Dynamic localization in quantum dots: analytical theory

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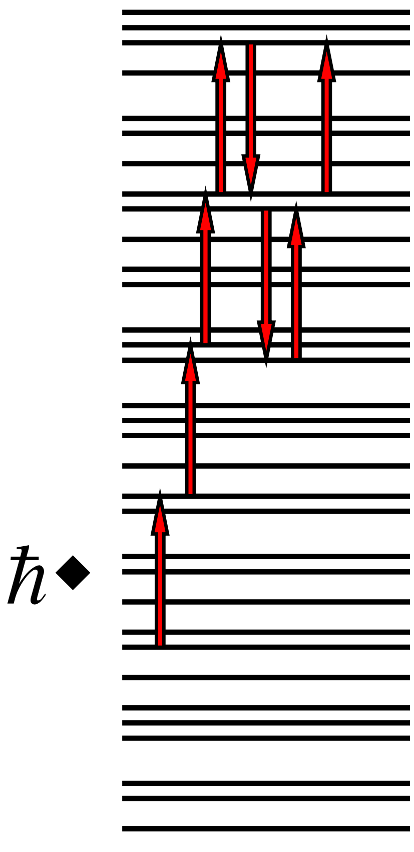
What everybody knows…



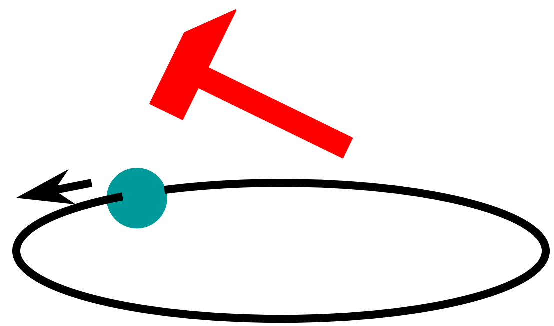
- $\hat{H} = \hat{H}_0 + \hat{V} \cos \omega t$
- (Quasi)continuous spectrum
- Absorption and emission of quanta ħ◆ random walk up and down
- Diffusive evolution of the electron distribution function

What some people know…

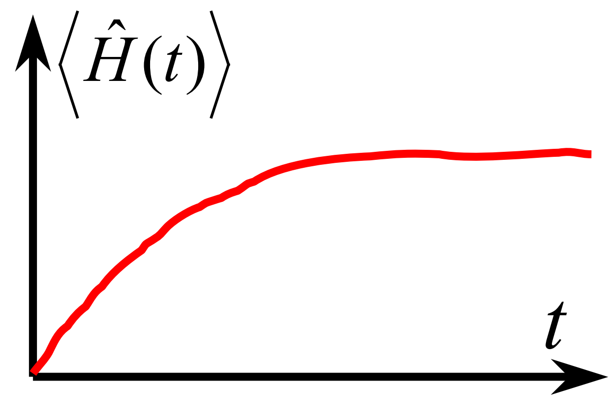
Kicked rotor:

$$\hat{H}(t) = -\frac{\partial^2}{\partial \theta^2} + V(\theta)\sum_{n=-\infty}^{\infty}\delta(t-nT)$$

$$\psi_m^{(0)}(\theta) = e^{im\theta}, \quad E_m^{(0)} = m^2$$



Dynamic localization in the energy space:
after some time the rotor stops absorbing



(G. Casati, B. V. Chirikov, J. Ford, and F. M. Izrailev, 1979)

Historical developments

1. Quantum interference – analogous to the Anderson localization (Fishman, Grempel, and Prange, 1982)
2. Incommensurate periods T_1, T_2, T_3 – 3D localization (Casati, Guarneri, Shepelyansky, 1989)
3. Particle in a box: just $\psi(0) = \psi(2\pi) = 0$ instead of the periodic $\psi(0) = \psi(2\pi)$ – no localization (Hu, Li, Liu, Gu, 1999)
4. Mapping to a quasi-*1d* σ-model (Altland, Zirnbauer, 1996)

What do these observations mean and how general are they?

Spatial localization

Quantum correction to the diffusion coefficient of electrons in disorder

$$D - D_0 \sim -\frac{D_0}{\nu}\int_0^{1/l}\frac{d^d\vec{k}}{D_0k^2 + 1/t_\varphi}$$

mean free path

density of states

dephasing time

Change variables $D_0k^2 = 1/t$:

$$D - D_0 \sim -\frac{1}{\nu}\int_\tau^{t_\varphi}\frac{D_0dt}{(D_0t)^{d/2}}$$

Localization: $d = 1: \quad L_{loc} \sim \nu D_0 \sim l$

$d = 2: \quad L_{loc} \sim l\exp(\nu D_0)$ (?)

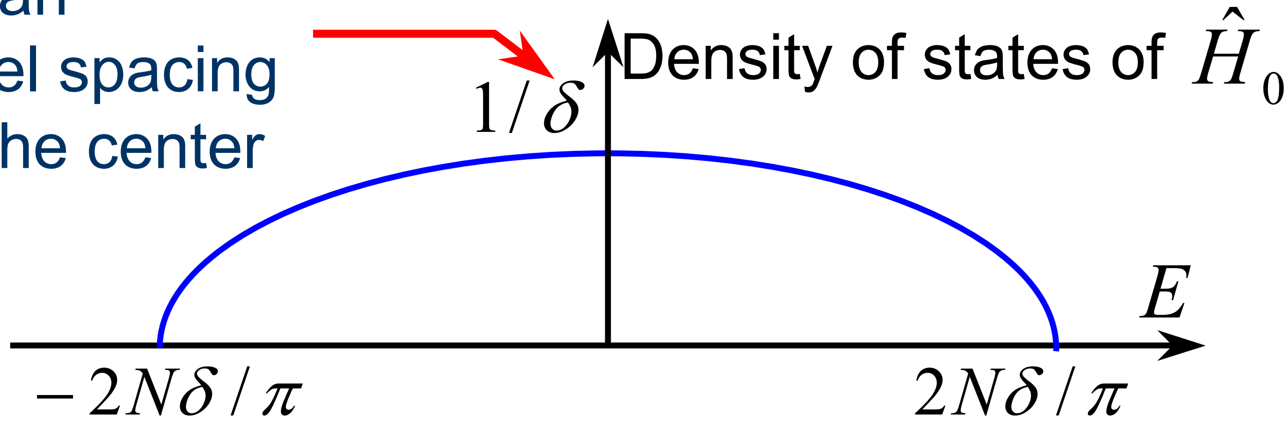
$d \geq 3:$ no localization in weak disorder

Random matrix theory

$$\hat{H}(t) = \hat{H}_0 + \hat{V}\phi(t)$$

real symmetric
$N \times N$ Gaussian
random matrices
with statistically independent elements

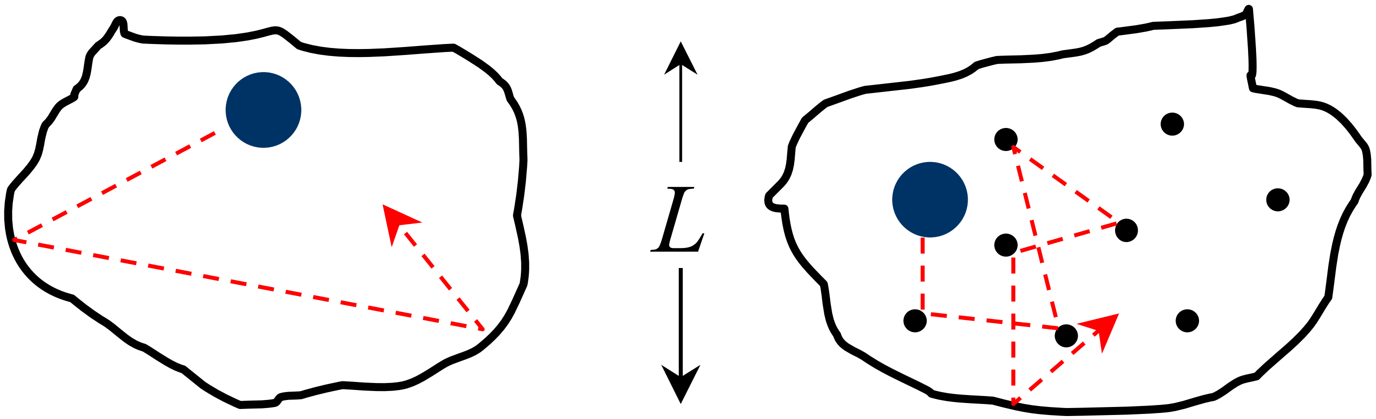
mean
level spacing
at the center



In the end let $N \to \infty$

Chaotic systems

Ballistic systems: Diffusive systems:

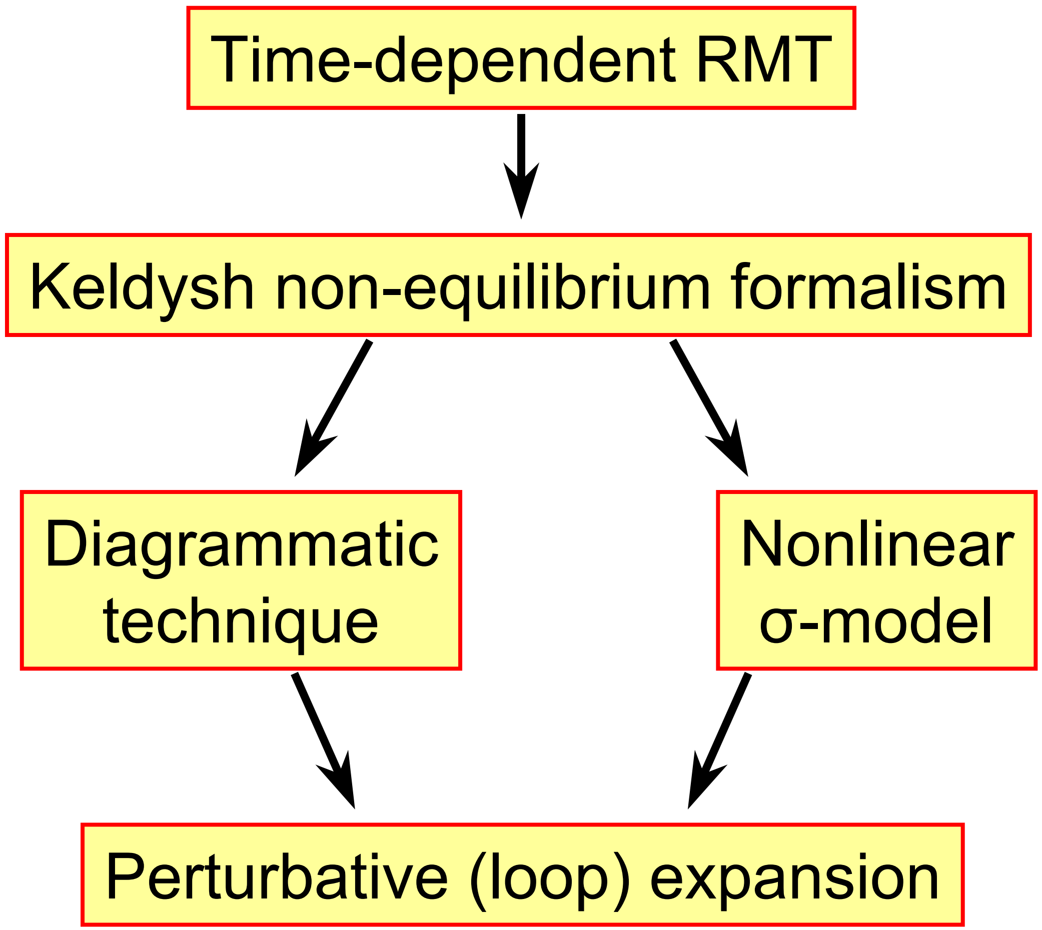


$\tau_{erg} = L / v_F$ ergodic time $\tau_{erg} = L^2 / D$

RMT is valid at low energies:

$E << E_{Th} = \hbar / \tau_{erg}$ (Thouless energy)

Technicalities



Zero order (diffusion)

$$\Gamma \equiv \left\langle V_{ll'}^2 \right\rangle / \delta$$ – one photon absorption rate (measure of perturbation strength)

Long-time, period-averaged dynamics:

$$\left[\frac{\partial}{\partial t} - D \frac{\partial^2}{\partial E^2} \right] f(E,t) = 0$$

time-dependent electron distribution (Wigner variables)

$$D = \Gamma \overline{(d\phi / dt)^2}$$ – energy diffusion coefficient

$$W_0 \equiv \frac{\partial}{\partial t} \int E\, f(E,t)\, dE = \frac{D}{\delta}$$ – energy absorption rate

One-loop correction

$$W(t) = \frac{D}{\delta} + \frac{\Gamma}{\pi}\int_0^t \dot{\phi}(t)\,\dot{\phi}(t-\tau)\,C_{t-\tau/2}(\tau,-\tau)\,d\tau$$

large zero-order

small (?) correction

Cooperon keeps track of the quantum interference:

$$C_t(\tau_1,\tau_2) \equiv \theta(\tau_1-\tau_2)\exp\left[-\int_{\tau_2}^{\tau_1}\frac{\Gamma}{2}[\phi(t+\tau/2)-\phi(t-\tau/2)]^2\,d\tau\right]$$

dephasing rate

Periodic perturbation

$$\phi(t) = \sum_{n=1}^{\infty} A_n \cos(n\omega t - \varphi_n) \qquad W_0 = \frac{\Gamma\omega^2}{2\delta}\sum_n n^2 A_n^2$$

$$C_t(\tau_1,\tau_2) \approx \exp\left[-\Gamma(\tau_1-\tau_2)\sum_{n=1}^{\infty} A_n^2 \sin^2(n\omega t - \varphi_n)\right]$$

If $\varphi_n = n\varphi$ the exponent can vanish at $t_k = \dfrac{\varphi + k\pi}{\omega}$

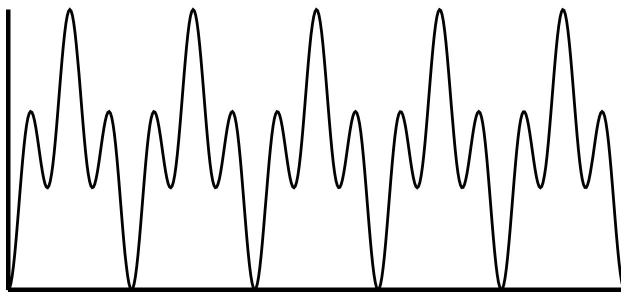
No-dephasing points give a large negative contribution to the integral:

$$W(t) - W_0 \sim -\omega^2\sqrt{\Gamma t}$$

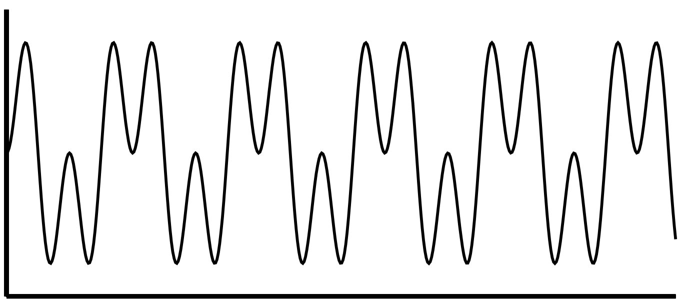
Time-reversal symmetry

$$\varphi_n = n\varphi \;\Leftrightarrow\; \phi(t - t_0) = \phi(-t - t_0)$$

Average dephasing rate versus time:



T-symmetry: yes



T-symmetry: no

Monochromatic perturbation: *T*-symmetry always –
a very special case

Two loops

There is a contribution from diffusons:

$$D_\tau(t_1,t_2) \equiv \theta(t_1 - t_2)\exp\left[-\int_{t_2}^{t_1} \Gamma\,[\phi(t+\tau/2) - \phi(t-\tau/2)]^2\,dt\right]$$

For a periodic perturbation:

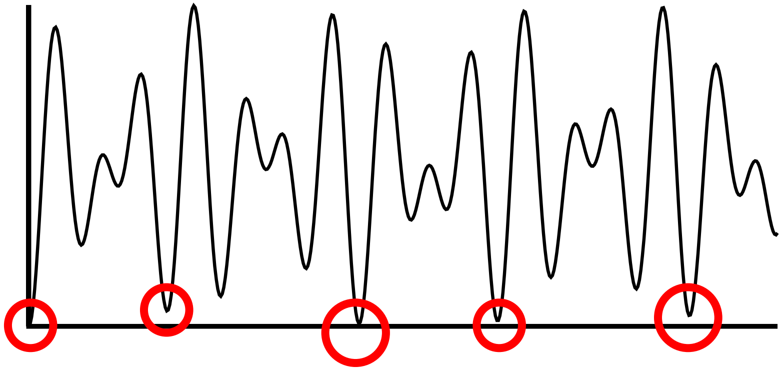
$$D_\tau(t_1,t_2) \approx \exp\left[-2\Gamma(t_1 - t_2)\sum_{n=1}^{\infty} A_n^2 \sin^2 n\omega\tau\right]$$

No-dephasing points are always present, regardless of the time-reversal symmetry…

Incommensurate periods

$$\phi(t) = \sum_{n=1}^{d} A_n \cos(\omega_n t - \varphi_n) \qquad W_0 = \frac{\Gamma}{2\delta} \sum_n \omega_n^2 A_n^2$$

dephasing rate:



Phase relationships do not matter that much

Almost-no-dephasing points contribute:

$$W(t) - W_0 \sim -\omega^2 \int_{1/\Gamma}^{t} \frac{\Gamma\, dt_1}{\sqrt{(\Gamma t_1)^d}}$$

– large for d<3

A glance at the reality

GaAs dot:

- size $L \sim 1\ \mu\mathrm{m}$
- mean level spacing $\delta \sim 1\ \mu\mathrm{eV}$
- Thouless energy $E_{Th} \sim 100 - 1000\ \mu\mathrm{eV}$
- dephasing time $t_{\varphi} \sim 1\,\mathrm{ns}$

Microwave field:

- *V*~ several μeV (field ~ several V/m)
- $\hbar\omega \sim 10 - 100\ \mu\mathrm{eV}\ (\sim 10^{10}\ \mathrm{Hz})$

Dynamic localization:

- $t_{loc} \sim 10\,\mathrm{ns},\ E_{loc} \sim \sqrt{D t_{loc}} \sim 100 - 1000\ \mu\mathrm{eV} \sim 1 - 10\,\mathrm{K}$

Conclusions…

1. A quantum-mechanical system under a time-dependent perturbation may be subject to dynamic localization in energy space.

2. It depends both on the model for the unperturbed system and the perturbation.

3. We have studied one-loop correction to the usual Fermi-Golden-Rule dissipation rate for a chaotic system described by RMT

…conclusions

4. For a perturbation with *d* incommensurate frequencies the correction can grow arbitrarily with time if *d=1,2* (analogously to spatial localization in *d*-dimensional disorder)
5. For commensurate frequencies phase relationships matter:
6. Time-reversal symmetry: the "dimensionality" is effectively lowered
7. No time-reversal: the correction is small