Interacting Electrons in Disordered Quantum Wires: Dephasing and Low-Temperature Transport

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ANDERSON LOCALIZATION

+ ELECTRON-ELECTRON INTERACTION

+ vanishing coupling to the external world (phonons, etc.)

finite $T \neq 0$: $\quad \sigma(T) = ?$

No interaction $\Longrightarrow$ $\sigma(T) \equiv 0$ for any T

$\sigma(T)$ is (possibly) nonzero due to e-e interaction only !

Electron-electron interaction: Quasi-1D and 2D

High T: $L_\varphi \ll \xi$ → singular conductivity corrections

- **Weak localization** (cut off by inelastic e-e scattering, L_φ)
- **Altshuler-Aronov corrections** (cut off by thermal smearing, L_T)

Low T: strong localization → $\sigma(T)$ unknown

Q: Variable-range hopping ? But energy conservation – ?

All excitations (plasmons, etc) are localized in disordered low-D systems...

Q: Activation? But no mobility edge + what will activate electrons?

Our answer: neither VRH nor Activation!

Quantum wires (1D)

- **Single channel, no interaction:** **Localization ($\xi \sim l$), no diffusion**
- **Single channel, no disorder:** **Luttinger-liquid (non-Fermi liquid)**
- **Luttinger liquid (LL) + impurities:** **Strong LL renormalization**
- **What is dephasing in Luttinger liquid ?**

Outline :

- **Dephasing & inelastic interactions in Luttinger liquid**
- **High T :** **Weak localization in Luttinger liquid**
- **Intermediate T :** **Power-Law Hopping (PLH)**
- **Low T :** **Anderson–Fock Glass (AFG)**

Model: Disordered Luttinger liquid

- Single-channel infinite wire: right(left) movers ψ_μ, $\mu = \pm$
- Spinless (spin-polarized, $\sigma = +$) or spinful ($\sigma = \pm$) electrons
- Linear dispersion, $\epsilon_k = k v_F$
- Short-range weak e-e interaction, $\alpha \equiv V(0)/2\pi v_F \ll 1$
- No e-e backscattering; g-ology with g_2 and g_4
- White-noise weak ($E_F\tau_0 \gg 1$) disorder, $\langle U(x)U(x')\rangle = \delta(x-x')/2\pi\nu_0\tau_0$.

$$H = \sum_{k,\mu,\sigma} v_F(\mu k - k_F)\psi^\dagger_{\mu\sigma}(k)\psi_{\mu\sigma}(k) + H_{\rm e-e} + H_{\rm dis}$$

$$H_{\rm e-e} = \frac{1}{2}\sum_{\mu,\sigma,\sigma'}\int dx\left\{\psi^\dagger_{\mu,\sigma}\psi_{\mu,\sigma}\, g_2\, \psi^\dagger_{-\mu,\sigma'}\psi_{-\mu,\sigma'} + \psi^\dagger_{\mu,\sigma}\psi_{\mu,\sigma}\, g_4\, \psi^\dagger_{\mu,\sigma'}\psi_{\mu,\sigma'}\right\}$$

$$H_{\rm dis} = \sum_{\sigma}\int dx\left\{\mathcal{U}\ \psi^\dagger_{+,\sigma}\psi_{-,\sigma} + \mathcal{U}^*\ \psi^\dagger_{-,\sigma}\psi_{+,\sigma}\right\} + H_f$$

Bosonization and disorder averaging

Giamarchi & Schulz '88

1. **Bosonization:** given realization of disorder, ψ (fermionic) $\rightarrow \phi$ (bosonic)

   $\longrightarrow$ Interaction term quadratic in ϕ; impurities: $\cos 2\phi$

2. **Disorder averaging.** Quenched disorder: Introduce replicas, ϕ_n

Bosonized replicated action (no spin), $u = v_F/K$, $K = (1+2\alpha)^{-1/2} \simeq 1-\alpha$:

$$S[\phi] = \frac{1}{2\pi v_F} \sum_n \int dx\, d\tau \, \left\{ [\partial_\tau \phi_n(x,\tau)]^2 - u^2 [\partial_x \phi_n(x,\tau)]^2 \right\}$$

$$- \frac{v_F k_F^2}{\pi^2 \tau_0} \sum_{n,m} \int dx\, d\tau\, d\tau' \cos[2\phi_n(x,\tau) - 2\phi_m(x,\tau')]$$

Bosonization and disorder averaging (cont'd)

- **powerful without impurities:**

  Gaussian action, interaction treated exactly

- **good for a single weak (or very strong) impurity**

- **inconvenient for disordered systems and for Anderson localization!**

Exercise:

put $K = 1$ (no interaction) in $S[\phi]$; calculate ac-conductivity:

how to obtain Drude and Berezinskii $\sigma(\omega)$ from bosonization?

Renormalization of disorder

Integrate out $T < \epsilon < \epsilon_F$ (cf. Giamarchi & Schulz '88) $\longrightarrow$ T-dependent static disorder (Mattis '74, Luther & Peschel '74 ...)

$$\tau(T) = \tau_0 (T/\epsilon_F)^{2\alpha} \quad \longrightarrow \quad \sigma_D(T) = \frac{e^2 n_e \tau(T)}{m} \propto T^{2\alpha}$$

Physically: Friedel oscillations, but beyond Hartree-Fock

$T\tau > 1$: independent renormalization of weak impurities

$T\tau \sim 1$: renormalization stops $\longrightarrow$ zero-T localization length

$$\xi(T=0) \propto \tau_0^{1-2\alpha}$$

BUT! $T\tau \sim 1 \neq$ onset of localization

Localization: $L_\varphi \sim \xi \longrightarrow$ 1D: $\tau/\tau_\varphi \sim 1$

Disordered Luttinger liquid is "Fermi-liquid"?

$T < \epsilon < \epsilon_F \rightarrow$ **integrated out:** $\quad \tau_0 \rightarrow \tau(T), \quad \epsilon_F \rightarrow T:$

all power-law singularities $\propto (E_F/T)^\gamma$ **now in** $\tau(T)$

Step 1: Luttinger liquid physics $\longrightarrow$ **renormalization of disorder**

interaction still important $\longrightarrow$ **inelastic scattering, dephasing**

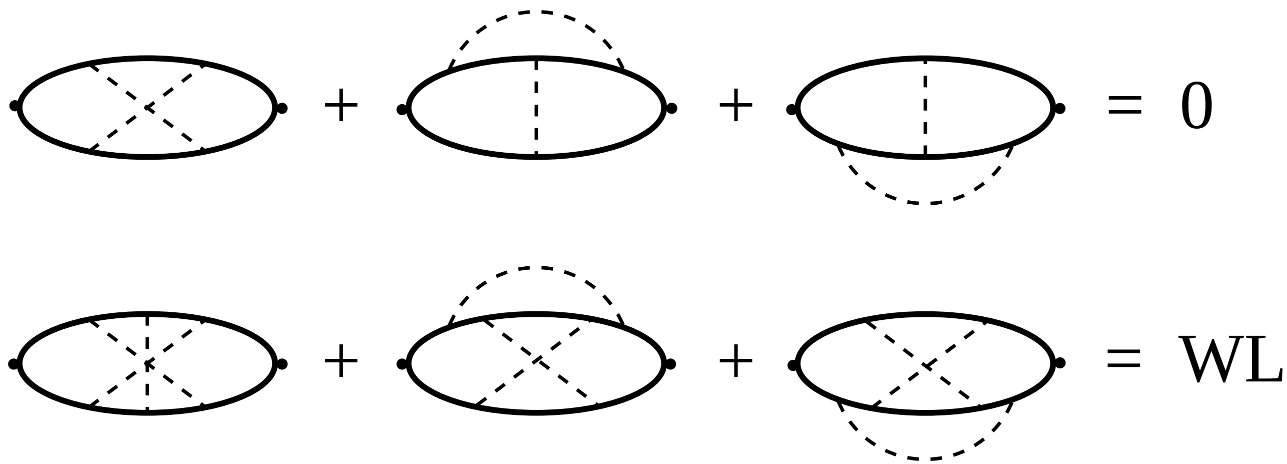
Step 2: Dephasing: Back to fermions !

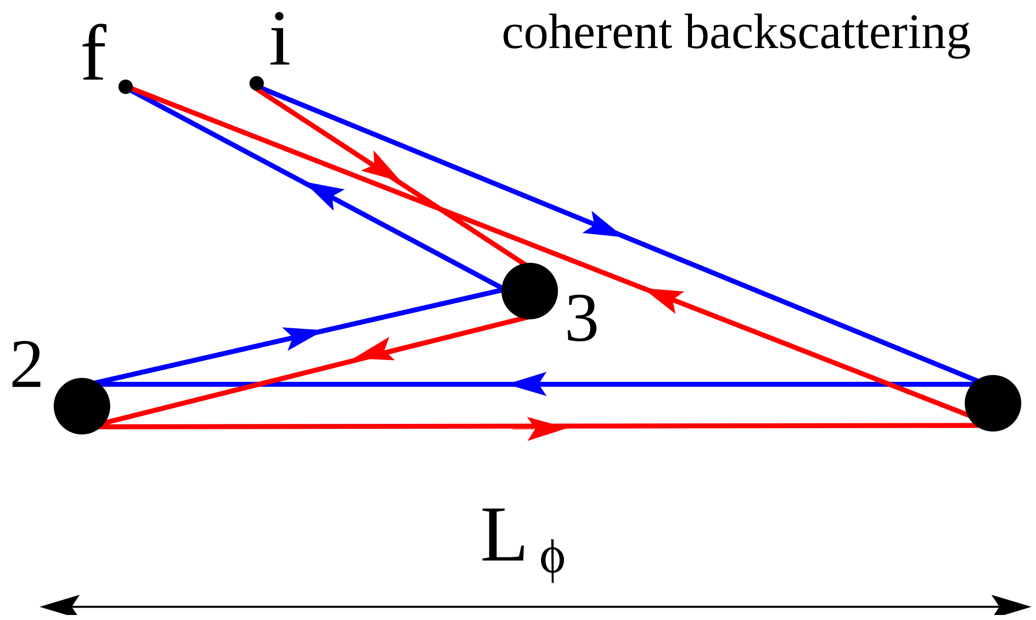
Apply the Fermi-liquid machinery...

Vocabulary

1D		2D
Luttinger (non-Fermi) liquid	$\longleftrightarrow$	Zero-bias anomaly in tunneling DoS
$\nu(\epsilon)/\nu_0 \sim (\epsilon/E_F)^\gamma \ll 1$	$\longleftrightarrow$	$\nu(\epsilon)/\nu_0 \sim \exp[-\frac{1}{8\pi^2 g}\ln^2\epsilon] \ll 1$
Renormalization of disorder	$\longleftrightarrow$	Temperature-dependent screening
$T \gg \Delta_1 \sim 1/\tau$	$\longleftrightarrow$	Ballistic regime, $T\tau \gg 1$
Strong coupling regime (Giamarchi-Schulz)	$\longleftrightarrow$	Strong Altshuler-Aronov corrections ($L_T \sim \xi$)
Anderson localization	$\longleftrightarrow$	Strong WL-corrections ($L_\varphi \sim \xi$)

Weak localization in 1D





Weak localization in LL: Spinless case

Hubbard-Stratonovich transformation $\longrightarrow$ path-integral $\longrightarrow$

electrons propagate in a fluctuating field created by other electrons;

similar to higher dimensions (Altshuler, Aronov & Khmelnitsky '82)

Systematic expansion of $\sigma(T)$ in $\tau_\varphi/\tau \ll 1$

Need disorder in RPA-interaction propagator!

$$\delta\sigma_{wl} = -\frac{\sigma_D}{\nu}\int_0^\infty dt\; W_3(t)\exp[-f(t/\tau_\varphi^{wl})]$$

$$= -\frac{\pi}{4}\sigma_D\int_0^\infty dt\;\frac{t}{\tau^2}\;\exp[-\alpha^2\frac{\pi T}{2\tau}t^2] \sim -\sigma_D\left(\frac{\tau_\varphi^{wl}}{\tau}\right)^2 \propto 1/\alpha^2 T$$

$\longrightarrow$ dephasing rate for weak localization in Luttinger liquid

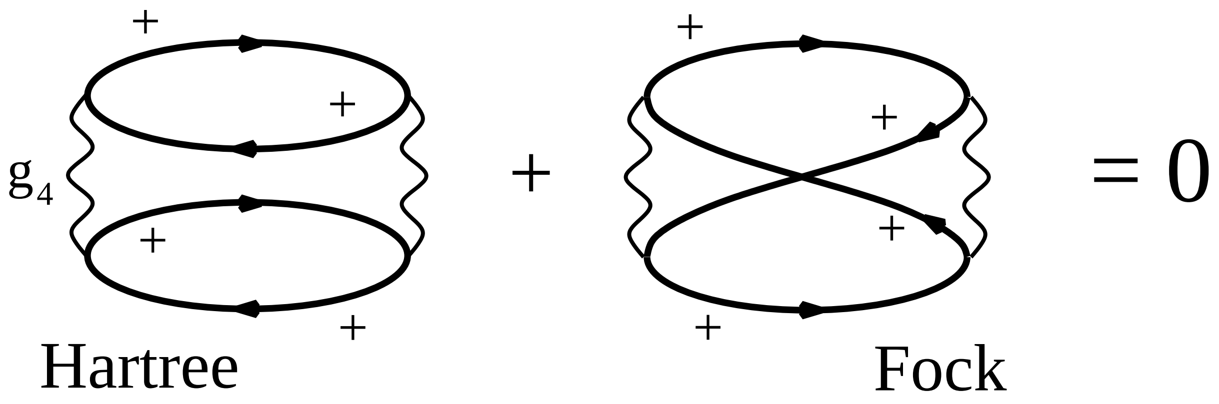
$$1/\tau_\varphi^{wl} = \alpha(T/\tau)^{1/2}$$

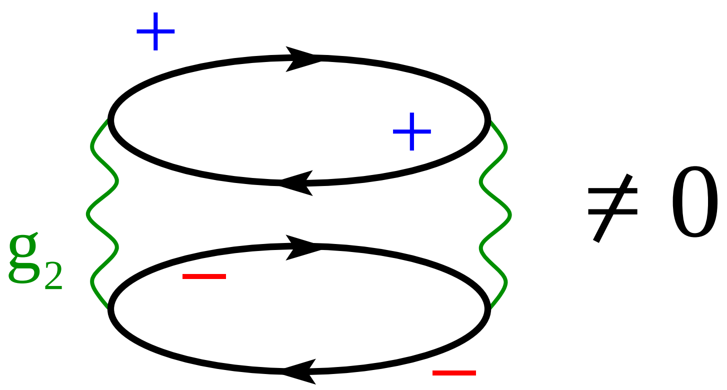
Perturbation theory in α: Second order, no spin

Clean Luttinger liquid

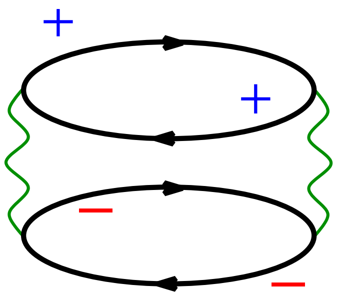
Hartree–Fock cancellation: intrabranch (++) inelastic scattering vanishes;

only scattering between left and right movers (interbranch, +−) contributes.





Golden rule: Inelastic scattering rate

Im 

$$= 1/\tau_{ee} = \alpha^2\, T\, \pi v_F \int d\omega \int dq\, \delta(\omega - v_F q)\delta(\omega + v_F q)$$

$$1/\tau_{ee} = \pi\alpha^2 T \ll T \qquad \text{(FL?)}$$

Relevant to Aharonov–Bohm oscillations in 1D $\quad (1/\tau_{ee} \sim 1/\tau_\varphi^{\mathrm{AB}})$

Single-particle Green function in LL, no spin, $\alpha \ll 1$:

$$G_+(x,t) \simeq \frac{(E_F/2\pi u)\,(\pi T/E_F)^{1+\alpha^2/2}}{\sinh^{1+\alpha^2/4}(\pi T[t - x/u])\,\sinh^{\alpha^2/4}(\pi T[t + x/u])}$$

$$G_+(x = ut, t) \propto \exp[-\pi\alpha^2 T t/2] = \exp[-t/2\tau_{ee}]$$

Golden rule: Dephasing & Weak localization

Soft inelastic scattering $\omega \ll 1/\tau_\varphi^{wl}$ does not produce dephasing in WL $\longrightarrow$
self-consistent cut-off at $\omega \sim 1/\tau_\varphi^{wl}$ (Altshuler, Aronov & Khmelnitsky '82).

Clean Luttinger liquid, i.e. no disorder in $\mathrm{Im}\Pi_\pm(\omega, q) = \pi\nu\omega\delta(\omega - v_F q)$:

$$\frac{1}{\tau_\varphi^{wl}} \sim \alpha^2 T v_F \int_{1/\tau_\varphi^{wl}} d\omega \int dq\ \delta(\omega - v_F q)\delta(\omega + v_F q) = 0$$

Disordered Luttinger liquid, $\delta-$functions get broadened by $1/\tau$ $\longrightarrow$

$$\frac{1}{\tau_\varphi^{wl}} \sim \alpha^2 T v_F \int_{1/\tau_\varphi^{wl}} d\omega \int dq\ \tilde{\delta}_\tau(\omega - v_F q)\tilde{\delta}_\tau(\omega + v_F q) \sim \alpha^2 \frac{T\tau_\varphi^{wl}}{\tau}$$

$$\longrightarrow \qquad 1/\tau_\varphi^{wl} \sim \alpha(T/\tau)^{1/2} \neq 1/\tau_{ee}$$ (cf. Path Integral result)

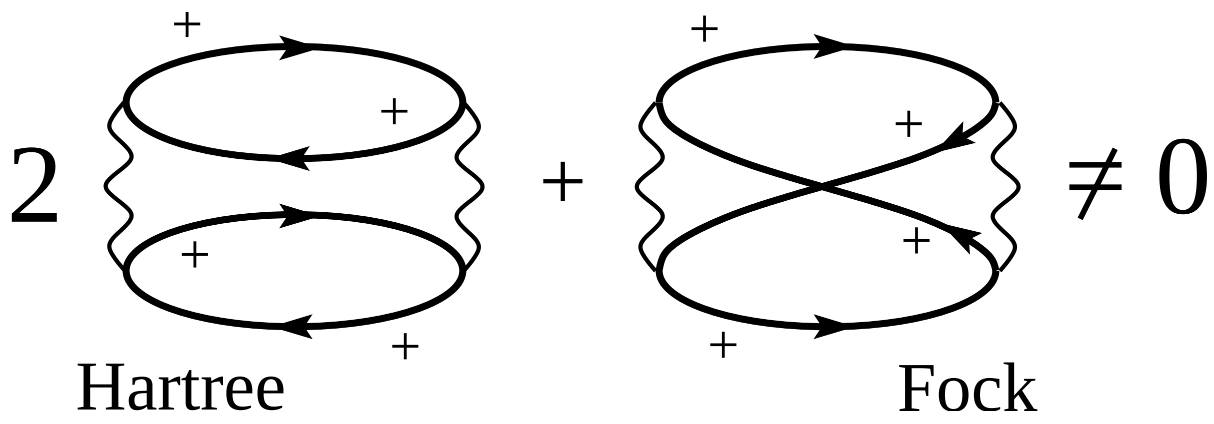
Golden Rule rules!

Perturbation theory: Second order + spin

Luttinger liquid: spinless/spinful crucially different!

No more HF-cancellation: intrabranch scattering is singular $\longrightarrow$

second-order contribution to the inelastic scattering rate **diverges!**



$$\tau_{ee}^{-1} \sim \alpha^2 T v_F \int d\omega \int dq\ \delta(\omega - v_F q)\delta(\omega - v_F q) \to \infty$$

Remedy for the divergencies: Golden Rule + RPA

Spinful case: Golden Rule + RPA

Clean Luttinger liquid: RPA exact

RPA: $v_F \to u = v_F(1+4\alpha)^{1/2}$ in effective interaction $\mathrm{Im}V(\omega,q)$:

delta-functions in $\mathrm{Re}D$ and $\mathrm{Im}V$ shifted $\longrightarrow$ divergence cured!

$$1/\tau_{ee}^{(s)} \sim \alpha^2\, T\, v_F \int d\omega \int dq\ \delta(\omega - v_F q)\delta(\omega - uq) \sim \pi\alpha^2 T v_F/|u - v_F|$$

$$1/\tau_{ee}^{(s)} = \pi|\alpha|T,$$

Single-particle Green function in LL with spin:

$$G_+^{(s)}(x,t) \simeq G_+(x,t)\frac{\sinh^{1/2}(\pi T[t - x/u])}{\sinh^{1/2}(\pi T[t - x/v_F])}$$

$$G_+(x = ut, t) \propto \exp[-\pi|\alpha|Tt/2] \times \exp[-\pi\alpha^2 Tt/2] \propto \exp[-t/2\tau_{ee}^{(s)}]$$

Intermediate T: Power-Law Hopping

Onset of localization: dephasing rate $\sim$ single-particle level spacing Δ_1

$$T_1 : \quad \delta\sigma_{wl} \sim -\sigma_D, \qquad 1/\tau_\varphi^{wl} \sim \Delta_1 \sim 1/\tau$$

no spin : $T_1 \sim 1/\alpha^2\tau$ spin : $T_1^{(s)} \sim 1/\alpha\tau$

$T_1 > T > T_3$: **Strong localization but**

$\sigma(T)$ power-law (not exponential) function of T

T_3 : $1/\tau_\varphi \sim$ three-particle level spacing Δ_3

Conductivity mechanism: diffusion over localized states

elementary step: inelastic scattering $\longrightarrow$ shift by ξ in space

$D \sim \xi^2/\tau_\varphi \quad \longrightarrow \quad \sigma(T) \sim \sigma_D \tau/\tau_\varphi$ with τ_φ from Golden Rule

Power-Law Hopping: Dephasing time

Boltzmann kinetic equation + Golden-Rule

$$1/\tau_\varphi = \int d\omega \int d\epsilon_1 K_\omega(\epsilon,\epsilon_1)\left\{ f^h_{\epsilon-\omega} f_{\epsilon_1} f^h_{\epsilon_1+\omega} + f_{\epsilon-\omega} f^h_{\epsilon_1} f_{\epsilon_1+\omega} \right\}$$

$K_\omega(\epsilon,\epsilon_1)$ – kernel of e-e collision integral, $f^h_\epsilon \equiv 1 - f_\epsilon$

Spinless case: $K_\omega \sim \alpha^2/\omega^2\tau$ for $\omega \gg 1/\tau \sim \Delta_1$

$T_1 = 1/\alpha^2\tau$: ω-transfer $\omega_0 \sim \tau^{-1} \sim \Delta_1 \longrightarrow 1/\tau_\varphi \sim \alpha^2 T$

Spinful case: intrabranch scattering dominates

$T_1^s = 1/\alpha\tau$: $\omega_0 \sim T \gg \Delta_1 \longrightarrow 1/\tau_\varphi = \alpha^2(\pi^2 T^2 + \epsilon^2)\tau$

Power-Law Hopping: Results

$T \gg \Delta_1$, cf. Gogolin, Mel'nikov & Rashba '75 (phonons)

$$\sigma(T) = \int d\epsilon \, (-\partial_\epsilon f_\epsilon) \; \sigma_{\rm ac}[\Omega = i/\tau_\varphi(\epsilon)]$$

$\sigma_{\rm ac}(\Omega) \sim -i\Omega \quad (\Omega\tau \ll 1)$ "zero-T" Berezinskii ac-conductivity

non-interacting electrons in a renormalized random potential $[\tau(T)]$

$$\sigma(T) = 4\zeta(3)\,\sigma_D(T)\,\tau(T)\,\langle\,\tau_\varphi^{-1}(T,\epsilon)\,\rangle_\epsilon\,, \qquad \Delta_3 \ll \tau_\varphi^{-1} \ll \tau^{-1} \ll \omega_0$$

Spinless: $\sigma(T) \sim \sigma_D\,\alpha^2\,T\,\tau \propto T^{1+4\alpha}$

Spinful: $\sigma(T) = c_s\,\sigma_D\,\alpha^2\,(T\,\tau)^2 \propto T^{2+6\alpha}$, $\quad c_s = (16/3)\pi^2\zeta(3)$

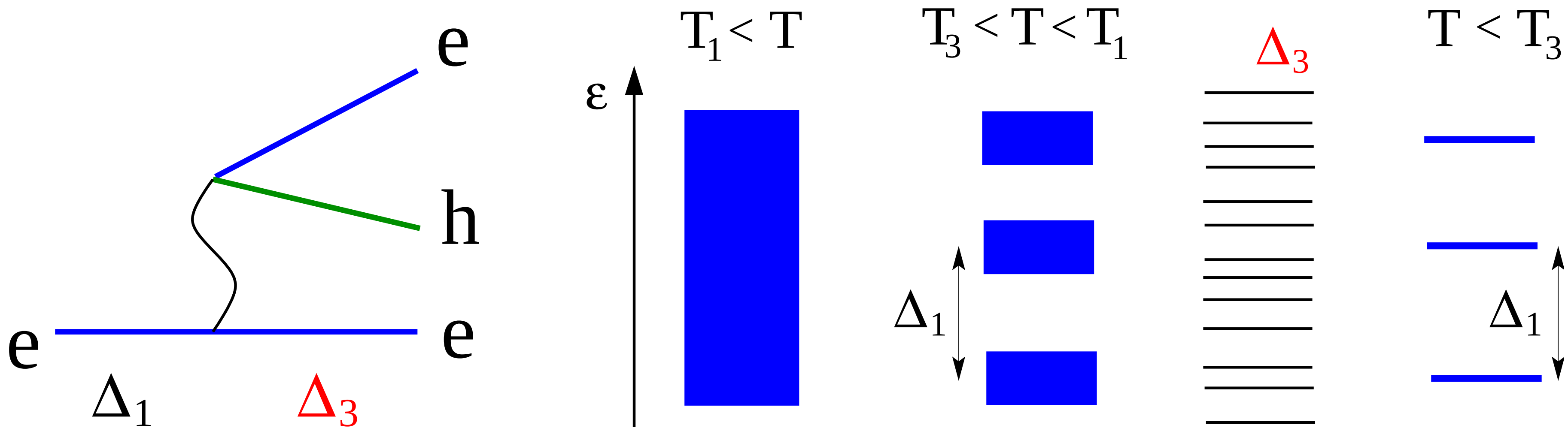
Low temperatures: $T < T_3$

PLH, $T > T_3$: creation of **e–h pairs** $\longrightarrow$ conductivity is **nonzero**

$\Longleftrightarrow$ **decay** of a single-particle state into 3-particle states (**electron** $\rightarrow$ 2 **electrons** + **hole**) is **possible**

$$T < T_3 \quad \longleftrightarrow \quad 1/\tau_\varphi < \Delta_3 \sim \Delta_1 \frac{\Delta_1}{\omega_0} \frac{\Delta_1}{T}$$

three-particle level spacing in loc. volume



$T_3 = 1/\alpha\tau$ (spinless) $\quad T_s = 1/\alpha^{1/2}\tau$ (spinful)

$T < T_3$: Localization in Fock space

cf. Altshuler, Gefen, Kamenev & Levitov '97 (quantum dots)

$\Delta_3(T_3) \sim 1/\tau_\varphi(T_3) \iff$ matrix element of interaction $|V_1| \sim \Delta_3$

No single-particle real transitions $\longrightarrow$ no conventional VRH

0D, AGKL: Localization in Fock space $\Longrightarrow$

No quasiparticles decay below E_3 ($\tau_{ee} = \infty$, DoS – δ-functions)

Quantum wires: Localization in Fock space $\Longrightarrow$

Localization in real space $\Longrightarrow$ $\sigma(T < T_3) \equiv 0$?

Activation? $\sigma(T < T_3) \propto \exp(-T_3/T)$? No!

Electrons with $\epsilon > T_3$ separated in space by $\xi \exp(T_3/T)$ and never meet!

"Non-equilibrium/non-ergodicity": T does not imply a "waiting" time $\propto \exp(\epsilon/T)$

Structure of perturbation theory: higher-order terms

cf. AGKL '97, Mirlin & Fyodorov '97, Silvestrov '97

n-th order coupling constant ($n \gg 1$ virtual electron–hole pairs)

$$|V_n|/\Delta_{2n+1} \sim \alpha^n \left[T\omega_0/\Delta_1^2(L_n)\right]^n M_n^{1/2}/(n!)^2$$

M_n – multiplicativity (number of paths in Fock space)

different paths – random signs of V_n $\longrightarrow$ $M_n^{1/2}$

$(n!)^2$ – n identical electrons and n holes (Fermi statistics)

$\Delta_1(L_n)$ – one-particle level spacing over "spreading" length L_n

1D vs 0D: different structures of Fock space

excitations spread in 1D-space $\longrightarrow$ $\Delta_1(L_n)$ decreases with n

How to obtain $\sigma(T < T_3) \neq 0$?

cf. Anderson '58

$|V_n|/\Delta_{2n+1} \ll 1$: shifts energy levels (ReΣ) but no real transitions

$|V_n|/\Delta_{2n+1} \gtrsim 1, \quad n > n_*$: ImΣ appear $\Leftrightarrow$ real transitions occur

Need $n!$-factors in $\Delta_1(L_n)$ and M_n to overcome $(T/T_3)^n/(n!)^2$

Typical ("diffusive") paths: $L_n \propto \ln^{1/2} n \longrightarrow$ "quantum dot":

$M_n \sim (n!)^3, \quad \Delta_1(L_n) \simeq \text{const}, \quad |V_n|/\Delta_{2n+1} \propto (n!)^{-1/2} \to 0$

$\Longrightarrow$ no real transitions $\Longrightarrow$ localization?

But "ballistic" paths: $L_n \sim n, \quad M_n \sim 1, \quad |V_n|/\Delta_{2n+1} \sim (T/T_3)^n$

Anderson–Fock Glass

Good news: $\sigma(T) \neq 0 \;\Leftrightarrow$ Optimal (quasi-ballistic) paths exist!

Model: "1D random granular metal", size of "grains" ξ, level spacing Δ_1

Introduce m: characteristic # of pairs excited in a grain and optimize

$$L_n \sim n/m, \quad M_n \sim (nm^2)^{n(m-1)/m}, \quad m_{opt} \sim 1$$

$$|V_n|/\Delta_{2n+1} \text{ grows as } n \text{ increases} \quad \Longrightarrow \quad \text{finite } \tau_\varphi^{-1}$$

$$n_* : \quad |V_{n_*}|/\Delta_{2n_*+1} \sim 1 \quad \longrightarrow \quad n_* \sim (T_3/T)^\nu, \quad \tau_\varphi^{-1} \propto \Delta_3 (T/T_3)^{n_*}$$

$$\text{spinless}: \quad \nu > 2, \qquad \text{spinful}: \quad \nu > 4$$

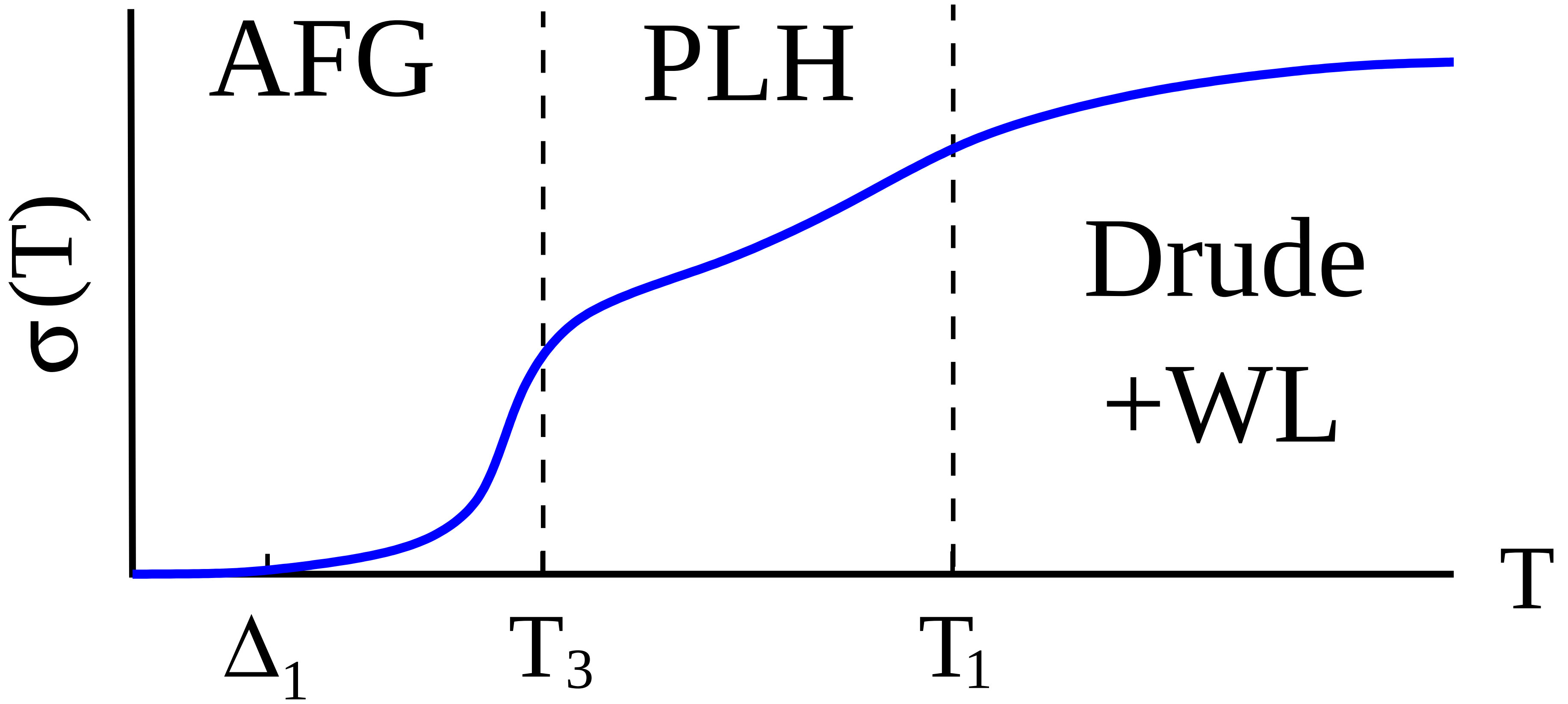
$$\sigma(T) \propto (T/T_3)^{(T_3/T)^\nu}$$

Anderson-Fock Glass: states well separated in Fock space

$T < \Delta_1$: no states with $\epsilon \lesssim T$ inside ξ

$$\sigma(T) \propto \exp[-\exp(\Delta_1/T)^\mu]$$

SUMMARY



Conclusions

1. Disordered 1D: $\sigma(T) \neq 0$ only because of dephasing

2. $T > T_1$: Weak localization in Luttinger Liquid

3. PLH (Power-Law Hopping), $T_3 < T < T_1$:
   $\sigma(T)$: Power law in T in the strongly localized regime

4. AFG (Anderson–Fock Glass), $T < T_3$:
   Neither activation nor variable-range hopping!
   Higher-order transitions between distant states in Fock space

Outlook:

Broad distribution of relaxation times in AFG

Quasi-1D: similar (N channels: more regimes but less LL);

Aharonov-Bohm in disordered Luttinger; 2D at low T, QHE; Coulomb;

Non-equilibrium; Field theory ("interacting 1D-σ-model")