2DEG under microwaves: oscillatory photoresistivity and zero-resistance states

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Dmitriev, ADM, Polyakov, PRL 91, 226802 (2003), cond-mat/0403598
Dmitriev, Vavilov, Aleiner, ADM, Polyakov, cond-mat/0310668

Andreev, Aleiner, Millis, PRL 91, 056803 (2003)
Vavilov, Aleiner, PRB 69, 035303 (2004)

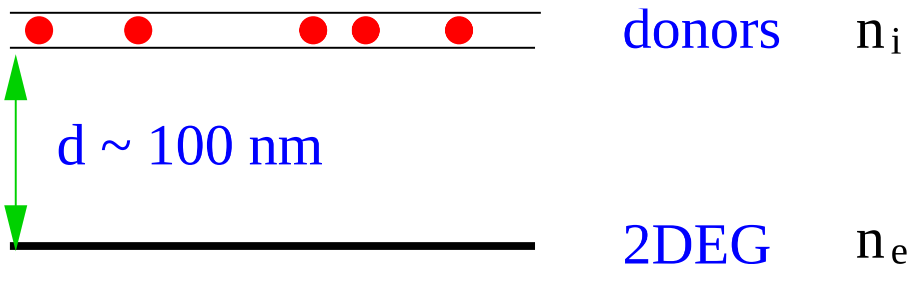
Outline:

- **Introduction: 2DEG magnetotransport**
- **Recent experiments:**
  Oscillatory PhotoConductivity and Zero-Resistance States
- **Microscopic theory:**
  - **Oscillatory ac conductivity due to interplay of Landau quantization and disorder**
  - **OPC due to microwave-induced oscillations in the distribution function**
- **From OPC to ZRS: spontaneous domain formation**
- **Theory vs. experiment**
- **Summary & Outlook**

2D electron gas in strong magnetic fields

Standard realization: GaAs/AlGaAs heterostructures

Disorder: charged donors



Typical experimental parameters:

$$n_e \sim \; n_i \sim (1 \div 3) \cdot 10^{11}\ \mathrm{cm}^{-2}, \qquad d \sim 100\ \mathrm{nm}$$

$\longrightarrow$ $k_F d \sim 10 \gg 1$ **weak long-range disorder**

$\longrightarrow$ **high mobility**

$$\frac{\tau}{\tau_q} \sim (k_F d)^2 \gg 1$$

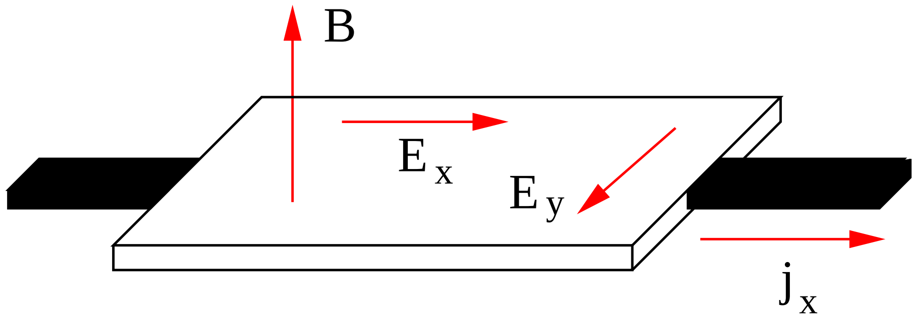
τ, τ_q – **transport and single-particle relaxation times**

Magnetotransport

resistivity tensor:

$\rho_{xx} = E_x / j_x$

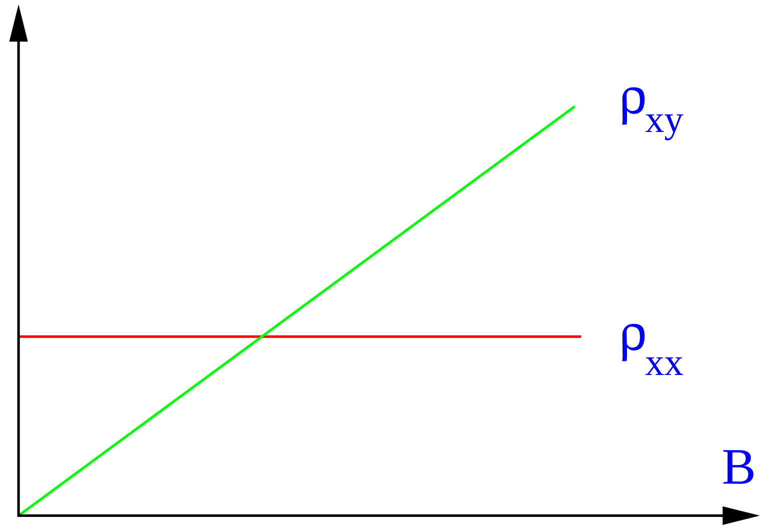
$\rho_{yx} = E_y / j_x$ **Hall resistivity**



classically (Drude–Boltzmann theory):

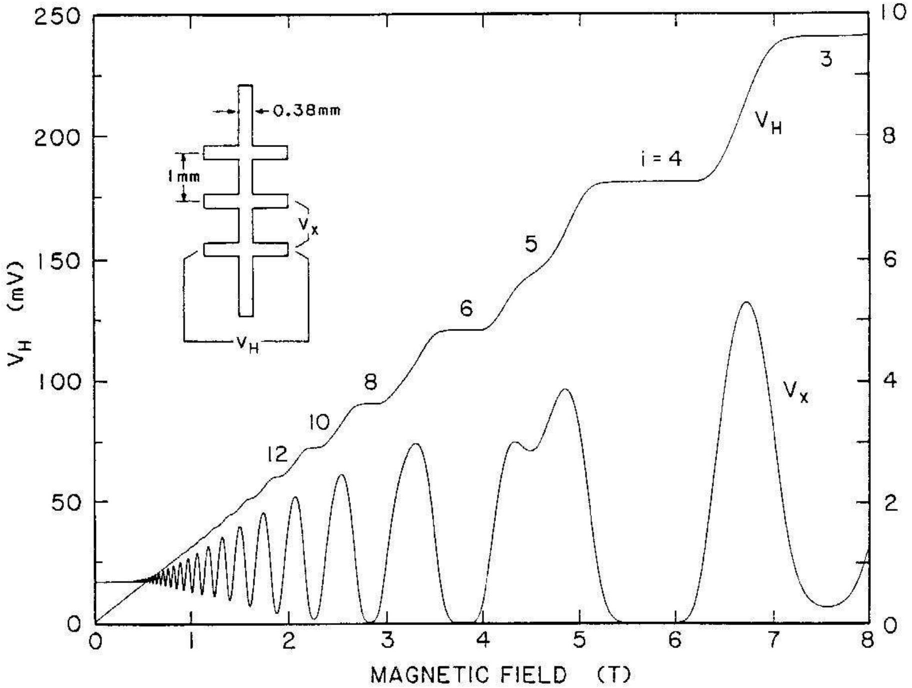
$$\rho_{xx} = \frac{m}{e^2 n_e \tau}$$ **independent of B**

$$\rho_{yx} = -\frac{B}{n_e e c}$$

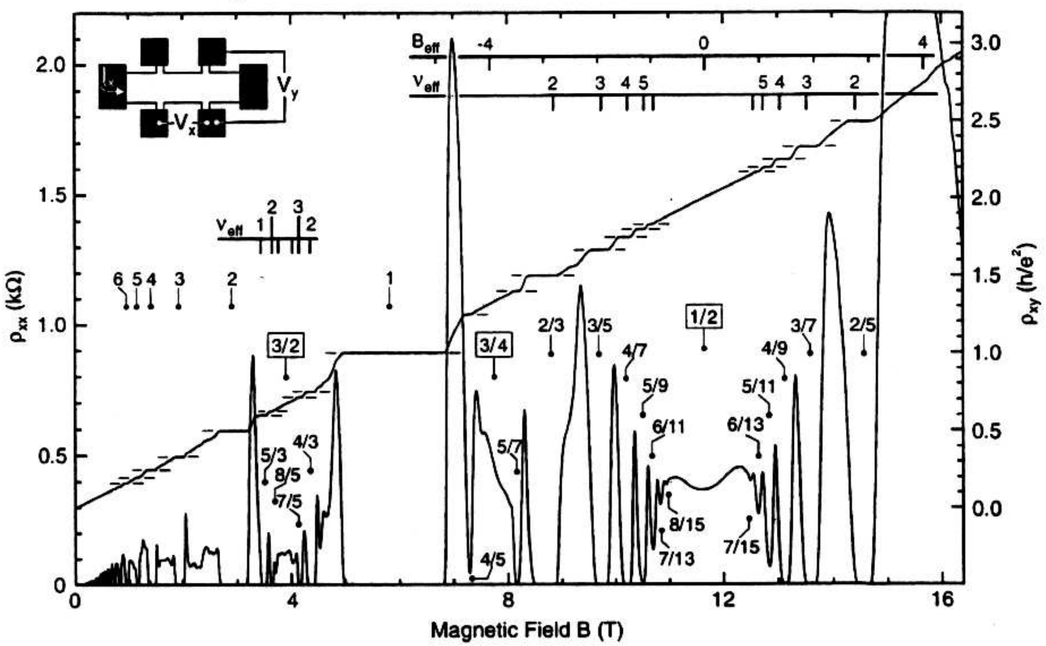


Quantum transport in strong magnetic fields

Shubnikov - de Haas Oscillations and Integer Quantum Hall Effect (IQHE)



Fractional Quantum Hall Effect (FQHE)



composite fermions:

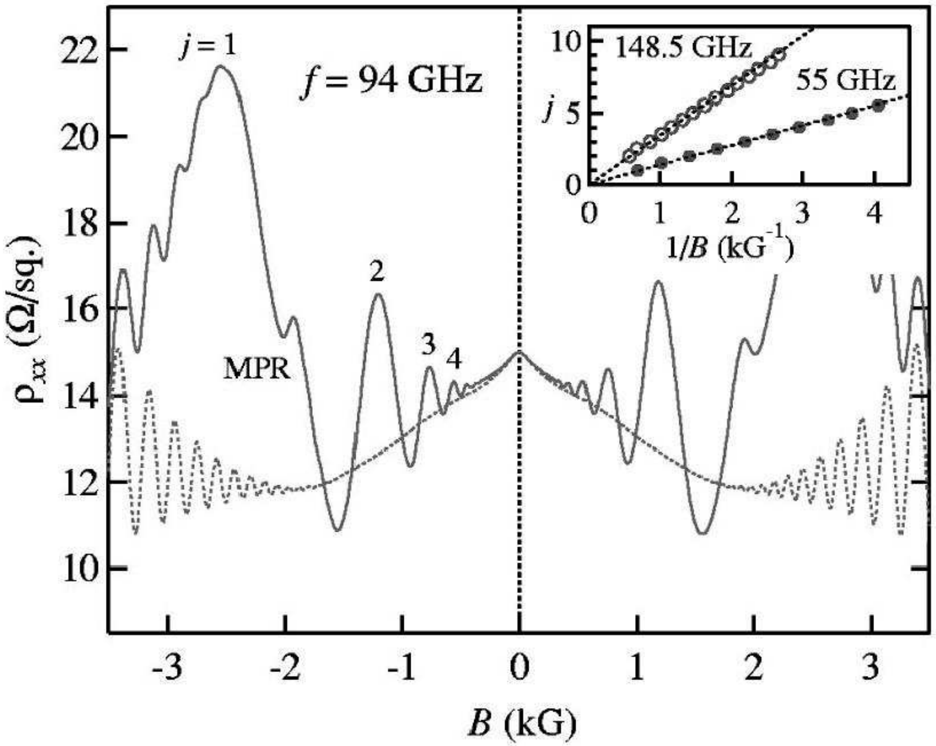
$e^- +$ 2 flux quanta

Experiment: Oscillatory photoresistance

Photoresistivity: DC response of a 2DEG subjected to microwave radiation

Zudov, Du, Simmons, and Reno, cond-mat/9711149; PRB (2001)

- High-mobility 2DEG $\quad \mu = 3 \times 10^6\ \mathrm{cm^2/Vs}$
- Microwave radiation at $\quad \omega/2\pi = 30 - 150$ GHz



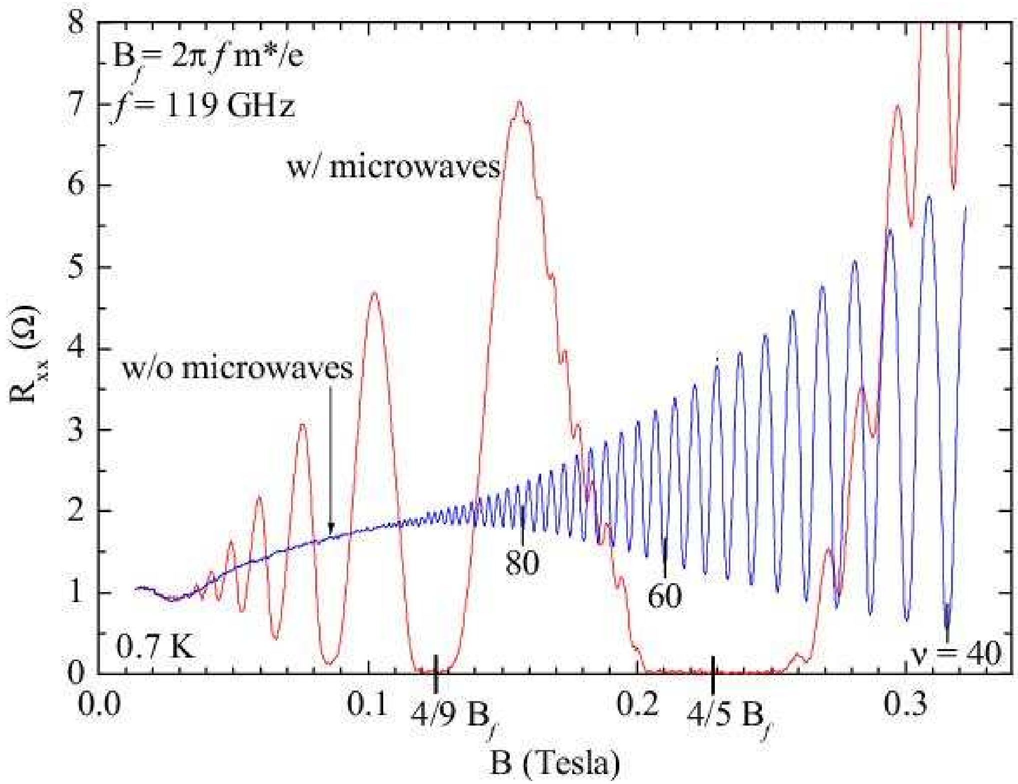
$\longrightarrow$ Oscillations governed by ω/ω_c

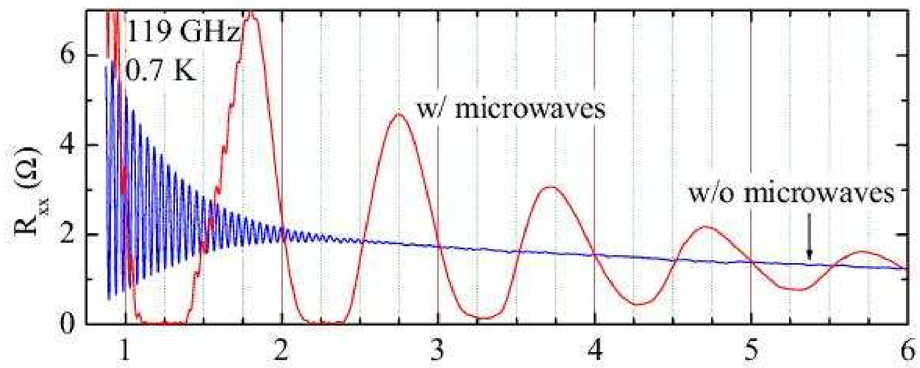
Experiment: Zero-resistance states

Mani, Smet, von Klitzing, Narayanamurti, Johnson, Umansky, Nature (2002); ...

Zudov, Du, Pfeiffer, West, PRL (2003); ...

- **ultra-high-mobility 2DEG** $\quad \mu = 1.5 \div 3 \times 10^7 \ \mathrm{cm^2/Vs}$



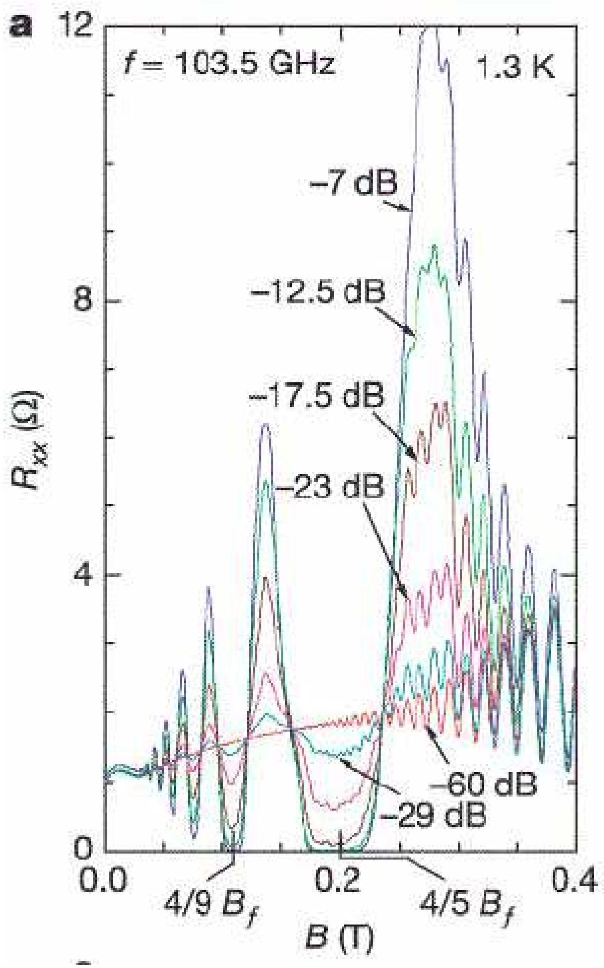


ω/ω_c

Experiment $\longrightarrow$

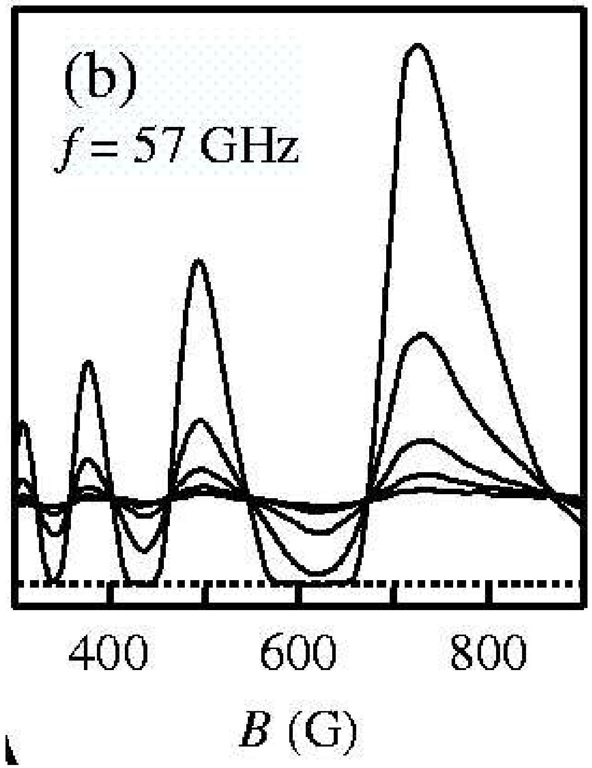
$$\Delta\rho \propto -\sin\frac{2\pi\omega}{\omega_c}\ \exp(-\pi/\omega_c\tau^*)$$

Experiment: Dependence on microwave power



Mani et al.

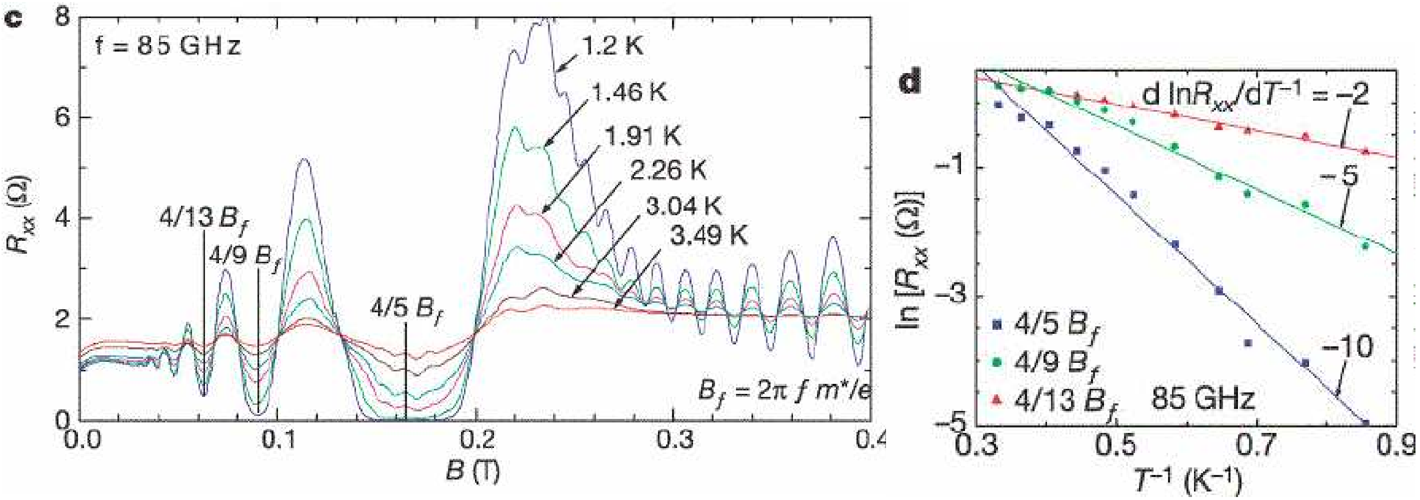
Zudov et al.



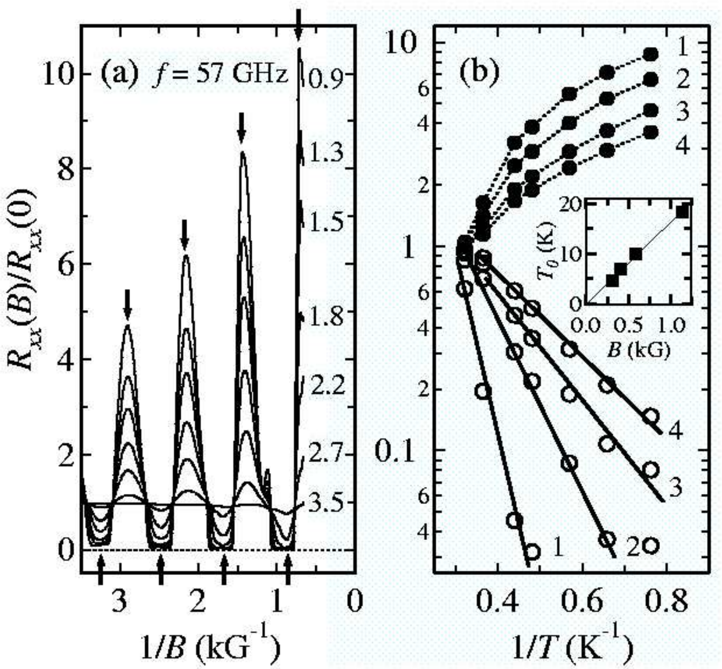
100 μW
30 μW
10 μW
3 μW
1 μW

- Maxima: $\delta\rho \propto P$
- Minima: $\rho \simeq 0$ at sufficiently strong P

Experiment: Temperature dependence



Mani et al.

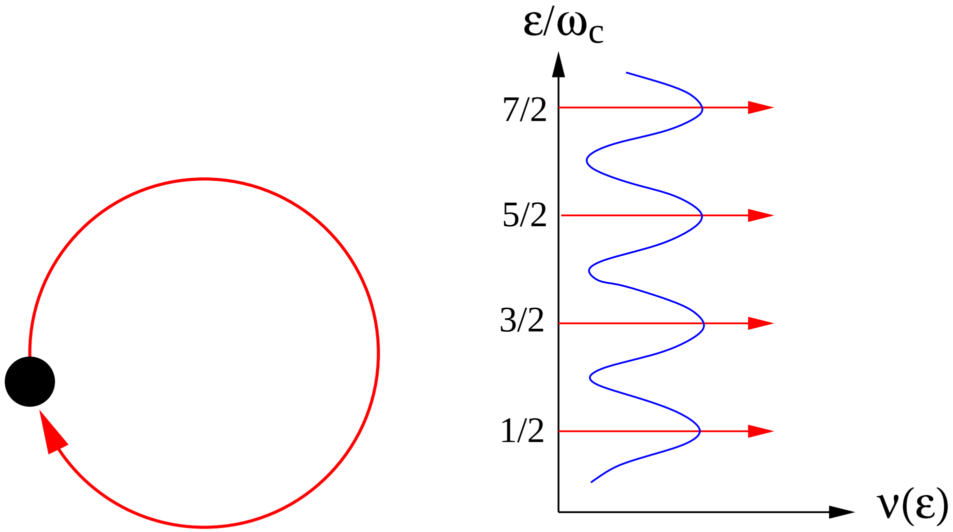


- Oscillations strongly enhanced with decreasing T
- Minima: $\rho \to 0$
  Zero-resistance regions

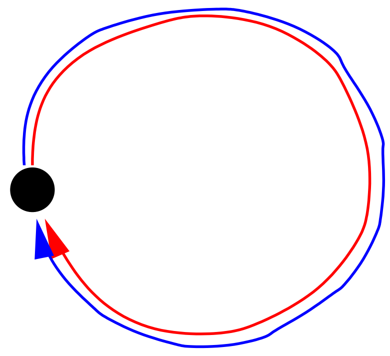
  seemingly activated behavior $e^{-T_0/T}$ with large $T_0 \sim 10$ K

Zudov et al.

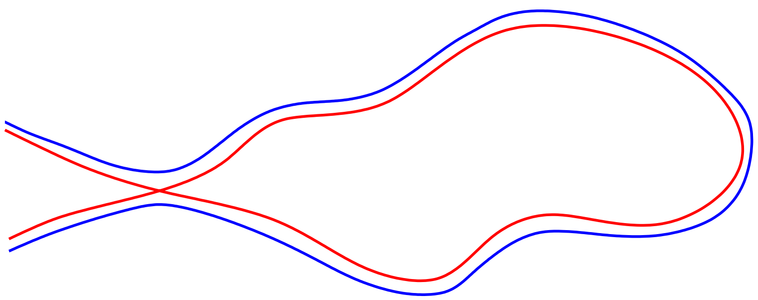
Beyond the Drude model: Return processes



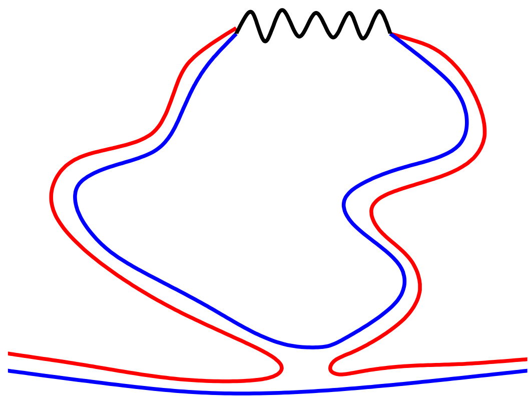
Landau quantization



quasiclassical memory effects



weak localization



interaction corrections

Oscillatory ac conductivity of a 2DEG

- sufficiently strong disorder $\longrightarrow$ constant DOS ν_0, no LLs $\longrightarrow$

  Drude theory: $\sigma_{xx}(\omega) = \sigma_+(\omega) + \sigma_-(\omega)$

$$\sigma_\pm^{(D)}(\omega) = \frac{e^2 \nu_0 v_F^2 \tau_{\mathrm{tr},0}}{4[1 + (\omega_c \pm \omega)^2 \tau_{\mathrm{tr},0}^2]}$$

$\longrightarrow$ only one cyclotron peak at $\omega_c = \omega$

- no disorder, sharp LLs $\longrightarrow$ Kohn theorem

  $\longrightarrow$ absorption at $\omega = \omega_c$ only.

- weak disorder $\longrightarrow$ oscillatory DOS

  $\longrightarrow$ LLs mixed by disorder

  $\longrightarrow$ cyclotron resonance harmonics at $\omega_c = \omega / n$.

- Disorder $U(r)$: $\langle U(\mathrm{r})U(\mathrm{r}')\rangle = W(|\mathrm{r}-\mathrm{r}'|)$

$B=0$: Quantum and transport relaxation times:

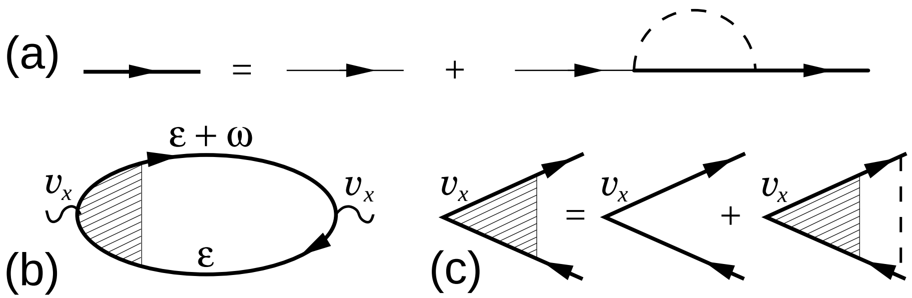
$$\left.\begin{matrix}\tau_{\mathrm{q},0}^{-1}\\ \tau_{\mathrm{tr},0}^{-1}\end{matrix}\right\} = 2\pi\nu_0 \int \frac{d\phi}{2\pi}\,\tilde{W}(2k_F \sin\frac{\phi}{2}) \times \left\{\begin{matrix}1\\ (1-\cos\phi)\end{matrix}\right.$$

White noise $\longrightarrow$ $W(r) = \frac{1}{2\pi\nu_0\tau_0}\delta(\mathrm{r})$ $\Longrightarrow$ $\tau_{\mathrm{q},0} = \tau_{\mathrm{tr},0} = \tau_0$

Smooth disorder $\longrightarrow$ correlation length $d \gg k_F^{-1}$ $\Longrightarrow$

$$\tau_{\mathrm{tr},0}/\tau_{\mathrm{q},0} \sim (k_F\, d)^2 \gg 1$$

- Self-Consistent Born Approximation:



validity conditions: $d \ll v_F\,\tau_{q,0}$, $d \ll \lambda_B = \left(\frac{c}{eB}\right)^{1/2}$

Oscillatory ac conductivity of a 2DEG

$$\sigma(\omega) = \sum_{\pm} \frac{e^2 v_F^2}{4\omega} \int d\varepsilon \, \frac{(f_\varepsilon - f_{\varepsilon+\omega})\, \nu(\varepsilon)\, \tau_{\rm tr}^{-1}(\varepsilon+\omega)}{(\omega \pm \omega_c)^2 + \frac{1}{2}\left[\tau_{\rm tr}^{-2}(\varepsilon) + \tau_{\rm tr}^{-2}(\varepsilon+\omega)\right]}$$

$\nu(\varepsilon)$ – DOS, $\tau_{\rm tr}(\varepsilon) = \dfrac{\tau_{\rm tr,0}\,\nu_0}{\nu(\varepsilon)}$ – transport time, depends on ε and B due to $\nu(\varepsilon)$

Overlapping Landau levels: LL width $> \omega_c$

- DOS $\nu = \nu_0\,[\,1 - 2\delta\cos 2\pi\varepsilon/\omega_c\,]$ $\delta = \exp(-\pi/\omega_c\tau_q) \ll 1$

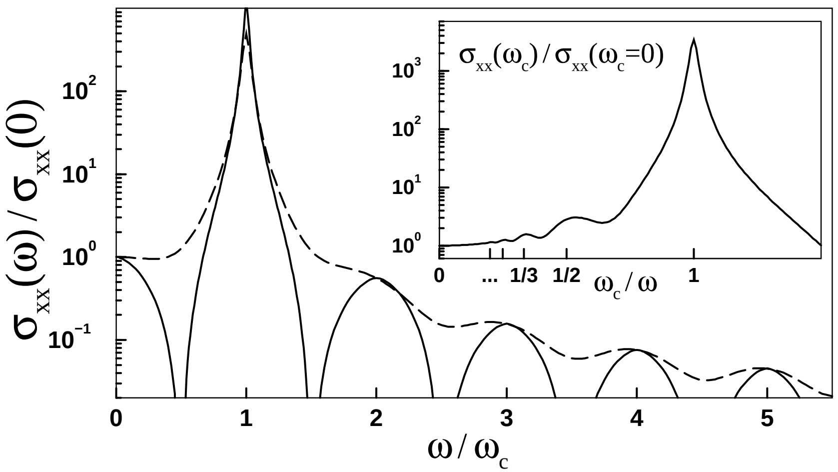
- Order δ^1: $\sigma(\omega) = \sigma^D(\omega)\left[1 - 4\delta\cos(2\pi\varepsilon_F/\omega_c)\,\dfrac{\sin(2\pi\omega/\omega_c)}{2\pi\omega/\omega_c}\right]$

$\omega = 0 \longrightarrow$ SdH oscillations

$T > \frac{\omega_c}{2\pi^2} \longrightarrow$ suppressed $\propto e^{-2\pi^2 T/\omega_c}$

- Order δ^2: survives at high T !

$$\frac{\sigma(\omega)}{\sigma^D(\omega)} = [\,1 + 2\,e^{-2\pi/\omega_c\tau_{\rm q}}\cos(2\pi\omega/\omega_c)\,]$$



- - - - - - - $\omega_c\tau_{\rm q} = \pi$

——— $\omega_c\tau_{\rm q} = 10$

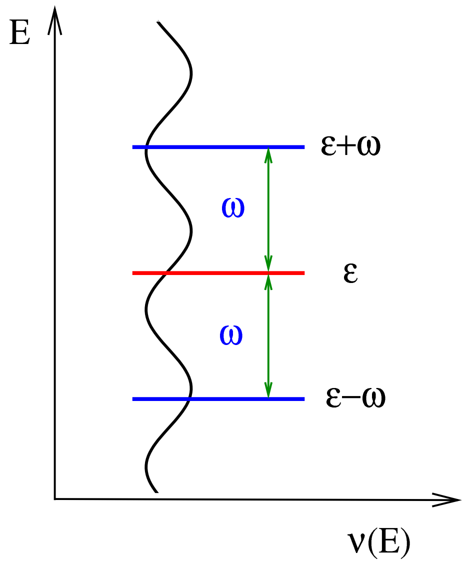
Photoconductivity theory: distribution function oscillations

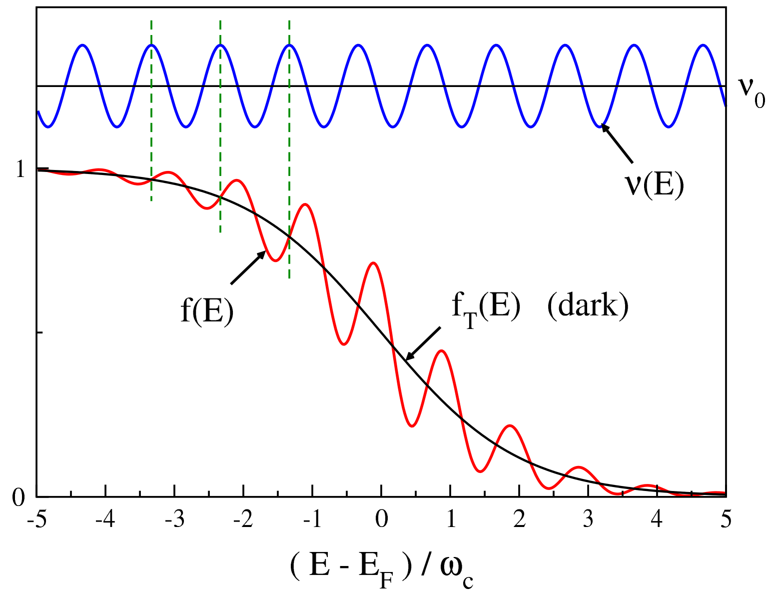
ac conductivity $\Longleftrightarrow$ absorbed power $\Longrightarrow$ change of distribution function f_ε

$$\sigma(\omega) \simeq \omega^2 |M(\omega)|^2 \int \frac{d\varepsilon}{\omega} (f_\varepsilon - f_{\varepsilon+\omega}) \nu(\varepsilon) \nu(\varepsilon+\omega) \qquad |M(\omega)|^2 = \frac{\sigma^{(D)}(\omega)}{\omega^2 \nu_0^2}$$

$$\delta f_\varepsilon = \tfrac{1}{2} |M(\omega)|^2 \left[(f_{\varepsilon-\omega} - f_\varepsilon)\nu_{\varepsilon-\omega} - (f_\varepsilon - f_{\varepsilon+\omega})\nu_{\varepsilon+\omega} \right] E_\omega^2 \, \tau_{\rm in}$$

$\tau_{\rm in}$ - inelastic relaxation time





$$\delta f_\varepsilon = \delta \cdot \tau_{\rm in} \cdot E_\omega^2 \, \sin\frac{2\pi\omega}{\omega_c} \, \sin\frac{2\pi\varepsilon}{\omega_c} \, \nu_0 \, \omega \, |M(\omega)|^2 \left(\frac{\partial f_T}{\partial \varepsilon} \right)$$

Microwave-induced oscillatory δf_ε

$$\delta f_\varepsilon \quad \Longrightarrow \quad \textbf{Photoconductivity}$$

$$\sigma_{\rm ph} - \sigma_{\rm dc} = \sigma^{\rm D}_{\rm dc} \int d\varepsilon \left(-\frac{\partial\, \delta f_\varepsilon}{\partial \varepsilon} \right) \frac{\nu^2(\varepsilon)}{\nu_0^2} = = -4\sigma^{\rm D}_{\rm dc}\, \delta^2 P_\omega \frac{\pi\omega}{\omega_c} \sin \frac{2\pi\omega}{\omega_c}$$

$$P_\omega = \frac{2\sigma^{\rm D}(\omega)}{\omega^2 \nu_0} E_\omega^2 \tau_{\rm in}$$ – dimensionless microwave power

Non-linear photoconductivity: Kinetic equation

Quantum kinetic equation for the distribution function $f(\varepsilon)$:

$$\mathcal{E}_\omega^2 \frac{\sigma^{\mathrm{D}}(\omega)}{2\omega^2\nu_0^2} \sum_{\pm} \nu(\varepsilon\pm\omega)[f(\varepsilon\pm\omega)-f(\varepsilon)] + \frac{\mathcal{E}_{\mathrm{dc}}^2\,\sigma_{\mathrm{dc}}^{\mathrm{D}}}{\nu_0^2\nu(\varepsilon)} \frac{\partial}{\partial\varepsilon}\left[\nu^2(\varepsilon)\frac{\partial}{\partial\varepsilon}f(\varepsilon)\right] = \frac{f(\varepsilon)-f_T(\varepsilon)}{\tau_{\mathrm{in}}}$$

Dimensionless units for the strength of ac and dc fields:

$$\mathcal{P}_\omega = \frac{\tau_{\mathrm{in}}}{\tau_{\mathrm{tr}}}\left(\frac{e\mathcal{E}_\omega v_F}{\omega}\right)^2 \frac{\omega_c^2+\omega^2}{(\omega^2-\omega_c^2)^2}, \qquad \mathcal{Q}_{\mathrm{dc}} = \frac{2\,\tau_{\mathrm{in}}}{\tau_{\mathrm{tr}}}\left(\frac{e\mathcal{E}_{\mathrm{dc}} v_F}{\omega_c}\right)^2\left(\frac{\pi}{\omega_c}\right)^2$$

Overlapping LLs $\longrightarrow$ $\delta = e^{-\pi/\omega_c\tau_{\mathrm{q}}} \ll 1$ $\longrightarrow$ look for a solution in the form

$$f(\varepsilon) = f_T(\varepsilon) + f_{\mathrm{osc}}(\varepsilon) + O(\delta^2), \qquad f_{\mathrm{osc}}(\varepsilon) \equiv \delta\,\mathrm{Re}\left[f_1(\varepsilon)\,e^{i\frac{2\pi\varepsilon}{\omega_c}}\right].$$

$\longrightarrow$ oscillations in non-equilibrium distribution function:

$$f_{\mathrm{osc}}(\varepsilon) = \delta\,\frac{\omega_c}{2\pi}\,\frac{\partial f_T}{\partial\varepsilon}\,\sin\frac{2\pi\varepsilon}{\omega_c}\,\frac{\mathcal{P}_\omega\frac{2\pi\omega}{\omega_c}\sin\frac{2\pi\omega}{\omega_c}+4\mathcal{Q}_{\mathrm{dc}}}{1+\mathcal{P}_\omega\sin^2\frac{\pi\omega}{\omega_c}+\mathcal{Q}_{\mathrm{dc}}}$$

$\longrightarrow$ oscillatory photoconductivity

Non-linear photoconductivity: Results

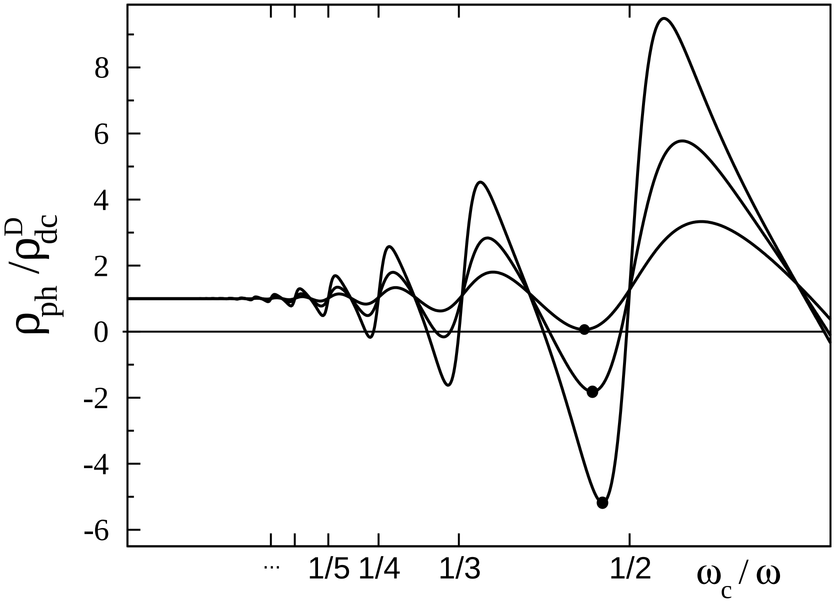
$$\frac{\sigma_{\rm ph}}{\sigma_{\rm dc}^{\rm D}} = 1 + 2\delta^2 \left[1 - \frac{\mathcal{P}_\omega \frac{2\pi\omega}{\omega_c} \sin\frac{2\pi\omega}{\omega_c} + 4\mathcal{Q}_{\rm dc}}{1 + \mathcal{P}_\omega \sin^2\frac{\pi\omega}{\omega_c} + \mathcal{Q}_{\rm dc}} \right]$$

- Linear response: $\mathcal{Q}_{\rm dc} \to 0$
- not too strong $\mathcal{P}_\omega \longrightarrow$ linear-in-$\mathcal{P}_\omega$ correction
- strong $\mathcal{P}_\omega \longrightarrow$ saturation: $\dfrac{\sigma_{\rm ph}}{\sigma_{\rm dc}} = 1 - 8\delta^2 \frac{\pi\omega}{\omega_c} \cot\frac{\pi\omega}{\omega_c}$, $\quad \mathcal{P}_\omega \sin^2\frac{\pi\omega}{\omega_c} \gg 1$

$\longrightarrow$ despite $\delta^2 \ll 1$, correction large near $\omega = k\omega_c$

$\longrightarrow$ $\sigma_{\rm ph} < 0$ around minima for $\mathcal{P}_\omega > \mathcal{P}_\omega^* = \left(4\delta^2 \frac{\pi\omega}{\omega_c} \sin\frac{2\pi\omega}{\omega_c} - \sin^2\frac{\pi\omega}{\omega_c} \right)^{-1}$

threshold power



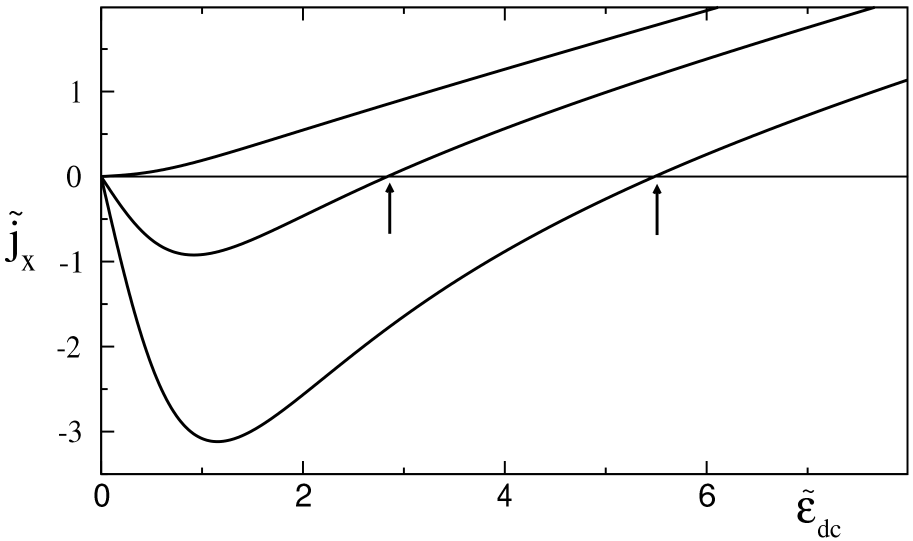
$\rho_{\rm ph}/\rho_{\rm dc}^{(D)}$ vs ω_c/ω

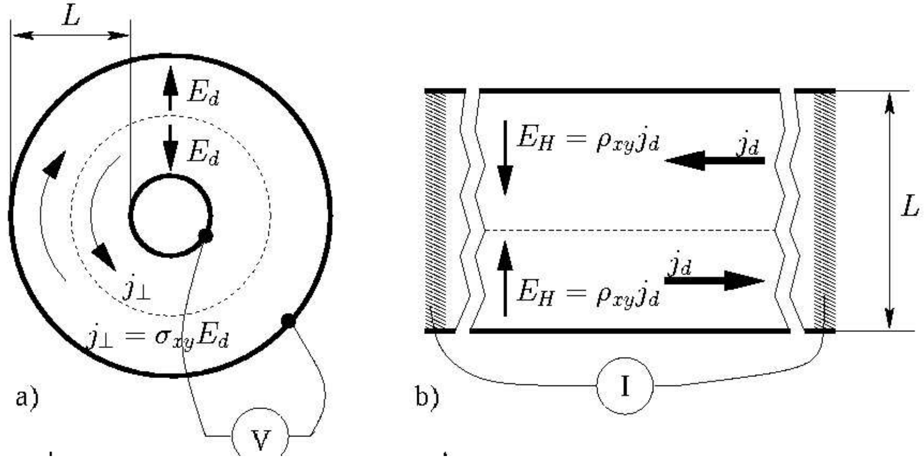
$\omega\tau_{\rm q,0} = 2\pi$

$\mathcal{P}_\omega^{(0)} \equiv \mathcal{P}_\omega(\omega_c = 0) = \{0.24,\ 0.8,\ 2.4\}$

I-V characteristics and Zero-Resistance States

Current-voltage characteristics at the minima:





Linear response $\sigma_{\rm ph}$ negative $\longrightarrow$ instability $\longrightarrow$ domains $\longrightarrow$ zero-resistance state

spontaneous field $\mathcal{E}^*_{\rm dc}$ in the domains: determined from $\sigma_{\rm ph}(\mathcal{E}^*_{\rm dc}) = 0$

$$e\mathcal{E}^*_{\rm dc} = \frac{\omega_c}{\pi R_c}\left(\frac{\tau_{\rm tr}}{2\tau_{\rm in}}\right)^{1/2}\left[\left(\frac{\mathcal{E}_\omega}{\mathcal{E}^*_\omega}\right)^2 - 1\right]^{1/2} \qquad \longrightarrow \quad \mathcal{E}^*_{\rm dc} \sim 1\ {\rm V/cm}$$

can be measured by a local probe: Willett, Pfeiffer, West, PRL 2004

Stability conditions and Zero-Resistance States

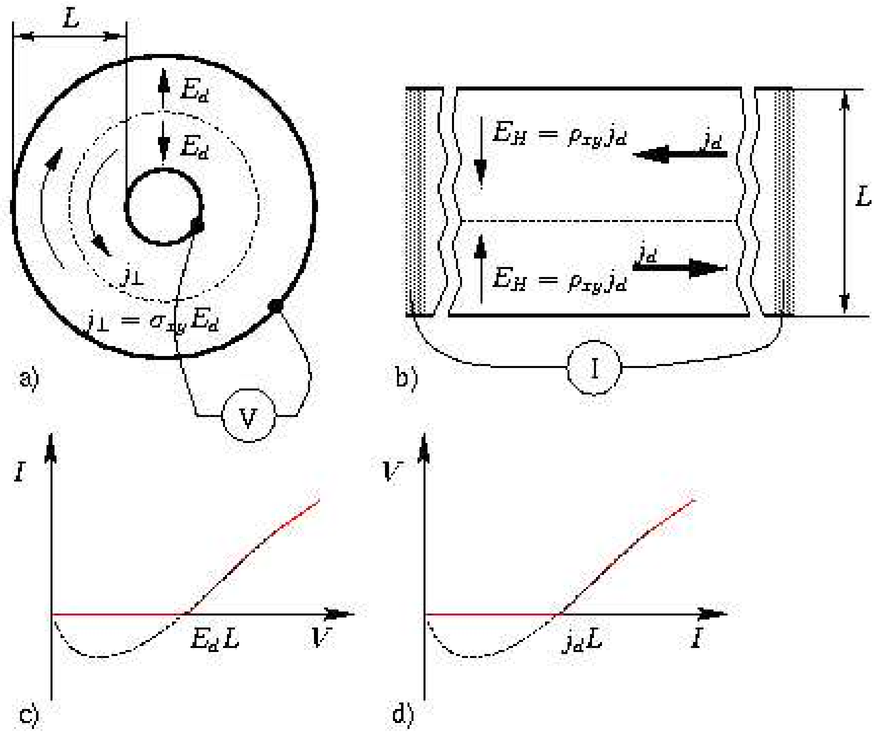
Andreev, Aleiner, Millis, PRL 2003

$$\frac{\partial n}{\partial t} = \nabla \mathbf{j} = 0 \qquad \text{continuity}$$

$$\mathbf{E} = -\nabla \phi = -\nabla U n \qquad \text{Poisson}$$

$$\mathbf{j} = \hat{\sigma}(E)\mathbf{E}$$

U – Coulomb interaction



⟶ Evolution equation for fluctuations $\delta n(\mathbf{r}, t)$:

$$\frac{\partial}{\partial t}\delta n = \nabla \left[\hat{\sigma} + E \frac{d\hat{\sigma}}{dE} \frac{\mathbf{E} \otimes \mathbf{E}}{E^2} \right] \nabla U \delta n$$

Stability conditions: Re (eigenvalues) > 0 ⟶

$$\sigma_{xx} = \frac{j_x}{E_x} \geq 0 \qquad \text{and} \qquad \sigma_{xx} + E\frac{d\sigma_{xx}}{dE} = \frac{\partial j_x}{\partial E_x} \geq 0$$

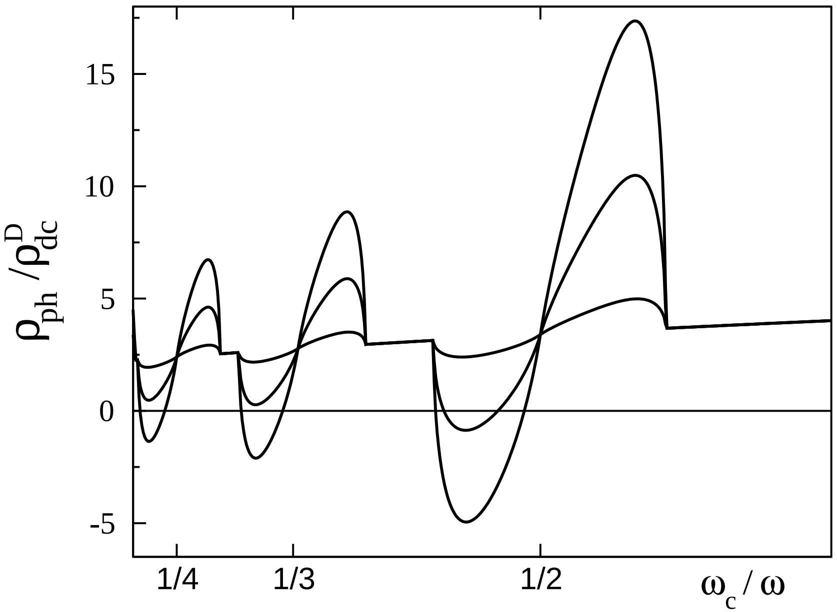
Photoresistivity: Separated LLs

$\frac{\omega_c \tau_q}{\pi} \gg 1$ $\longrightarrow$ separated LLs, width $2\Gamma = 2\left(\frac{2\omega_c}{\pi\tau_q}\right)^{1/2}$

Linear response ($\mathcal{Q}_{dc} \to 0$) photoconductivity:

$$\frac{\sigma_{ph}}{\sigma_{dc}^{D}} \simeq \frac{16\omega_c}{3\pi^2\Gamma}\left[1 - \mathcal{P}_\omega \frac{\omega\omega_c}{\Gamma^2}\sum_n \Phi\left(\frac{\omega - n\omega_c}{\Gamma}\right)\right]$$

$$\Phi(x) = \frac{3x}{4\pi}\mathrm{Re}\left[\arccos(|x|-1) - \frac{1-|x|}{3}\sqrt{|x|(2-|x|)}\right]$$



Linear-response $\sigma_{ph} < 0$ around minima $\longrightarrow$ ZRS

strong $\mathcal{E}_{dc}$: $\sigma_{ph}(\mathcal{E}_{dc})$ positive only if inter-LL elastic impurity scattering efficient

$\longrightarrow$ stronger spontaneous field $\mathcal{E}_{dc}^{*} \simeq \left(\frac{\tau_{tr}}{\tau_q}\right)^{1/2}\frac{\omega_c^2}{ev_F}$

Temperature dependence: Inelastic relaxation time

For not too strong microwave power:

$$\sigma_{\rm ph} - \sigma_{\rm dc} \propto \mathcal{P}_\omega \propto \tau_{\rm in}$$

Dominant relaxation mechanisms:

smooth part of the distriubution function $f_T(\varepsilon) \longrightarrow$ phonons

oscillatory part $f_{\rm osc}(\varepsilon) \longrightarrow$ e-e collisions

$$\frac{1}{\tau_{ee}} = \frac{\pi T^2}{4\epsilon_F} \ln \frac{\epsilon_F}{\max\left[T, \omega_c(\omega_c\tau_{\rm tr})^{1/2}\right]} \qquad \text{overlapping LLs}$$

$$\frac{1}{\tau_{\rm ee}} \sim \frac{\omega_c}{\Gamma} \frac{T^2}{\epsilon_F} \ln \frac{\epsilon_F}{\max\left[T, \Gamma\left(\omega_c\tau_{\rm tr}\right)^{1/2}\right]} \qquad \text{separated LLs}$$

$$\Longrightarrow \qquad \sigma_{\rm ph} - \sigma_{\rm dc} \propto T^{-2}$$

Two contributions to photoconductivity

- $\sigma_{\text{ph}}^{(1)}$: related to the change of the distribution function

  $\Longrightarrow \quad \propto \tau_{\text{in}}$, strongly T–dependent, independent of polarization

  $\longrightarrow$ discussed above

- $\sigma_{\text{ph}}^{(2)}$: influence of microwave on impurity scattering,

  T–independent, polarization-dependent

Durst, Sachdev, Read, Girvin, PRL 2003

Vavilov, Aleiner, PRB 2004

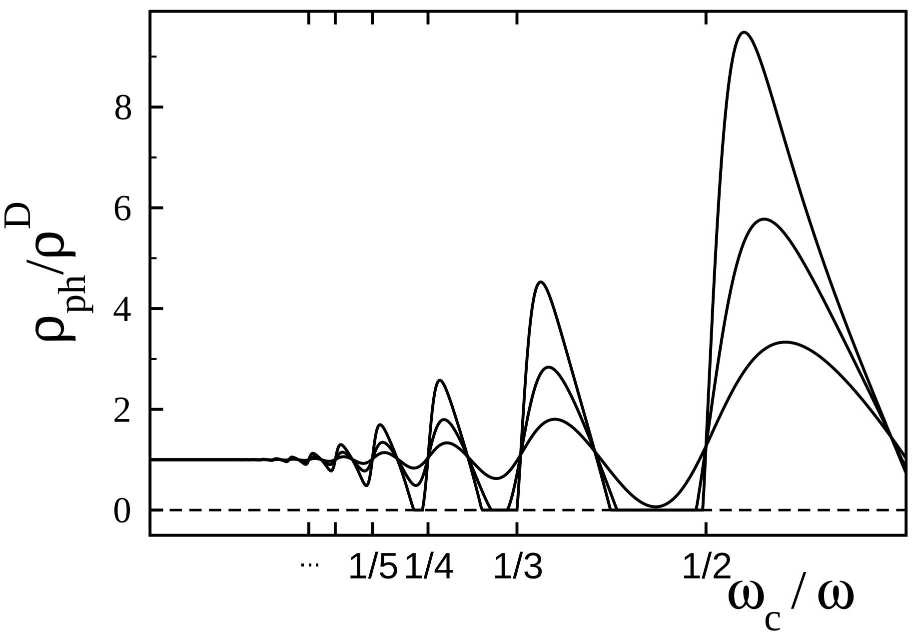
$$\frac{\sigma_{\text{ph}}^{(1)}}{\sigma_{\text{ph}}^{(2)}} \sim \frac{\tau_{\text{in}}}{\tau_q} \gg 1 \qquad \text{for relevant } T$$

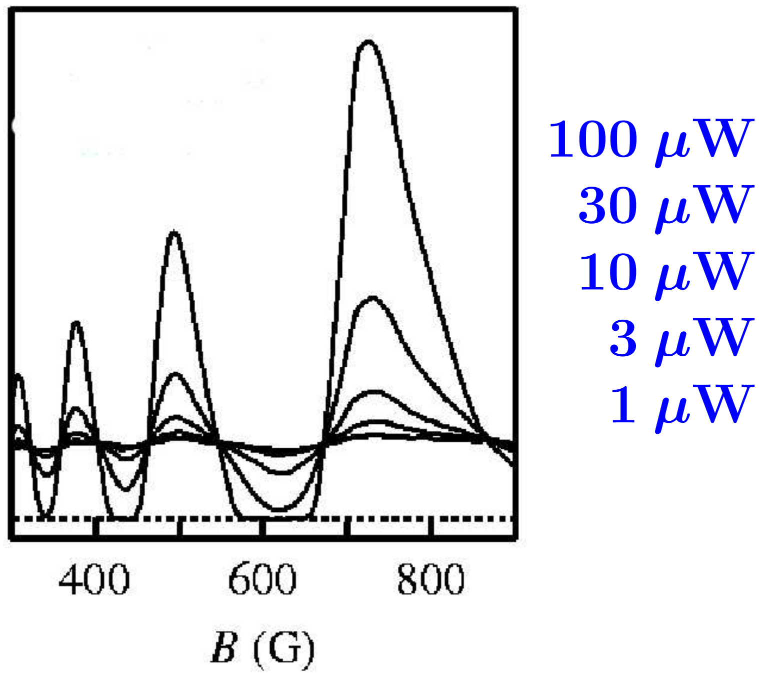
Order-of-magnitude estimate of energy scales:

$E_F \sim 100$ K	$\tau_{\text{q}}^{-1} \sim 1$ K	$T \sim 1$ K
$\omega \sim 1$ K	$\tau_{\text{tr}}^{-1} \sim 10$ mK	$\tau_{\text{in}}^{-1} \sim \frac{T^2}{E_F} \sim 10$ mK

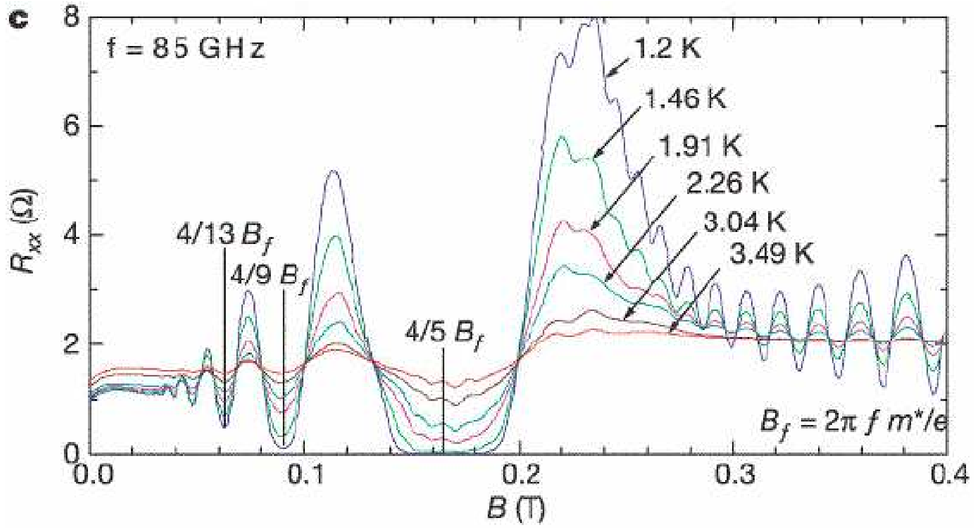
Theory vs experiment

$$\frac{\omega}{2\pi} \simeq 50 - 100 \text{ GHz}, \quad \tau_q \simeq 10 \text{ ps} \quad \longrightarrow \quad \frac{\omega\tau_q}{2\pi} \simeq 0.5 - 1 \quad \text{(overlapping LLs)}$$





- **Period, phase, shape – OK**
- **Independence of polarization – OK**
- **Temperature dependence at maximum**
  $\propto \tau_{\rm in} \propto T^{-2}$ **– reasonably OK**



Summary

- Microwaves
  - ⟶ magnetooscillations in the 2DEG conductivity
  - ⟶ negative linear-response conductivity
  - ⟶ instability
  - ⟶ domains
  - ⟶ zero-resistance states

- Parametrically largest contribution:
  Microwave-induced non-equilibrium distribution function.

- Magnitude of the effect proportional to $\tau_{\text{in}} \propto T^{-2}$

- Future research (experimental and theoretical):
  – detailed study of the domain physics
  – – effect of finite T on ZRS (experiment: activation?)
  – – noise in ZRS