Electron Correlations in Quantum Wires (Transport in a Luttinger Liquid)

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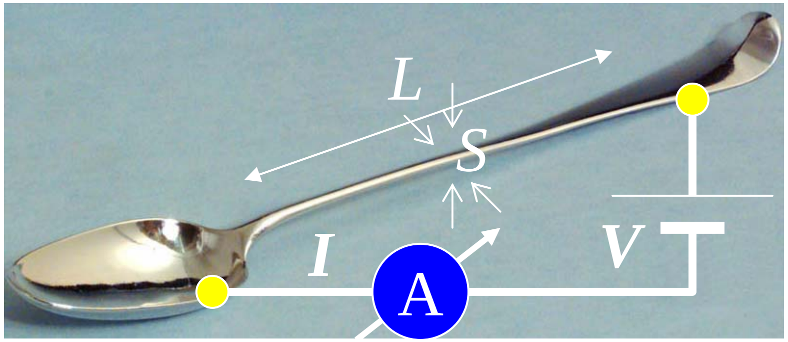
Outline

- Electrical resistance of a quantum wire in the absence of correlations
- Interaction effects: scattering of electron waves off a Friedel oscillation
- An alternative view– the flow of electron fluid
- Dealing with many modes: spin-charge separation, drag effect, etc
- 1D superconductors and magnets, quantum dots, and other applications

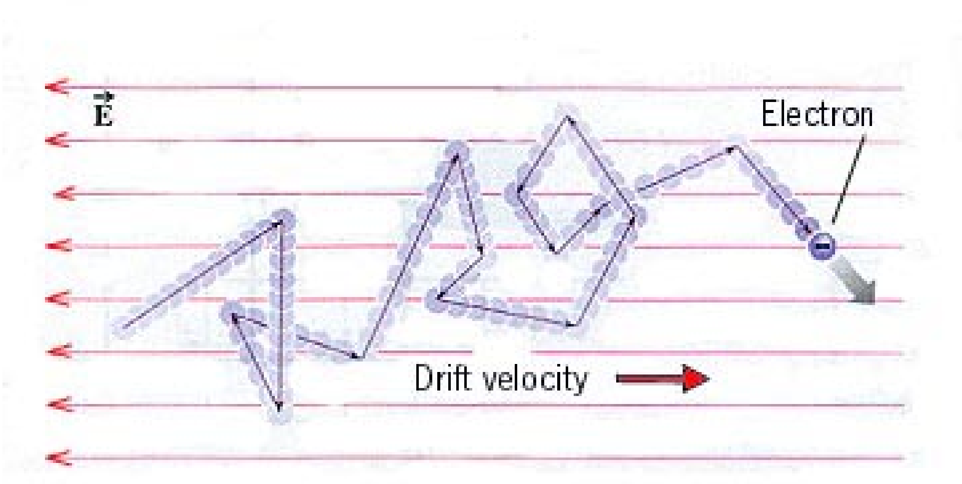
Resistance, Conductance, Conductivity



Ohm's law: $V = IR$

Conductance: $G = 1/R = \sigma \cdot S/L$

Metals–high conductivity [Cu: $\sigma \sim 10^8 \, (\Omega \cdot m)^{-1}$]



Drude conductivity:

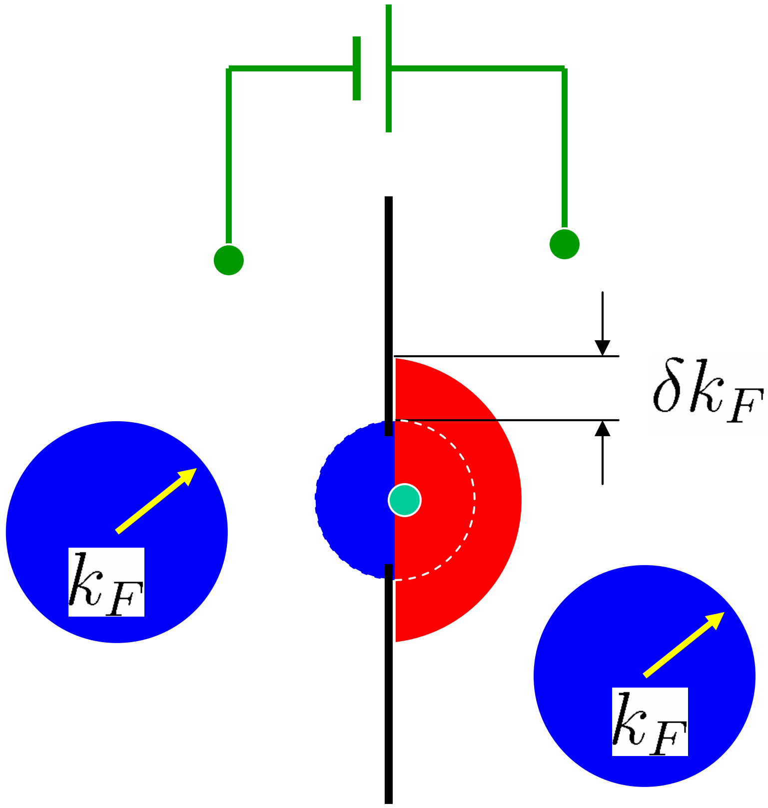
$$\sigma = \frac{n_e e^2 \tau}{m_e}$$

Ballistic Electron Conductance

Conductance: $G \propto S/L$

Does it always hold？

Point contact (Sharvin,1965)



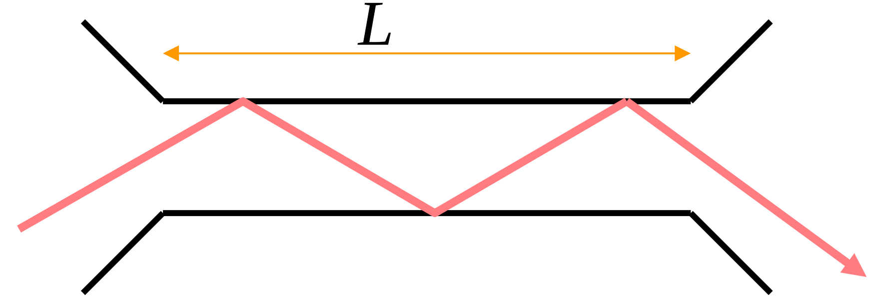
$$\delta k_F = eV/\hbar v_F$$

$$\delta n \sim k_F^2 \cdot \delta k_F$$

$$I \sim e v_F \delta n S$$

$$G = \frac{e^2}{\hbar} \frac{S k_F^2}{4\pi^2}$$

Ballistic channel - same thing



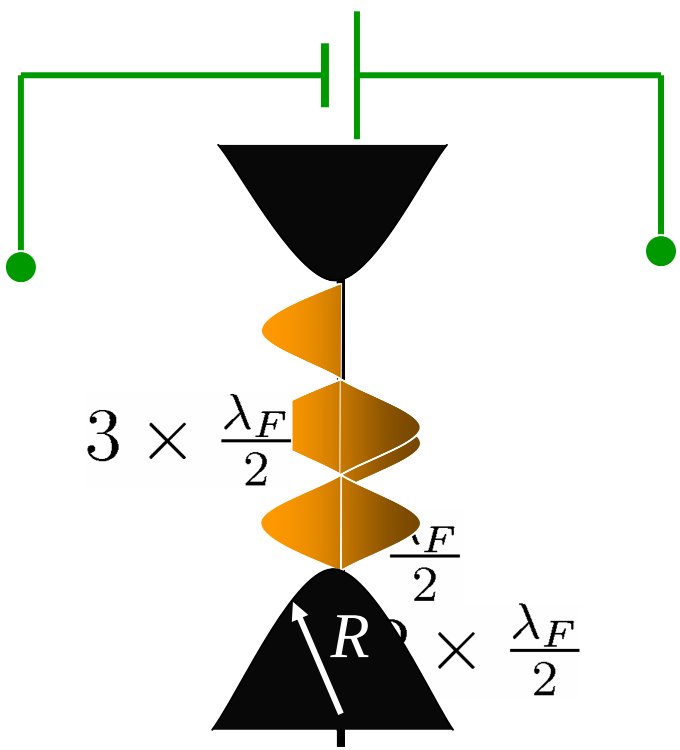
$G = \frac{e^2}{\hbar} \frac{S k_F^2}{4\pi^2}$, independent of L

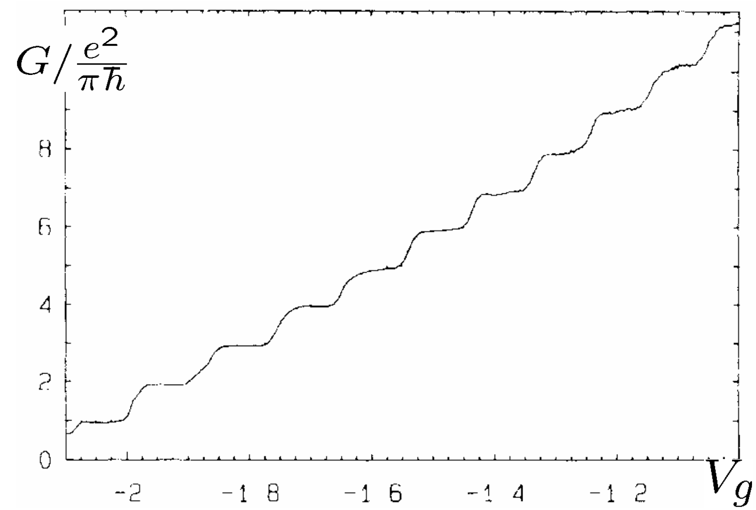
Quantum Ballistic Electron Conductance

Conductance: $G \propto S$

Does it always hold ?

Quantum point contact
(van Wees et al,1988)

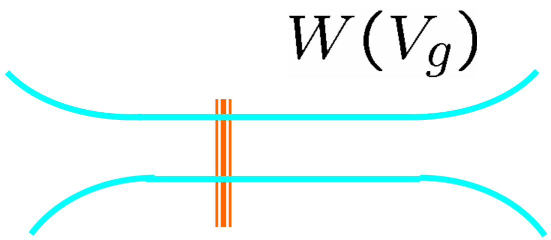




$R \gtrsim \lambda_F$ is crucial for the conductance quantization
(L.G.,Lesovik,Khmelnitskii,Shekhter 1988)

$$G = \frac{e^2}{\pi\hbar}\frac{Wk_F}{\pi} \longrightarrow G = \frac{e^2}{\pi\hbar}N$$

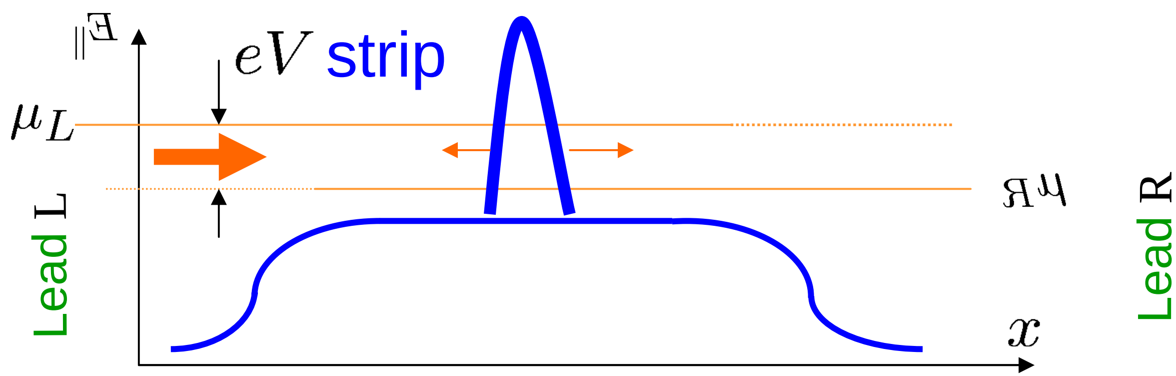
Conductance of a 1D channel, free electrons



Ideal, adiabatic channel:
quantized conductance

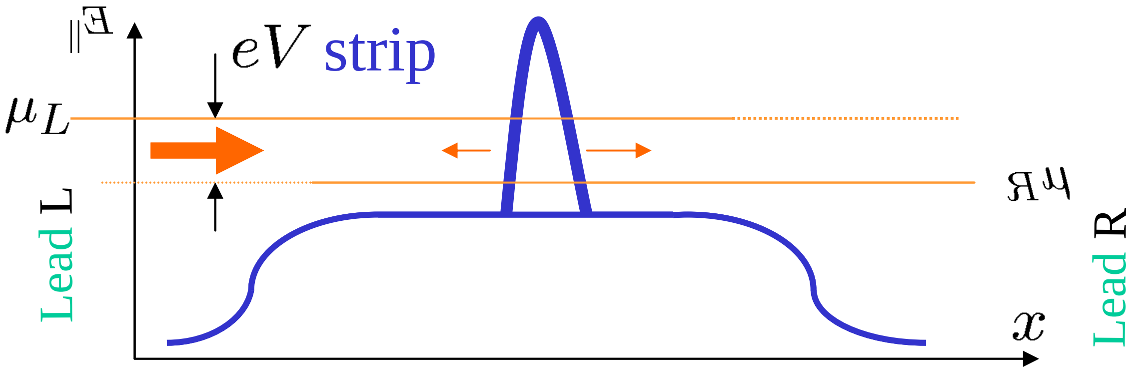
~~Ballistic~~ conductance (~~no~~ with scatterers) is less than

$\frac{e^2}{2\pi\hbar}$ per mode per spin



$$I = e\int \frac{dp}{2\pi\hbar} v(p) = \frac{e}{2\pi\hbar}\int_{\text{strip } eV} dE = \frac{e}{2\pi\hbar}(\mu_L - \mu_R) = \frac{e^2}{2\pi\hbar}V$$

Conductance of a 1D channel, free electrons



$$I = \frac{e}{2\pi\hbar} \int_{\text{strip } eV} T_0(E)\,dE$$

$T_0(E)$ - transmission coefficient of the barrier

Current = sum of partial currents at different energy "slices"

For each "slice" $[E, E + dE]$, the partial current depends on T_0 at the same energy E only.

The linear conductance

$$G = \frac{e^2}{2\pi\hbar} T_0(E_F)$$

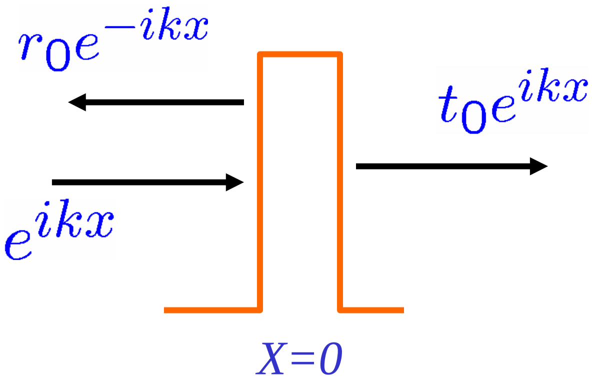
(Landauer formula).

Friedel oscillation (Friedel, 1952)

Reflection at the barrier changes all electron states, including those with energy $E<E_{F.}$

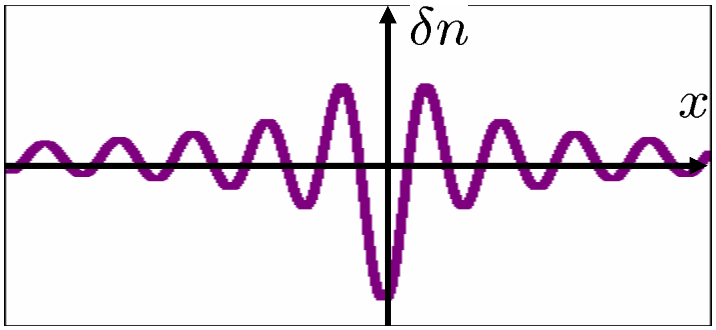
r_0 - reflection amplitude

t_0 - transmission amplitude



$$x<0$$
$$\psi_k(x) \sim \left(e^{ikx} + r_0 e^{-ikx}\right)$$

$$\delta n(x) \sim -\frac{|r_0|}{|x|}\sin[2k_F|x| - \delta], \quad |x| \gg \lambda_F$$



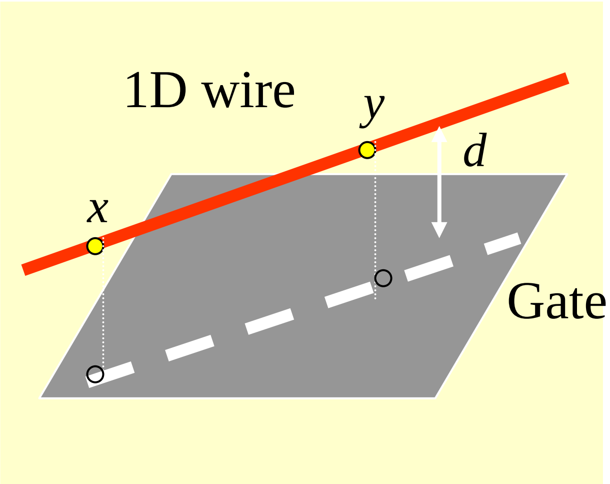
Friedel oscillation: Hartree potential

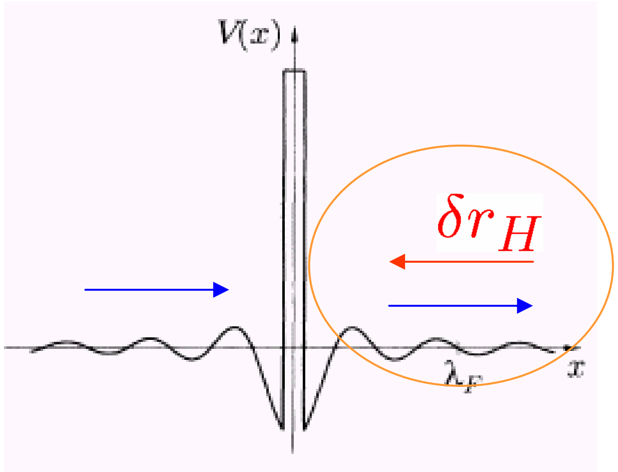
Hartree potential

$V_H(x) = \int dx_1 V(x-x_1)\delta n(x_1)$

oscillates with the period $\lambda_F/2$; at $|x| \gg d$

$$V_H(x) \sim |r_0| V(2k_F) \frac{\sin[2k_F|x| - \delta]}{|x|}$$





Scattering off the Friedel oscillation:

$$\delta r_H \sim r_0 \frac{V(2k_F)}{\hbar v_F} \int_d^\infty dx e^{2ikx} \frac{\sin(2k_F x)}{x}$$

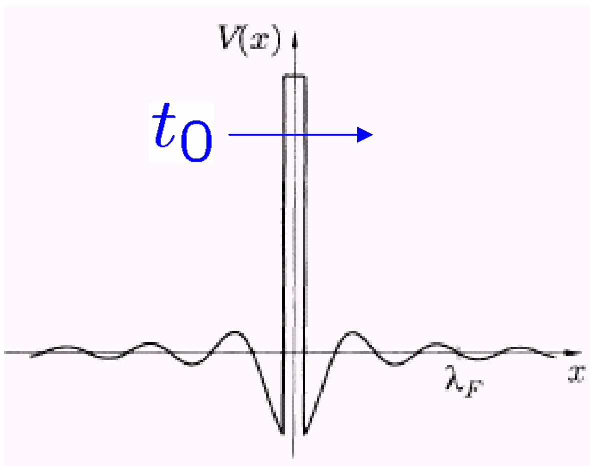
$$\sim r_0 \frac{V(2k_F)}{\hbar v_F} \ln \frac{1}{|k_F - k| d}$$

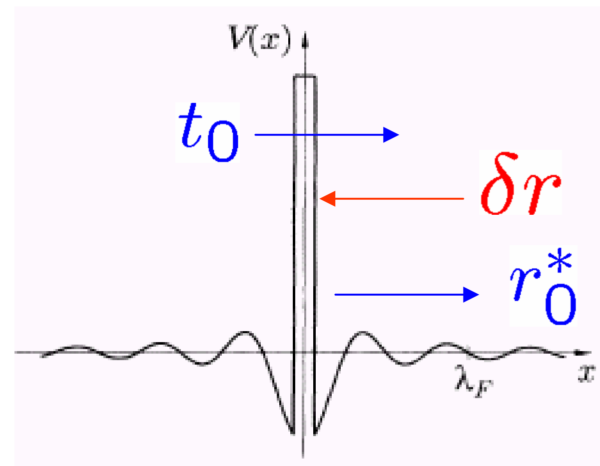
Exchange contribution — similar

Transmission modified by the Friedel oscillation

Transmission coefficient of a "composite" barrier:

$$T = T_0 + 2T_0 \mathrm{Re}(r_0^* \delta r)$$





correction to T_0:

Hartree

exchange

$$\delta T = T_0(1 - T_0)\frac{V(2k_F) - V(0)}{\pi\hbar v_F} \ln \frac{1}{|k_F - k|d}$$

First-order interaction correction to the transmission coefficient

Transmission coefficient becomes **energy-dependent** :

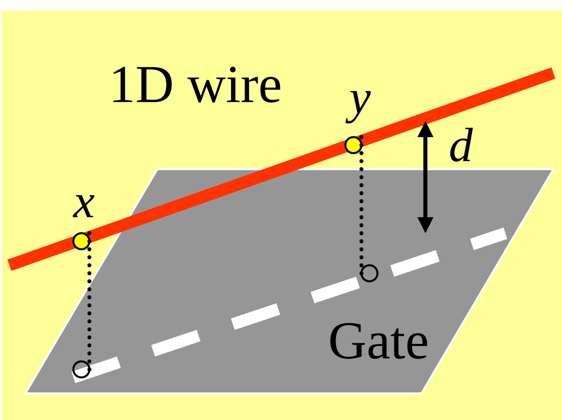
$$\delta T(\varepsilon) = -2\alpha T_0(1-T_0)\ln\left|\frac{D_0}{\varepsilon}\right|$$

$$\varepsilon = \hbar v_F(k-k_F)$$

$$D_0 = \hbar v_F/d$$

$$\alpha = \frac{1}{2\pi\hbar v_F}[V(0)-V(2k_F)]$$

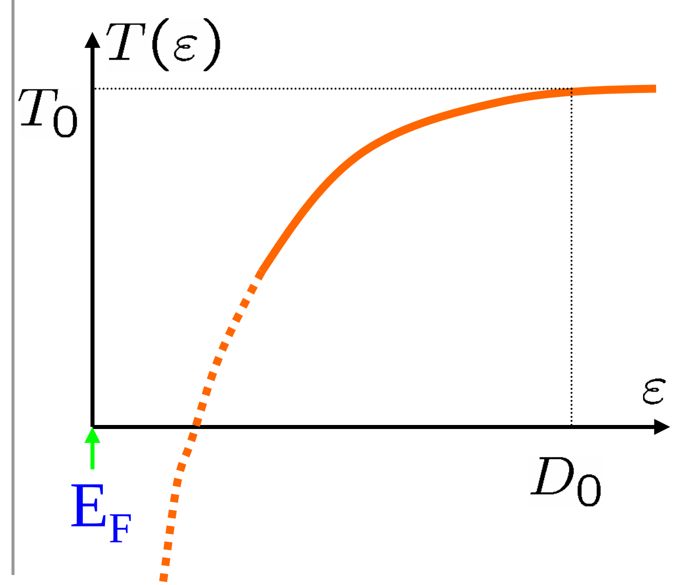
suppression of the transmission (from $V(0)$); enhancement of the transmission (from $V(2k_F)$)



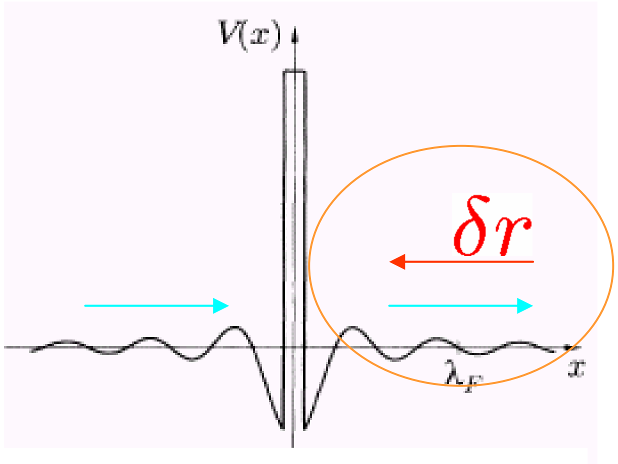
$$\alpha \approx \frac{e^2}{\hbar v_F}\ln(k_F d)$$

at $k_F d \gg 1$

The first-order correction diverges at low energies:



Cure: the leading—logarithm approximation



$$\delta r \sim r_0 \alpha \int_d^{\infty} \frac{dx}{x} \sin(2k_F x) e^{2ikx}$$

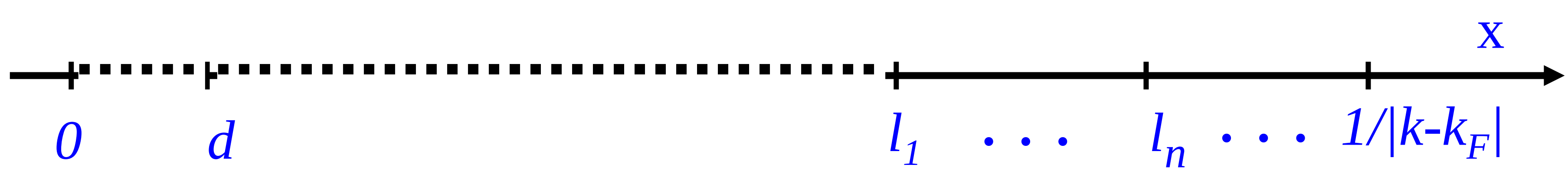
$$= \underbrace{r_0 \alpha \int_d^{\frac{1}{|k-k_F|}} \frac{dx}{x} \sin(2k_F x) e^{2ikx}} + r_0 \alpha \int_{\frac{1}{|k-k_F|}}^{\infty} \frac{dx}{x} \sin(2k_F x) e^{2ikx}$$

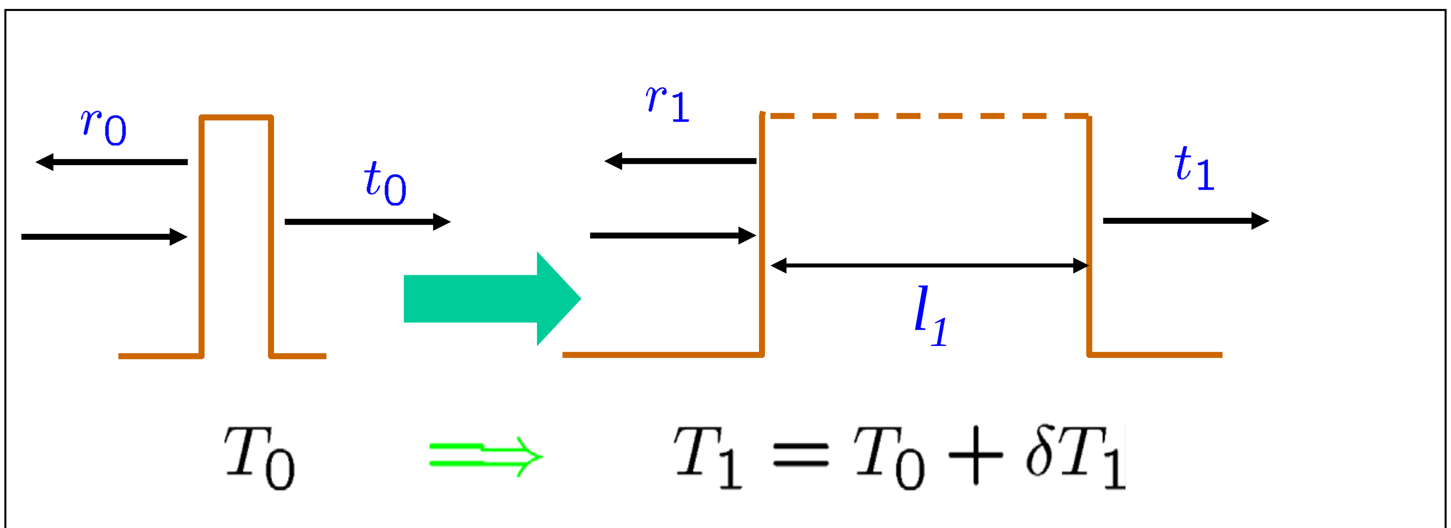
$$\propto \alpha \ln \frac{1}{|k - k_F| d} = \alpha \ln \frac{D_0}{\varepsilon}$$

Leading—log: sums up the most divergent terms, $\left[\alpha \ln \left|\frac{D_0}{\varepsilon}\right|\right]^n$, of the perturbation theory

Real-space RG

Split the important interval $[d,\ 1/|k-k_F|]$ on smaller pieces, so that $l_n - l_{n-1} \gg d$, but $\alpha \int_{l_{n-1}}^{l_n} dx/x \ll 1$





Transmission in the leading-log. approximation

RG equation:

$$T_n - T_{n-1} = -2\alpha T_{n-1}(1 - T_{n-1}) \ln \frac{l_n}{l_{n-1}}$$

$$1 \leq n \leq D_0/|\varepsilon|$$

Solution of the RG equation:

$$T(\varepsilon) = \frac{T_0 \left|\frac{\varepsilon}{D_0}\right|^{2\alpha}}{R_0 + T_0 \left|\frac{\varepsilon}{D_0}\right|^{2\alpha}} \qquad R_0 \equiv 1 - T_0$$

$$\alpha = \frac{V(0) - V(2k_F)}{2\pi\hbar v_F} \ll 1$$

$$\varepsilon = \hbar v_F (k - k_F)$$

$$D_0 = \hbar v_F / d$$

After the divergency is cured

Sum up the most divergent terms, $\alpha^n \left\{\ln\left|\frac{D_0}{\varepsilon}\right|\right\}^n$, of the perturbation theory

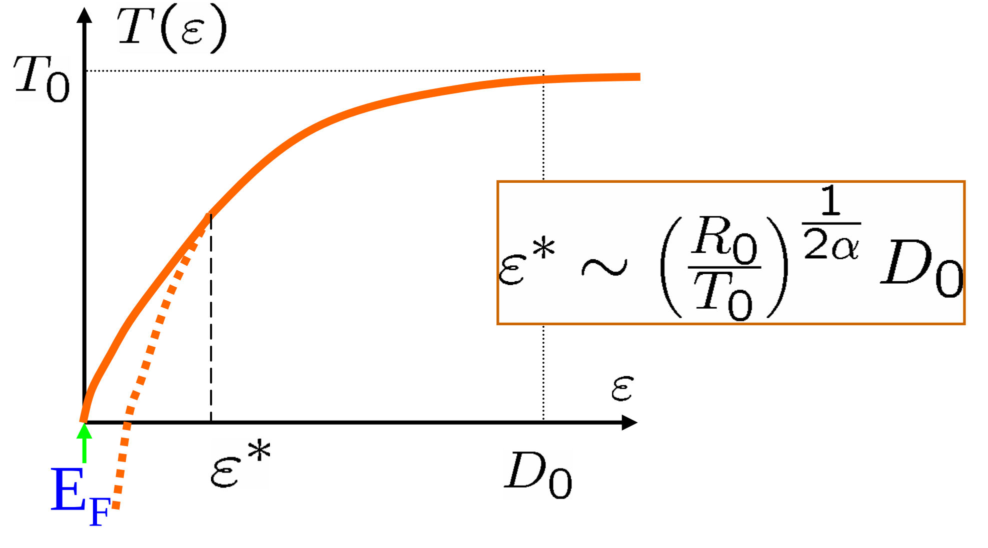
Use Renormalization Group (RG) technique for book-keeping

$$T(\varepsilon) = \frac{T_0 \left|\frac{\varepsilon}{D_0}\right|^{2\alpha}}{R_0 + T_0 \left|\frac{\varepsilon}{D_0}\right|^{2\alpha}}$$

$$R_0 \equiv 1 - T_0$$

Matveev, Yue, L.G. 1993

The leading-log result does not diverge



Conductance in the leading-log. approximation

scattering remains elastic ➡ Landauer formula works

$$G(k_BT) = \frac{e^2}{2\pi\hbar}\int d\varepsilon \left(-\frac{df_F}{d\varepsilon}\right) T(\varepsilon)$$

Within log-accuracy:

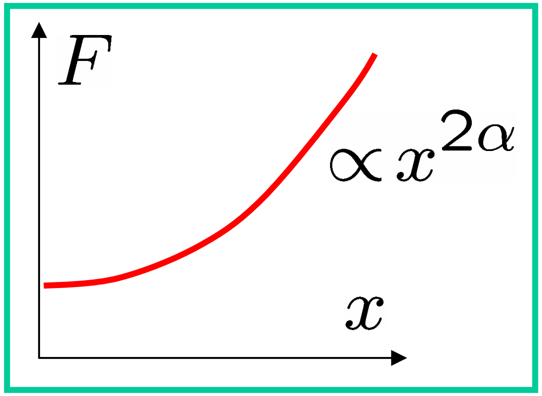
$$G(k_BT) = \frac{e^2}{2\pi\hbar}\frac{T_0\left|\frac{k_BT}{D_0}\right|^{2\alpha}}{R_0 + T_0\left|\frac{k_BT}{D_0}\right|^{2\alpha}}$$

At low energies

$$\left[k_BT, eV \ll \varepsilon^* \sim \left(\frac{R_0}{T_0}\right)^{\frac{1}{2\alpha}} D_0\right]$$

$$\boxed{\frac{dI}{dV} = (k_BT)^{2\alpha} F\left(\frac{eV}{k_BT}\right)}$$

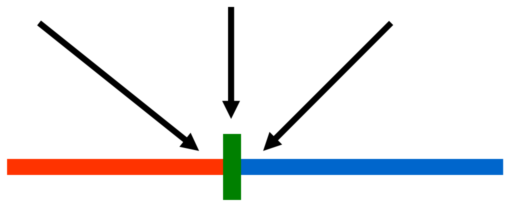
scaling $F(x)$



Effects of interaction – Friedel oscillation picture

1. Tunneling across a barrier is modified, $\left.\frac{dI}{dV}\right|_{T=0} \propto (eV)^{2\alpha}$

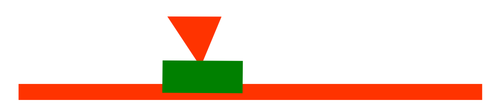
$$\frac{dI}{dV} \propto \nu_{\text{end}}(\varepsilon) \cdot T_0 \cdot \nu_{\text{end}}(\varepsilon) \propto T_0 \cdot \varepsilon^{\alpha} \cdot \varepsilon^{\alpha}\big|_{\varepsilon=eV} \propto (eV)^{2\alpha}$$



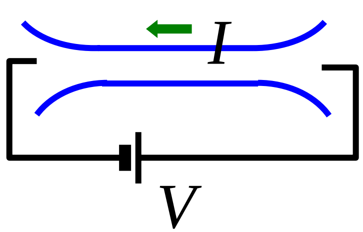
"Edge" tunneling density of states: $\nu_{\text{end}}(\varepsilon) \propto \varepsilon^{\alpha}$

2. No barrier ⇨ no Friedel oscillation; properties of an ideal 1D channel are not modified **?**

"Bulk" tunneling density of states: $\nu_{\text{bulk}}(\varepsilon) \propto \nu_0$



$I = GV$



Two-terminal conductance remains quantized $G = \frac{e^2}{2\pi\hbar}$

1D electron liquid: phenomenology

Dynamical variable:
displacement of a unit 1D volume $u(x,t)$

Lagrangian:

$$\delta L = \delta K - \delta U = \frac{1}{2} n_0 m \left(\frac{\partial u}{\partial t}\right)^2 - \frac{1}{2}\frac{\partial \mu}{\partial n} n_0^2 \left(\frac{\partial u}{\partial x}\right)^2$$

Kin. energy   Potential energy

External field: $\delta U_{\text{ext}} = e n_0 E(x,t) u$

Wave equation: $$\frac{\partial^2 u}{\partial t^2} - v^2 \frac{\partial^2 u}{\partial x^2} = -\frac{e}{m} E(x,t)$$

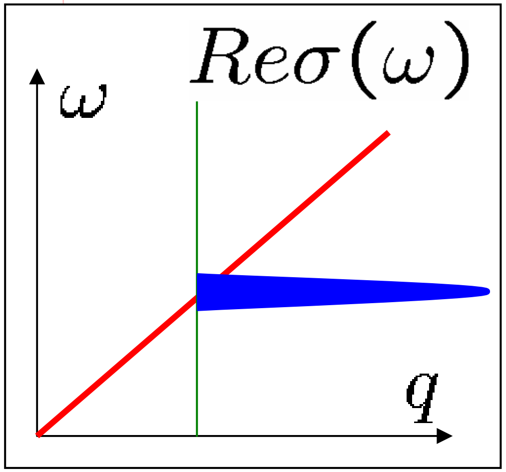
Conductivity & charge waves

$$\frac{\partial^2 u}{\partial t^2} - v^2 \frac{\partial^2 u}{\partial x^2} = -\frac{e}{m} E(x,t)$$

$$v = \left[\frac{n_0}{m}\frac{\partial \mu}{\partial n}\right]^{\frac{1}{2}} = \left[v_F^2 + \frac{2}{\pi} V(0) v_F\right]^{\frac{1}{2}} > v_F$$

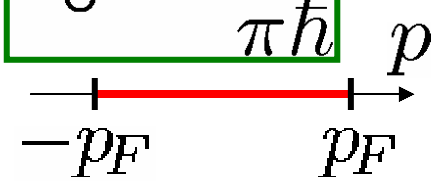
Velocity of 1D plasmon wave

Fermi gas rigidity + Coulomb repulsion



current: $$I(x,t) = e n_0 \frac{\partial u(x,t)}{\partial t}$$

$$n_0 = \frac{p_F}{\pi \hbar}$$



$$\sigma(q,\omega) = \frac{e^2 v_F}{\pi \hbar} \frac{-i\omega}{(qv)^2 - \omega^2 - i0 \cdot \omega}$$

From conductivity to the conductance

Ideal (homogeneous) wire

$$I(x,\omega) \qquad\qquad V = \int_0^L E(x,\omega)$$

$$I(x,\omega) = \int \frac{dq}{2\pi} e^{iqx} \sigma(q,\omega) \int dx_1 e^{-iqx_1} E(x_1,\omega)$$

$$\omega \to 0 \Rightarrow q = \omega / v \to 0 \ \text{[pole in } \sigma(q,\omega)\text{]}$$

$$\lim_{\omega \to 0} I(x,\omega) = I$$ independent of x

$$\frac{I}{V} = G = \frac{e^2}{2\pi\hbar} g; \ g = \frac{v_F}{v}$$ (C.L. Kane, M.P.A. Fisher, 1992)

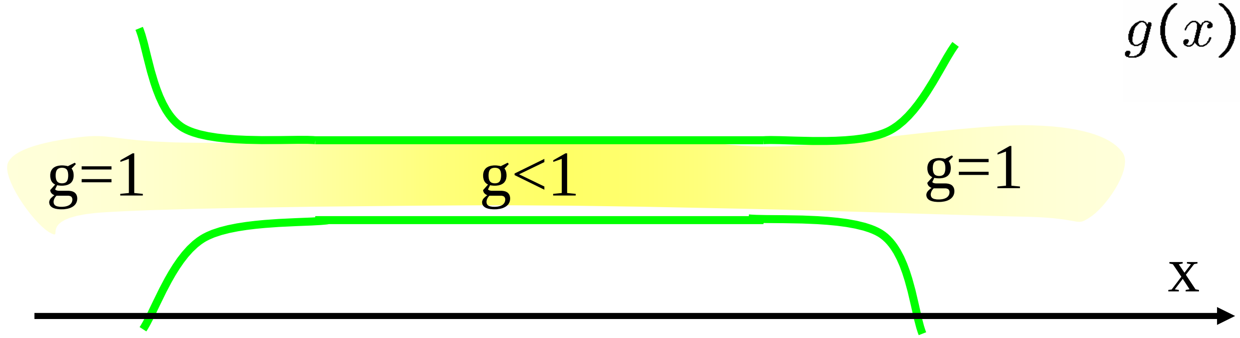
Conductance G☎◆➂=emission of plasmon waves of wavelength

Conductance of a finite channel

Dissipative conductance=emission of plasmons of wavelength ~v/ω.

Finite-length channel: *g(x)* slowly varies in space

$$g(x) = \frac{v_F}{v(x)}$$



$$G(\omega) = \frac{e^2}{2\pi\hbar} g(x \sim v/\omega)$$

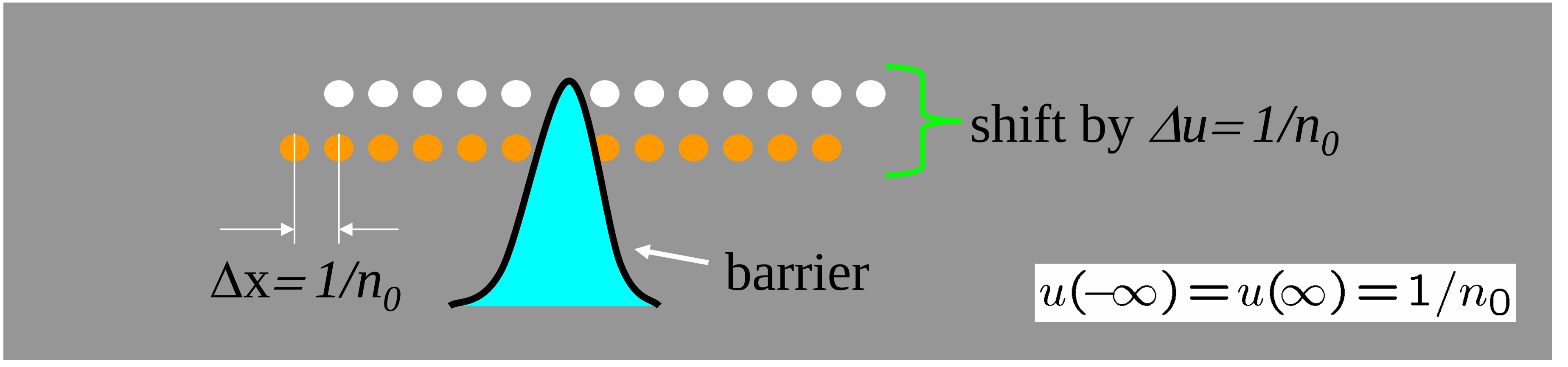
Outside the channel: $g(x \to \pm\infty) \to 1 \Rightarrow$ $G_{\text{dc}} = \frac{e^2}{2\pi\hbar}$

regardless the interaction strength within the channel

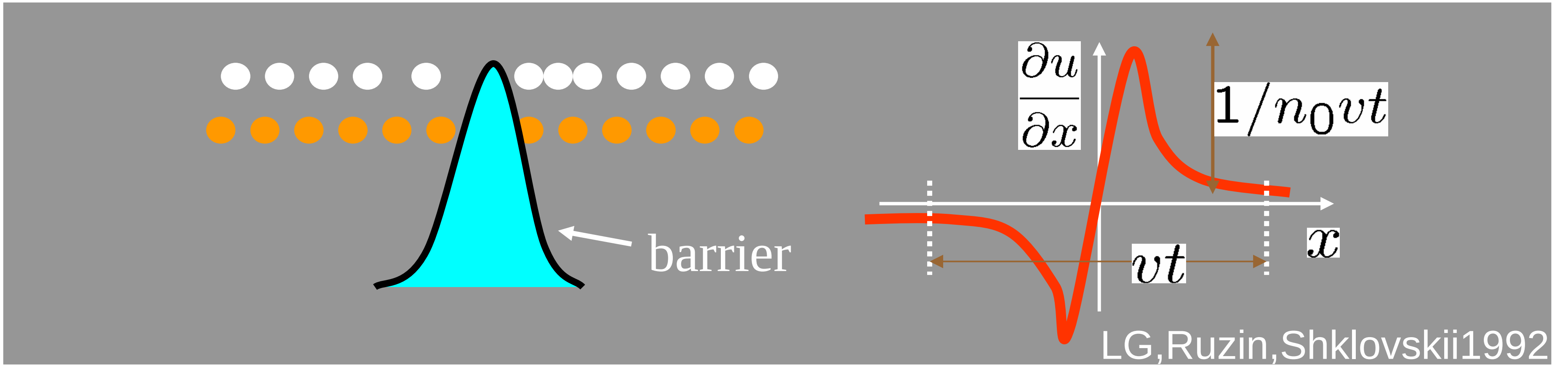
(Matveev, L.G. 1993; D.Maslov, M.Stone; I.Safi, H.Schulz; D.Ponomarenko, 1995)

Tunneling across a barrier

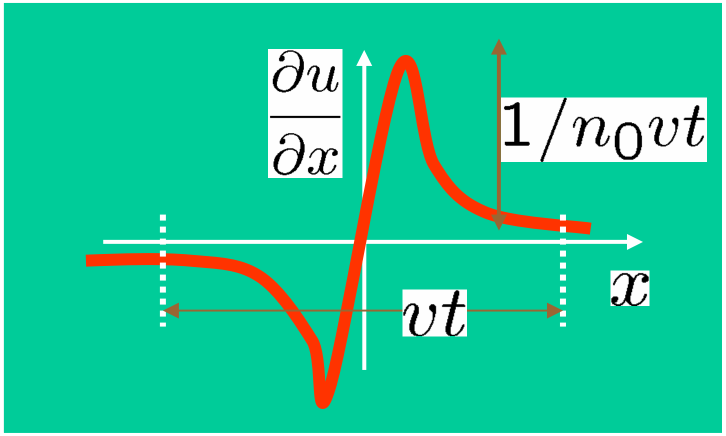
A barrier reveals the discreteness of the fermions; the Hamiltonian must be invariant only under the discrete shifts



Each particle changed state – zero overlap of the old and new ground states (Orthogonality Catastrophe, Anderson 1967)



Tunneling amplitude



Energy deficit:

$$E(t) \propto \int_{-v|t|}^{v|t|} dx \left(\frac{\partial u}{\partial x}\right)^2 \propto \frac{v}{v_F|t|}$$

WKB tunneling action:

$$S(t) \sim \int_{t_0}^{t} d\tau E(\tau) \sim i\frac{1}{g}\ln\frac{|t|}{t_0}$$

Tunneling amplitude:

$$A(\varepsilon) \propto \int dt e^{-i\varepsilon t} \exp\left[iS(t)\right] \propto |\varepsilon|^{\frac{1}{g}-1}$$

Tunneling rate:

$$T(\varepsilon) \propto |\varepsilon|^{2\left(\frac{1}{g}-1\right)}$$

$$g = v_F/v$$

Weak interaction:

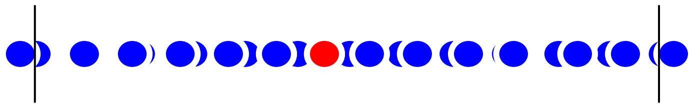
$$\frac{1}{g} - 1 \to \alpha$$

fits perturbation theory

$$\alpha = V(0)/2\pi\hbar v_F$$

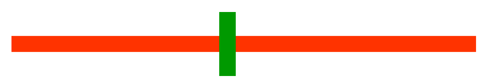
Tunneling density of states

New particle ⇨ a finite shift of the liquid

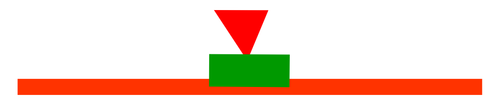


$$-u(-\infty)=u(\infty)=1/2n_0$$

$$\nu_{\text{end}} \propto |\varepsilon|^{[\frac{1}{g}-1]}$$



$$\nu_{\text{bulk}} \propto |\varepsilon|^{\frac{1}{2}(g+\frac{1}{g}-2)}$$



$$g = v_F/v < 1$$

C.L. Kane, M.P.A. Fisher (1992)

$$\nu_{\text{bulk}}(\varepsilon) = -\frac{1}{\pi}\text{Im}G_R(x,x,\varepsilon)$$

Earlier related work:
Luther & Peschel;
Luther & Emery (1974)

$$G_+(x,t) = \frac{1}{2\pi}\frac{1}{x - v_F t + i\delta}\left[\frac{x - v_F t + i\lambda}{x - vt + i\lambda}\right]^{\gamma}$$

$$\times\left[(x - vt + i\lambda)(x + vt - i\lambda)/\lambda^2\right]^{-\gamma}$$

Dzyaloshinskii & Larkin (1973)

$t > 0;\ \gamma = (g-1)^2/8g$

Many modes – many fluids

Example: spin&charge $u_{\uparrow,\downarrow} = u_\rho \pm u_s$

rigidity: Fermi+Coulomb

u_ρ +-+-+-+-+-+-+-+-+-+-

rigidity: Fermi only

u_s ↑↓↑↓↑↓↑↓↑↓↑↓↑↓↑↓

$v_\rho > v_F$

$v_s = v_F$

Tunneling density of states

Inserting electron: finite shift $\Delta u = 1/n_0$

Many (N>>1) modes: $\Delta u = \sum \Delta u_i$

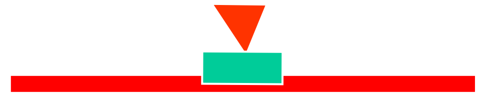
$\nu(\varepsilon) \propto |\varepsilon|^\alpha$ $\alpha \propto \sum (\Delta u_i)^2 \propto 1/\sqrt{N}$

LG, Matveev 93

Carbon nanotubes – tunneling experiments

Single-wall nanotubes – 4-mode (incl. spin) Luttinger liquids

tunneling density of states:



+ data scaling

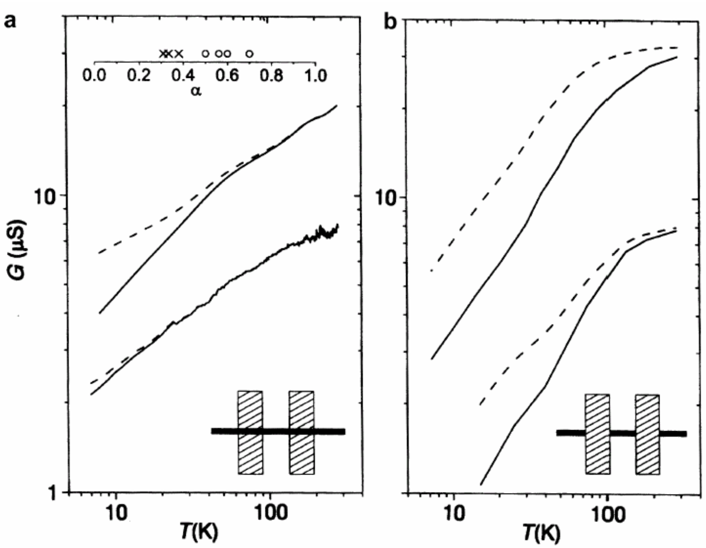
$$\nu_{\text{end}} \propto |\varepsilon|^{\alpha_{\text{end}}}$$

$$\nu_{\text{bulk}} \propto |\varepsilon|^{\alpha_{\text{bulk}}}$$

$$\frac{dI}{dV} = V^{\alpha} f\left(\frac{V}{T}\right)$$

$$\alpha_{\text{end}} = \frac{1}{4}\left[\frac{1}{g} - 1\right]$$

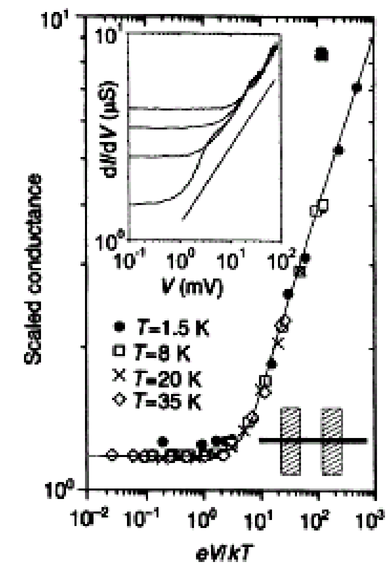
$$\alpha_{\text{bulk}} = \frac{1}{8}\left[g + \frac{1}{g} - 2\right]$$



$$\alpha_{\text{bulk}} \approx 0.3$$

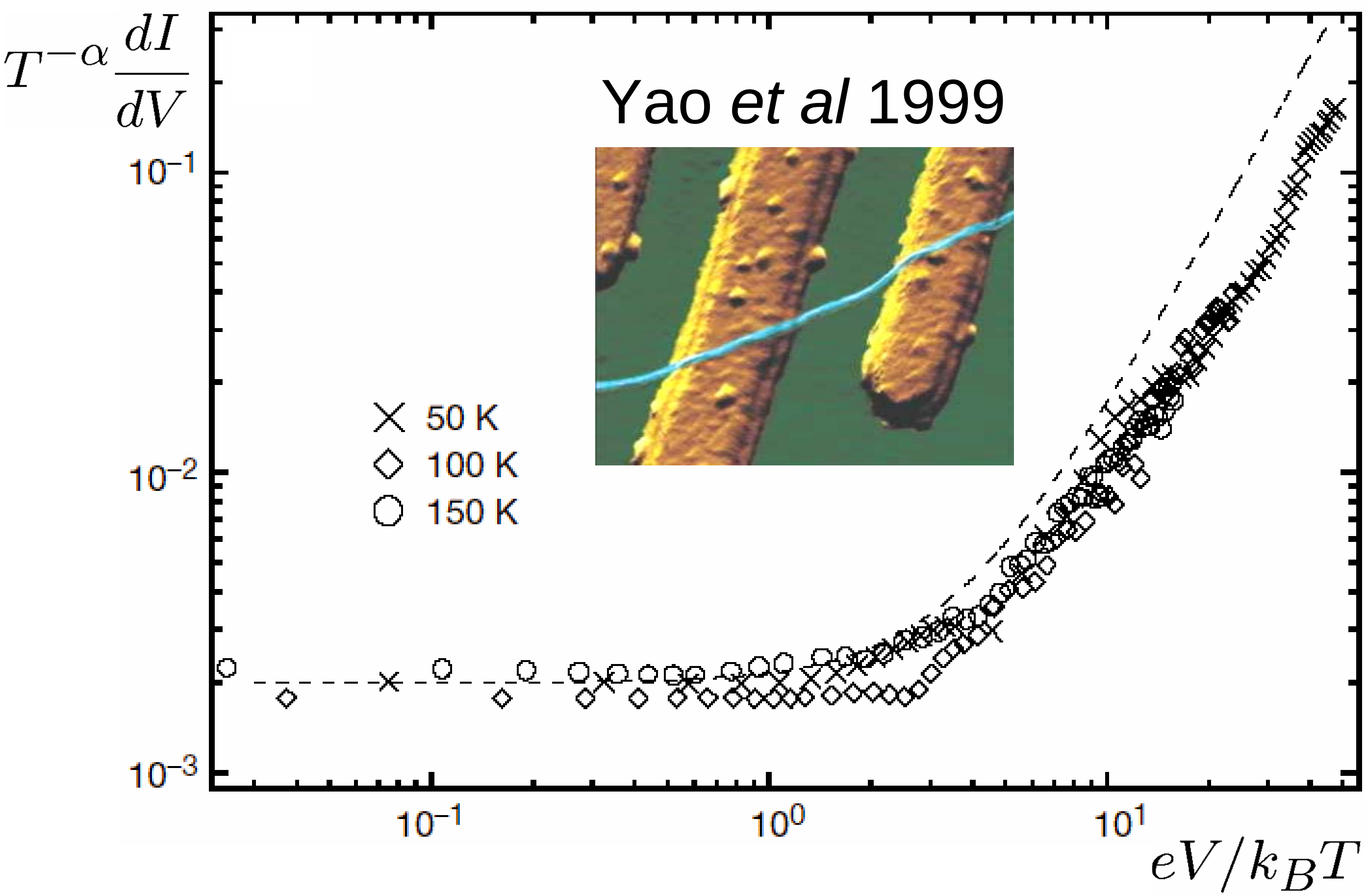
$$\alpha_{\text{end}} \approx 0.6$$

Bockrath *et al* 1999



Carbon nanotubes – tunneling experiments

corroborating experiment



$$\alpha_{\text{end}} \approx 2\alpha_{\text{bulk}}$$

corresponds to $g \approx 0.22$

caution…

Carbon nanotubes – tunneling experiments

In a multi-wall nanotube dI/dV is also a power-law…

…instead of a different function (incl. disorder):

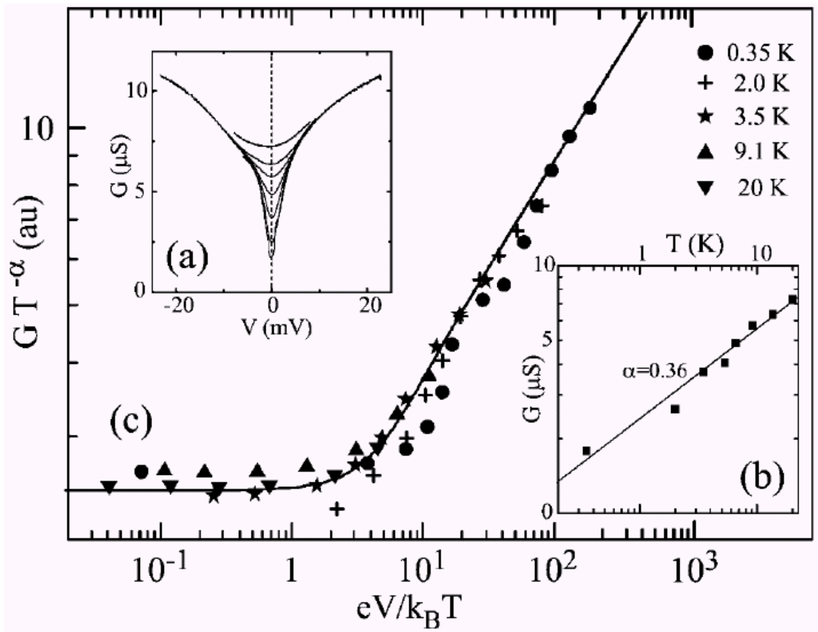


FIG. 2. (a) $G(V,T = \text{const}) = dI/dV$ of a second MWNT for $T = 0.35,\ldots,20$ K. (b) The linear conductance $G(0,T)$ in a double logarithmic plot demonstrating power-law scaling. (c) $G(V,T)T^{-\alpha}$ versus eV/k_BT. Similar to the T dependence, $G \propto V^{\alpha}$ for $eV \gg k_BT$ with power $\alpha = 0.36$.

$$\nu(\epsilon, T) \propto \exp\left\{-\sqrt{\frac{\epsilon^*}{T}}\, F\left(\frac{\epsilon}{\sqrt{\epsilon^* T}}\right)\right\}$$

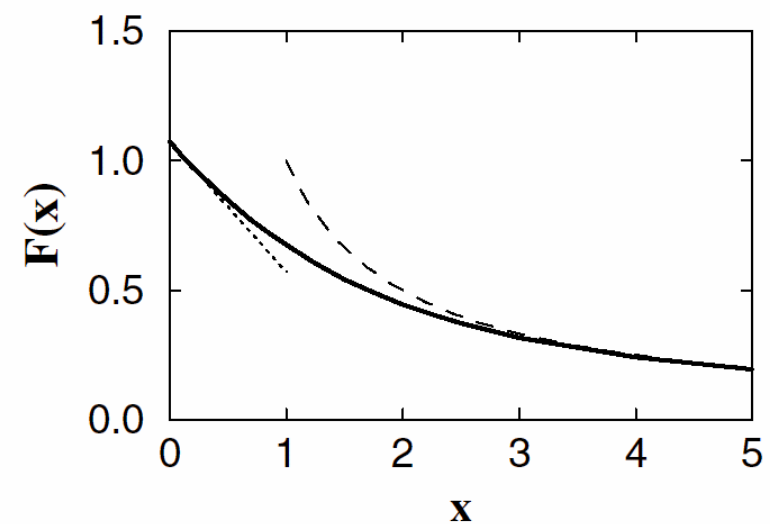


FIG. 1. The scaling function $F(x)$ and its asymptotics: $F(x) = 1.07 - x/2$ for $x \ll 1$ (dotted line), and $F(x) \sim 1/x$ for $x \gg 1$ (dashed line).

[Bachtold et al (2001)]

Easy to confuse with …

Mischenko, Andreev, L.G. (2001)

Zero-bias anomaly
(aka “dynamic Coulomb blockade”)

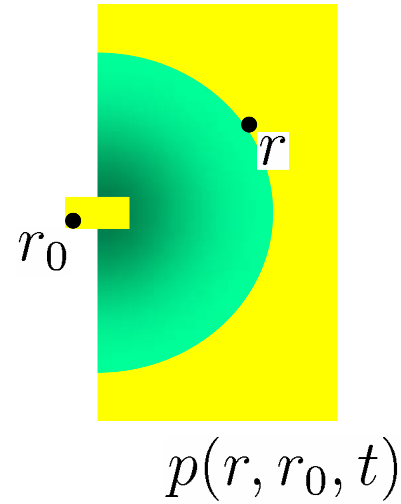
$$\frac{\delta\nu(\varepsilon, r_0)}{\nu_0} \propto -\int_{\varepsilon/\hbar}^{\infty} \frac{d\omega}{\pi\hbar} \mathrm{Im} \int d^3r d^3r' p(r_0, r, \omega) U(r, r', \omega) p(r', r_0, \omega)$$

Altshuler, Aronov 1979

Two-point impedance:

$$e^2 z(r_0, r', \omega) = \int d^3r\, p(r_0, r, \omega) U(r, r', \omega)$$

meaning: $z(r_0, r', \omega) = V(r', \omega)/I(r_0, \omega)$



Propagation probability

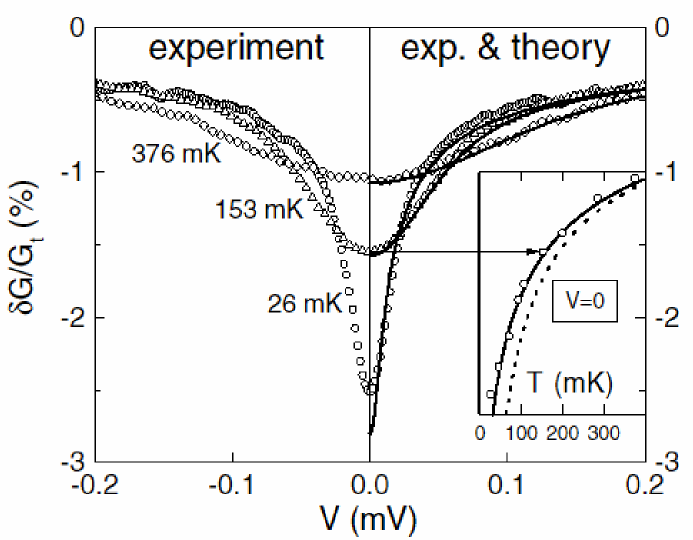
$$Z_{\mathrm{eff}}(r_0, \omega) = \frac{\int d^3r\, z(r_0, r', \omega) p(r', r, \omega)}{\int d^3r\, p(r_0, r, \omega)}$$

$1/i\omega$

Zero-bias anomaly
(aka “dynamic Coulomb blockade”)

$$\frac{\delta\nu(\varepsilon, r_0)}{\nu_0} = -\frac{2e^2}{h}\int_{\varepsilon/\hbar}^{\infty}\frac{d\omega}{\omega}\mathrm{Re}Z_{\mathrm{eff}}(r_0,\omega)$$

A ref.: Pierre et al, PRL2001



Low ω : real and constant Z_{eff}

$$\frac{\delta\nu(\varepsilon, r_0)}{\nu_0} \propto -\frac{2e^2}{h}Z_{\mathrm{eff}}\ln\frac{\varepsilon_0}{\varepsilon} \quad \rightarrow \quad \alpha$$

In many cases, **exponentiation is possible**

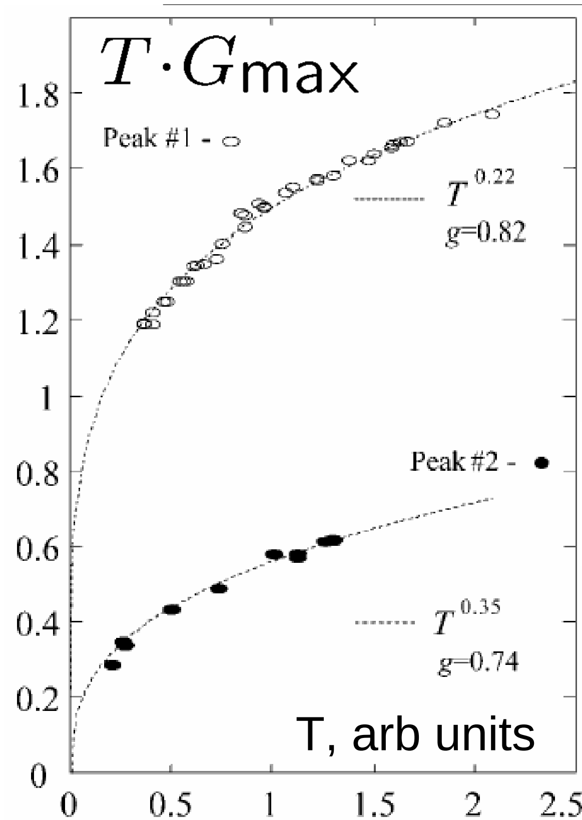
$$Z_{\mathrm{eff}}^{\mathrm{end}} = 2Z_{\mathrm{eff}}^{\mathrm{bulk}}$$

Dangerous resemblance to a Luttinger liquid behavior

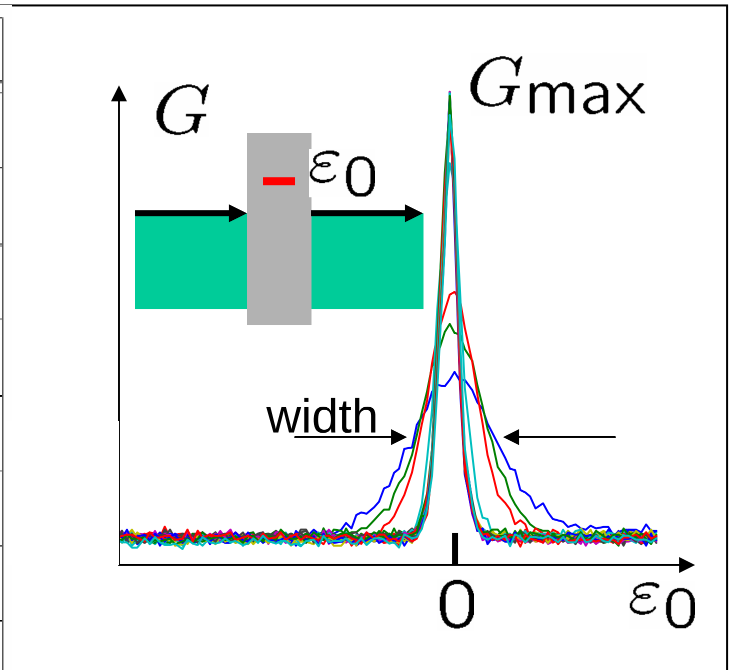
$$\alpha_{\mathrm{end}} = 2\alpha_{\mathrm{bulk}}$$

More tunneling experiments

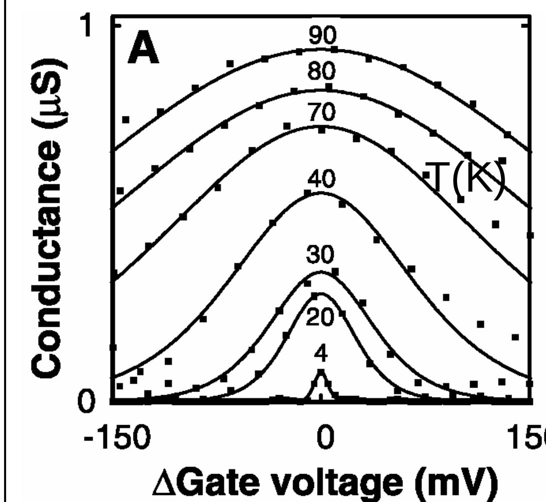
resonant tunneling



Yacoby 2000



Dekker 2001



no

yes

? ?

Theory:

1. $T \gg \Gamma_0$: width $\propto T$, $T \cdot G_{max} \propto T^{\alpha_{end}}$
(Furusaki 1998)

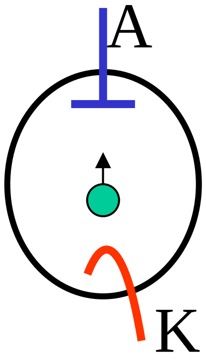
2. $T \ll \Gamma_0$: (CL Kane, MPA Fisher 1992)

3. Weak interaction: full lineshape, any T; (Nazarov,LG 2003; Gornyi,Polyakov 2003)

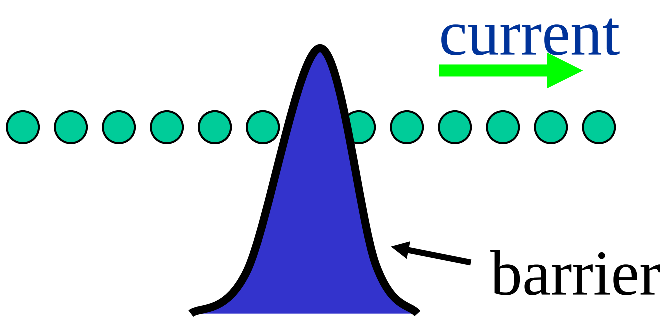
Current noise

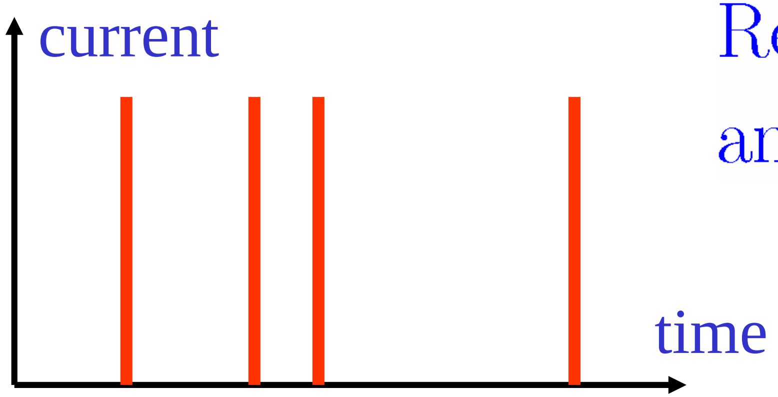
Charge discreteness leads to shot noise in current

Vacuum diode



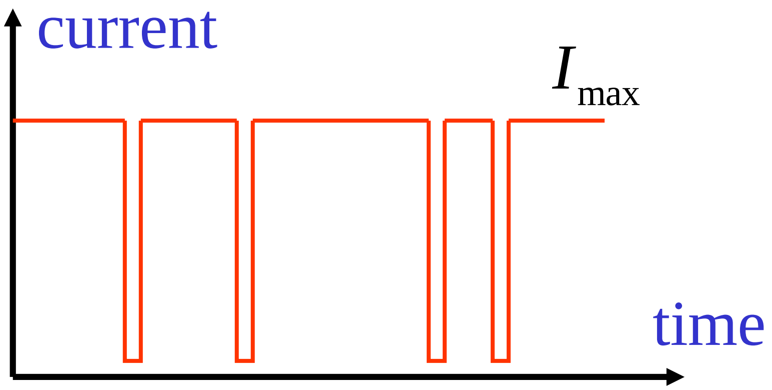
Rare tunneling events

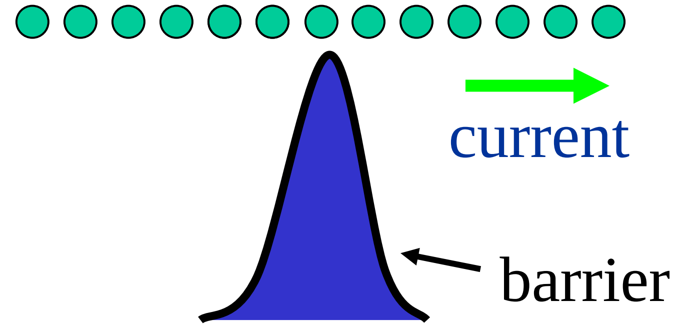




Relation between noise power and current: $S_I(\omega = 0) = e\langle I \rangle$

Current noise





Backscattered current: $I_{back} = I_{max} - I$

Current noise power: $S_I \propto \langle \hat{I}_{back} \cdot \hat{I}_{back} \rangle$

Average "backscattered current" $\langle \hat{I}_{back} \rangle = I_{max} - \langle I \rangle.$

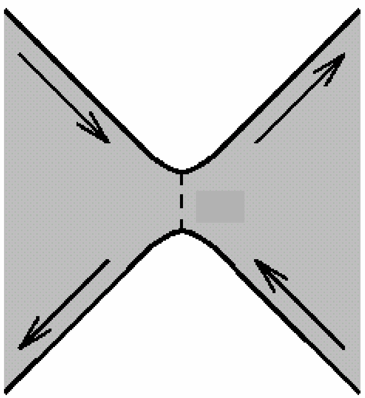
Relation between noise power and current:

$$S_I(\omega = 0) = (ge) \langle I_{back} \rangle$$

[C.L. Kane, M.P.A. Fisher (1994)]

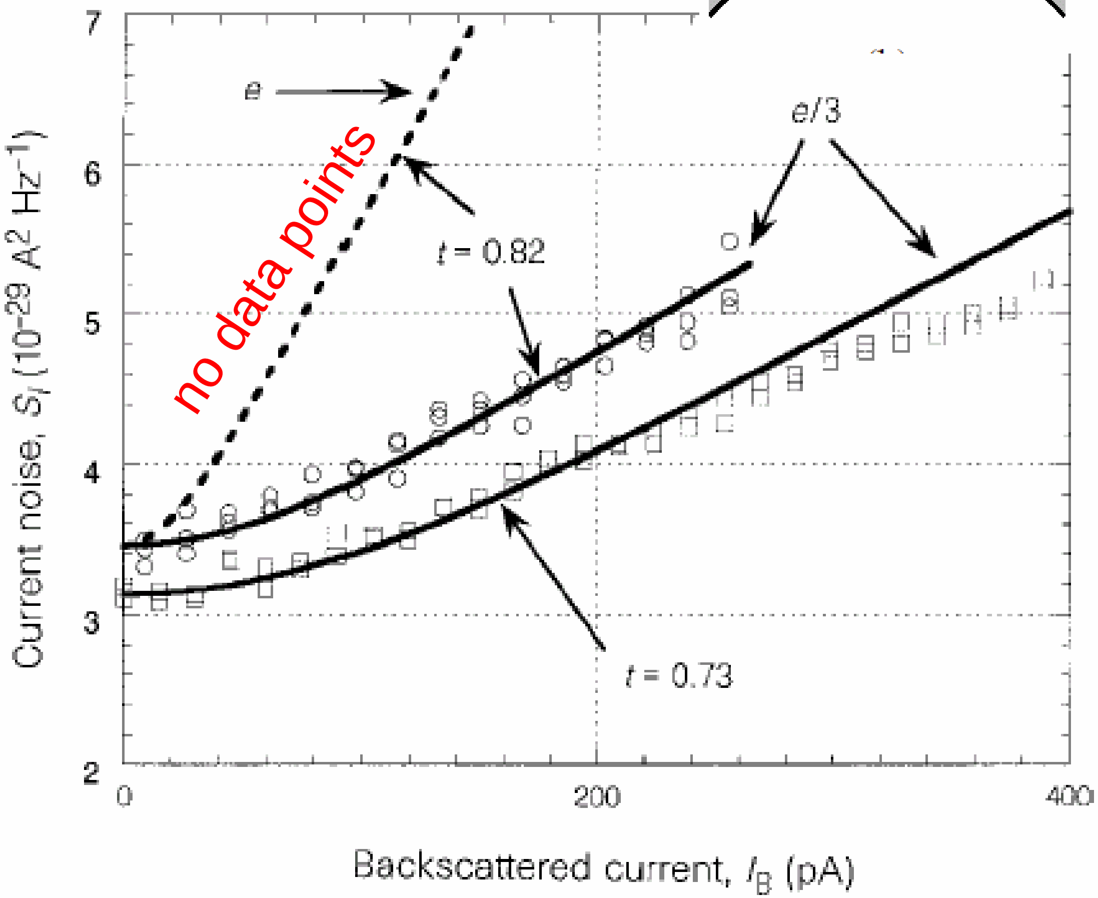
Edge states – experiments: Current noise

1. Phenomenology: States at the edges of 2D electron system in the conditions of the fractional (ν<1) quantum Hall effect are chiral Luttinger liquids with charge e*=νe and interaction parameter g= ν.



1.1. Noise must reveal fractional charge

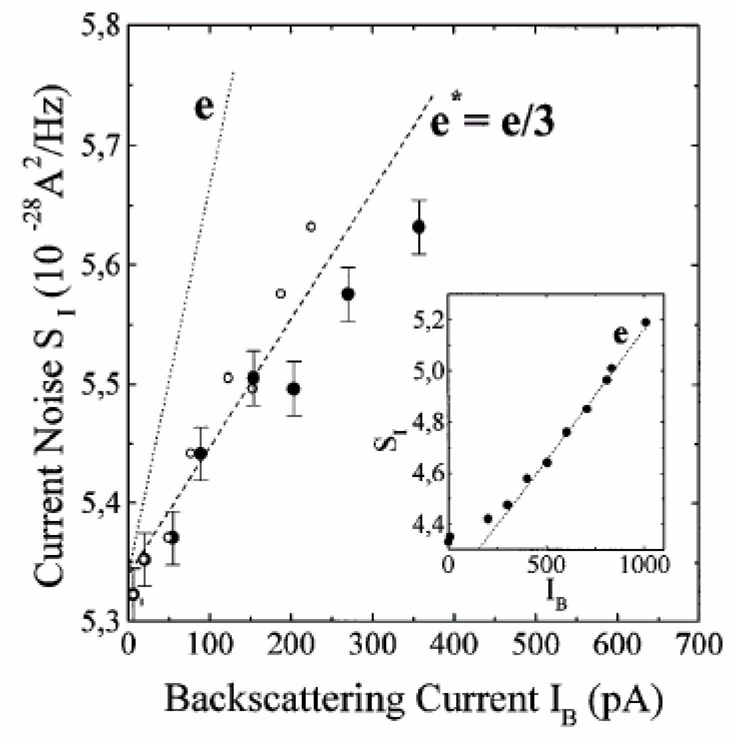
$$S_I = (\nu e)\langle I_{back}\rangle$$
$$\nu = 1/3$$

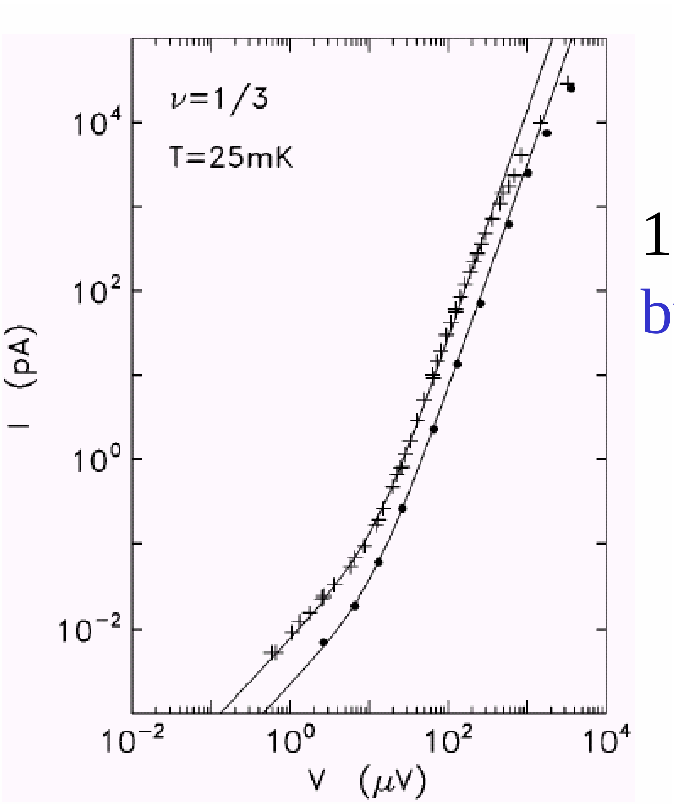


R. De-Picciotto et al (1997)

Edge states – experiments

Corroborating observation of the noise power vs. current ratio:
[L. Saminadayar et al (1997)]





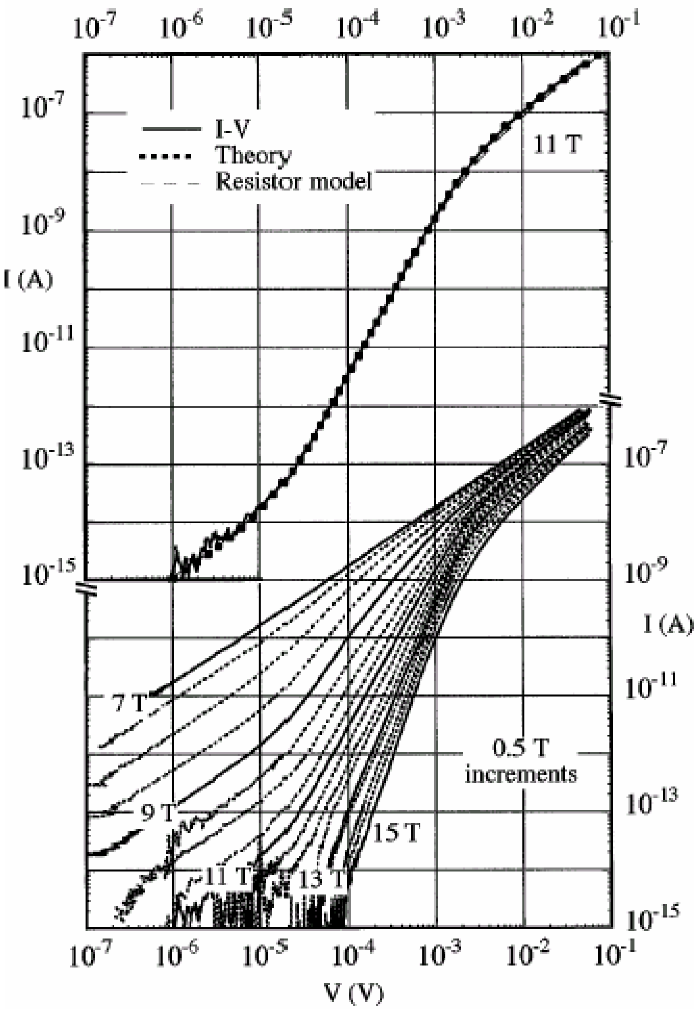
1.2. Tunneling into an edge state is characterized by $I(V) \sim V^{1/\nu}$ for a discrete set of ν.

$$I(V) \propto V^{\frac{1}{g}}$$

Confirmed by Chang et al in 1996 for ν=1/3

but…

Edge states – experiments

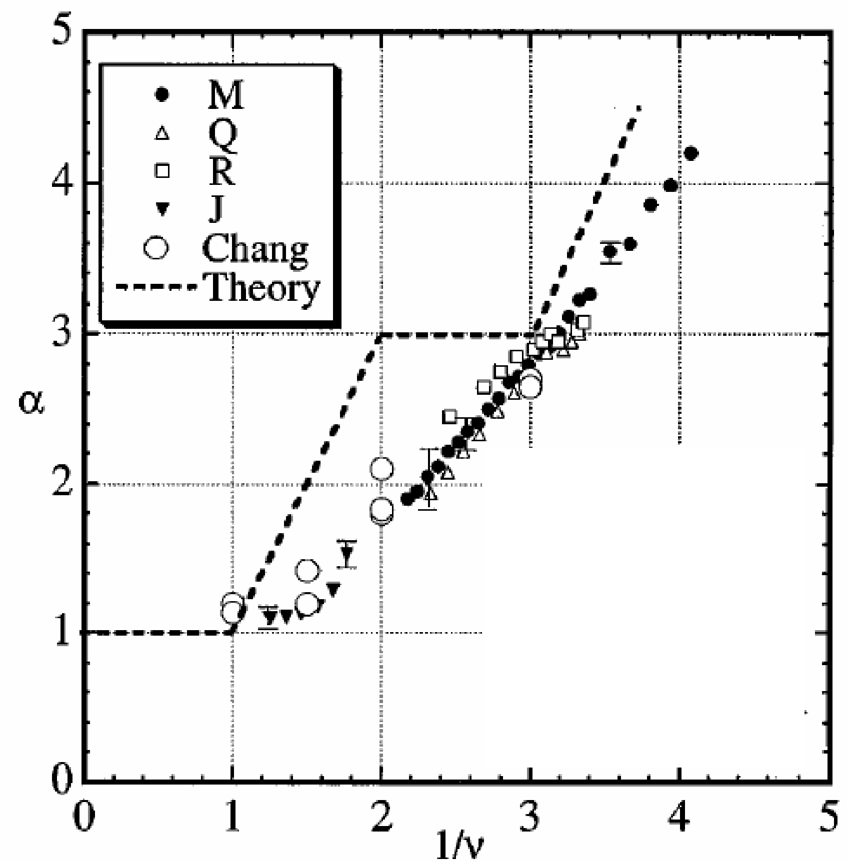


There is no predicted qualitative difference between the compressible and incompressible states; continuous evolution of current-voltage characteristics with ν.

[M. Grayson et al (1998)]

$I(V) \sim V^{\alpha}$

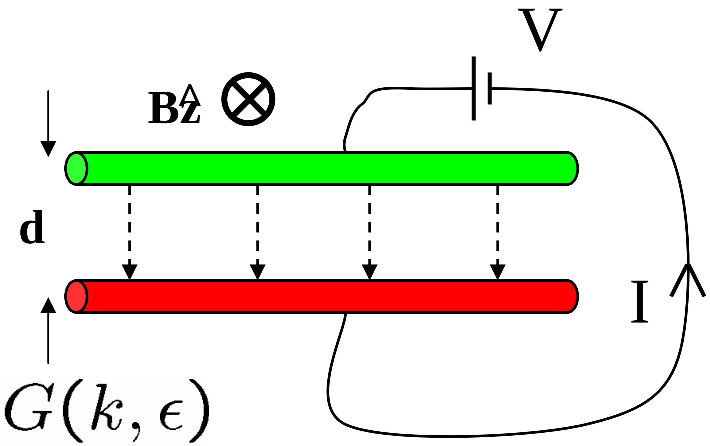
continuous set of α



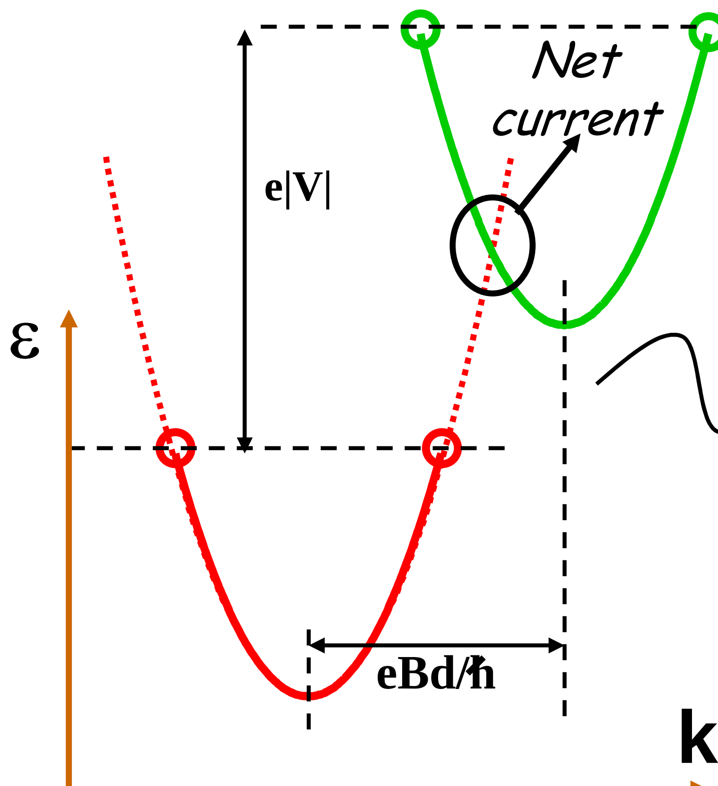
Tunneling with the momentum conservation

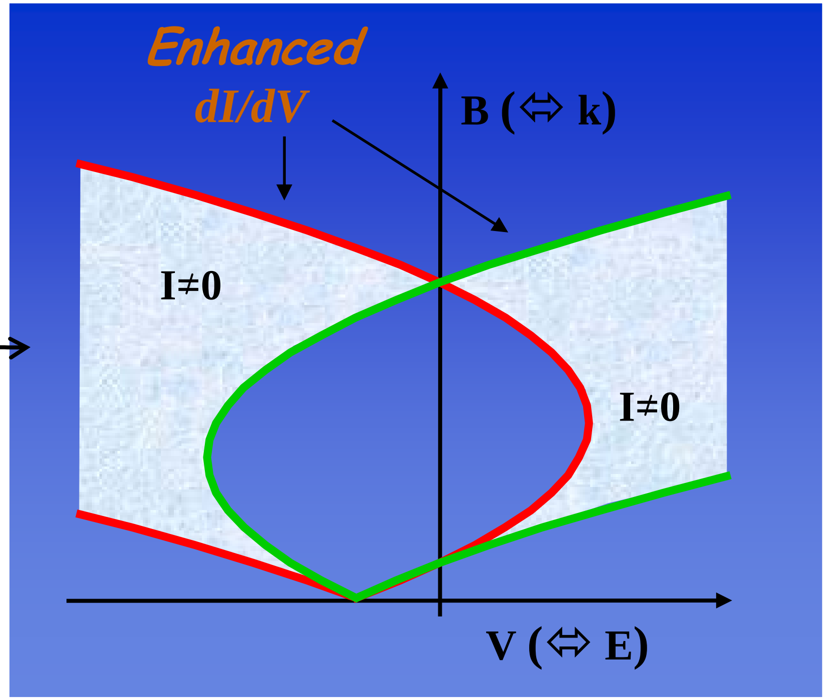
(courtesy of Auslender&Yacoby)

$$\frac{dI}{dV} \propto A(k,\epsilon) \star A(k,\epsilon)$$



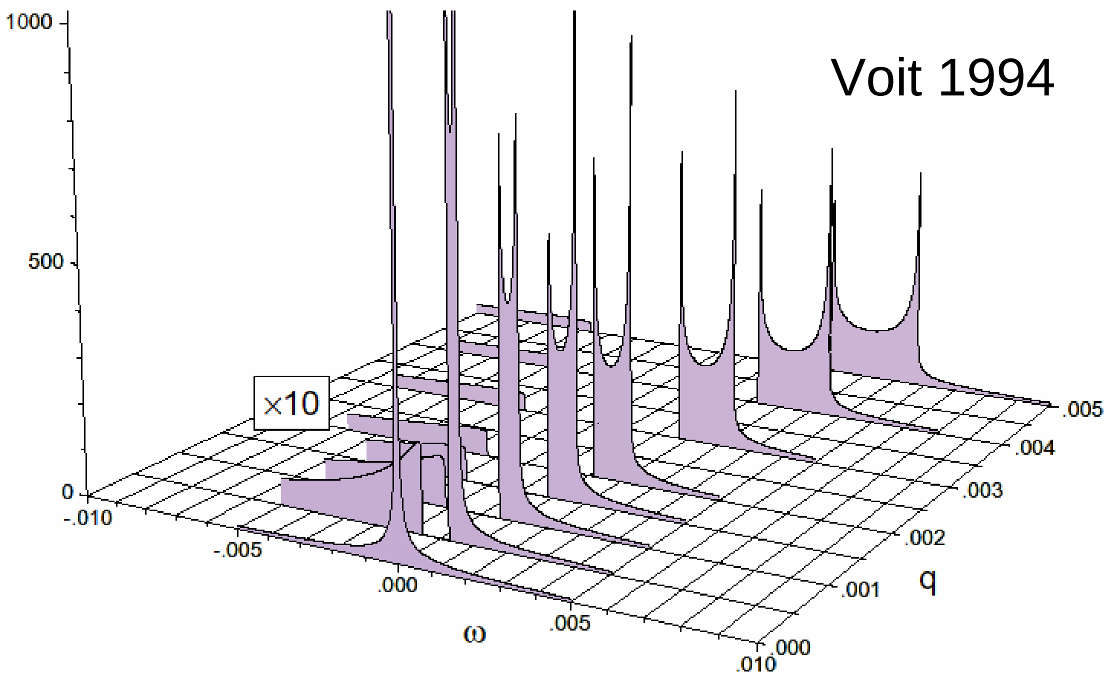
$$A(k,\epsilon) = \mathrm{Im}G(k,\epsilon)$$





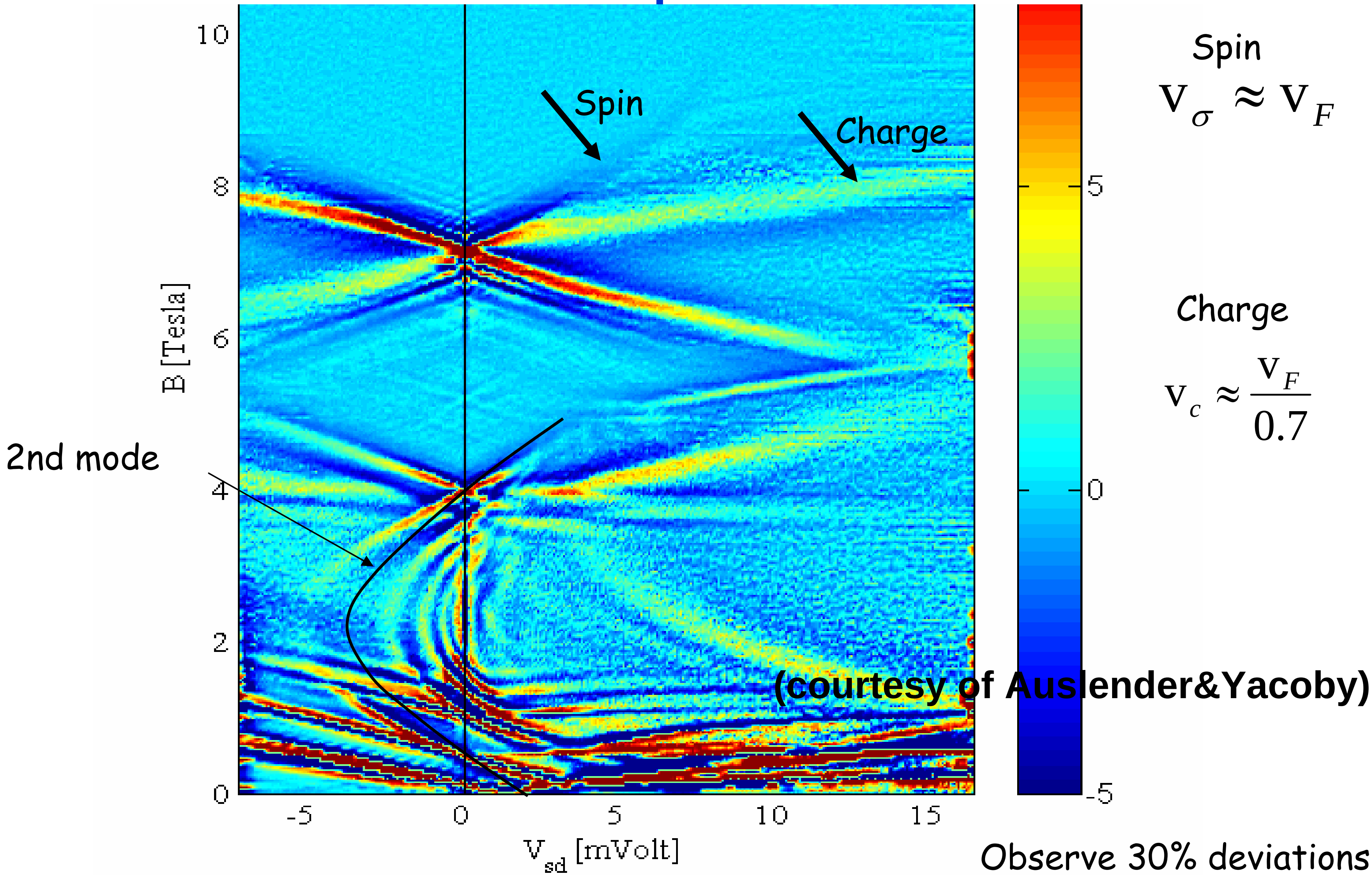
Spectral density of a Luttinger liquid

$$A(q, \omega) = \mathrm{Im} G(k_F + q, \epsilon_F + \omega)$$



Double-peak structure – spin&charge modes for the linearized spectrum

Comparison With the Non-Interacting Dispersion



Spin

$$v_\sigma \approx v_F$$

Charge

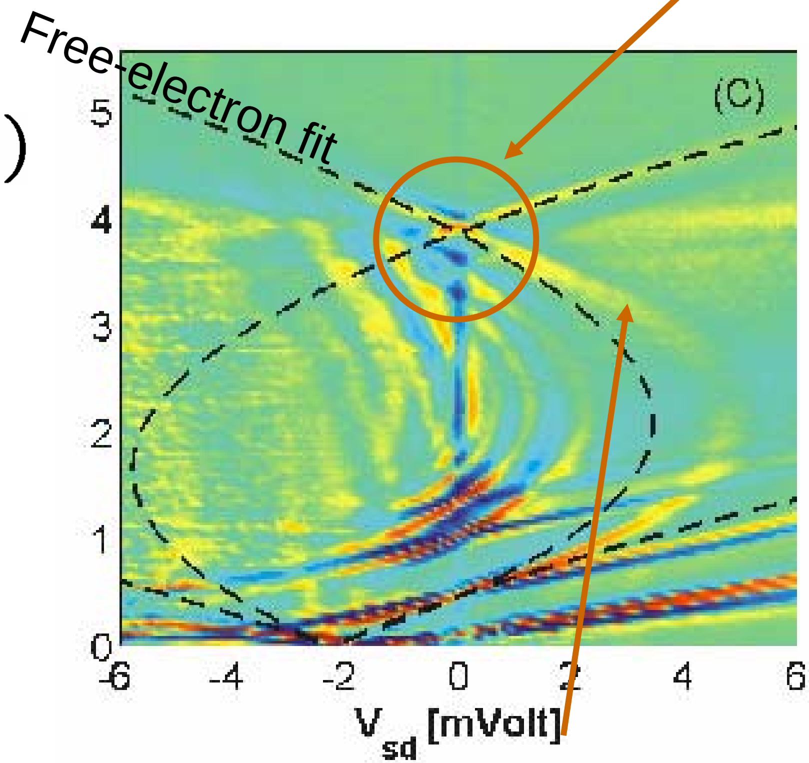
$$v_c \approx \frac{v_F}{0.7}$$

Observe 30% deviations

(data: courtesy of Auslender&Yacoby)

this is a charge mode of a Luttinger liquid

$$A(q,\omega) = \mathrm{Im}G(k_F + q, \epsilon_F + \omega)$$



what is this bright line?

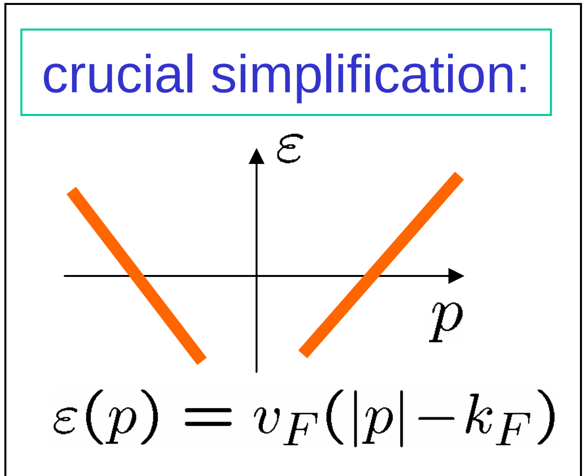
In the presence of interactions, the spectral density is known only for the linearized spectrum

Luttinger liquid vs Tomonaga-Luttinger model

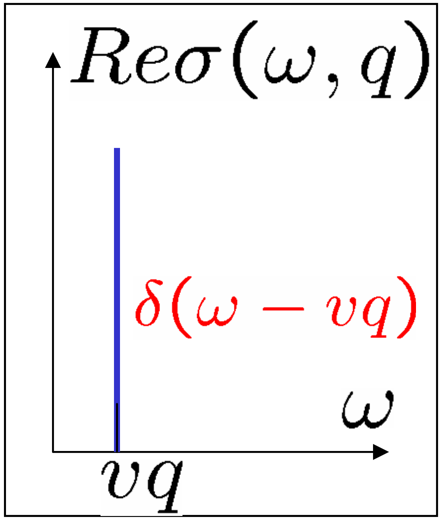
D>1:
Fermi gas (exactly solvable) ➡ Fermi liquid (low-energy theory)

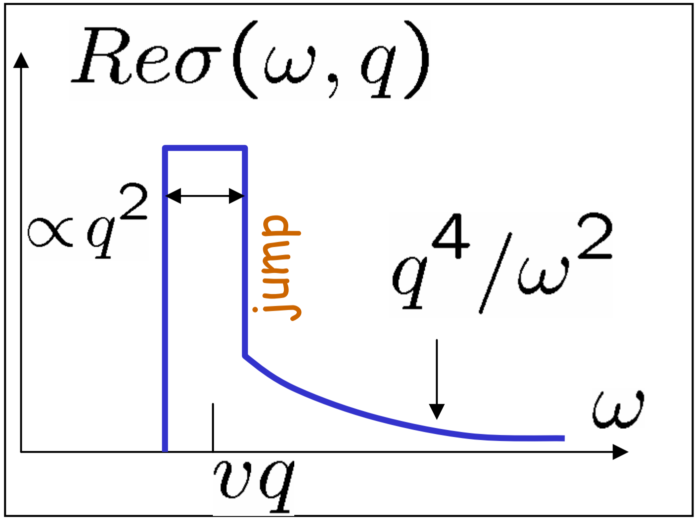
$$\frac{c}{T} \propto \text{const} \Rightarrow \frac{c}{T} \propto \text{const} + V^2 T$$ **non-analytic correction**

D=1:
Tomonaga-Luttinger model (exactly solvable) ➡ Luttinger liquid (low-energy theory)



Tomonaga (1950);
Luttinger (1963)



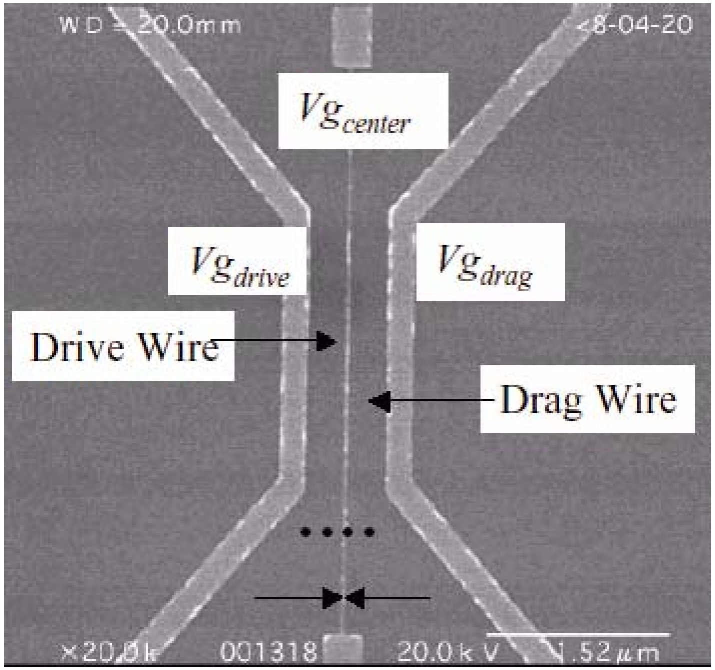


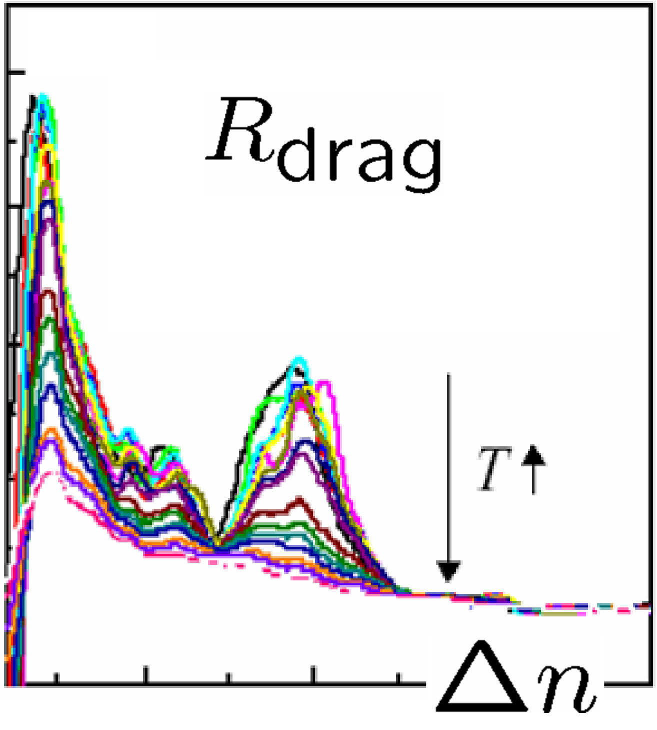
Pustilnik,LG 2003+current

Corrections to Luttinger liquid theory are important if particle-hole asymmetry plays role

- Thermopower – no experiments
- Coulomb drag effect – inconclusive experiments

4mm-long wires; Tarucha group 2001



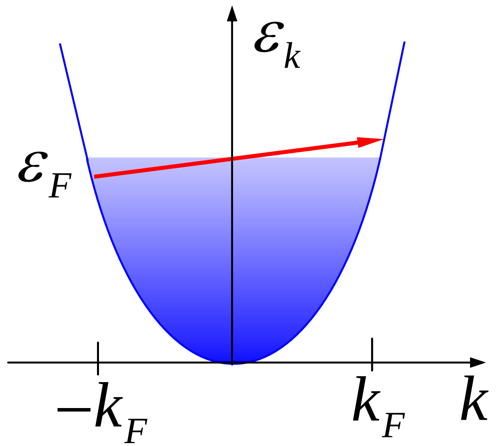


Drag between quantum wires

Drag between identical wires
at low temperature,

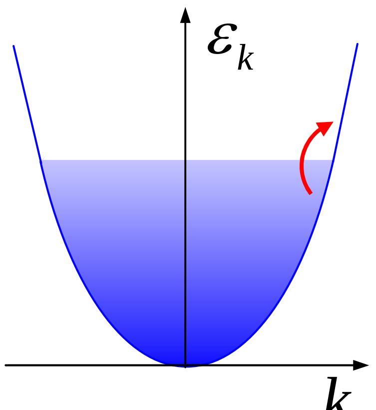
$T \ll \varepsilon_F \exp(-4\gamma k_F d)$

is dominated by $2k_F$ processes.
(Nazarov&Averin, 1998,
Klesse&Stern, 2000)



Drag at higher temperatures, or
drag between non-identical wires,

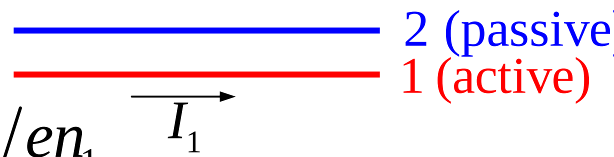
$$|k_F^{(1)} - k_F^{(2)}| \neq 0$$



is controlled by small-momentum transfer, $\Delta p \ll k_F$

Drag between quantum wires

In 1D system, even weak short-range interaction results in a singular perturbation (Fermi liquid ,Luttinger liquid).



$$\mathrm{v}_d = I_1 / e n_1$$

Derivation of the drag resistivity:

1. Weak interaction between wires – perturbation theory in $U_{12}(q)$.
2. No disorder – Galilean invariance of charge fluctuations,

Structure factor: $\mathring{S}_1(x,t) = S_1(x - \mathrm{v}_d t, t)$

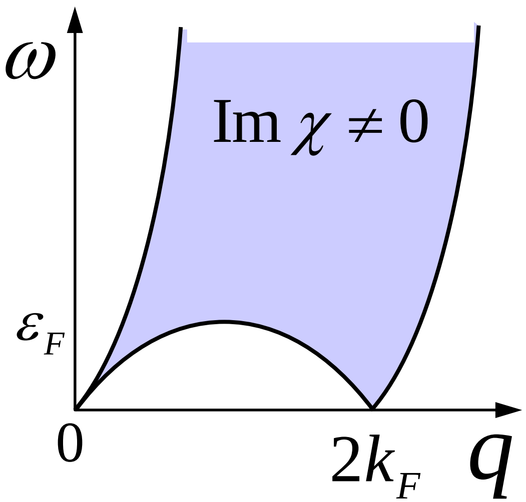
$$\Rightarrow \quad r \propto \frac{1}{n_1 n_2} \int_0^\infty d\omega \int_0^\infty dq \; q^2 U_{12}^2(q) \frac{\mathrm{Re}\,\sigma_1(q,\omega)\,\mathrm{Re}\,\sigma_2(q,\omega)}{T \sinh^2(\omega/2T)}$$

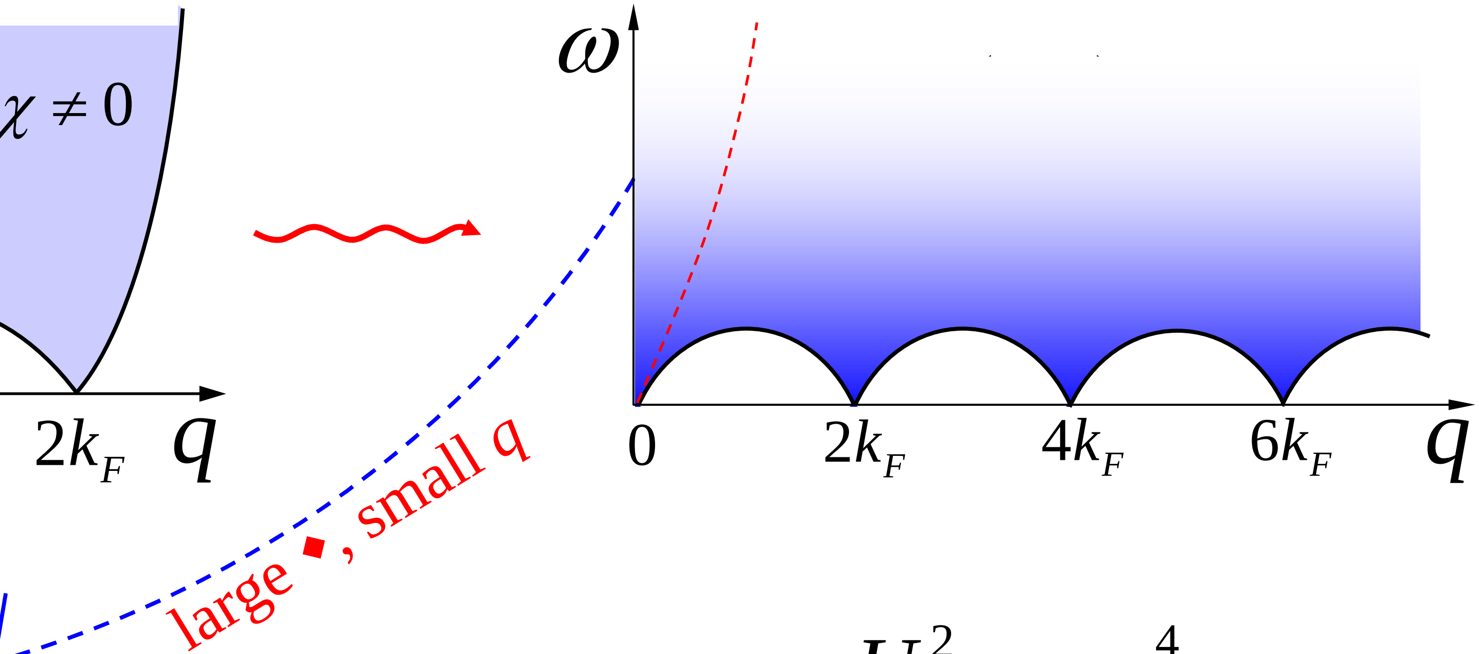
$$\mathrm{Re}\,\sigma_i = -\mathrm{Im}\,\chi_i = \frac{1}{2}\left(1 - e^{-\omega/T}\right) S_i$$

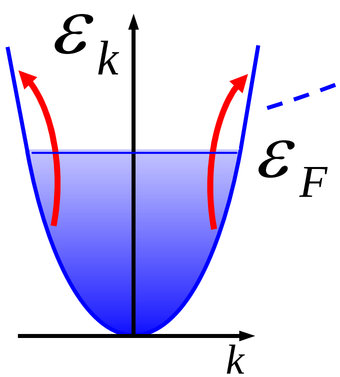
FDT

accounts fully for the intra-wire interaction

Interacting electrons: beyond RPA



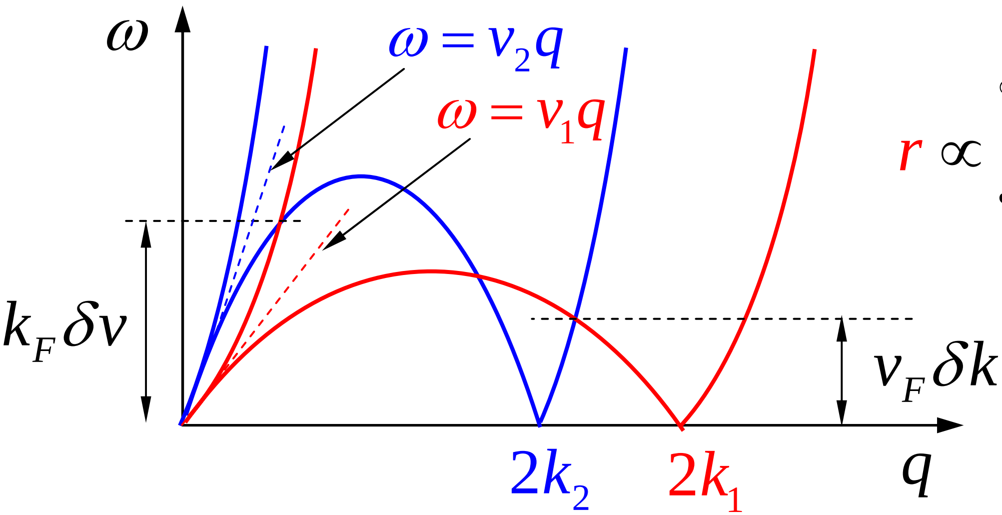




$$\delta\,\mathrm{Im}\,\chi(q,\omega) \propto \frac{U^2}{m^2}\frac{q^4}{\omega^2 - v_F^2 q^2}$$

$$v_F q + \frac{q^2}{2m} = \quad \omega = \quad \varepsilon_F$$

Drag between non-identical wires



$$r \propto \int_0^\infty d\omega dq [\cdots] \frac{\operatorname{Im}\chi_1(q,\omega)\operatorname{Im}\chi_2(q,\omega)}{\sinh^2(\omega/2T)}$$

Free-electron and the Calogero-Sutherland models:

$$r \propto l_0^{-1} \exp(-k_F \delta v/T) + l_{2k_F}^{-1} \exp(-v\delta k_F/T)$$

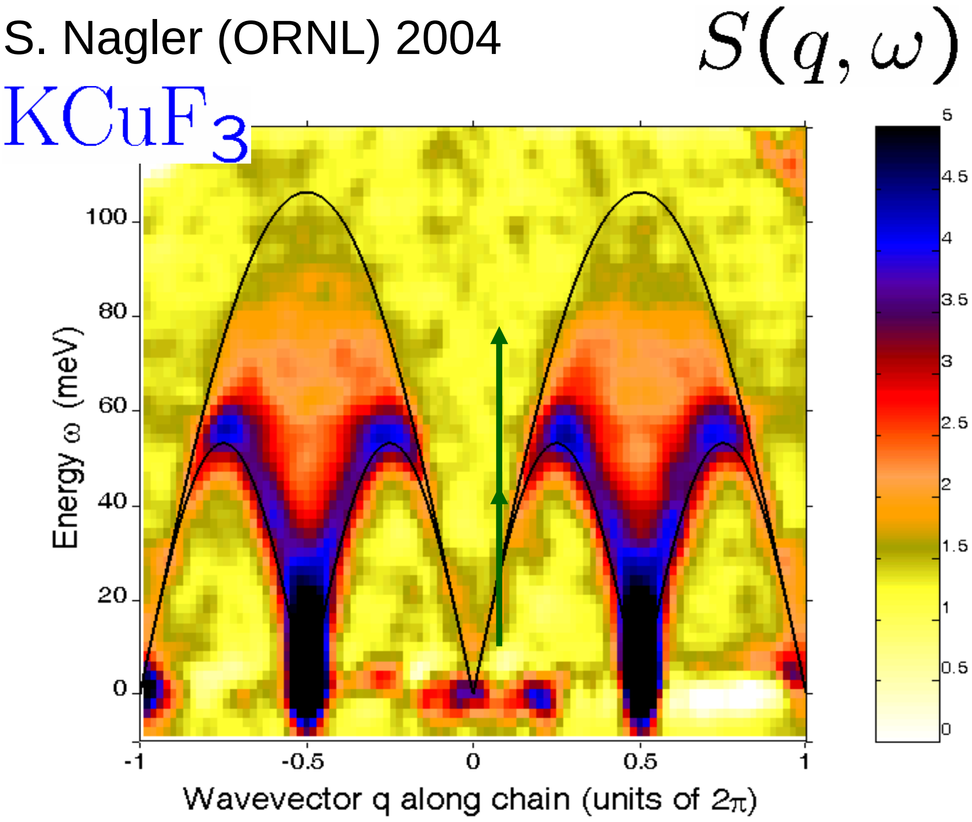
Generic electron-electron interaction:

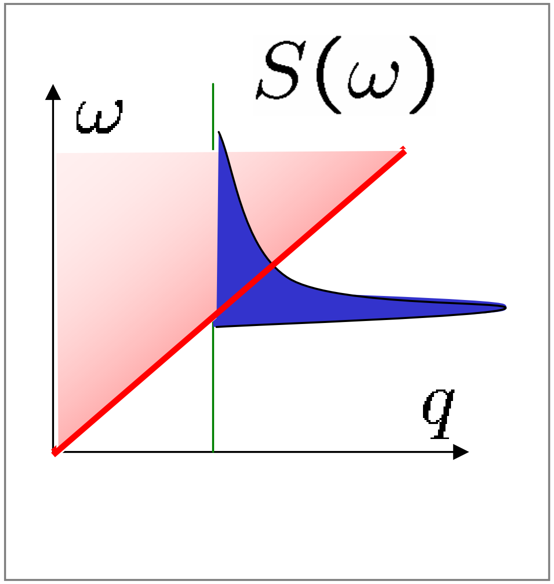
$$r \propto l_0^{-1} \frac{T^5}{\varepsilon_F^4 |k_F \delta v|} + l_{2k_F}^{-1} \exp(-v\delta k_F/T)$$

In an experiment, $\delta v \propto \delta k_F \sim |n_1 - n_2|$.

1D AF-magnets: spin liquid

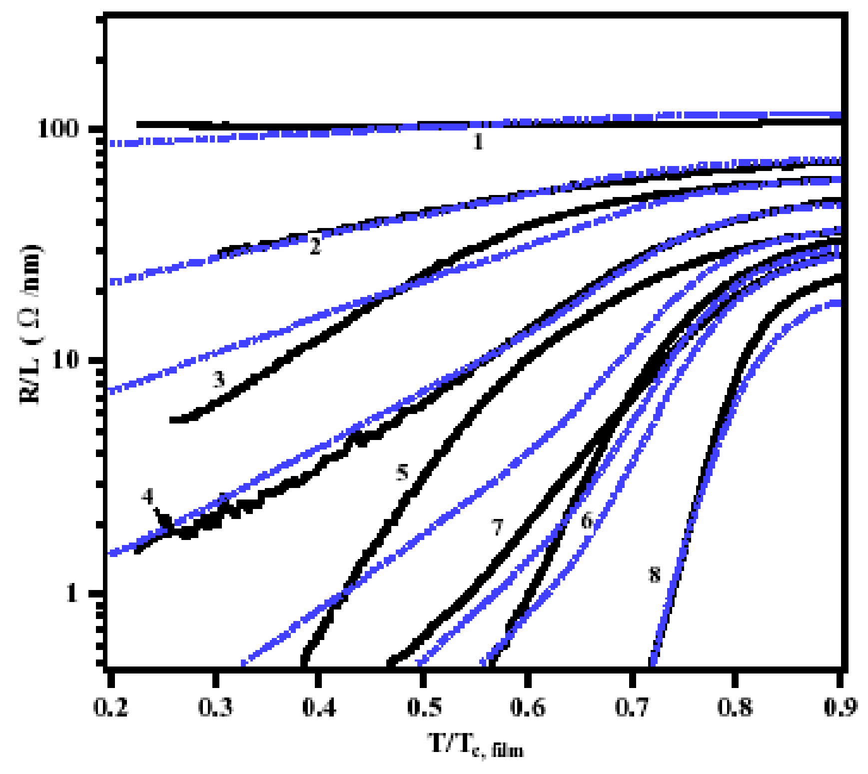
S. Nagler (ORNL) 2004



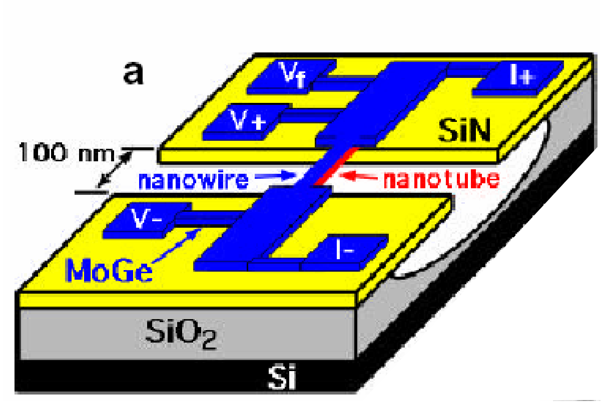


beyond Luttinger

Superconductor-insulator transition in nanowires



MoGe nanowire [Bezryadin, Bockrath, Markovic, Lau, Tinkam (2000, 2001)]



One gapless mode – phase (φ) – Luttinger liquid

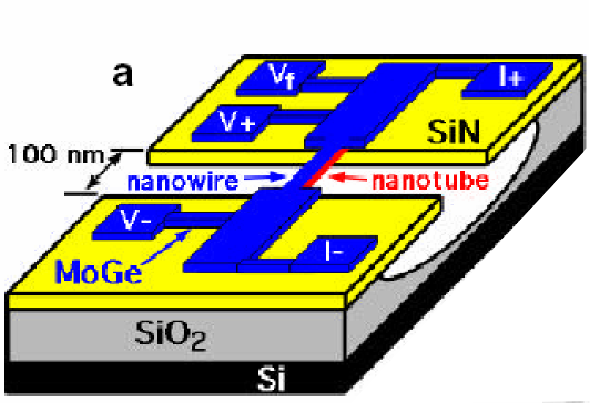
$$\mathcal{L} = \frac{\hbar^2 n_s}{2m}\left(\frac{\partial\varphi}{\partial x}\right)^2 + \frac{1}{e^2 \ln(D/R)}\left(\frac{\partial\varphi}{\partial\tau}\right)^2$$ +core energy

Superconductor-insulator transition in nanowires

Wires with cross-sections $S \lesssim S_0 = \lambda_F^2 \left(\frac{\xi_{sc}}{l} \right)^2$

are insulating, but localization length is huge

$$\ln \left[\frac{\xi^*_{loc}}{\xi_{loc}} \right] \propto \frac{\xi_{sc}}{l} \gg 1$$

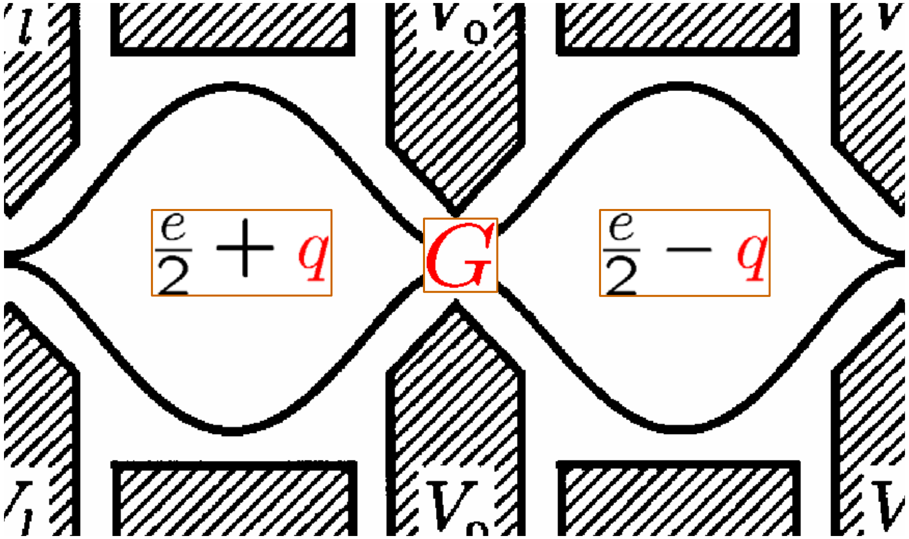


Tinkham group 2000

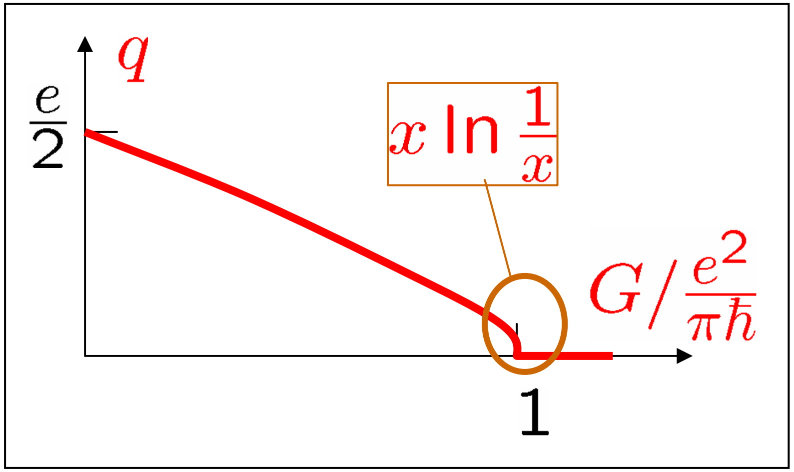
Where is the proximity effect in experiments?

Other applications: Charge of a quantum dot

How one electron is split between two quantum dots?



Fluctuations of the charge "liquid" can be mapped onto a Tomonaga-Luttinger model (Flensberg 93, Matveev 95)



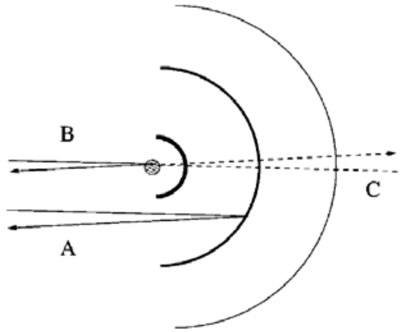
related problems for $G/\frac{e^2}{\pi\hbar} \to 1$

mesoscopic fluctuations:
theory Aleiner & LG 1998;
experiment: Marcus group 1999;
Kondo effect:
theory LG,Hekking,Larkin 1999

Matveev, LG, Baranger;
Golden&Halperin, 1996

Scattering off Friedel oscillation in D=2

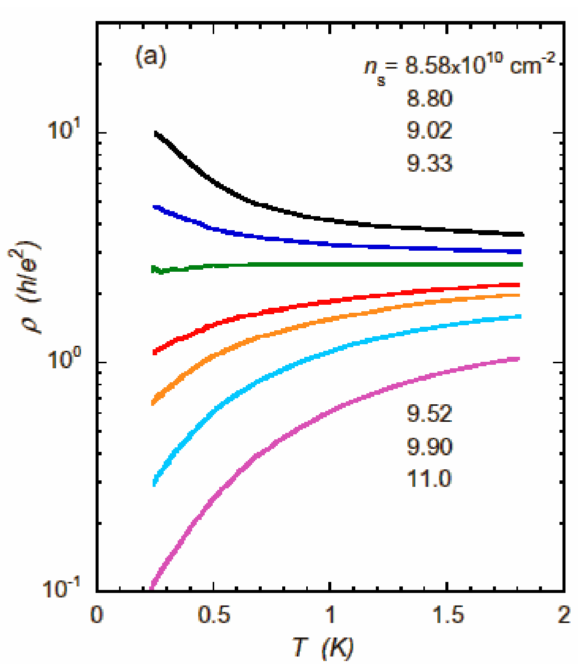
Interference
of waves A&B



Origin of Altshuler-Aronov corrections

Rudin, Aleiner, LG, 1997 – correction to the DOS

Zala, Narozhny, Aleiner, 2001 – correction to conductivity



Kravchenko et al 1990s

dρ/dT sign change near Stoner instability; possibly explains the <u>2D metal-insulator "transition"</u>

Conclusions

- Several complementary tools exist for treating interactions in 1D electron systems, a number of specific predictions are made, some attractive problems remain open

- A consistent set of experimental manifestations of the Luttinger liquid behavior perhaps is still to come