further description**

### (4) Low shape parameter

We find that the most probable posterior estimates for shape parameter  $\xi$  are approximately -0.5 for many of the analyses reported in this work and appear relatively stable with respect to threshold choice. These estimates are considerably lower than those quoted in previous work such as Jonathan and Ewans

(2007)<sup>[8]</sup>. Data from a single location often contains many moderate values combined with just a few larger values corresponding to close passage of a hurricane event. Previous estimates of shape parameter from location-by-location analysis may well yield long tails and correspondingly large values of shape parameter for this reason. Conversely, the STM analysis is based on storm peak hurricane values from across the Gulf of Mexico. The shape parameter estimate from STM therefore reflects the tail decay of the spatial maximum of storm peak significant wave height observed anywhere in the region. This is a preliminary application of the STM-E method to a hurricane-dominated region. Nevertheless, it suggests that the relationship between shape parameter estimates from STM and location-by-location analysis deserves further analysis.

#### **ACKNOWLEDGMENTS**

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