Mesoscopic NS rings : from persistent current to Josephson current

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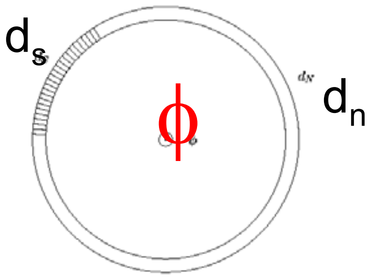


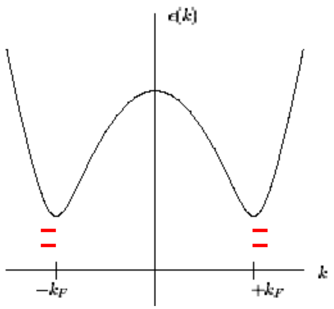
$I(\varphi)$ $\quad \varphi = \phi / \phi_0$ $\quad \xi_0$ coherence length

$d_s = 0$ → persistent current in normal ring

$d_s >> \xi_0$ → Josephson current through SNS junction : $\Delta\chi = 4\pi\, \phi/\phi_0$

$\Phi_0 = h/e$

$h/2e$



Flux dependent spectrum ?
Current ?
Any d_s and d_n

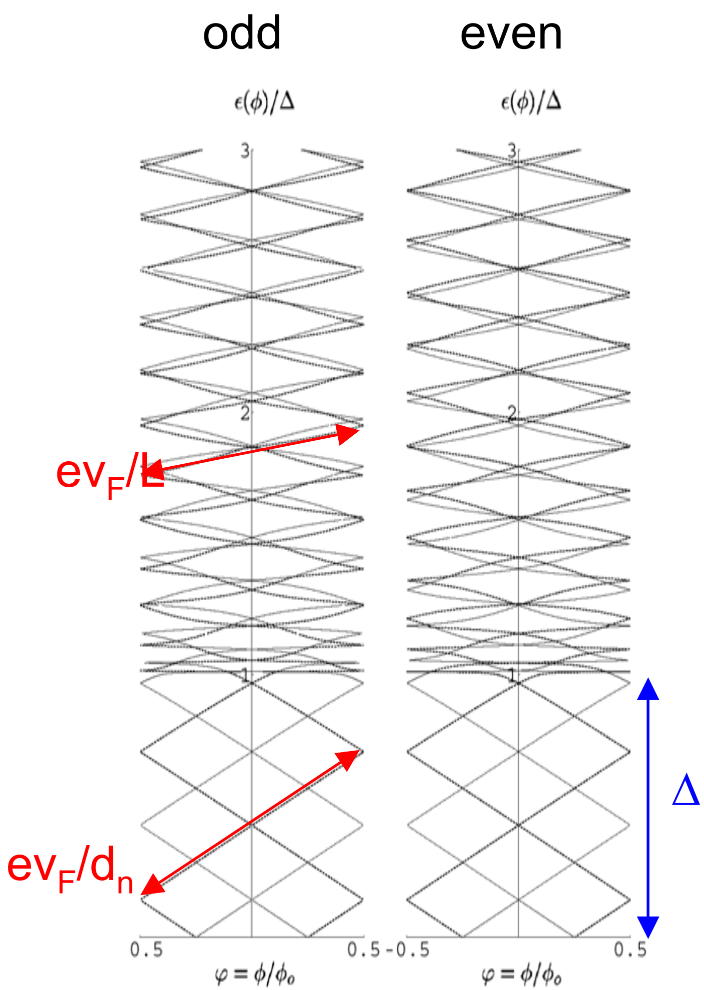
$I = -dE/d\phi$

The current is an equilibrium quantity, but it can be calculated from the excitation spectrum (BdG equations) (Beenakker-Van Houten 91)

$I = \Sigma\, d\varepsilon_n/d\phi$

Büttiker, Klapwijk (85) : $d_n >> \xi_0$, many Andreev levels

Spectrum of a 1D NS ring

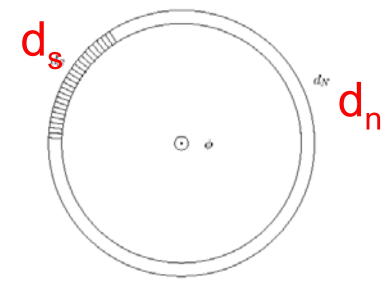


$d_n = 10\ \xi_0$, $d_s = 20\ \xi_0$

1D normal ring :

Interlevel spacing $\delta = h\ v_F / L$

Persistent current $\delta / \phi_0 = e\ v_F / L$



$L = d_n + d_s$

Number of Andreev levels

$\Delta / (ev_F/d_n) = d_n / \xi_0$

B.K. : d_n large (many Andreev levels), vary d_s

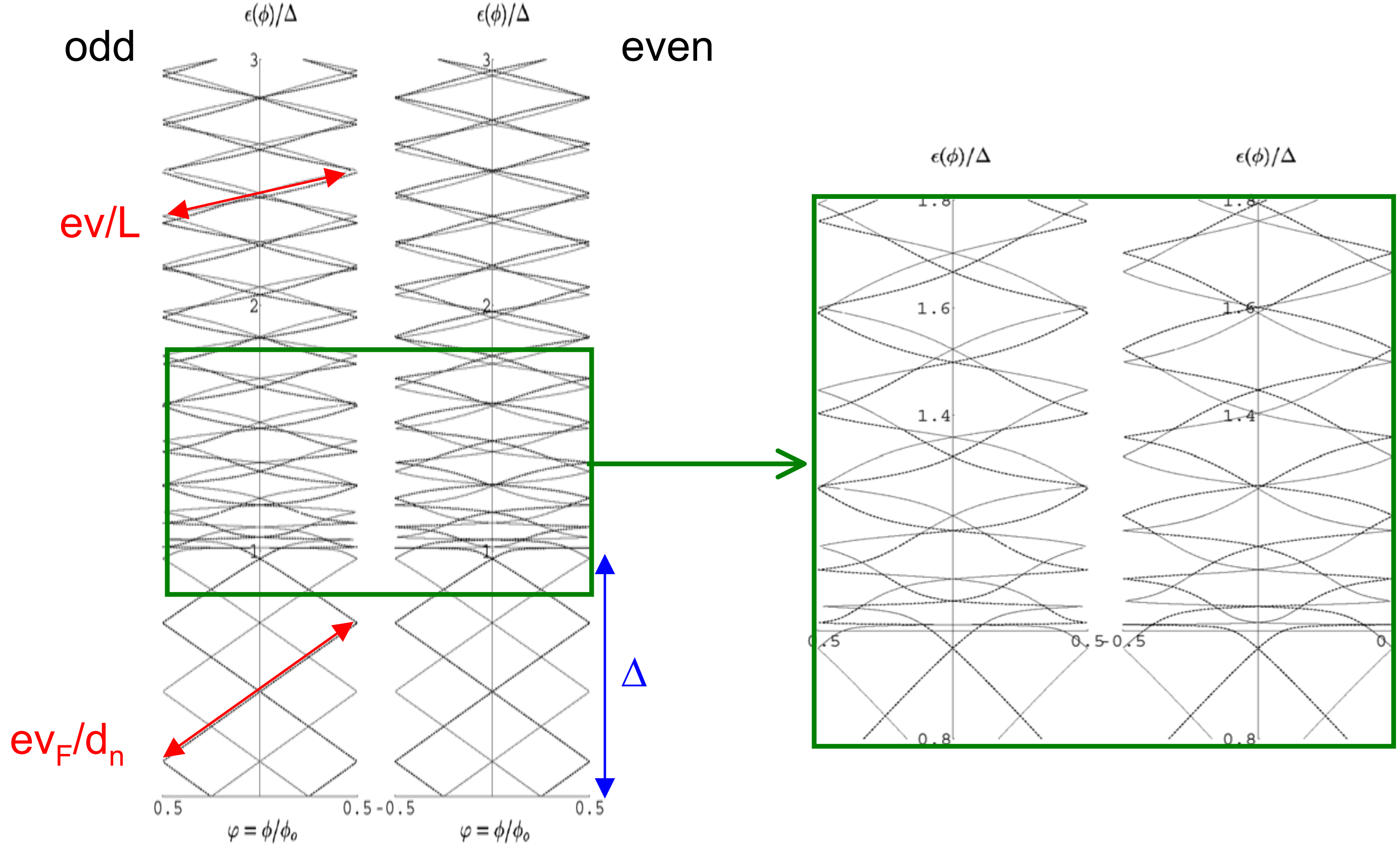
Assume levels above the gap do not contribute to the current

d_s large : problem equivalent to SNS junction

(Bardeen, Johnson) levels above the gap form a continuum

Goals: Treat any d_s and d_n, and cross-overs,
Levels above the gap,
$d_n \rightarrow 0$, short junction → one single Andreev level
Non linear spectrum

d_s finite : levels above the gap are discrete

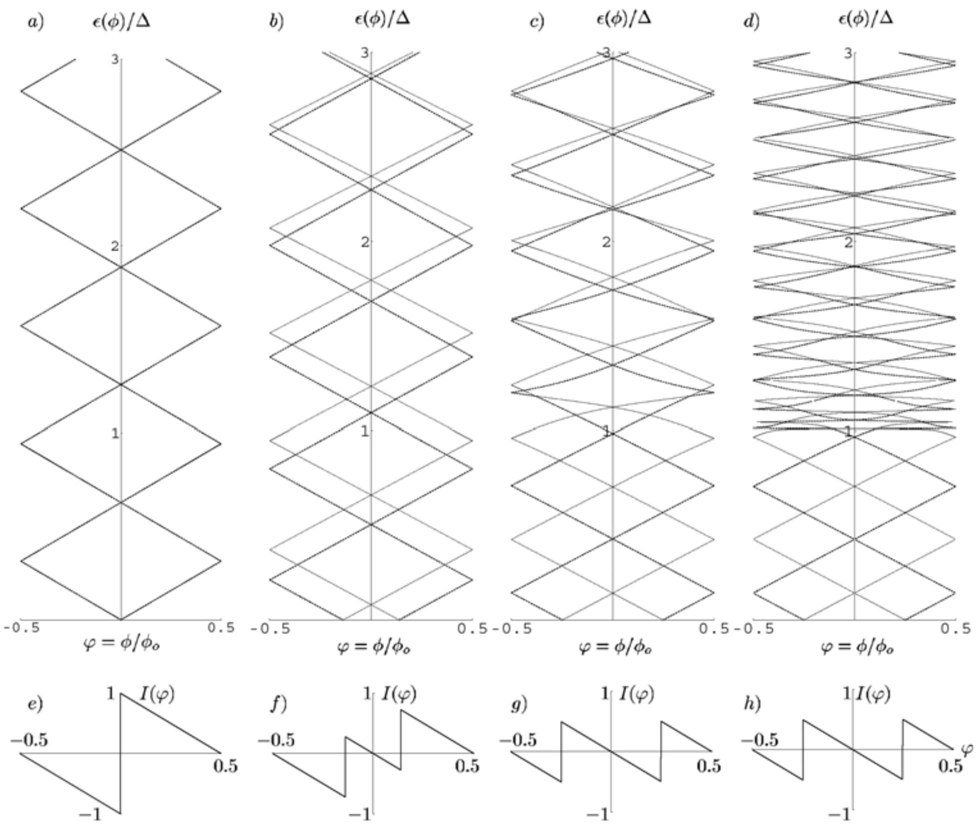


d_n large : linear flux dependence of the Andreev states

Questions

- How to calculate the current ? $I = -dE/d\phi$
- Linearized spectrum → No current !
- How to get properly a current from a linearized spectrum ?
- Current of the last level ? How many levels contribute ?
- How to estimate the current of the states near and above the gap?

d_n large : from $d_s = 0$ to d_s large (BK)



$d_s=0$ $d_s=\xi_0$ $d_s=5\xi_0$ $d_s=20\xi_0$

$d_n=10\ \xi_0$ $ev_F/(d_n+d_s)$

$d_s = 0$ → Phase shift $0, \pi$

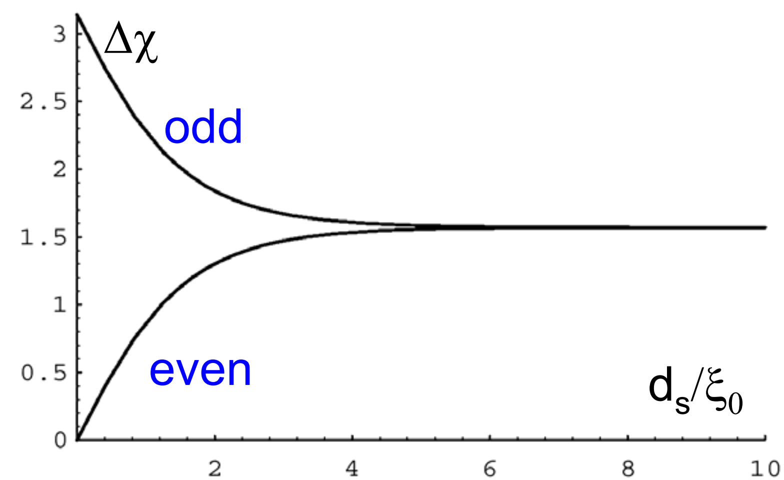
$d_s >> \xi_0$ → Phase shift $\pi/2$

$d_s \sim \xi_0$ $$\Delta\chi = ArcCos \frac{(-1)^N}{Cosh\ d_s / \xi_0}$$

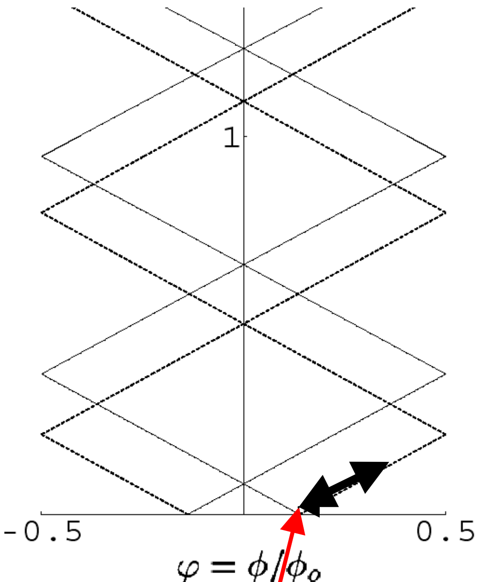
Parity effect is lost, for Andreev states

$\Delta\chi(E)$ Non linear levels near the gap

How to calculate the current ?



Persistent current



Harmonics expansion $$I(\varphi) = \sum_{1}^{\infty} I_p \sin 2\pi p \varphi$$

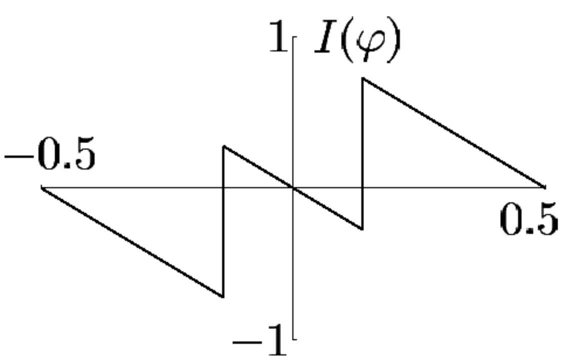
$$\epsilon_n(\varphi) \longrightarrow \text{unfolded} \quad \epsilon(y) \quad , \quad \varphi \in [-\frac{1}{2}, \frac{1}{2}] \longrightarrow y \in [y_0, \infty]$$

$$I_p = \frac{2}{\pi p} \frac{1}{\phi_o} \left[\epsilon'(y_o) \cos p\Delta\chi - \int_{y_o}^{\infty} dy \; \epsilon''(y) \cos 2\pi p y \right]$$

$$y_0 = \frac{1}{2\pi} \Delta\chi$$

piecewise linear I(φ)

Non-linear correction



- Each harmonics is an integral over the complete spectrum. It is the sum of :
- a boundary term evaluated at zero energy + a curvature term integrated over all the spectrum
- This curvature term can be evaluated and bounded

Example : persistent current of a normal 1D ring

$$I_p = \frac{4}{\pi p}\frac{1}{\phi_o}\left[\epsilon'(y_o)\cos p\Delta\chi - \int_{-\infty}^{y_o} dy\ \epsilon''(y)\cos 2\pi p y\right]$$

$$\epsilon_n(\varphi) = \frac{h^2}{2mL^2}(n+\varphi)^2 \longrightarrow \epsilon(y) = \frac{h^2}{2mL^2}\ y^2$$

$$y_0 = \frac{k_F L}{2\pi}$$

$$\epsilon'(y_0) = \frac{h v_F}{L} \qquad \epsilon''(y) = \frac{h^2}{mL^2} = Cte$$

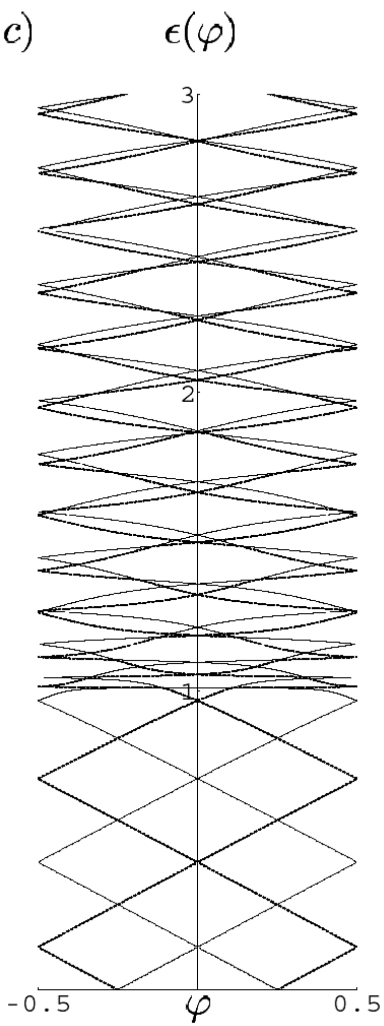
$$\boxed{I(\varphi) = \frac{4}{\pi}\frac{e v_F}{L}\sum_1^{\infty}\left(\frac{\cos p k_F L}{p} - \frac{\sin p k_F L}{p^2 k_F L}\right)\sin 2\pi p \varphi}$$

Cheung, Gefen, Riedel (89)

piecewise linear I(φ)

Non-linear $1/k_F L$ correction

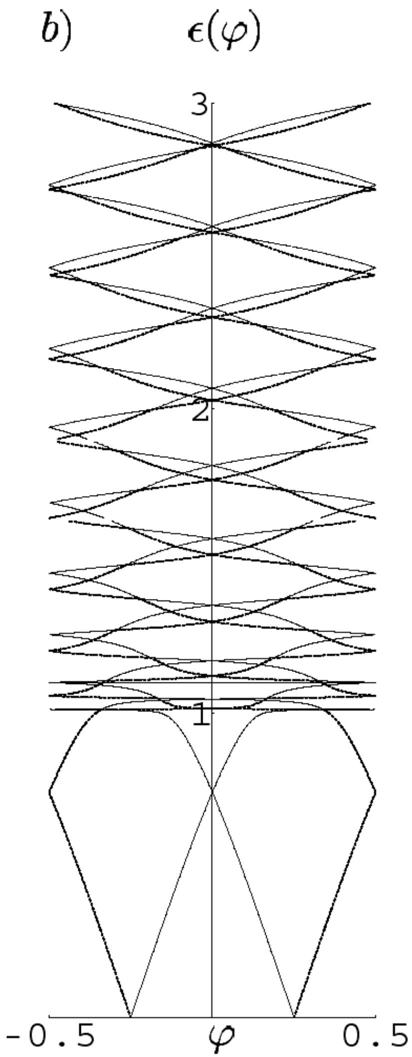
Curvature term



$d_n = 10\ \xi_0$

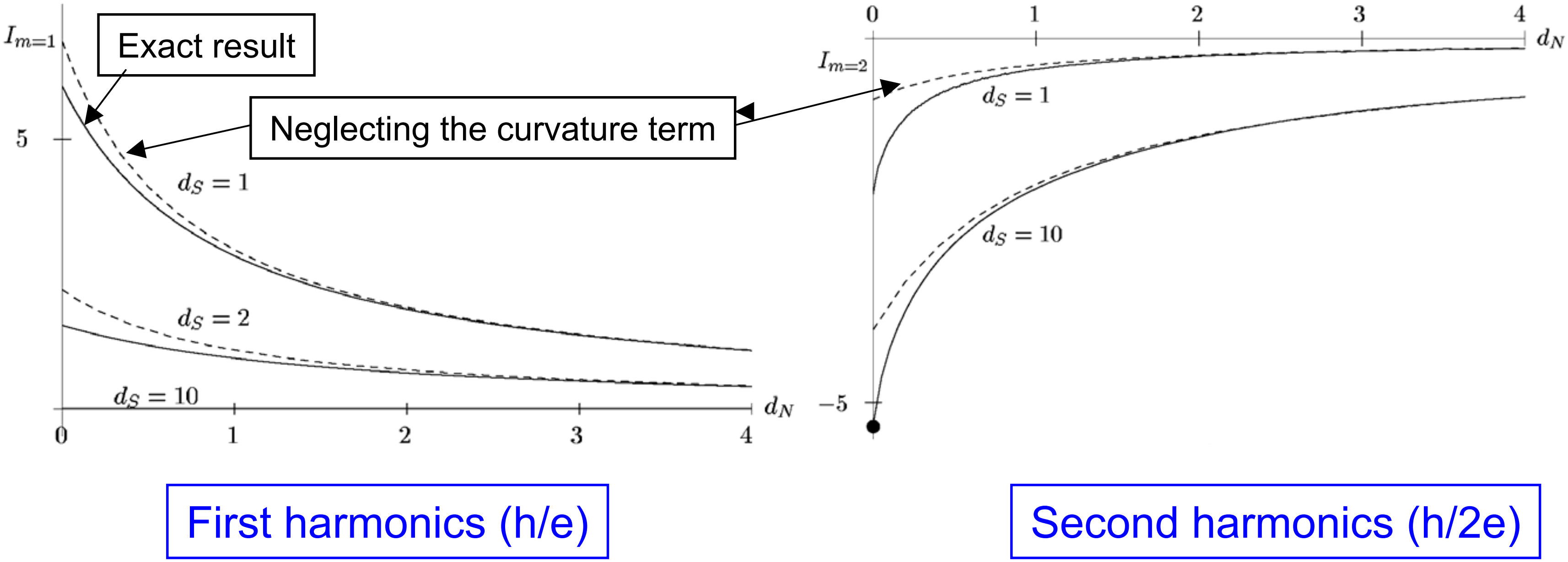
d_n large
Andreev levels are linear
States above the gap are non-linear
they do not form a continuum
However non linearities compensate
and the contribution of high energy levels vanishes

d_n small
Andreev levels are non-linear
States near the gap are non-linear
Non-linear flux dependent current



$d_n = \xi_0$

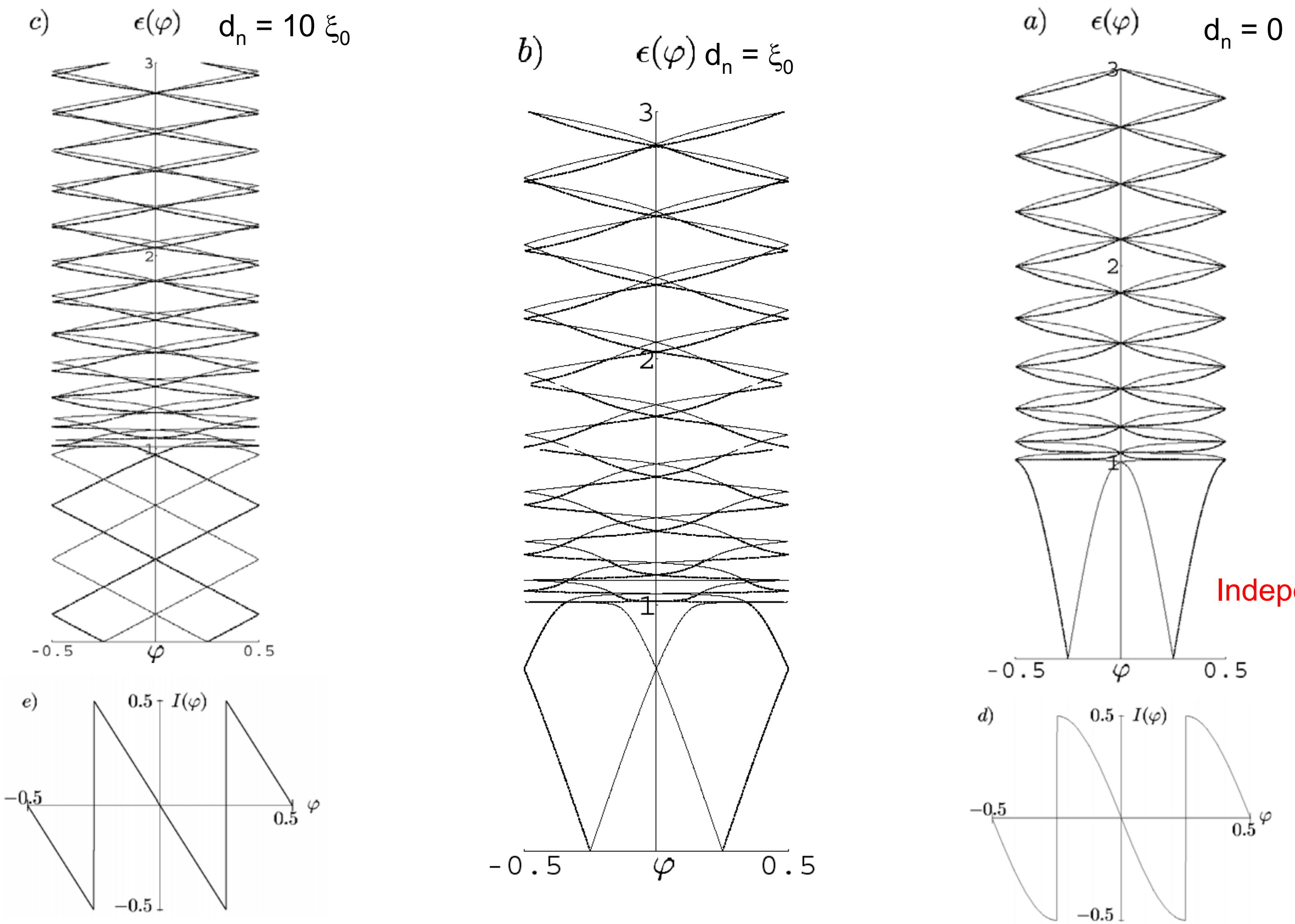
Non-linearities



Non-linearities disappear for $d_n > 2\ \xi_0$

First harmonic disappear for d_s large
Second harmonics increases with d_s

d_s large : from d_n large to $d_n = 0$



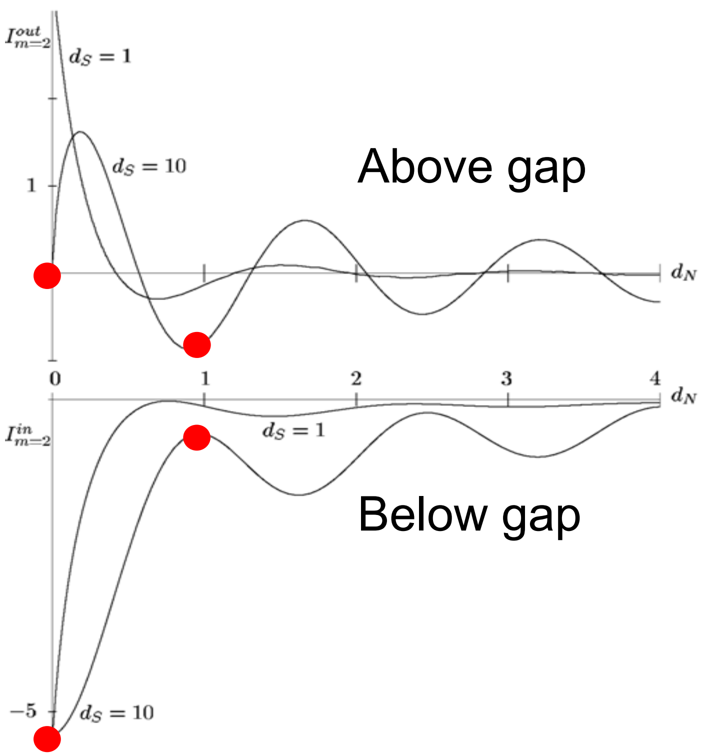
Independant of d_s

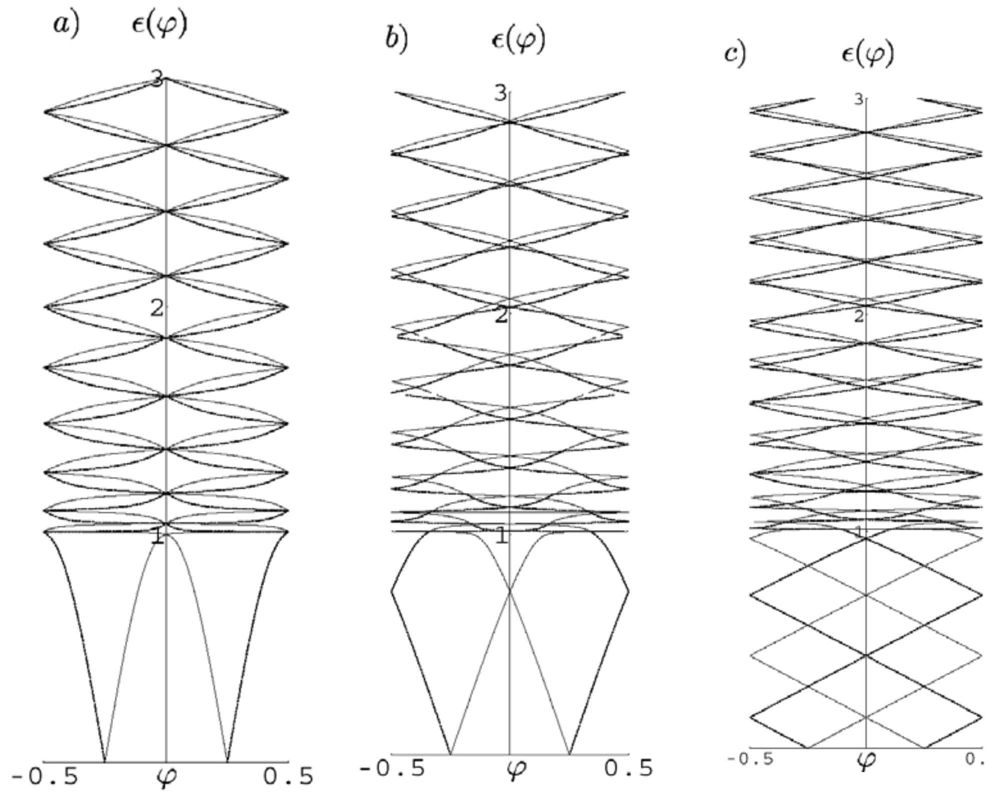
Which levels carry the current ?

Beenakker, Van Houten, 91

Which levels carry the current ?

h/2e





$d_n = 0$ $\quad$ $d_n = \xi_0$ $\quad$ $d_n = 10\,\xi_0$

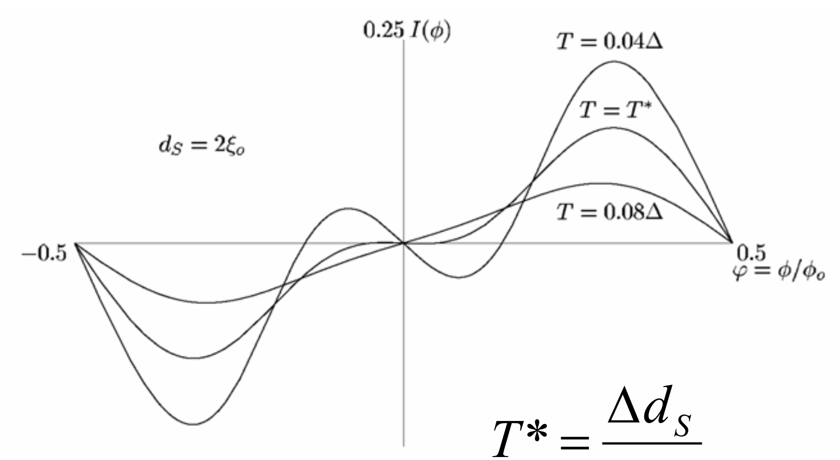
The current is carried by levels above and below the gap (even if d_s is large)

Oscillations as function of d_n with period $\pi\xi_0/2$

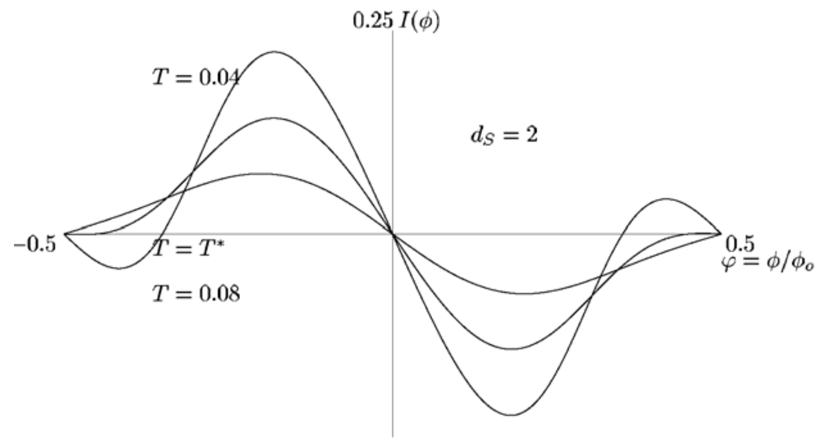
Ensemble average

Rings with even N
paramagnetic when d_s=0
diamagnetic when d_s finite

Rings with odd N
always diamagnetic



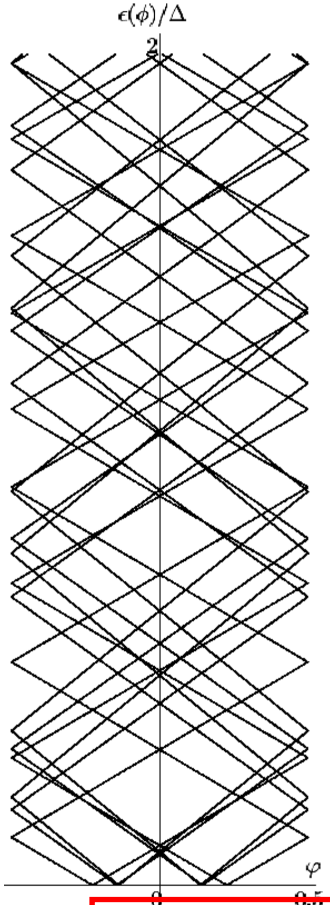
$$T^* = \frac{\Delta d_S}{\pi d_n}$$



An ensemble of normal rings has a paramagnetic magnetization
When $d_s \sim 0{,}5\ \xi_0$, the magnetization becomes diamagnetic

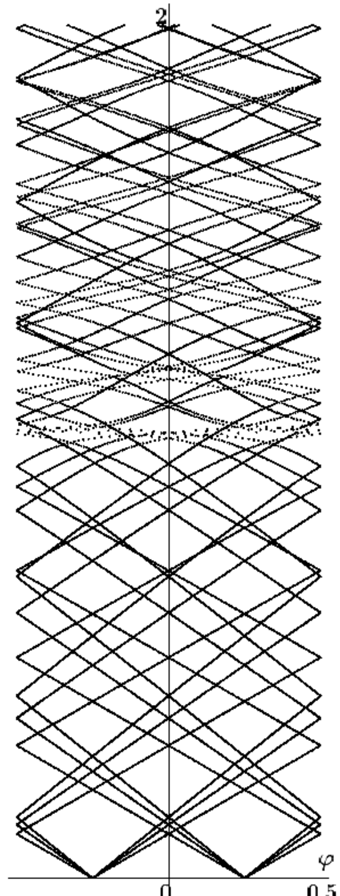
Multichannel NS rings

d_n large
d_s small

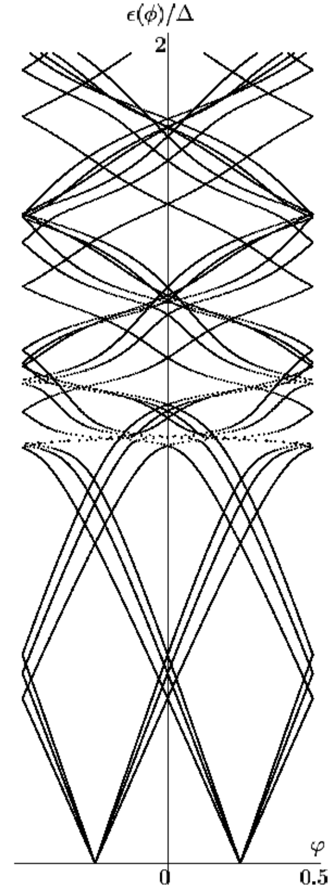


d_n large
d_s large

M e v_F/d_n

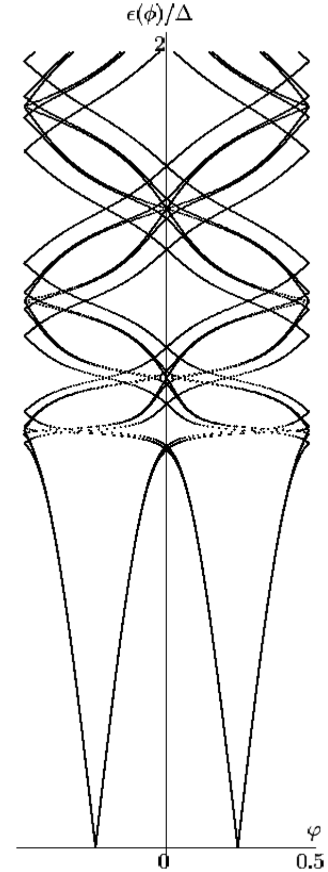


d_n small
d_s large



d_n = 0
d_s large

M e v_F/ξ_0



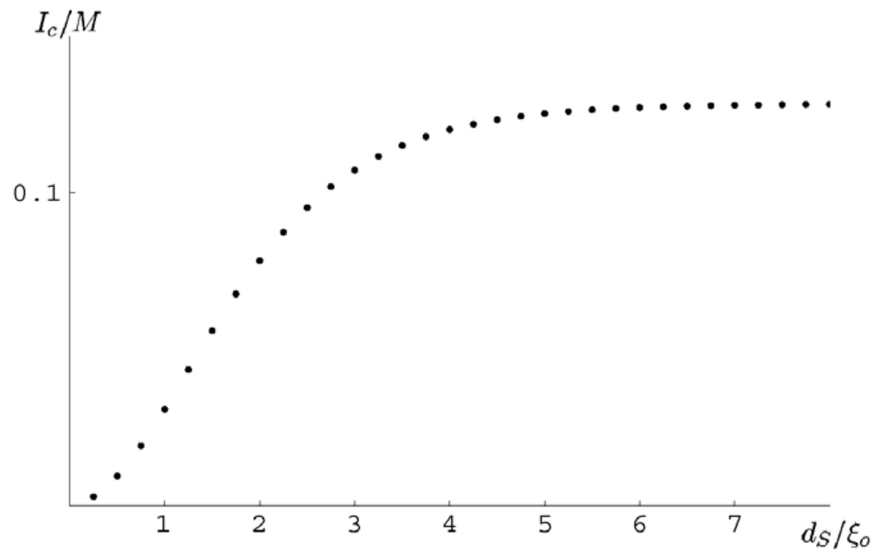
$$\boxed{\Delta\chi = ArcCos\frac{Cos\,k_{F_x}L}{Cosh\,d_s/\xi_0}}$$

$$\boxed{\Delta\chi = k_{F_x}L}$$ (d_s = 0)

$d_s < \xi_0$ → «random» $\Delta\chi$, current averages to 0

$d_s > \xi_0$ → locked $\Delta\chi$, current of M channels

Current of a multichannel ring (d_n large)



Persistent current ~ 0 $\longrightarrow$ Josephson current ~ M e v_F/d_n

Conclusions

- NS 1D and multichannel rings, without disorder
- Full spectrum
- States above the gap do not form a continuum and carry a current
- Persistent current has contributions from linear and non-linear regions of the spectrum
- Cross-over from normal current to Josephson current, vs. d_s and d_n
- Non interacting average current 0 $\rightarrow$ M I_0 , I_0=e v_F/ d_n if $d_n >> \xi_0$
- Disorder and interactions ? I_0=e v_F/ ξ_0 if $d_n << \xi_0$

Parameters

d_n d_s	$d_n = \xi_0$ Short junction	$d_n \sim \xi_0$	any d_n	$d_n >> \xi_0$ long junction
$d_s = 0$				Normal ring Riedel et al.
$d_s \sim \xi_0$				
any d_s	Levels above the gap contribute	Levels above the gap contribute		Buttiker-Klapwijk
$d_s >> \xi_0$	Beenakker-Van Houten	Levels above the gap contribute		Bardeen-Johnson

unfolding

$$\boxed{\epsilon_n(\varphi) \longrightarrow \text{unfolded} \quad \epsilon(y) \quad , \quad \varphi \in [-\frac{1}{2}, \frac{1}{2}] \longrightarrow y \in [y_0, \infty]}$$