# Theory of Half Metal-Superconductor Heterostructures

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- Half Metals
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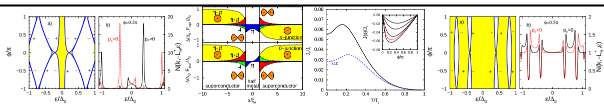


Lancaster, January 2003

# Introduction

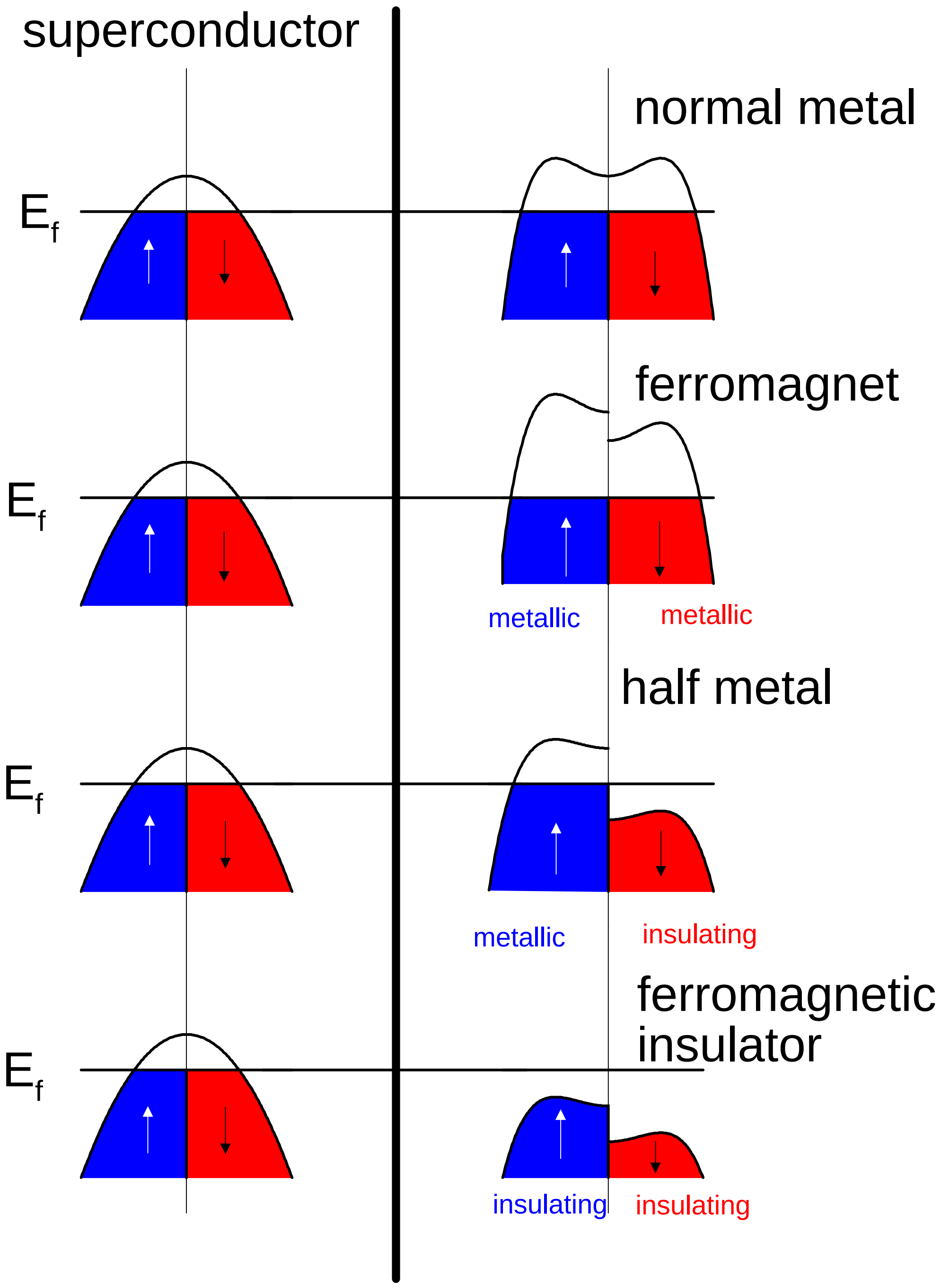
- Heterostructures including spin polarized materials are important for the emerging field of spin electronics.
- In ferromagnets exchange splitting $h$ leads to suppression of superconducting proximity effect $\sim \Delta/h$, and decay length $\sim \sqrt{\hbar D/h}$
- But: Superconductor/Ferromagnet heterostructures showed an unusual long-range proximity effect

  M. Giroud it et al., Phys. Rev. B **58**, R11872 (1998); cond-mat/0204140 (2002)

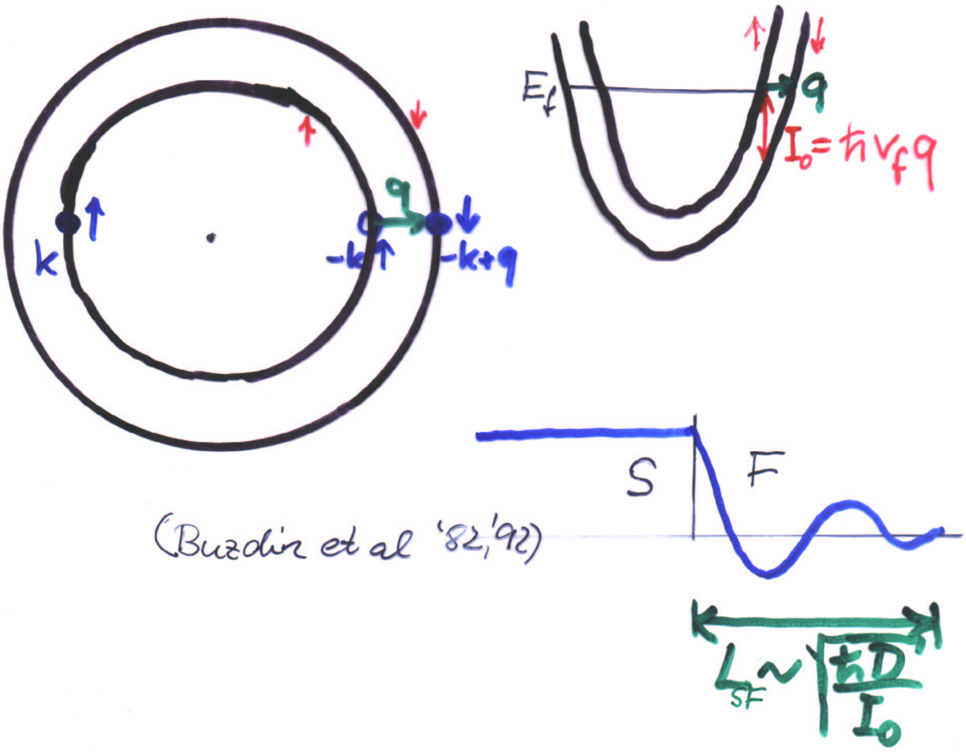
  V.T. Petrashov *et al.*, Phys. Rev. Lett. **83**, 3281 (1999);

  see however J. Aumentado and V. Chandrasekhar, Phys. Rev. B **64**, 054505 (2001).
- Question: What is the nature of the proximity effect at interfaces between superconductors and strongly spin polarized materials?
- for applications one would like to have ideally 100 % spin polarization → half metals.

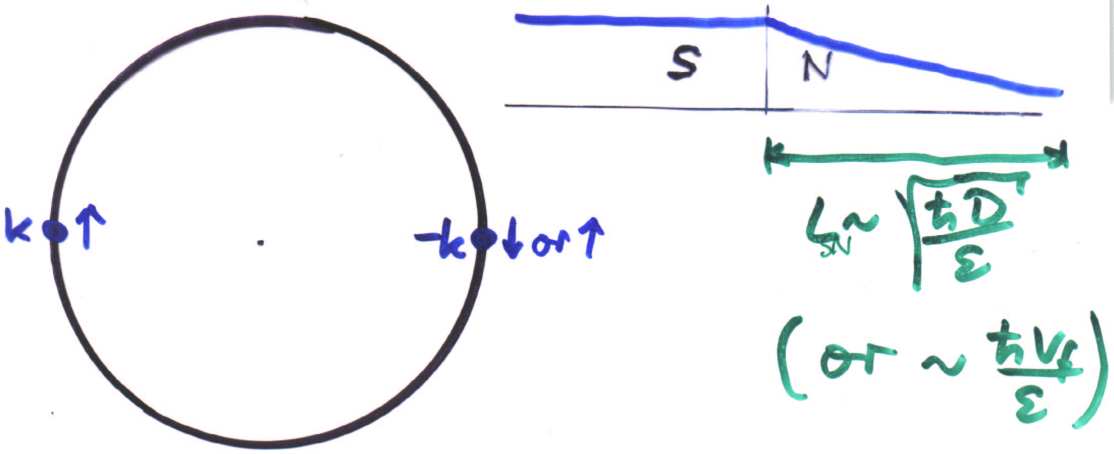
# Introduction



# Introduction: S/F and S/N junctions



(Buzdin et al '82,'92)

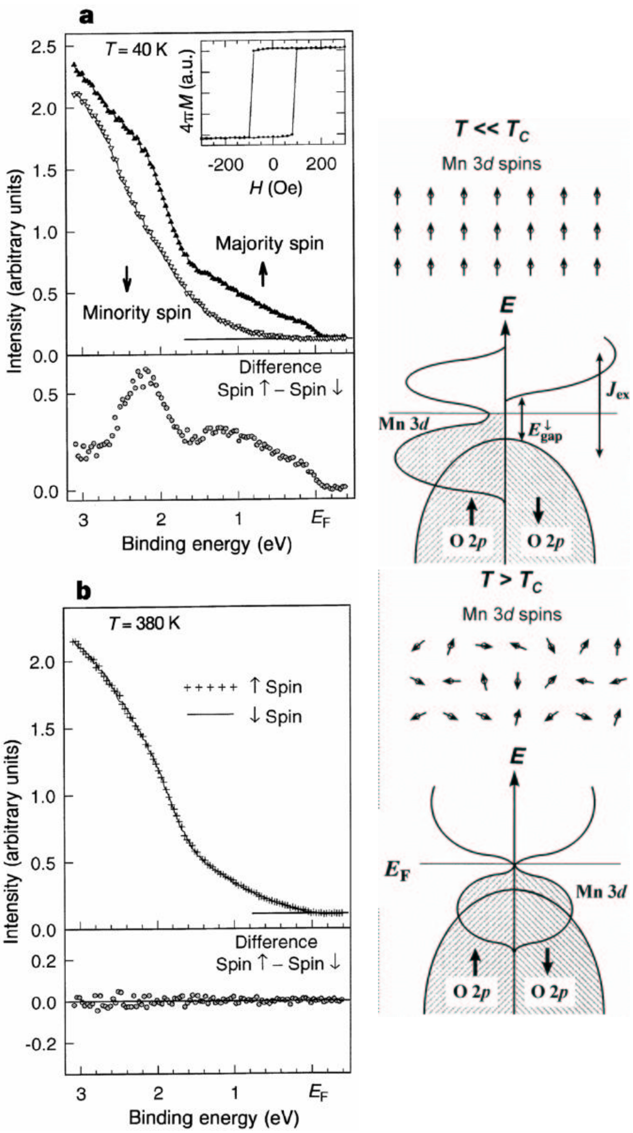


# Half Metals: $La_{0.7}Sr_{0.3}MnO_3$

J.-H. Park, *et al.*,

"*Direct evidence for a half-metallic ferromagnet*",

Nature **392** 794



Spin-resolved photoemission spectra of a thin film of $La_{0.7}Sr_{0.3}MnO_3$.

# Half Metals: $CrO_2$

Y. Ji, *et al.*,

"*Determination of the Spin Polarization of Half-Metallic $CrO_2$ by Point Contact Andreev Reflection*"

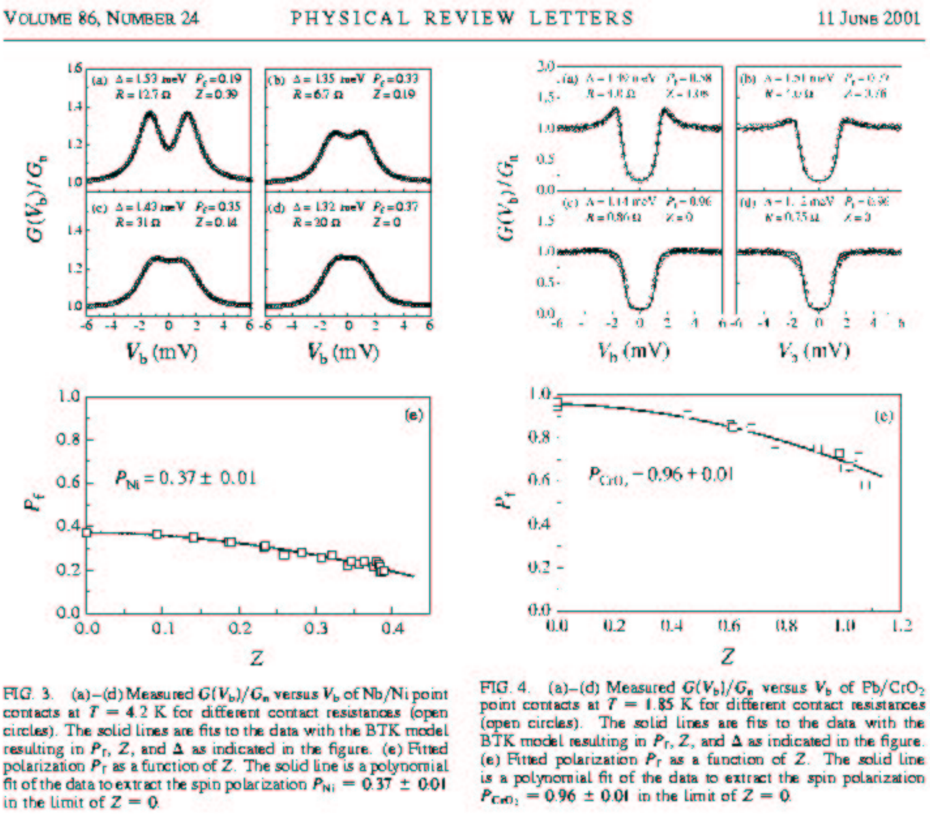


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FIG. 3. (a)–(d) Measured $G(V_b)/G_n$ versus $V_b$ of Nb/Ni point contacts at $T = 4.2$ K for different contact resistances (open circles). The solid lines are fits to the data with the BTK model resulting in $P_f$, $Z$, and $\Delta$ as indicated in the figure. (e) Fitted polarization $P_f$ as a function of $Z$. The solid line is a polynomial fit of the data to extract the spin polarization $P_{Ni} = 0.37 \pm 0.01$ in the limit of $Z = 0$.

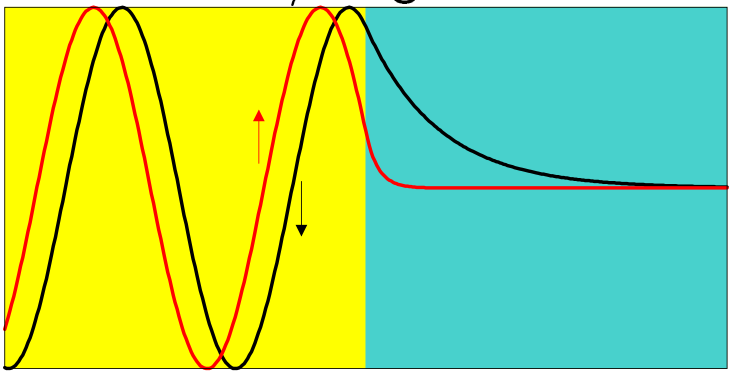
FIG. 4. (a)–(d) Measured $G(V_b)/G_n$ versus $V_b$ of Pb/$CrO_2$ point contacts at $T = 1.85$ K for different contact resistances (open circles). The solid lines are fits to the data with the BTK model resulting in $P_f$, $Z$, and $\Delta$ as indicated in the figure. (e) Fitted polarization $P_f$ as a function of $Z$. The solid line is a polynomial fit of the data to extract the spin polarization $P_{CrO_2} = 0.96 \pm 0.01$ in the limit of $Z = 0$.

Nb/Ni: $P_f \approx 0.37$

Pb/$CrO_2$: $P_f \approx 0.96$

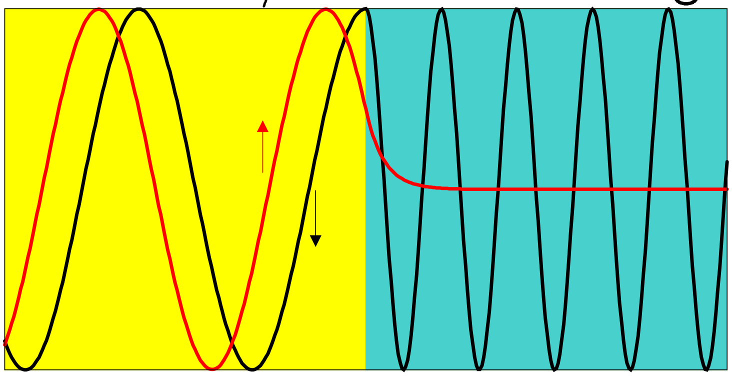
# Spin-mixing effect

Normal Metal/Magnetic Insulator:



T. Tokuyasu, J.A. Sauls, and D. Rainer, Phys. Rev. B **38**, 8823 (1988)

Normal Metal/Half-metallic Magnet:



relative scattering phase between $\Psi_\uparrow$ and $\Psi_\downarrow$

spin-mixing angle: $\phi_\uparrow - \phi_\downarrow \equiv \theta \neq 0$.

# Singlet-Triplet Mixing

Consider reflection of two quasiparticles with spin up and down with respect to the quantization axis of the half metal:

$$\begin{aligned} |\uparrow\rangle_{-k} &= e^{+i\theta/2}|\uparrow\rangle_k \\ |\downarrow\rangle_{-k} &= e^{-i\theta/2}|\downarrow\rangle_k \end{aligned}$$

How does a singlet Cooper pair transform?

$$|\uparrow\rangle_k|\downarrow\rangle_{-k} - |\downarrow\rangle_k|\uparrow\rangle_{-k}$$

$$\downarrow$$

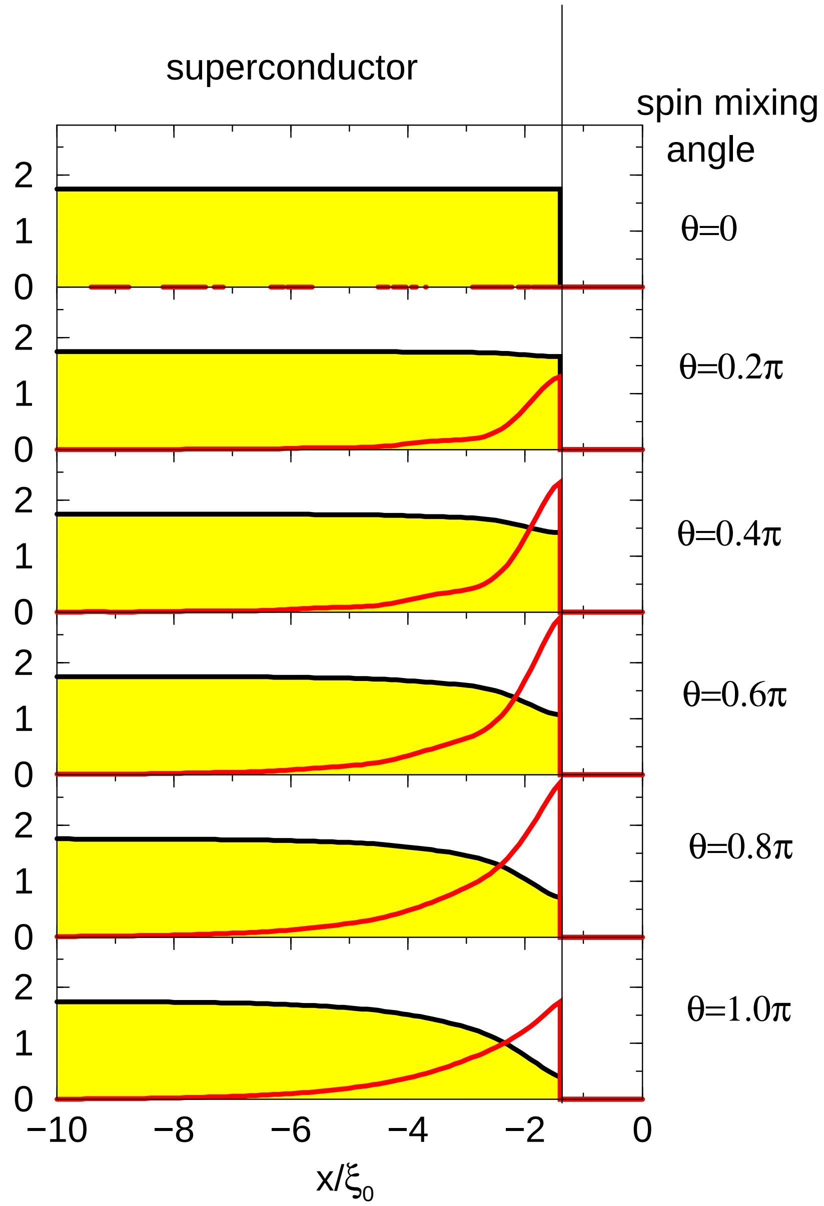
$$e^{i\theta}|\uparrow\rangle_k|\downarrow\rangle_{-k} - e^{-i\theta}|\downarrow\rangle_k|\uparrow\rangle_{-k}$$

$$= \cos\theta\left(|\uparrow\rangle_k|\downarrow\rangle_{-k} - |\downarrow\rangle_k|\uparrow\rangle_{-k}\right)$$
$$+ i\sin\theta\left(|\uparrow\rangle_k|\downarrow\rangle_{-k} + |\downarrow\rangle_k|\uparrow\rangle_{-k}\right)$$

Pairing states near S/HM interface
are singlet-triplet mixtures.

# Spin-mixing effect

## Superconductor/Magnetic Insulator:



- -Triplet correlations in the superconductor
- -Suppression of the singlet order parameter

# Spin flip scattering and indirect proximity effect

- No singlet ($f_{\uparrow\downarrow-\downarrow\uparrow}$) correlations in half metal possible. Only triplet correlations of form $f_{\uparrow\uparrow}$.

- Two transfer channels from superconductor to half metal: $\tau_{\uparrow\uparrow}$ and $\tau_{\downarrow\uparrow}$.

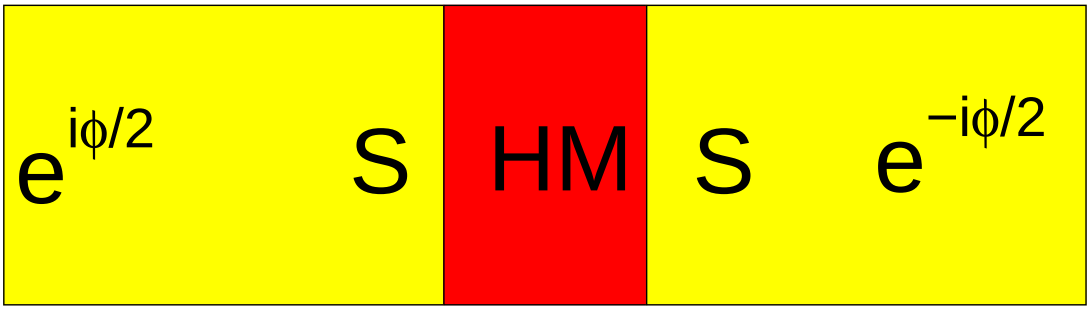
Note: even with spin flip scattering, $\tau_{\downarrow\uparrow}$, the singlet component does not induce $f_{\uparrow\uparrow}$ correlations in the HM: $f_{\uparrow\downarrow-\downarrow\uparrow}$ and $f_{\uparrow\uparrow}$ have different orbital symmetry.

Main source for proximity effect in half metals: spin mixing effect, leading to triplet $f_{\uparrow\downarrow+\downarrow\uparrow}$ correlations in superconductor → indirect proximity effect

$$\begin{tikzcd}
f_{\uparrow\downarrow-\downarrow\uparrow} \arrow[r] \arrow[dr] & \tau_{\uparrow\uparrow}, \tau_{\downarrow\uparrow} \arrow[r] & 0 \\
\text{SC} & \text{spin mixing} \arrow[dl] & \text{HM} \\
f_{\uparrow\downarrow+\downarrow\uparrow} \arrow[r] & \tau_{\uparrow\uparrow}, \tau_{\downarrow\uparrow} \arrow[r] & f_{\uparrow\uparrow}
\end{tikzcd}$$

# Josephson effect

Is there a Josephson effect in an S/HM/S structure ?



Consider a clean half metal piece of length $L_{HM}$ with Fermi velocity $v_F$ and minority gap $E_g$:

a) $L_{HM} < \frac{\hbar v_F}{E_g}$:
direct Josephson effect via singlet component

b) $L_{HM} \gg \frac{\hbar v_F}{E_g}$:
no direct Josephson effect. However:

Indirect Josephson effect via triplet correlations

# Theory

We use a Green's functions technique within the framework of Quasiclassical Theory of Superconductivity.

## Internal degrees of freedom:

matrix structure of the Nambu-Gor'kov propagator:

$$\hat{g} = \begin{pmatrix} g & f \\ \tilde{f} & \tilde{g} \end{pmatrix}$$

-2x2 spin degree of freedom

-2x2 particle-hole degree of freedom

## External degrees of freedom:

-motion of quasiparticles with Fermi velocity $\mathbf{v}_f$ along trajectories, parameterized by the Fermi momentum $\mathbf{p}_f$.

Transport equation for $\hat{g}(\mathbf{p}_f, \mathbf{R}, \epsilon)$:

(Eilenberger, Larkin and Ovchinnikov, 1968)

$$\left[\epsilon\hat{\tau}_3 - \hat{\Sigma}, \hat{g}\right] + i\mathbf{v}_f \cdot \nabla\hat{g} = 0$$

Normalization condition

(Eilenberger 1968)

$$\hat{g}\;\hat{g} = -\pi^2\hat{1}$$

# Boundary conditions

half space propagator:

$$\hat{g}^0_{out} \quad = \quad \hat{S}\hat{g}^0_{in}\hat{S}^\dagger$$

with *scattering matrix* $\hat{S} = e^{i\frac{\theta}{2}\sigma_z}\hat{1}$.

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$t$ matrix equations:

$$\begin{aligned} \hat{t}_{in} &= \hat{\tau}\underline{\hat{g}}^0_{out}\,\hat{\tau}^\dagger(\hat{1}+\hat{g}^0_{in}\;\hat{t}_{in}) \qquad \hat{t}_{out}=\hat{S}\hat{t}_{in}\hat{S}^\dagger \\ \underline{\hat{t}}_{out} &= \hat{\tau}^\dagger\hat{g}^0_{in}\hat{\tau}(\hat{1}+\underline{\hat{g}}^0_{out}\,\underline{\hat{t}}_{out}) \qquad \underline{\hat{t}}_{in}=\underline{\hat{S}}^\dagger\underline{\hat{t}}_{out}\underline{\hat{S}} \end{aligned}$$

with *transfer matrix* $\hat{\tau}$.

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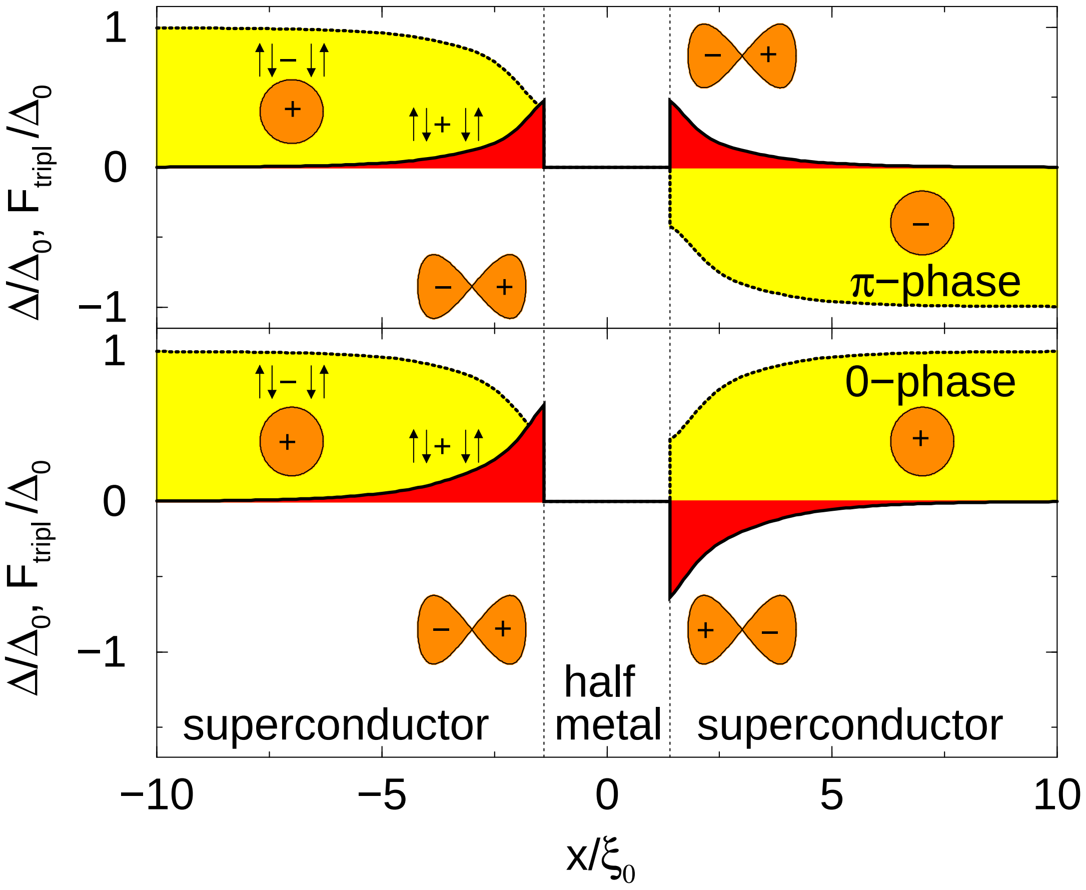
full propagators:

$$\begin{aligned} \hat{g}_{in} &= \hat{g}^0_{in} + \{\hat{g}^0_{in} + i\pi\hat{1}\}\,\hat{t}_{in}\,\{\hat{g}^0_{in} - i\pi\hat{1}\} \\ \hat{g}_{out} &= \hat{g}^0_{out} + \{\hat{g}^0_{out} - i\pi\hat{1}\}\,\hat{t}_{out}\,\{\hat{g}^0_{out} + i\pi\hat{1}\} \end{aligned}$$

Note that the normalization conditions $\hat{g}^2_{in} = -\pi^2\hat{1}$ and $\hat{g}^2_{out} = -\pi^2\hat{1}$ are conserved by our the boundary conditions.

# Triplet pairing correlations

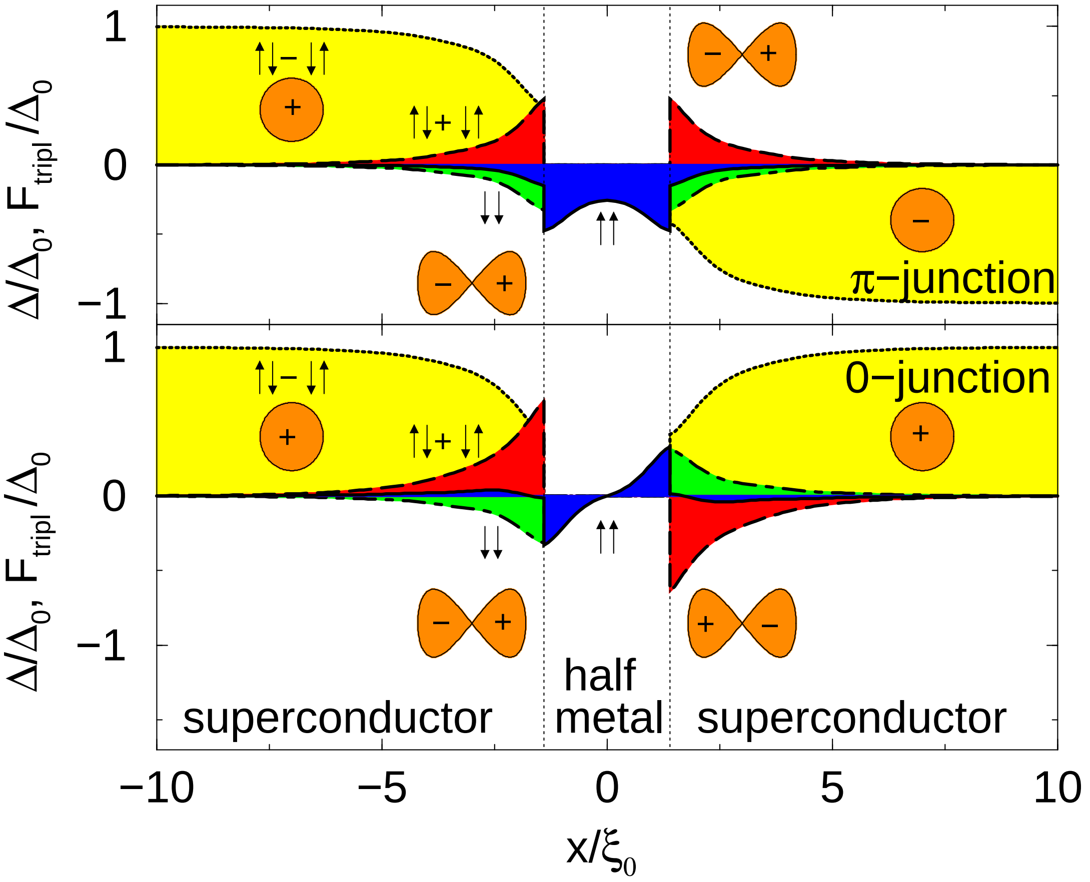


$$\Delta(x) = \lambda \int_{-\epsilon_c}^{\epsilon_c} \frac{d\epsilon}{2\pi i} \, \langle f(\hat{\mathbf{k}}, x, \epsilon) \rangle_{\hat{\mathbf{k}}} \; \tanh\left(\frac{\epsilon}{2T}\right)$$

$$F_{tripl}(x) = \int_{-\epsilon_c}^{\epsilon_c} \frac{d\epsilon}{2\pi i} \, \langle (\hat{\mathbf{k}} \cdot \hat{\mathbf{n}}) f(\hat{\mathbf{k}}, x, \epsilon) \rangle_{\hat{\mathbf{k}}} \; \tanh\left(\frac{\epsilon}{2T}\right)$$

# Triplet pairing correlations

parameters: $\theta = 0.75\pi \cos\Psi$, $\tau_{\uparrow\downarrow}/\tau_{\uparrow\uparrow} = 0.7$, $2\pi\tau_{\uparrow\uparrow} = 1.0 \cos\Psi$



$\pi$-junction has lower free energy and thus is stable.

$$\Delta(x) = \lambda \int_{-\epsilon_c}^{\epsilon_c} \frac{d\epsilon}{2\pi i} \langle f(\hat{\mathbf{k}}, x, \epsilon)\rangle_{\hat{\mathbf{k}}} \tanh\left(\frac{\epsilon}{2T}\right)$$

$$F_{tripl}(x) = \int_{-\epsilon_c}^{\epsilon_c} \frac{d\epsilon}{2\pi i} \langle (\hat{\mathbf{k}} \cdot \hat{\mathbf{n}}) f(\hat{\mathbf{k}}, x, \epsilon)\rangle_{\hat{\mathbf{k}}} \tanh\left(\frac{\epsilon}{2T}\right)$$

# Singlet-triplet current conversion

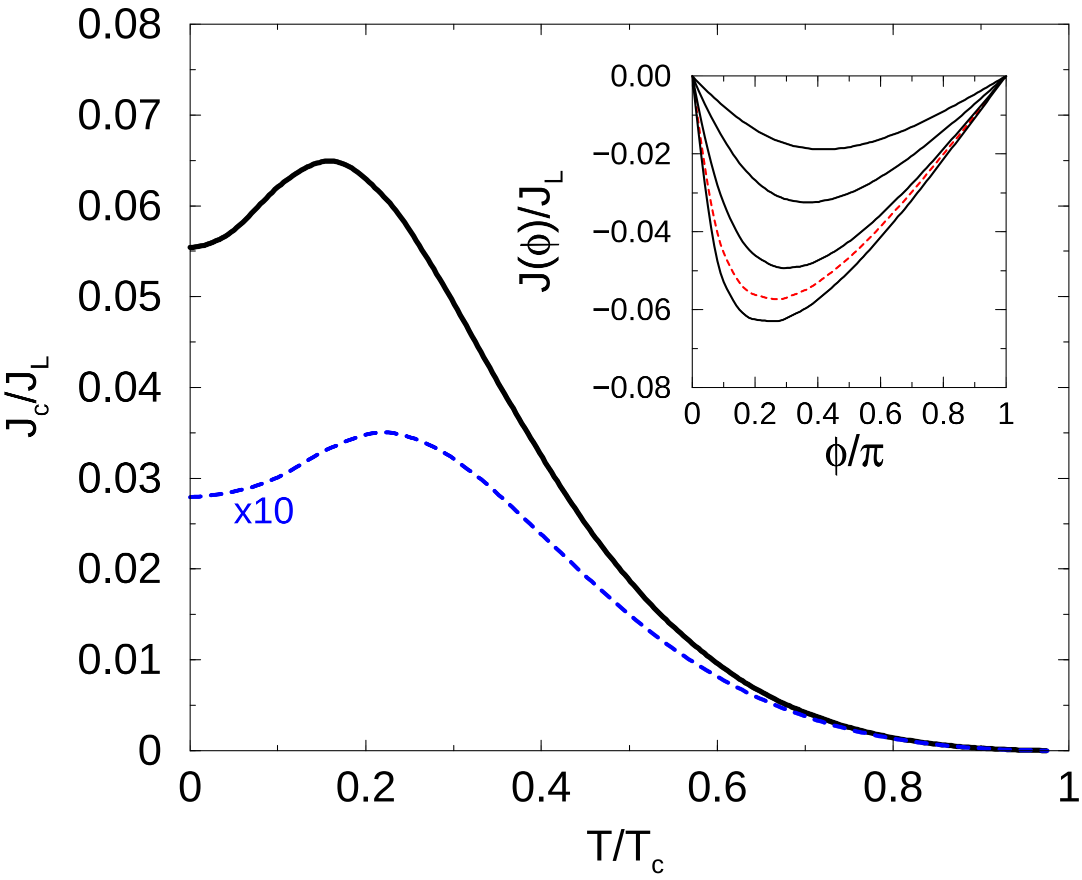
What about self consistency of the order parameter?

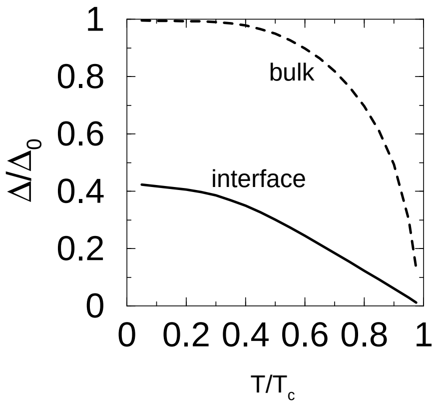
- Superconductor/normal metal interfaces usually well described without necessity of self consistent order parameter near the interface.
  This is true also for SC/FM interfaces if exchange field $h$ is small compared to Fermi energy.
- Near interfaces between superconductors and strong ferromagnets or half metals strong triplet correlations are present.
- Spatial variation of singlet order parameter in accordance with the triplet correlations, which decay into the superconductor.
- Current conversion between singlet and triplet components.

Near interfaces between a SC and a strong FM or HM self consistency is essential to ensure current conservation.

# Critical Josephson Current

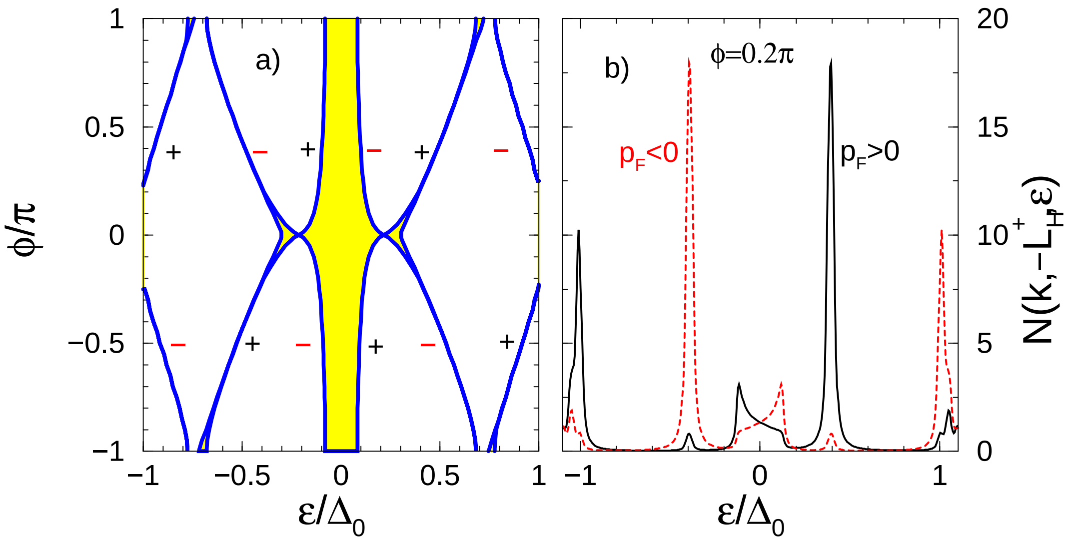
mixing angle: $\theta = 0.75\pi \cos\Psi$



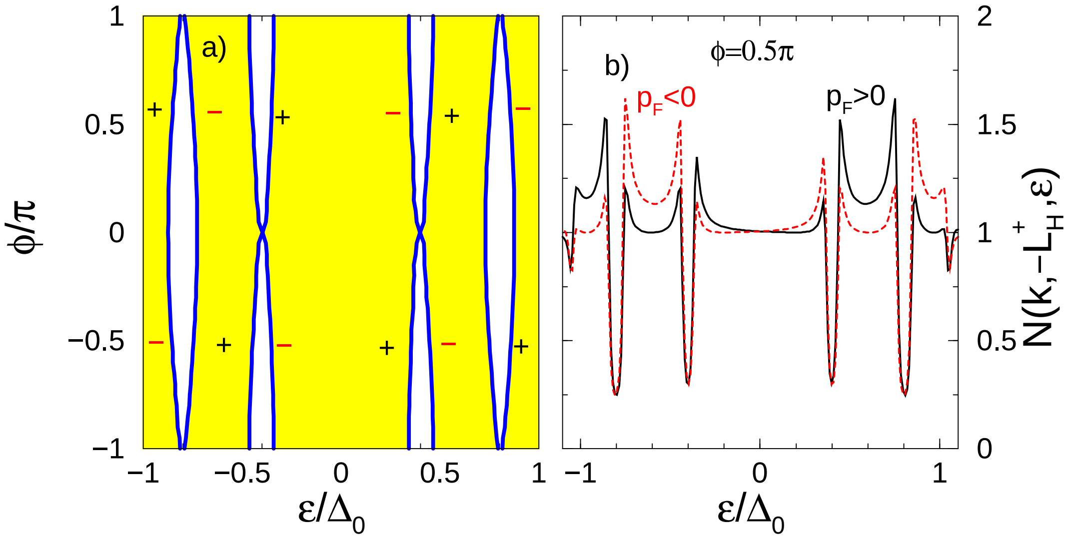


# Quasiparticle density of states

## $\tau_{\uparrow\downarrow}/\tau_{\uparrow\uparrow} = 0.7$



## $\tau_{\uparrow\downarrow}/\tau_{\uparrow\uparrow} = 0.1$



$$J \sim \sum_b \frac{\partial E_b}{\partial \phi} n_f(E_b)$$

# Conclusions

- We have presented a theory for half metal-superconductor heterostructures.
- We propose an indirect mechanism for a Josephson coupling between two singlet superconductors separated by a half-metallic magnet.
- We investigate within the framework of quasiclassical theory quantitatively this indirect Josephson effect.
- We found a low temperature anomaly in the temperature behavior of the critical Josephson current.
- We explain the temperature variation of the Josephson current in terms of the Andreev excitation spectrum.