Cooper pair transport in an array of Josephson nanojunctions with dice lattice

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Outline

✓ Introduction
  ■ localisation effect in the dice lattice
✓ classical superconducing arrays :
  ▪ $T_c, I_c$ suppression, glassy vortex state
✓ quantum arrays
  ▪ S-I transition, metallic phase
✓ the dice family of JJ arrays
  ▪ exotic superconducting phase

Collaborators

- E. Serret, F. Balestro, C. Abilio
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- O. Buisson, K. Hasselbach
- Th. Fournier

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- "CEA-LETI-PLATO"

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localisation effect in the « dice » lattice

- non-interacting tight binding electrons \((Vidal, PRL81, 1998, p5888)\)
  
  \[ f = \frac{\phi}{\phi_0} \]
  
  \(\phi_0\) : flux quantum

  - localisation due to quantum interferences (AB cages)
  - cage effect suppressed by: disorder, edge states, interaction.

- GaAs quantum wires (electrons: fermions e) \(C. Naud et al. PRL86, 5104 (2001)\)
- \(h/e, h/2e\) magnetoresistance oscillations \rightarrow Aharonov Bohm cages

- Superconducting arrays (Cooper pairs: bosons 2e)
- wire networks: Schrödinger equation (1 particle) \(\equiv\) linearized GL equations for the macroscopic superconducting state (fluctuations neglected)
Josephson Junction arrays

- classical dice JJ array
  
  highly frustrated state with thermal fluctuations: \( \cos(\phi_i - \phi_j - A_{ij}) \Rightarrow J(f) S_i S_j \)
  
  vortices on the Kagomé (dual) lattice

- quantum JJ array
  
  Josephson coupling + Coulomb blockade
  
  \[
  H = -E_J \sum \cos(\phi_i - \phi_j - A_{ij}) + \frac{(2e)^2}{2} \sum n_i C_{ij}^{-1} n_j
  \]

  hopping of Cooper pairs

- role of elementary rhombuses ("dimers")
  
  SQUID: \( E_J \) suppressed at \( f=1/2 \)
  
  Rhombus: additional degree of freedom:
  
  phase of intermediate island

  \[
  E_{\text{class}} = -2 \cos \theta_{ij} |\cos \pi f| \\
  E_{\text{class}} = -2 \left| \cos \left( \frac{\theta}{2} - \frac{\pi f}{2} \right) \right| - 2 \left| \cos \left( \frac{\theta}{2} + \frac{\pi f}{2} \right) \right|
  \]
Landau levels of the ‘dice’ array

degenerate discrete levels $\epsilon = \pm \sqrt{6}$
Superconducting Al wire network

- Critical temperature
- Critical Current

\[ T_c(0) = 1.234 \text{ K} \]

Theory: \( \xi_0 = 157 \text{ nm} \)

\[ 1 - \frac{T_c(f)}{T_c(0)} \]

at \( f = 1/2 \) => « suppression » of superconducting order

C. Abilio et al. PRL83, (1999) 5102
The superconducting ground state at $f=1/2$

- Classical spins on Kagomé lattice disordered at $T=0$
  
  \[ H_{\text{Kagomé}} \]  

\[ H \text{PRB45, 1992 p7536} \]

- Josephson « dice » array:
  highly degenerate metastable states

Theoretical Prediction:
S. Korshunov PRB, 63, 134503 (2001)

Ground state $\Rightarrow$ vortex triads with zero energy domain walls

\[ S = (N+M)\ln 2 : \text{non-extensive entropy} \]

Vortex glass phase at $T < T_{\text{KTB}}$
(Cataudello and R. Fazio, 2002)

\[ T_{\text{KTB}} = 0.03 \, E_J \]
thermal hysteresis slow dynamics
Magnetic imaging: Observed Configurations at $f=1/2$

Magnetic decoration of vortices


- Field cooled epitaxial Nb wire array
- $T_c = 9\text{K}$
- $B = 11.93\text{ Gauss}$ for $f=1/2$

- Observed configuration (reconstructed on the Kagomé lattice)

- No commensurate phase

⇒ disordered configuration?

3 456 sites containing 1 725 vortices

$f=1/2 - 0.001$
Magnetic imaging: Correlation function calculation

\[ C_{\alpha, \beta, \gamma}(r) = \langle V_i \cdot V_{i+r} \rangle \]

\( V_i \) : « vortex » variable
- \( = 1 \) if a vortex is in the i cell
- \( = -1 \) if not

Long range order: \( C(r) > 0.8 \) until \( r = 40 \)

No long range order: \( \xi \approx 1.5 \)

Collaboration: P. Butaud
Nanofabrication:

Samples

(Array containing more than 127,000 junctions)

Chain of «cages»

Josephson junctions
Transport:

- Estimation of $E_J$ with $R_n$ measured at 4K
- Estimation of $E_c$ with the offset voltage:

$$V_{\text{offset}} = N \frac{e}{2C}$$

i.e. about 50fF/µm²

Estimation of $E_J$ with $R_n$ measured at 4K

$V_{\text{offset}} = (40\pm10) \text{ mV}$

Samples overview

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0,06</td>
<td>4,93</td>
<td>130</td>
<td>3</td>
<td>27</td>
<td>4,9</td>
<td>Classical</td>
</tr>
<tr>
<td>B</td>
<td>0,023</td>
<td>20,4</td>
<td>32</td>
<td>1,3</td>
<td>61</td>
<td>0,5</td>
<td>Quantum</td>
</tr>
<tr>
<td>C</td>
<td>0,015</td>
<td>53,3</td>
<td>12</td>
<td>0,5</td>
<td>160</td>
<td>0,05</td>
<td>Charge</td>
</tr>
</tbody>
</table>

Cell area = 5,57 µm² => $f=1 \equiv B=0,3716$ mT
Transport:

Classical array with $E_J/E_C=4.9$

Study of the KTB transition:

If $T > T_{KTB}$,

\[
\frac{R(T)}{R_n} = b_1 \exp \left[ -\frac{-b_2}{\sqrt{\tau - \tau_{KTB}}} \right]
\]

with $\tau = \frac{k_B T}{E_J(T)}$, and $b_1, b_2 \approx 1$

\[
\tau_{KTB}
\]

with $\tau_{KTB}$, measured

<table>
<thead>
<tr>
<th>$f$</th>
<th>$\tau_{KTB}$ theoretical</th>
<th>$\tau_{KTB}$ measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.8</td>
<td>1.68</td>
</tr>
<tr>
<td>1/3</td>
<td>0.36</td>
<td>0.57</td>
</tr>
<tr>
<td>1/2</td>
<td>0.108</td>
<td>0.155</td>
</tr>
</tbody>
</table>

✦ At $f=1/2$, the array undergoes a KTB transition

✦ Relatively good agreement with theory (uncertainty on $\Delta(T)$)

Is the $f=1/2$ state a vortex glass?
Transport: Classical array with $E_J/E_C=4.9$

Study of the superconducting phase at $f=1/2$

$\Rightarrow$ vortex configuration pinning force measurement

1. Max of $I_d$ at $f = 0, 1/13, 1/9, 1/6, 1/3, 2/3, 5/6,...$

2. Max of $I_d$ at $f=1/2 \neq$ wire arrays

Commensurate state at $f=1/2$
Transport:

Charge array with $E_j/E_C = 0.05$

Metal-Insulator transition induced by $B$ at $f_{c,1} = 0.23$ and $f_{c,2} = 0.76$

At $f=1/2$: insulator ($R > 50M\Omega$)

At $f=0$: low resistance state
Transport:

Quantum array with $E_J/E_C=0.5$

At $f=0$: KTB transition

fit $R_0(T)$ => $\tau_{KTB,mes}=1.58$

theory with quantum fluctuations => $\tau_{KTB,th}=1.47$

At $f=1/2$: saturation of $R_0(T)$

Both near $f=0$ and $1/2$:

differential Resistance is:

- $T$- independent
- proportional to $f$
Transport:

Quantum array with $E_J/E_C=0.5$

Study of the resistive phase at $f=1/2$

$\Rightarrow$ Behavior comparison between $f=0$ and $f=1/2$

- If $T>T_{cr}$, thermal activation behavior:
  
  Same energy barrier at $f=0$ and $1/2$:
  
  $E_b = 73 \text{ mK} = 0.2E_J$
  
  (theory: $0.19E_J$)

- Theoretical prediction for $T_{cr}$:
  
  $T_{cr,th} \approx \sqrt{E_b E_c} = 230 \text{ mK}$

  Observed $T_{cr}$ is smaller
  (dissipation effect)

At $f=1/2$, resistive phase at $T\to0$:

Evidence of a vortex liquid induced by the quantum fluctuations
Suppression of the quantum fluctuations in the dice array at f=0!

Disagreement between dice and square phase diagrams

- $E_{c,eff}$ measure with CBT for sample C ⇒ $E_{c,CBT} = E_c /10$
- Fabrication and measurements of other samples: $E_{c,CBT} = E_c$
  and for $1/x=4$ et $5$ ⇒ superconducting at $f=0$

**Phase diagram:**

At $f=0$

Suppression of the quantum fluctuations in the dice array at $f=0$!
Phase diagram:

At $f = 1/2$

$R(T)$ thermally activated

$T_{cr} \propto \sqrt{E_J E_c}$

Area S: vortex glass

Area R: vortex liquid induced by quantum fluctuations

$E_c / E_J$
Conclusions

**Imaging:**  at f=1/3 observation of a commensurable state
  at f=1/2 very short range order (triades)

**Transport at f=0:** suppression of the quantum fluctuations in the dice array compared to the square array

**Transport at f=1/2:**
  - charge array:
    observation of an insulating phase
  - classical array:
    evidence of a commensurate phase at f=1/2
    no thermal hysteresis
    no ordering induced by electrical excitation (≠ f=1)
  - quantum array:
    evidence of a vortex liquid induced by quantum fluctuations