

Ballistic transport in bulk 1D electronic/magnetic systems

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an observation

approach linear response theory

theoretical/experimental status

Idea

Commonly used prototype models for the description of (quasi-) 1D materials exhibit **ballistic transport** at **all temperatures**.

Hubbard model (electronic/magnetic)

spin 1/2 Heisenberg (magnetic)

nonlinear σ -model (magnetic - $S = 1$)

" $t - J$ " ($t = J$) (doped antiferromagnets)

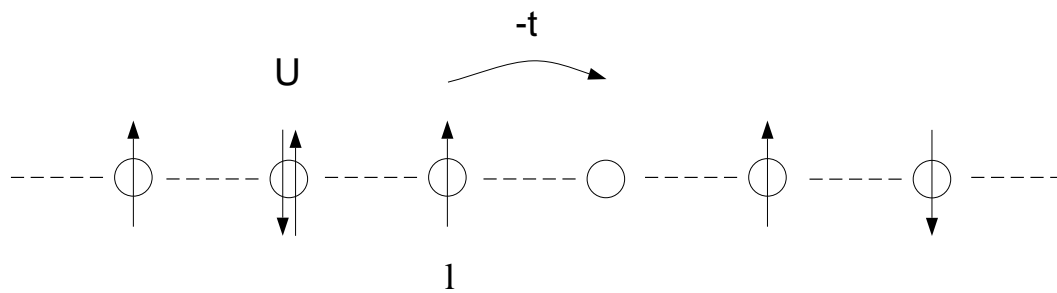
sine-Gordon (?)

1D integrable quantum models

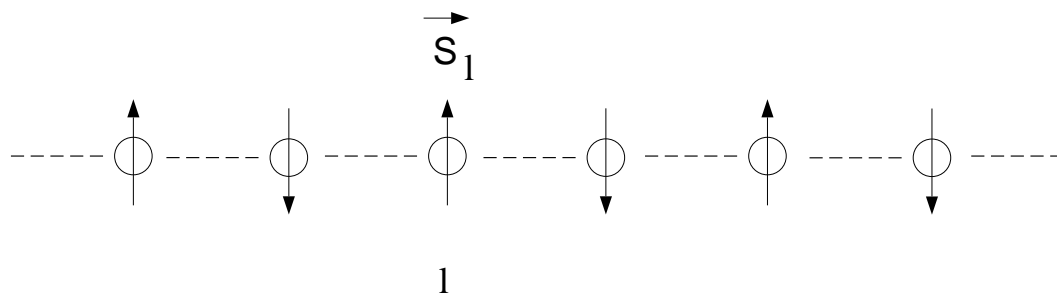
physics analogous to **classical nonlinear integrable systems**; widely studied in mathematics - physics (theory/ experiment) - for technological applications (e.g. optical fibers)

Approach

Hubbard model (electronic)



Heisenberg model, spin 1/2 (magnetic)



$$H_{Heis} = J \sum_l \vec{S}_l \cdot \vec{S}_{l+1}$$

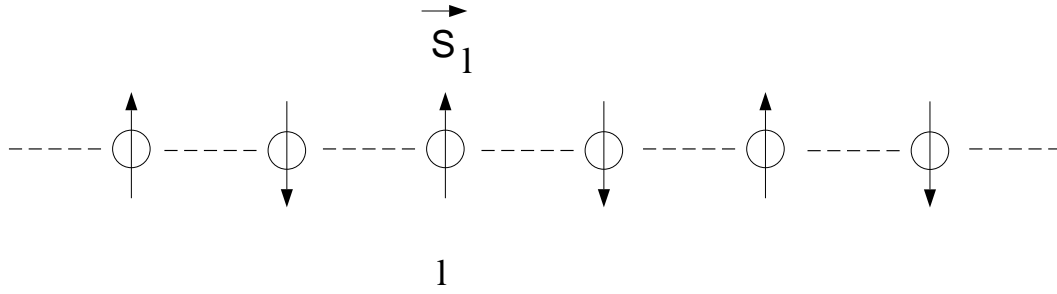
integrable quantum systems

Bethe ansatz \rightarrow thermodynamic properties

dynamic and transport properties ?

electrical/thermal/magnetic conductivity

spin 1/2 Heisenberg model (magnetic)



$$H_{Heis} = \sum_l h_l = J \sum_{l=1}^L (S_l^x S_{l+1}^x + S_l^y S_{l+1}^y + \Delta S_l^z S_{l+1}^z)$$

$J > 0$, antiferromagnet, $S = 1/2$, p.b.c.

$\Delta > 1$ “easy axis”, $\Delta < 1$ “easy plane”

spin current j^z (continuity equation)

$$S^z = \sum_l S_l^z, \quad [H, S^z] = 0, \quad \frac{\partial S_l^z}{\partial t} + \nabla j_l^z = 0$$

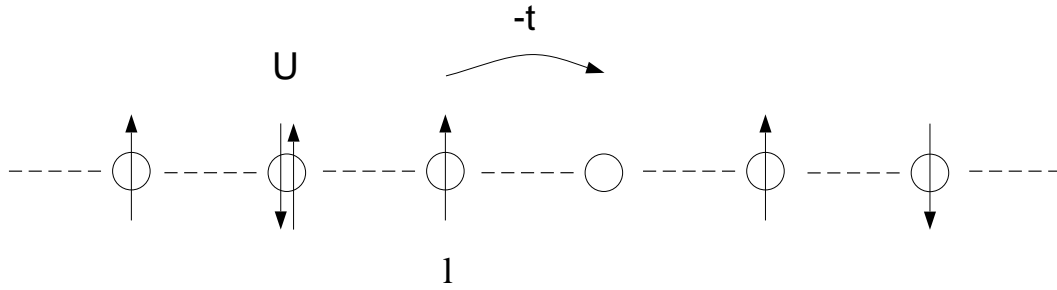
$$j^z = \sum_l (S_l^y S_{l+1}^x - S_l^x S_{l+1}^y), \quad [j^z, H] \neq 0$$

energy current j^E

$$\frac{\partial h_l}{\partial t} + \nabla j_l^E = 0$$

$$j^E = J \sum_l (S_{l-1}^x S_l^z S_{l+1}^y - S_{l-1}^y S_l^z S_{l+1}^x) + \Delta (\text{perm.} - x, y, z)$$

Hubbard model (electronic)



$$H = (-t) \sum_{\sigma,l} (c_{l\sigma}^\dagger c_{l+1\sigma} + h.c.) + U \sum_l n_{l\uparrow} n_{l\downarrow}$$

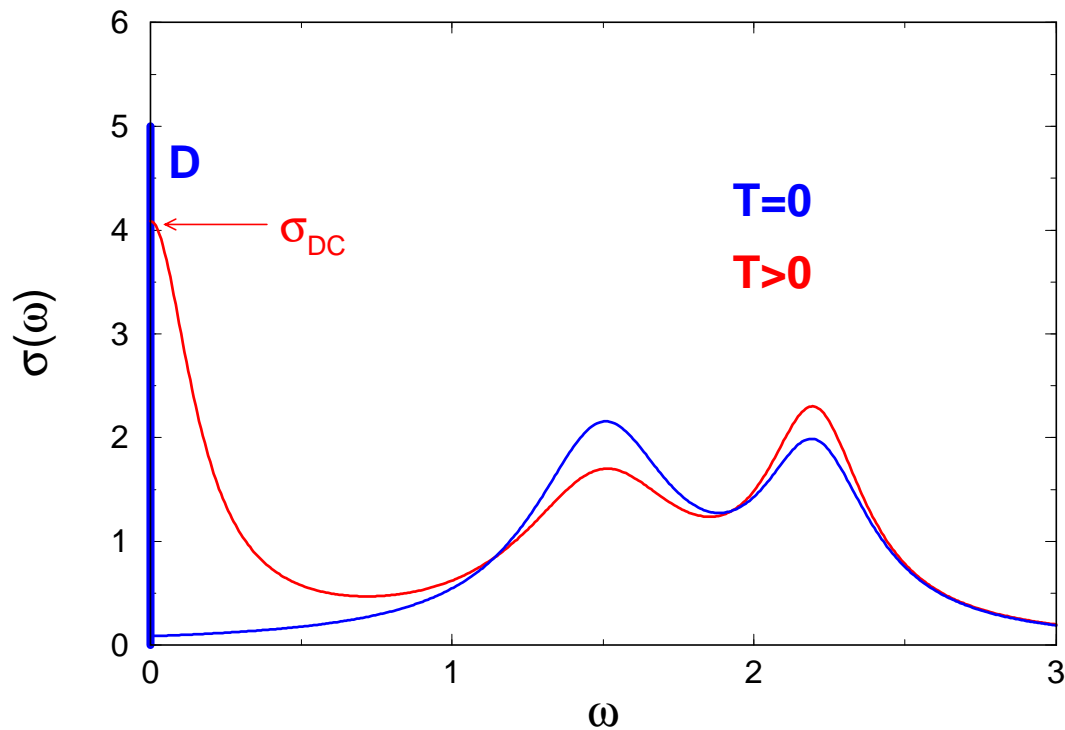
charge current j

$$\frac{\partial n_l}{\partial t} + \nabla j_l = 0$$

$$j = (-t) \sum_{\sigma,l} (i c_{l\sigma}^\dagger c_{l+1\sigma} + h.c.)$$

Finite temperature transport properties

“common sense”



conductivity

$$\sigma'(\omega) = 2\pi D\delta(\omega) + \sigma_{reg}(\omega > 0)$$

$$\sigma'_{reg}(\omega) = \frac{1 - e^{-\beta\omega}}{\omega L} \pi \sum_{n,m \neq n} p_n |\langle n | \hat{j} | m \rangle|^2 \delta(\omega - \epsilon_m + \epsilon_n)$$

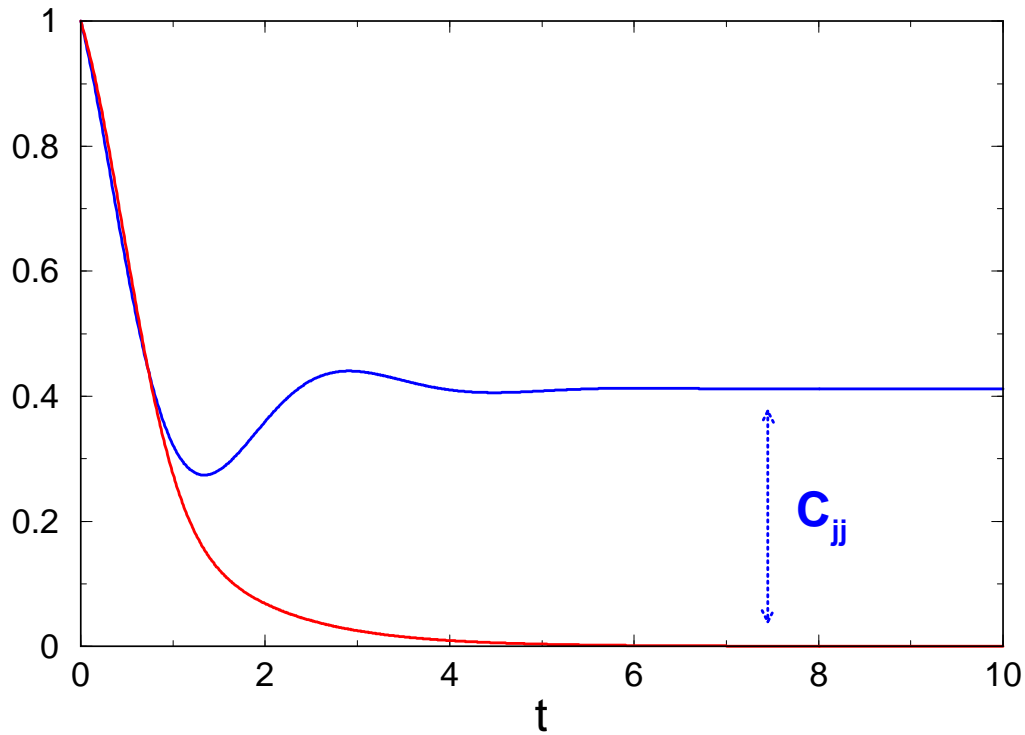
$$\sigma''(\omega) = 2D/\omega|_{\omega \rightarrow 0}$$

D = “Drude weight”

$$\int_{-\infty}^{\infty} \sigma'(\omega) d\omega = \frac{\pi}{L} \langle -\hat{T} \rangle$$

$$D = \frac{1}{L} \sum_n p_n D_n = \frac{1}{L} \sum_n p_n \frac{1}{2} \frac{\partial^2 \epsilon_n(\phi)}{\partial \phi^2} \Big|_{\phi \rightarrow 0}, \quad (W.Kohn 1964)$$

$\langle j(t)j(0) \rangle$



$$D = \frac{\beta}{2L} \langle j(t)j(0) \rangle_{t \rightarrow \infty} = \frac{\beta}{2L} C_{jj}$$

$$C_{jj} = \sum_n p_n | \langle n | j | n \rangle |^2$$

generic behavior

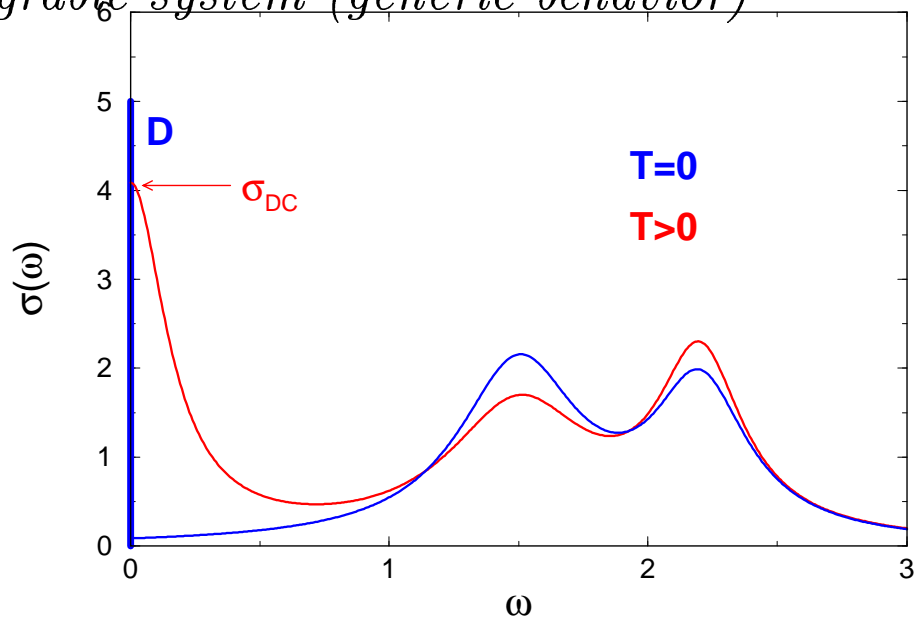
- $T > 0, D = 0 \rightarrow$ normal conductor (dissipation)
- $T = 0, D > 0 \rightarrow$ *ideal conductor*

Proposal (*Phys. Rev. Lett.* **74**, 972 (1995))

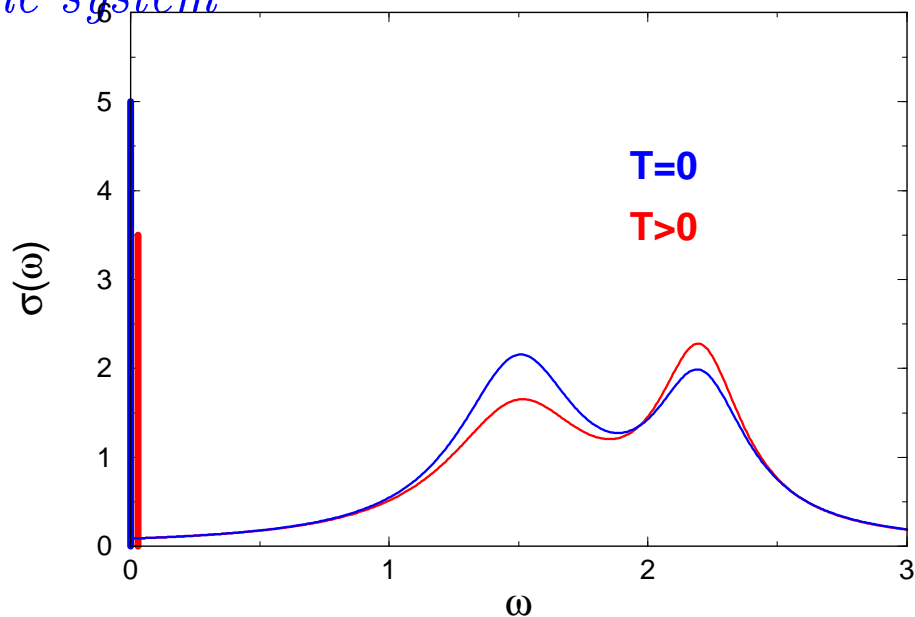
numerical simulation \rightarrow analytical confirmation

integrable quantum systems present
ideal conduction (without dissipation)
at *all temperatures*.

nonintegrable system (generic behavior)



integrable system



Mazur inequality and conservation laws

(Phys. Rev. B55 11029 (1997))

integrable system: macroscopic number of Q_n

$$[Q_n, Q_m] = 0, \quad [Q_n, H] = 0$$

$$C_{AA} = \langle A(t)A \rangle_{t \rightarrow \infty} \geq \sum_n \frac{\langle A Q_n \rangle^2}{\langle Q_n^2 \rangle}$$

$\langle \rangle$ thermal average, $\langle Q_n Q_m \rangle = \langle Q_n^2 \rangle \delta_{n,m}$, $A^\dagger = A$,
 $\langle A \rangle = 0$.

Heisenberg model

$Q_3 = j^E \rightarrow$ *ideal thermal conductivity*

$\langle j^E(t)j^E(0) \rangle = \text{const.}$

$$D(T) \geq \frac{\beta}{2L} \frac{\langle j^z Q_3 \rangle^2}{\langle Q_3^2 \rangle}$$

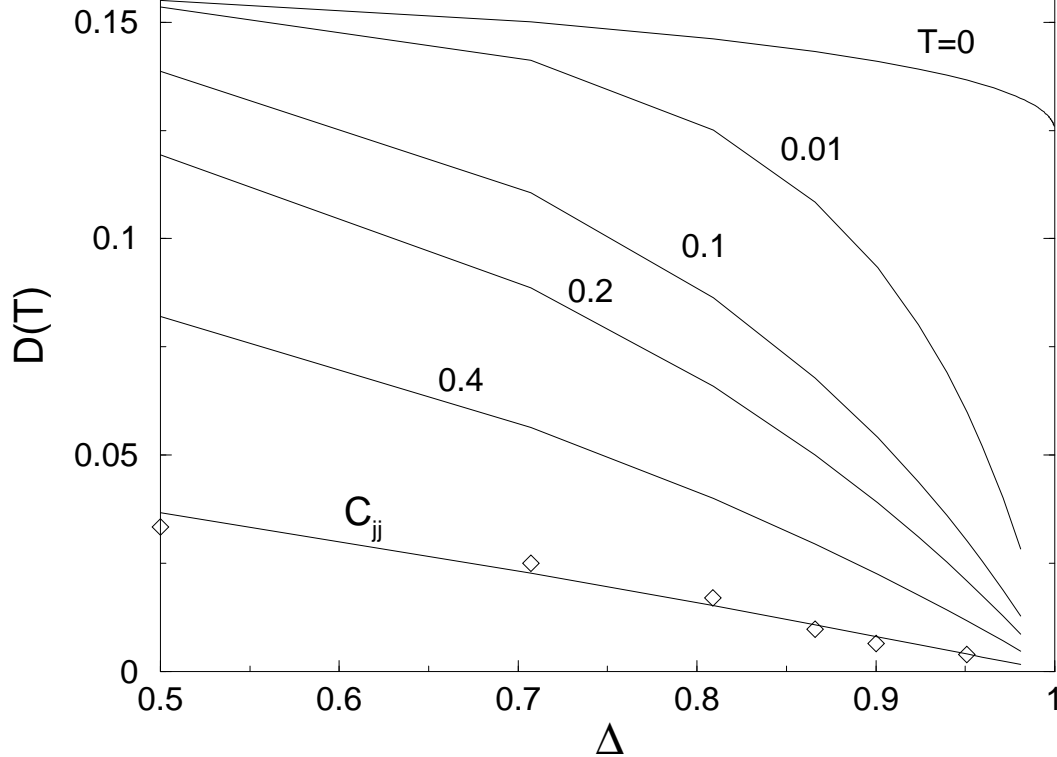
high temperature limit

$$D(T) \geq \frac{1}{2k_B T} \frac{8\Delta^2 m^2 (1/4 - m^2)}{1 + 8\Delta^2 (1/4 + m^2)}, \quad m = \langle S_l^z \rangle$$

BA calculation of $D(T)$

(Hubbard model: S. Fujimoto and N. Kawakami,
J. Phys. A. **31**, 465 (1998))

Heisenberg model (*Phys. Rev. Lett.* **82**, 1764 (1999))



$T \rightarrow 0$

$$D(T) = D(T = 0) - \text{const.} T^\alpha$$

$$\alpha = \frac{2}{\nu - 1}, \quad \Delta = \cos\left(\frac{\pi}{\nu}\right)$$

Experimental developments

NMR

M. Takigawa et al. (IBM/Tokyo, 1996):

Sr_2CuO_3 : $S=1/2$, $D_{exp}^s \simeq 10^3 D_{th-cl}^s$

$AgVP_2S_6$: $S=1$, diffusive behavior, nonlinear- σ model ?

thermal conductivity (magnetic materials)

H.R. Ott (ETHZ), Sr_2CuO_3 , $SrCuO_2$, $(La, Sr, Ca)_{14}Cu_{24}O_{41}$

B. Büchner (Aachen), $(La, Ca)_{14}Cu_{24}O_{41}$

F. Steglich (MPI Dresden), Yb_4As_3

optical absorption ?

F. Degiorgi, G. Grüner (ETHZ, UCLA, 1996)

Bechgaard salts (“zero mode”)

Perspectives

theory

- transport in integrable systems:
quantum/classical systems
- conservation laws and finite frequency conductivity
- stability to perturbations:
linear vs. nonlinear transport

experiment

- (quasi-) 1D magnetic materials
- (quasi-) 1D electronic materials
- superlattices
- self-assembled structures on surfaces

In collaboration with:

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