Nonequilibrium superfluid state of atomic Fermi gas

Leonid Levitov

Collaboration:

Roman Barankov (MIT), Boris Spivak (U. Washington)

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Cold Fermi Gases

Fermions in magneto-optical traps:
(i) evaporatively cooled to degeneracy;
(ii) control of interaction strength & sign
near magnetically tuned resonance

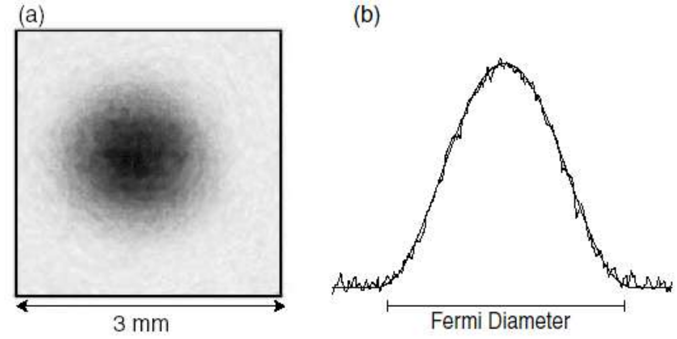


FIG. 2. Large and ultradegenerate Fermi sea. (a) Absorption image of 3×10^7 ^{6}Li atoms released from the trap and imaged after 12 ms of free expansion. (b) Axial (vertical) line density profile of the cloud in (a). A semiclassical fit (thin line) yields a temperature $T = 93$ nK $= 0.05T_F$. At this temperature, the high energy wings of the cloud do not extend visibly beyond the Fermi energy, indicated in the figure by the momentum-space Fermi diameter.

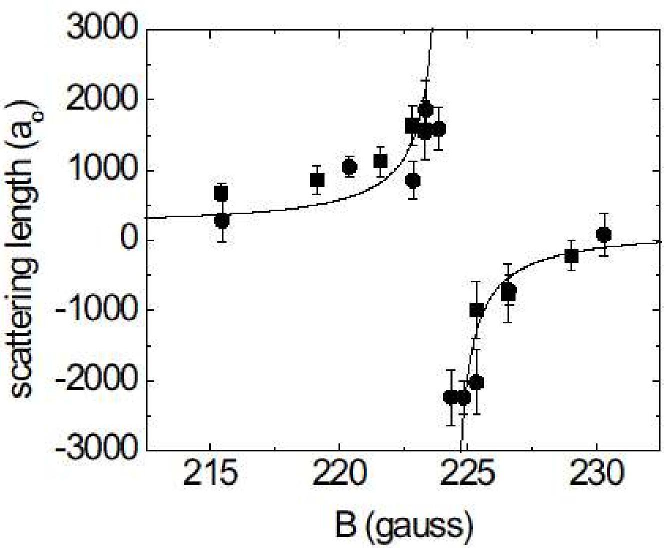
Z. Hadzibabic, *et al.* '03

$^6Li, |F = 3/2, M_F = 3/2 >$

Feshbach resonance

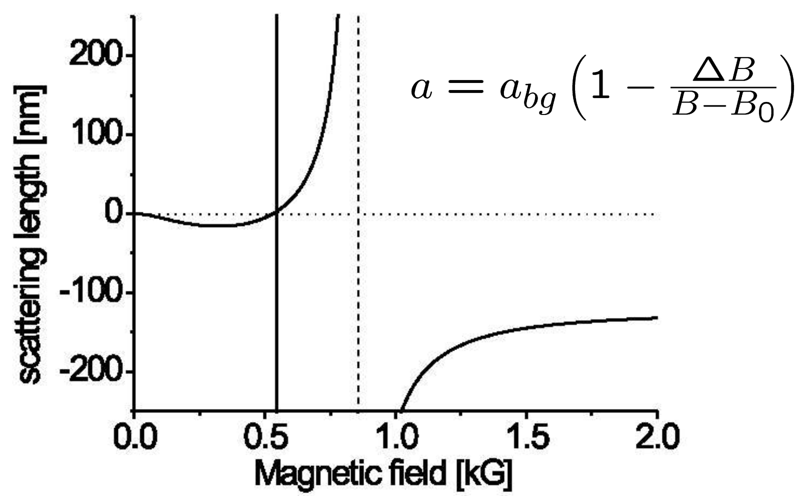
A. Regal, et al. '03

$^{40}K, |9/2, -9/2 > + |9/2, -5/2 >$



T. Bourdel, et al. '03

$^6Li, |1/2, -1/2 > + |1/2, -1/2 >$



Nonadiabatic vs. adiabatic dynamics

- External control of interaction, *a(B)*
- Fast on fermion time scales
- Time-resolved state evolution, normal-to-BCS ?

Bardeen-Cooper-Schrieffer Hamiltonian

$$\mathcal{H} = \sum_{\mathbf{p},\sigma} \xi_{\mathbf{p}} a^{+}_{\mathbf{p},\sigma} a_{\mathbf{p},\sigma} - \lambda(t) \sum_{\mathbf{p},\mathbf{q},\mathbf{d}} a^{+}_{\mathbf{p}+\mathbf{d}/2\uparrow} a^{+}_{-\mathbf{p}+\mathbf{d}/2\downarrow} a_{-\mathbf{q}+\mathbf{d}/2\downarrow} a_{\mathbf{q}+\mathbf{d}/2\uparrow}$$

$$\lambda(t) = \frac{4\pi\hbar^2|a|}{m}\theta(t)$$

Abrupt switching of pairing interaction

$$\xi_{\mathbf{p}} = \frac{\mathbf{p}^2}{2m} - \mu$$

BCS ground state at t>>0

Describe the transition?

Time scales in a superconductor

Time of change of the order parameter τ_Δ

Quasiparticle energy relaxation time τ_ϵ

$$\tau_\epsilon^{-1} \simeq max\{T^2, \epsilon^2\}/E_F, \ \tau_\Delta^{-1} \simeq \Delta_0$$

$$\tau_\Delta \ll \tau_\epsilon$$

True not too close to critical temperature

~~Time-dependent Ginzburg-Landau eqn~~

Short time of interaction switching $\tau_0 \ll \tau_\Delta, \tau_\epsilon$

Nonadiabatic time evolution

BCS parameters for trapped gas

Nonretarded BCS pairing: $T_c = 0.5 E_F e^{-1/\lambda}$ $\lambda = \frac{2}{\pi} k_F |a|$

Jila experiment: $n \approx 1.8 \times 10^{13}\, cm^{-3}$ $E_F \approx 0.35\, \mu\mathrm{K}$

$a \approx -50\,\mathrm{nm}$ $T_c \approx 0.01 E_F$

Time scales: $\tau_\Delta \simeq \hbar/\Delta_0 \approx 2\,\mathrm{ms}$ Slow relaxation

$\tau_\epsilon \simeq \hbar E_F/\Delta_0^2 \approx 200\,\mathrm{ms} = 100\tau_\Delta$

BCS correlation length (Cooper pair size) vs. gas sample size: $\xi = \hbar^2 k_F / m\Delta_0$

$\xi \simeq 24\,\mu\mathrm{m}$ ... comparable to $L \approx 18\,\mu\mathrm{m}$

Zero-dimensional limit (no spatial dependence)

Reduced BCS Hamiltonian

$$\mathcal{H} = \sum_{\mathbf{p},\sigma} \xi_{\mathbf{p}} a^{+}_{\mathbf{p},\sigma} a_{\mathbf{p},\sigma} - \lambda(t) \sum_{\mathbf{p},\mathbf{q}} a^{+}_{\mathbf{p}\uparrow} a^{+}_{-\mathbf{p}\downarrow} a_{-\mathbf{q}\downarrow} a_{\mathbf{q}\uparrow}$$

Mean field theory provides exact solution

BCS state

$$|\Psi(t)\rangle = \prod_{\mathbf{p}} \left(u_{\mathbf{p}}(t) + v_{\mathbf{p}}(t) a^{+}_{\mathbf{p},\uparrow} a^{+}_{-\mathbf{p},\downarrow} \right) |0\rangle$$

Pairing amplitude

$$\Delta(t) = \lambda \sum_{\mathbf{p}} u_{\mathbf{p}}(t) v^{*}_{\mathbf{p}}(t)$$

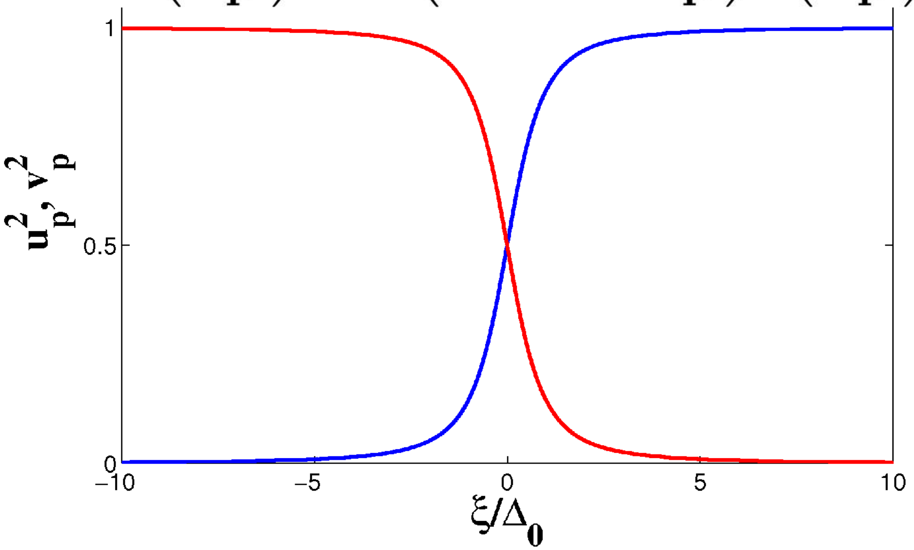
Bogoliubov-deGennes eqs

$$\epsilon \begin{pmatrix} u_{\mathbf{p}} \\ v_{\mathbf{p}} \end{pmatrix} = \begin{pmatrix} \epsilon_{\mathbf{p}} & \Delta \\ \Delta^{*} & -\epsilon_{\mathbf{p}} \end{pmatrix} \begin{pmatrix} u_{\mathbf{p}} \\ v_{\mathbf{p}} \end{pmatrix}$$

Equilibrium values

$$u^2_{\mathbf{p}} = \frac{1}{2}\left(1 + \frac{\xi_{\mathbf{p}}}{\sqrt{\xi^2_{\mathbf{p}} + \Delta^2_0}}\right)$$

$$v^2_{\mathbf{p}} = \frac{1}{2}\left(1 - \frac{\xi_{\mathbf{p}}}{\sqrt{\xi^2_{\mathbf{p}} + \Delta^2_0}}\right)$$



Pairing instability

Normal state $u_{\mathbf{p}}^{(0)}(t) = e^{-i\xi_{\mathbf{p}}t}\theta(\xi_{\mathbf{p}}),\ v_{\mathbf{p}}^{(0)}(t) = e^{i\xi_{\mathbf{p}}t}\theta(-\xi_{\mathbf{p}})$

Linear stability analysis of
B-dG equations
Abrahams and Tsuneto '66

$$\Delta(t) \propto e^{\gamma t}e^{-i\omega t}$$

Eqn for instability growth rate

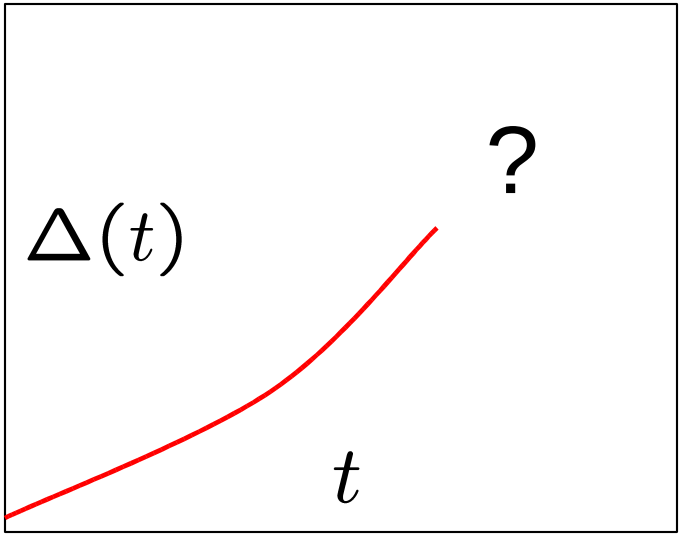
$$1 = \lambda \sum_{\mathbf{p}} \frac{\mathrm{sgn}\,\xi_{\mathbf{p}}}{2\xi_{\mathbf{p}} - \omega - i\gamma}$$

$$\gamma \approx 2\omega_c \exp(-1/g),$$
$$g = \nu_0 G/2 \ll 1$$

Characteristic time scale

$$\tau_{\Delta}^{-1} = \gamma \approx \Delta_0$$

Nonlinear dynamics



Time-dependent B-dG equations

$$i\partial_t \begin{pmatrix} u_{\mathbf{p}} \\ v_{\mathbf{p}} \end{pmatrix} = \begin{pmatrix} \xi_{\mathbf{p}} & \Delta \\ \Delta^* & -\xi_{\mathbf{p}} \end{pmatrix} \begin{pmatrix} u_{\mathbf{p}} \\ v_{\mathbf{p}} \end{pmatrix}$$

Selfconsistency eqn for pairing amplitude

$$\Delta(t) = \lambda \sum_{\mathbf{p}} u_{\mathbf{p}}(t) v^*_{\mathbf{p}}(t)$$

$$w_{\mathbf{p}} = \begin{cases} u_{\mathbf{p}}/v_{\mathbf{p}}, & \xi_{\mathbf{p}} > 0 \\ v_{\mathbf{p}}/u_{\mathbf{p}}, & \xi_{\mathbf{p}} < 0 \end{cases}$$

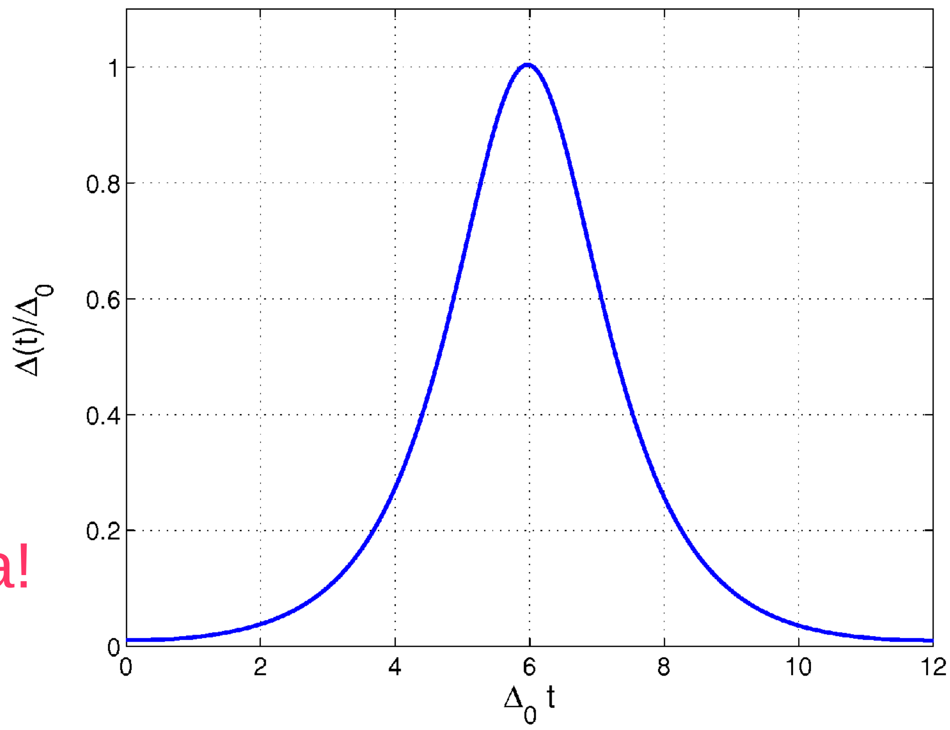
$$i\partial_t w_{\mathbf{p}} = 2\xi_{\mathbf{p}} w_{\mathbf{p}} + \Delta(t) - \Delta^*(t) w_{\mathbf{p}}^2$$

$$\Delta(t) = \lambda \sum_{\xi_{\mathbf{p}}>0} \frac{w_{\mathbf{p}}(t)}{1+|w_{\mathbf{p}}(t)|^2} + \lambda \sum_{\xi_{\mathbf{p}}<0} \frac{w^*_{\mathbf{p}}(t)}{1+|w_{\mathbf{p}}(t)|^2}$$

Soliton solution

Ansatz

$$w_{\xi_{\mathbf{p}}>0}(t) = 2\xi_{\mathbf{p}} f(t) - i\dot{f}(t),$$
$$f(t) \equiv \frac{1}{\Delta^*} = e^{-i\omega t}\alpha(t)$$



Same equation for all momenta!

$$\alpha\ddot{\alpha} = \dot{\alpha}^2 + 1$$

$$\alpha(t) = \frac{1}{\gamma}\cosh\gamma(t - t_0)$$

$$\Delta(t) = \frac{\gamma e^{-i\omega t}}{\cosh\gamma(t-t_0)}$$

Self-consistency condition
of the same form as
in the linear analysis

$$1 = \lambda \sum_{\mathbf{p}} \frac{\operatorname{sgn}\xi_{\mathbf{p}}}{2\xi_{\mathbf{p}}-\omega-i\gamma}$$

Pseudospins in BCS theory

$$H = -\sum_{\mathbf{p}} \xi_{\mathbf{p}} \sigma^z_{\mathbf{p}} - \lambda/4 \sum_{\mathbf{p},\mathbf{q}}{}' \left(\sigma^x_{\mathbf{q}}\sigma^x_{\mathbf{p}} + \sigma^y_{\mathbf{q}}\sigma^y_{\mathbf{p}}\right)$$

$$\sigma^z_{\mathbf{p}} = 1 - n_{\mathbf{p}\uparrow} - n_{-\mathbf{p}\downarrow},$$
$$\sigma^+_{\mathbf{p}} = a_{-\mathbf{p}\downarrow} a_{\mathbf{p}\uparrow},$$
$$\sigma^-_{\mathbf{p}} = a^\dagger_{\mathbf{p}\uparrow} a^\dagger_{-\mathbf{p}\downarrow}$$

Cooper pair states

$$empty = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$
$$full = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\sigma^z_{\mathbf{p}},\ \sigma^{\pm}_{\mathbf{p}} = (\sigma^x_{\mathbf{p}} \pm i\sigma^y_{\mathbf{p}})/2$ – Pauli matrices

P W Anderson '58

Conservation of particle number <==> Total spin Z-component conservation

$$\sigma^z_{tot} = \sum_{\mathbf{p}} \sigma^z_{\mathbf{p}}$$

Interaction of infinite range => mean field theory exact

Mean field analysis

$$\mathcal{H} = \sum_{\mathbf{p}} \mathcal{H}_{\mathbf{p}} = \sum_{\mathbf{p}} \mathbf{b}_{\mathbf{p}} \cdot \sigma_{\mathbf{p}}$$

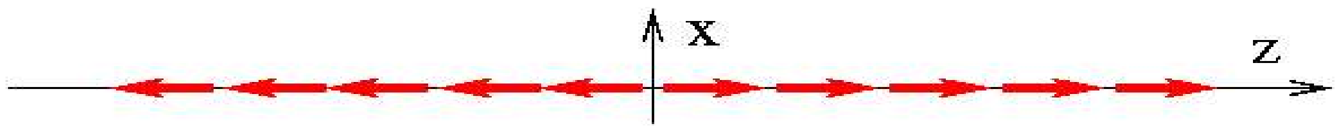
Pairing amplitude

$$\Delta \equiv \Delta_x + i\Delta_y = \lambda \sum_{\mathbf{q}} \langle \sigma_{\mathbf{q}}^{+} \rangle$$

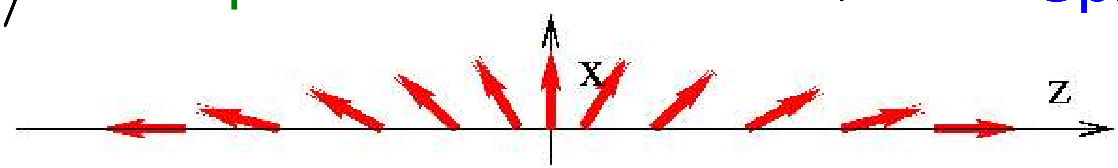
‘Magnetic field’ $\mathbf{b}_{\mathbf{p}} = (\Delta_x, \Delta_y, \xi_{\mathbf{p}})$

(x,y-components the same for all $\mathbf{p}$; z-component $\mathbf{p}$-dependent)

Normal state $\lambda = 0$



$\langle \sigma_{\mathbf{p}} \rangle$ Superfluid state $\lambda > 0$ Spin texture



Gap equation

Spin rotation <==> Bogoliubov angle

$$1 = \frac{\lambda}{2} \sum_{p}{}' \frac{1}{\sqrt{\Delta_0^2 + \xi_p^2}}$$

$$\Delta_0 \approx 2\omega_c \exp(-1/g), \; g = \nu_0 G/2$$

Dynamical equations for pseudospins

$$\mathcal{H} = \sum_{\mathbf{p}} \mathcal{H}_{\mathbf{p}} = \sum_{\mathbf{p}} \mathbf{b}_{\mathbf{p}} \cdot \sigma_{\mathbf{p}}$$

Bloch dynamics

$$\dot{\sigma}_{\mathbf{p}} = i[\mathcal{H}_{\mathbf{p}}, \sigma_{\mathbf{p}}] = 2\mathbf{b}_{\mathbf{p}} \times \sigma_{\mathbf{p}}$$

Ordinary differential equations for expectation values

$$\dot{\mathbf{r}}_{\mathbf{p}} = 2\mathbf{b}_{\mathbf{p}} \times \mathbf{r}_{\mathbf{p}} \qquad r_i = \langle \sigma^i_{\mathbf{p}} \rangle$$

$$\mathbf{b}_{\mathbf{p}} = (\Delta_x, \Delta_y, \xi_{\mathbf{p}})$$

$$\Delta \equiv \Delta_x + i\Delta_y = \lambda \sum_{\mathbf{q}} \langle \sigma^+_{\mathbf{q}} \rangle$$

Simulate ODE numerically

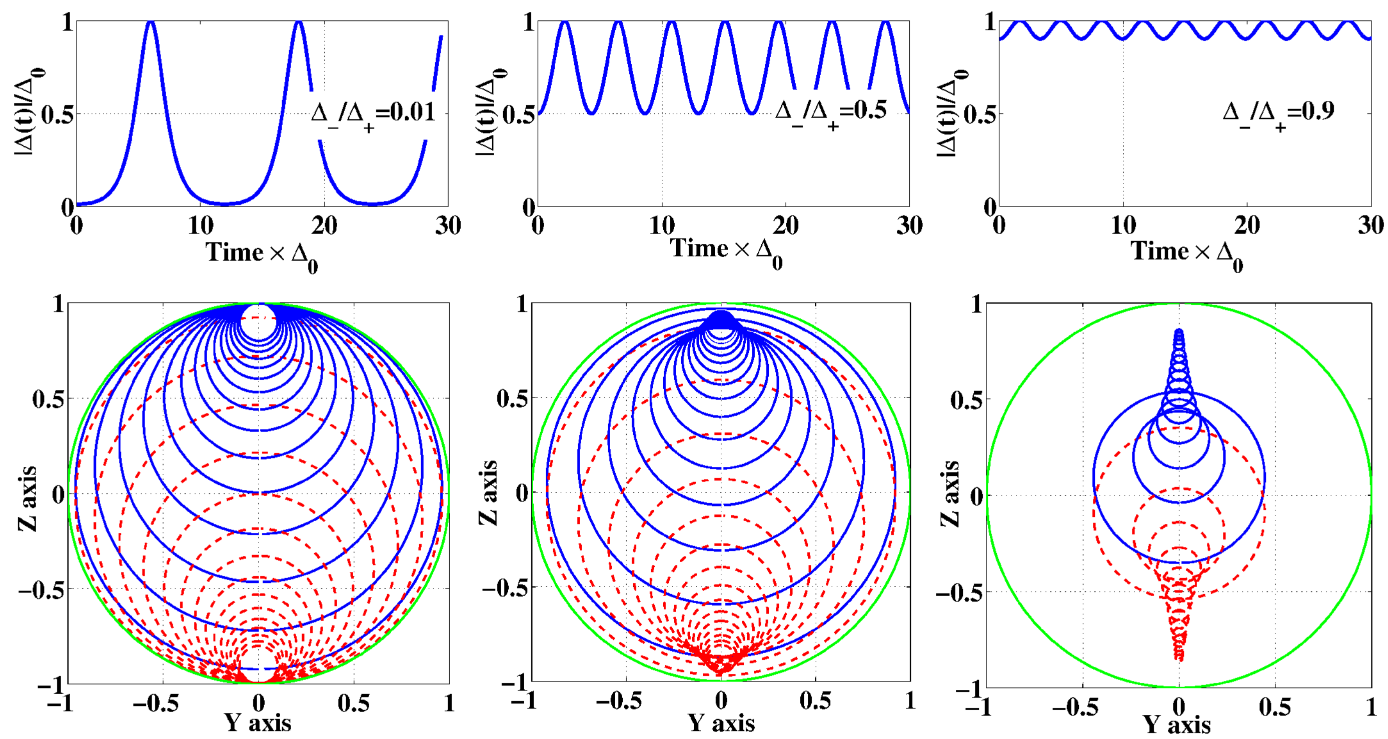
Analytic solution?

Integrability: infinitely many integrals of motion

Collective Rabi oscillations

Synchronized spin dynamics $\dot{\mathbf{r}}_\mathbf{p} = 2\mathbf{b}_\mathbf{p} \times \mathbf{r}_\mathbf{p}$

All spins complete a 2π Rabi cycle at the same time



Multisoliton solutions

Bloch equation in
a rotating `Larmor' frame

$$\tilde{\xi}_{\mathbf{p}} = 2\xi_{\mathbf{p}} - \omega$$

$$\Delta(t) = e^{-i\omega t}\Omega(t)$$

$$\dot{r}_1 = -\tilde{\xi}_{\mathbf{p}} r_2,$$
$$\dot{r}_2 = \tilde{\xi}_{\mathbf{p}} r_1 + 2\Omega r_3,$$
$$\dot{r}_3 = -2\Omega r_2$$

Ansatz $r_1 = A_{\mathbf{p}}\Omega,\ r_2 = B_{\mathbf{p}}\dot{\Omega},\ r_3 = C_{\mathbf{p}}\Omega^2 - D_{\mathbf{p}}$

$$\dot{\Omega}^2 + (\Omega^2 - \Delta_-^2)(\Omega^2 - \Delta_+^2) = 0, \quad \Delta_- \leq \Delta_+$$

Self-consistency relation

$$1 = \lambda \sum_{\mathbf{p}} \frac{\tilde{\xi}_{\mathbf{p}} \operatorname{sgn} \tilde{\xi}_{\mathbf{p}}}{\left((\tilde{\xi}_{\mathbf{p}}^2 + \Delta_-^2 + \Delta_+^2)^2 - 4\Delta_-^2\Delta_+^2\right)^{1/2}}$$

Oscillatory time dynamics

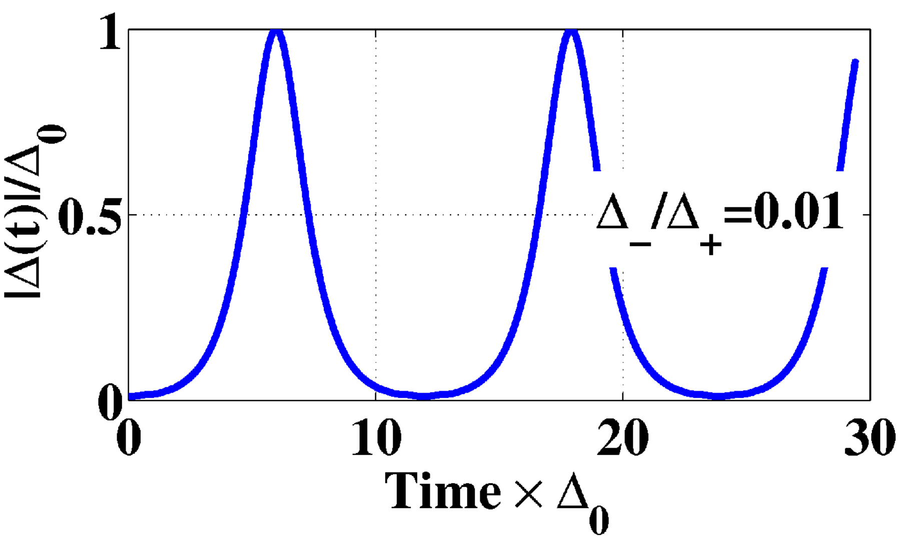
Soliton train

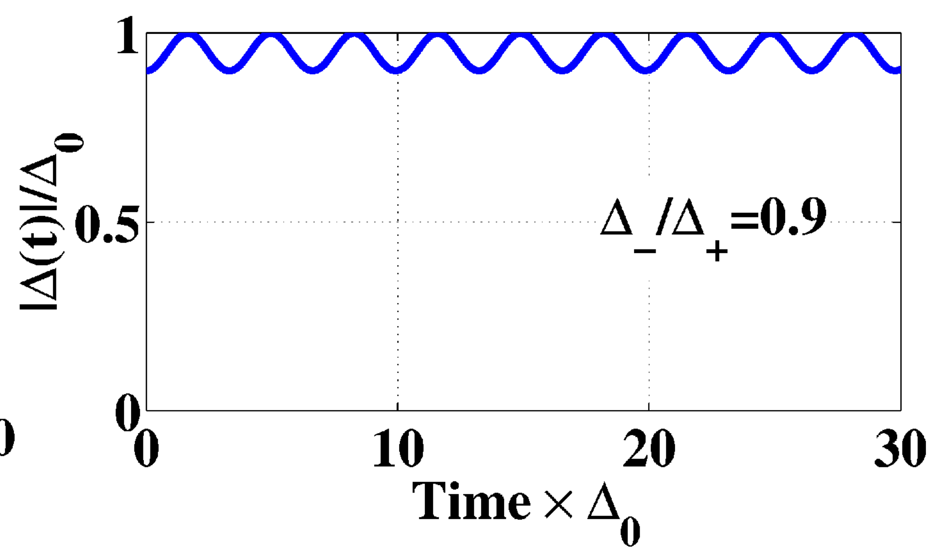
$$\Delta(t) = e^{-i\omega t}\Delta_0 dn\left[\Delta_0(t-t_0); k^2\right]$$

$$\Delta_0 = 2\omega_c \exp(-1/g) \qquad k^2 = 1 - \Delta_-^2/\Delta_0^2 \leq 1$$

Limiting cases:

$$k^2 \approx 1,\ T \approx \frac{1}{\Delta_0}\ln\left(\frac{16}{1-k^2}\right) \qquad k^2 \ll 1,\ T \approx \frac{\pi}{\Delta_0}(1 + k^2/4\ldots)$$





Damping, relaxation, noise

Damped Bloch dynamics $\dot{\mathbf{r}}_{\mathbf{p}} = 2\mathbf{b}^{\text{eff}}_{\mathbf{p}} \times \mathbf{r}_{\mathbf{p}}$

Heuristic model of energy relaxation $\mathbf{b}_{\mathbf{p}} \to \mathbf{b}^{\text{eff}}_{\mathbf{p}} = \mathbf{b}_{\mathbf{p}} - \gamma \mathbf{b}_{\mathbf{p}} \times \mathbf{r}_{\mathbf{p}}$

Damping constant $\gamma \sim 1/(\Delta_0 \tau_\epsilon)$

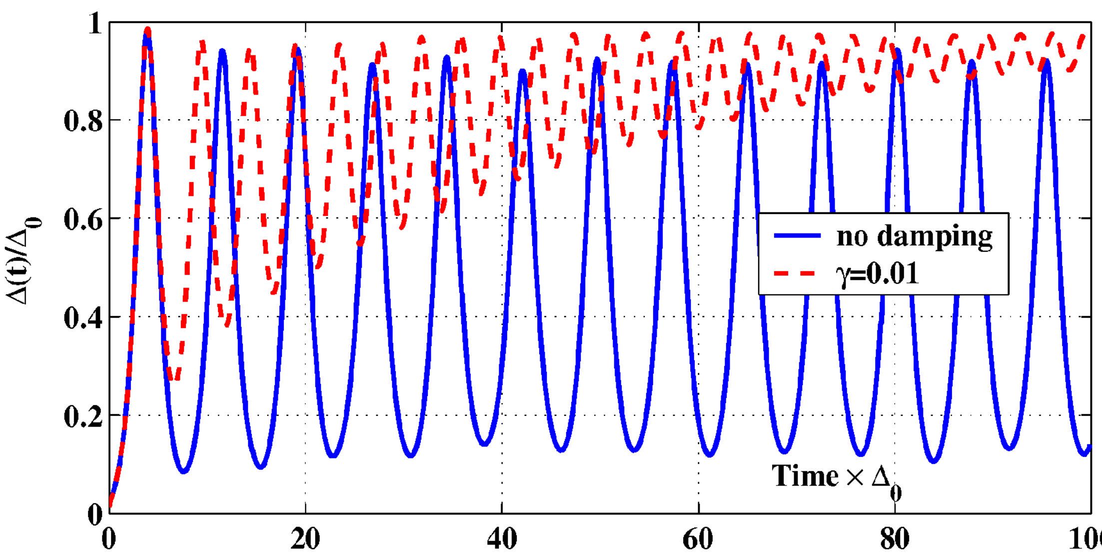
$$r_{3,\vec{p}} = \tanh(\beta\xi_{\mathbf{p}}/2)$$

$$(r_1 + ir_2)_{\mathbf{p}} = \frac{e^{i\phi_{\mathbf{p}}}}{\cosh(\beta\xi_{\mathbf{p}}/2)}$$

Noise in initial conditions:

$$0 < \phi_{\mathbf{p}} < 2\pi$$

(random, uniform, uncorrelated)



Soliton trains robust

Summary

Nonadiabatic regime:
dissipationless, nonlinear, relevant for cold gases

- Exact solution of the BCS pair formation problem
- Single soliton and soliton train solutions
- Robustness with respect to noise