Coulomb drag in high Landau levels

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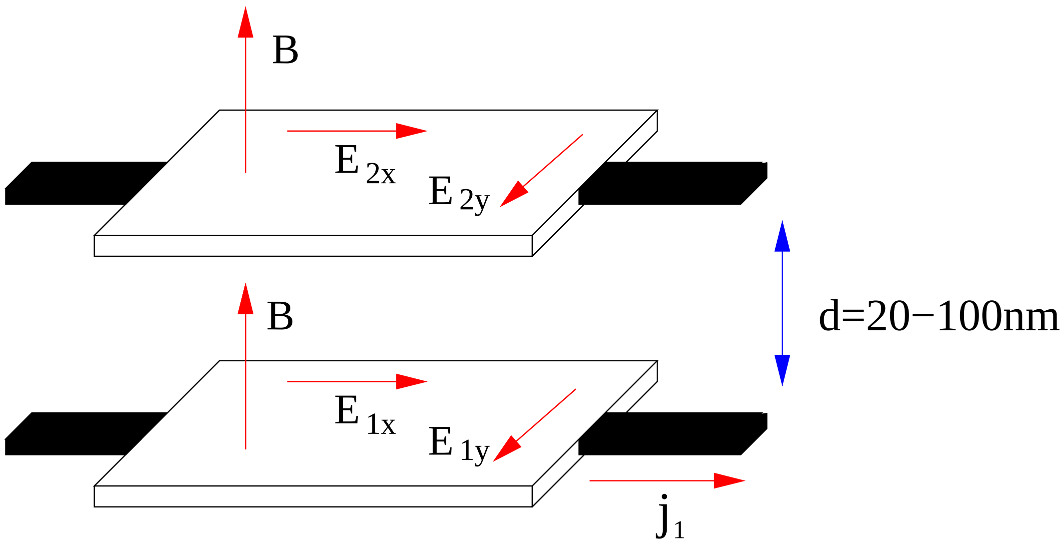
Outline:

- **Introduction & Background:** Coulomb Drag
- **Experiments:** Coulomb drag in strong magnetic fields
- **Qualitative considerations:** particle–hole asymmetry enhanced by Landau quantization.
- **Microscopic theory:** Strong-B diagrammatics
- Theory vs Experiment
- **Conclusions & Outlook**

Coulomb drag: Setup and standard theory

Response of the "passive" layer (2) to a current in the "active" layer (1) mediated by the Coulomb Interaction.



transresistivity (drag resistivity):

$$\rho^D_{\alpha\beta} = -E_{2\alpha}/j_{1\beta} \simeq \rho^{(1)}_{\alpha\gamma}\sigma^D_{\gamma\delta}\rho^{(2)}_{\delta\beta}$$

transconductivity:

more convenient for diagrammatics

$$\sigma^D_{\alpha\beta} = -j_{2\alpha}/E_{1\beta}$$

$$\rho^D_{xx} = \frac{\hbar^2}{e^2 n_1 n_2}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{1}{2T\sinh^2(\omega/2T)}\int\frac{d^2q}{(2\pi)^2}\,q_x^2\,|U(\omega,\mathrm{q})|^2\,\mathrm{Im}\Pi_1(\omega,\mathrm{q})\,\mathrm{Im}\Pi_2(\omega,\mathrm{q})$$

Zheng, MacDonald '93; Jauho, Smith '93; Kamenev, Oreg '95; Flensberg *et al.* '95

$U(\omega,\mathrm{q})$ – interaction, $\Pi_i(\omega,\mathrm{q})$ – density-density response function

$\longrightarrow$ ρ^D_{xx} positive, Fermi liquid: $\rho^D_{xx} \propto T^2$

“Coulomb drag”: why interesting

- no drag without interaction between electrons
- provides information about inelastic scattering
- correlations between layers: “drag” as indicator of phase-coherent phenomena
- Relation to particle-hole asymmetry

Important recent eperiments:

- **Drag in 1D**: Correlations and inelastic scattering in quantum wires: Luttinger liquid signatures? Debray *et al.*’00-01
- Correlations between two Quantum Hall layers with $\nu \simeq 1/2$
  Eisenstein *et al.* ’99-04, Lok *et al.* ’03
  - $T^{4/3}$ dependence: Gauge-field interaction between composite fermions
  - new strongly correlated ground state: Excitionic condensate
- Oscillatory anomalous drag in high Landau levels
  Feng *et al.*’98, Lok *et al.* ’01-03

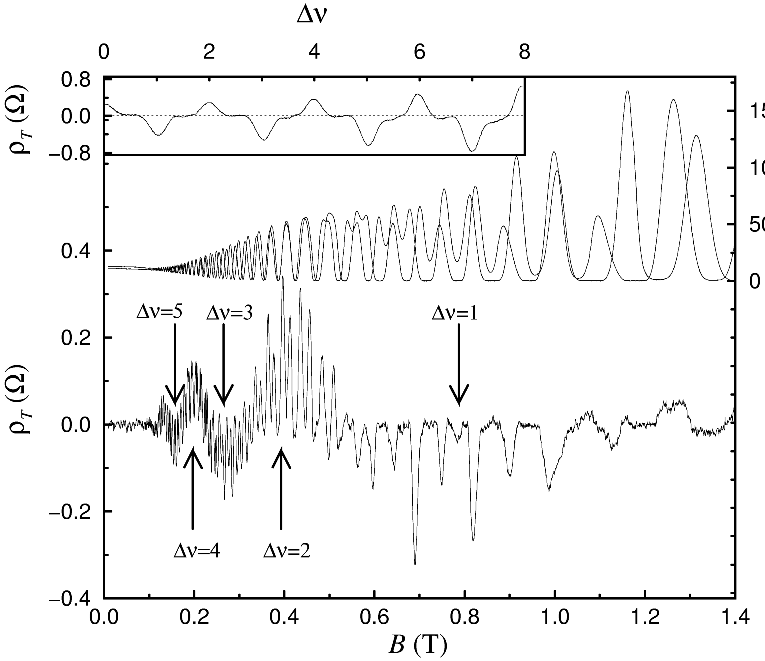
earlier theories: Bonsager, Flensberg, Hu, Jauho’98; Khaetskii,Nazarov’99,...:

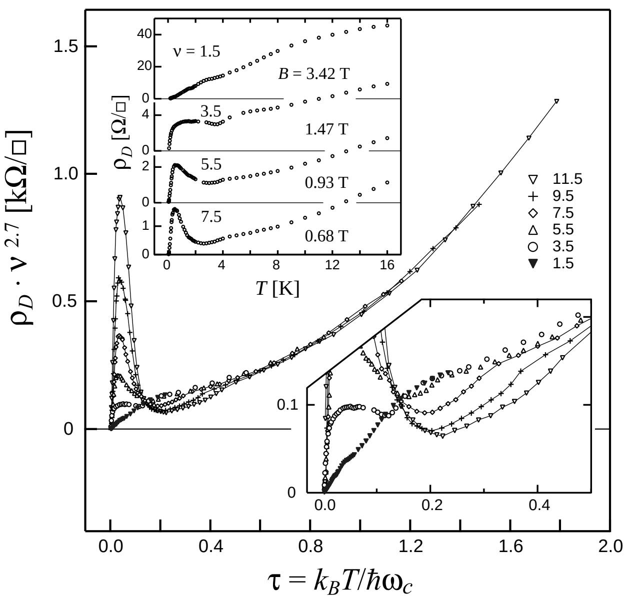
conventional formula for ρ_{xx}^{D} valid also in strong magnetic fields.

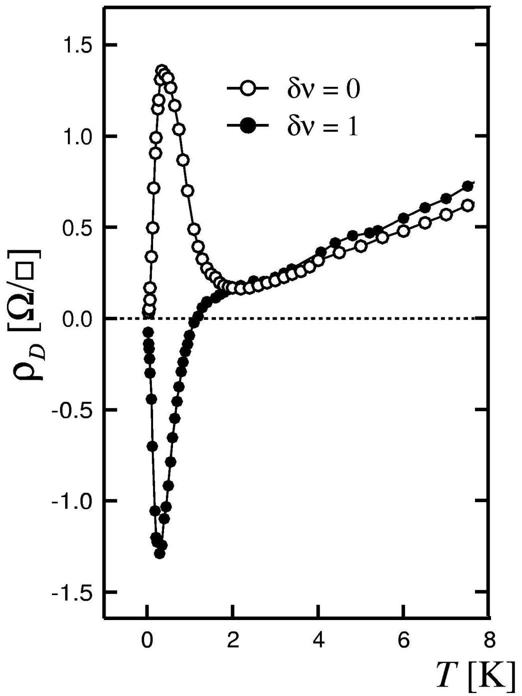
Experiment: Oscillatory Coulomb drag in strong B

Feng *et al.* '98, Lok *et al.* '01, Muraki et al '03:

- **GaAs double quantum wells,** $B \simeq 0.2 \div 1$ T
- **Oscillatory transresistivity; double-peak structure within a LL;**
- **Non-monotonic temperature dependence;**
- **Drag positive for matched $(\delta\nu = 0)$ and negative for mismatched $(\delta\nu = 1)$ densities.**

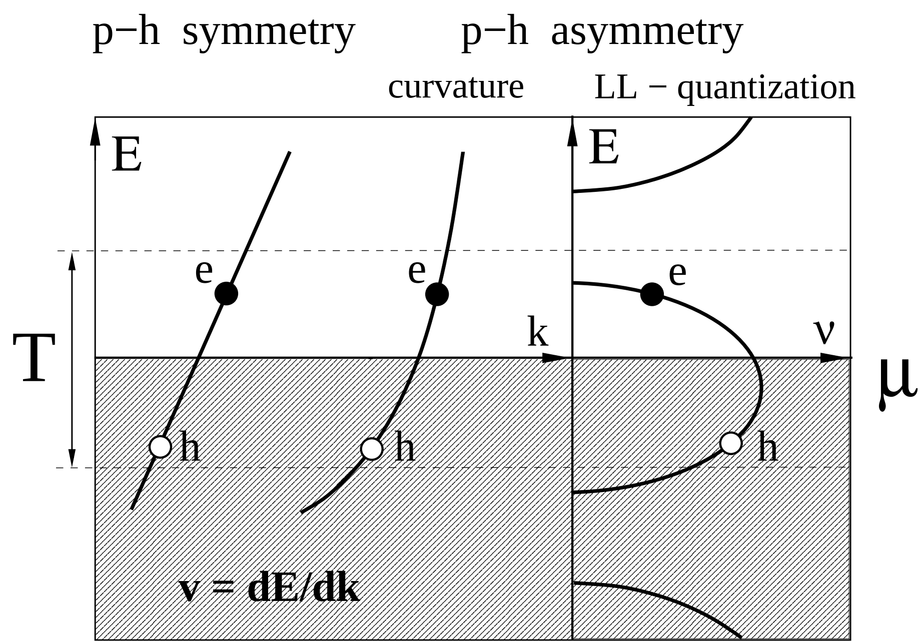


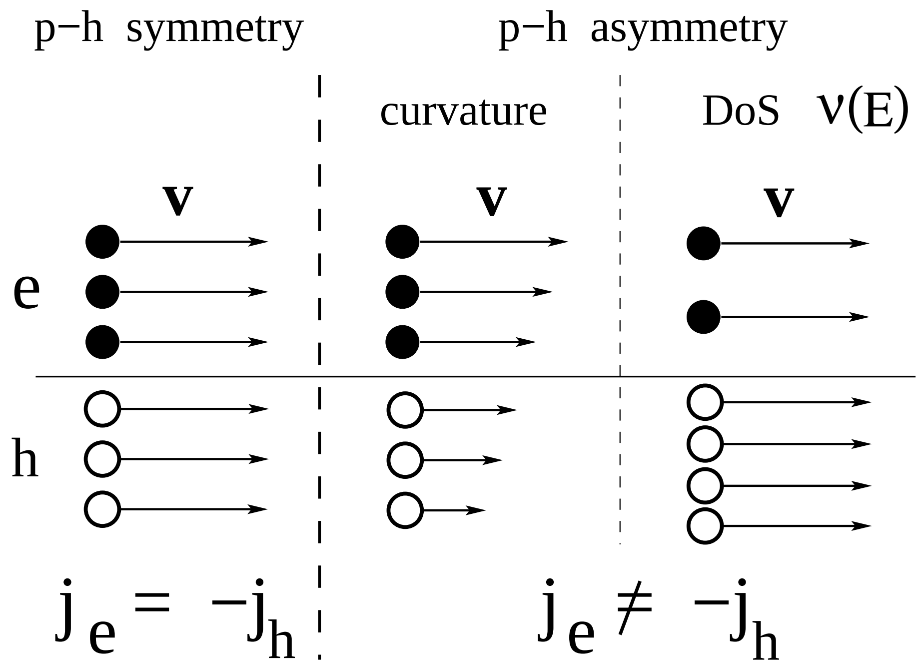




Magnetodrag: Qualitative considerations

Particle-hole asymmetry is necessary for the Coulomb drag.





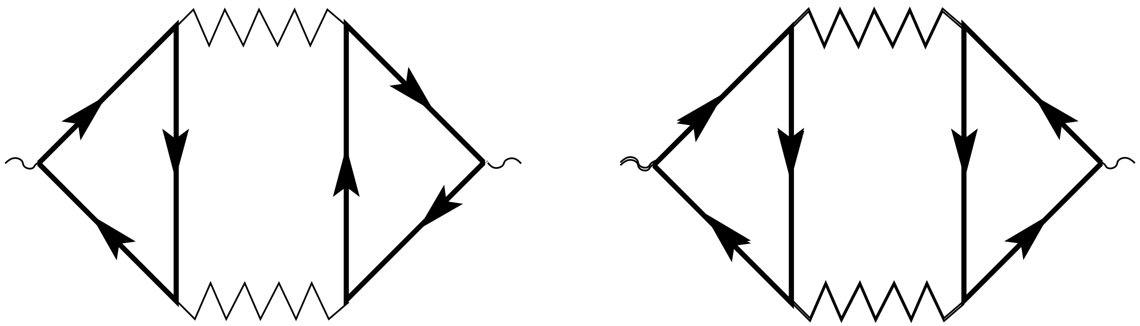
Strong magnetic fields: two sources of the particle-hole asymmetry:

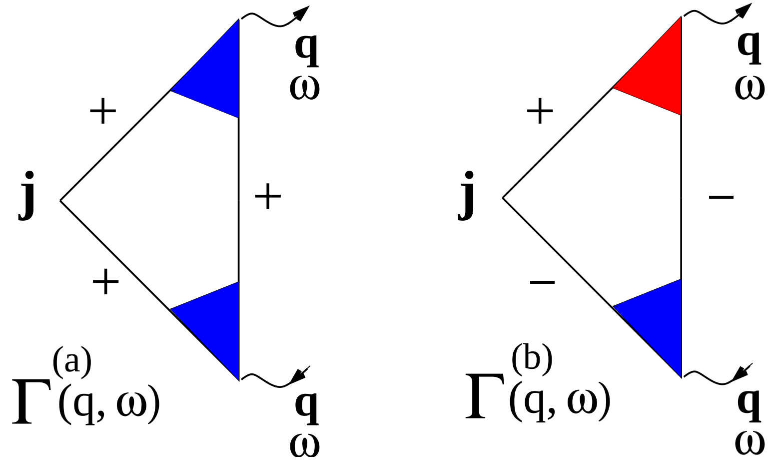
(i) curvature of the zero-B spectrum

$\longrightarrow$ "normal" positive drag at high T

(ii) LL DOS $\longrightarrow$ "anomalous" drag $\longrightarrow$ oscillatory sign at low T

Microscopic theory: Matsubara diagrammatics





$$\sigma_{ij}^{D}=\frac{e^2}{16\pi TS}\sum_{\mathrm{q}}\int_{-\infty}^{\infty}\frac{d\omega}{\sinh^2(\omega/2T)}\Gamma_i^{(1)}(\mathrm{q},\omega,B)\Gamma_j^{(2)}(\mathrm{q},\omega,-B)|U(\mathrm{q},\omega)|^2$$

Triangle ("rectification") vertex: $\Gamma=\Gamma^{(a)}+\Gamma^{(b)}$

$$\Gamma^{(a)}(\mathrm{q},\omega)=\frac{\omega}{2\pi i}\mathrm{tr}\left\{\mathrm{v}\mathcal{G}^{+}(\epsilon)e^{i\mathrm{qr}}\mathcal{G}^{+}(\epsilon)e^{-i\mathrm{qr}}\mathcal{G}^{+}(\epsilon)-(\mathcal{G}^{+}\longrightarrow\mathcal{G}^{-})\right\}$$

$$\Gamma^{(b)}(\mathrm{q},\omega)=\frac{\omega}{\pi i}\mathrm{tr}\left\{\mathrm{v}\mathcal{G}^{-}(\epsilon)e^{i\mathrm{qr}}[\mathcal{G}^{-}(\epsilon)-\mathcal{G}^{+}(\epsilon)]e^{-i\mathrm{qr}}\mathcal{G}^{+}(\epsilon)\right\}$$

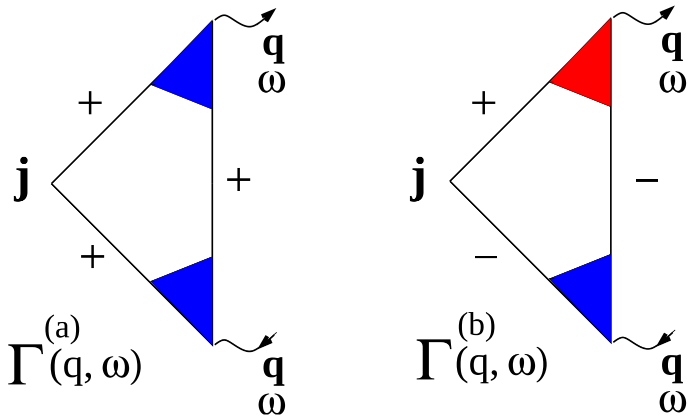
$\Gamma^{(b)}$ – quasiclassical

$\Gamma^{(a)}$ – quantum, negligible at $B\to 0$ but important in strong B

Triangle vertex

$$\Gamma_\alpha(\mathrm{q},\omega) \simeq \hat{q}_\alpha \frac{2\omega R_c}{\pi^2\ell^2} J_0(qR_c)J_1(qR_c)$$

$$\times\ \mathrm{Im}[G_N^+\gamma^{++}]\ \mathrm{Re}[G_N^+(\gamma^{++}-\gamma^{+-})]$$



$q \sim 1/a$, a – interlayer distance, γ^{++}, γ^{+-} – vertex corrections

- diffusive regime, $qR_c \ll 1$ $\longrightarrow$ $\gamma^{+-} \gg \gamma^{++}$ $\longrightarrow$ $\Gamma^{(b)}$ dominates

$$\Gamma(\mathrm{q},\omega) = \frac{d\hat{\sigma}}{d(en)}\,\mathrm{q}\ \mathrm{Im}\Pi(\mathrm{q},\omega)$$

quasiclassical local rectification coefficient von Oppen, Simon, Stern '01

- exper. relevant: ballistic regime, $qR_c \gg 1$ $\longrightarrow$ $\gamma^{+-} \simeq \gamma^{++} \simeq 1$

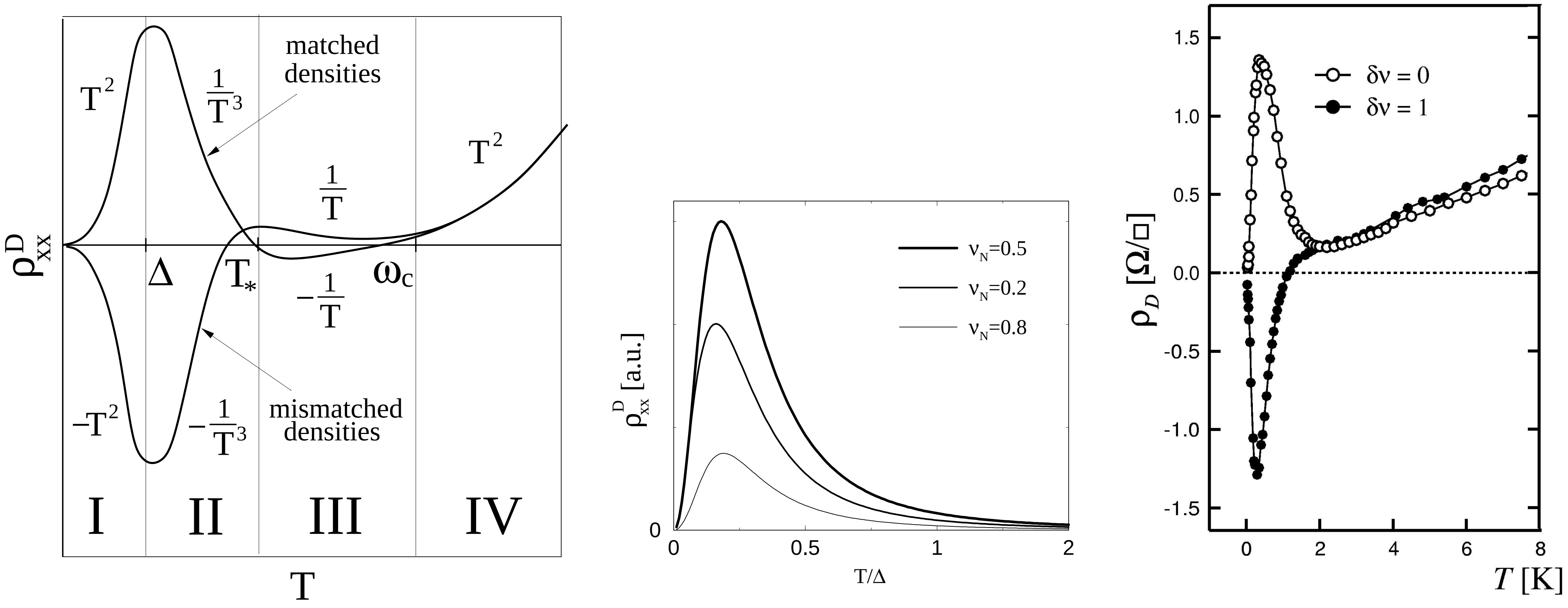
$\longrightarrow$ leading contribution vanishes:

cancellation between $\Gamma^{(a)}$ and $\Gamma^{(b)}$ in the leading order!

two sources of the particle-hole-asymmetry

$\longrightarrow$ $\mathcal{O}(\Gamma/\omega_c)$ and $\mathcal{O}(q/k_F)$ corrections

Magnetodrag in the ballistic regime: Results



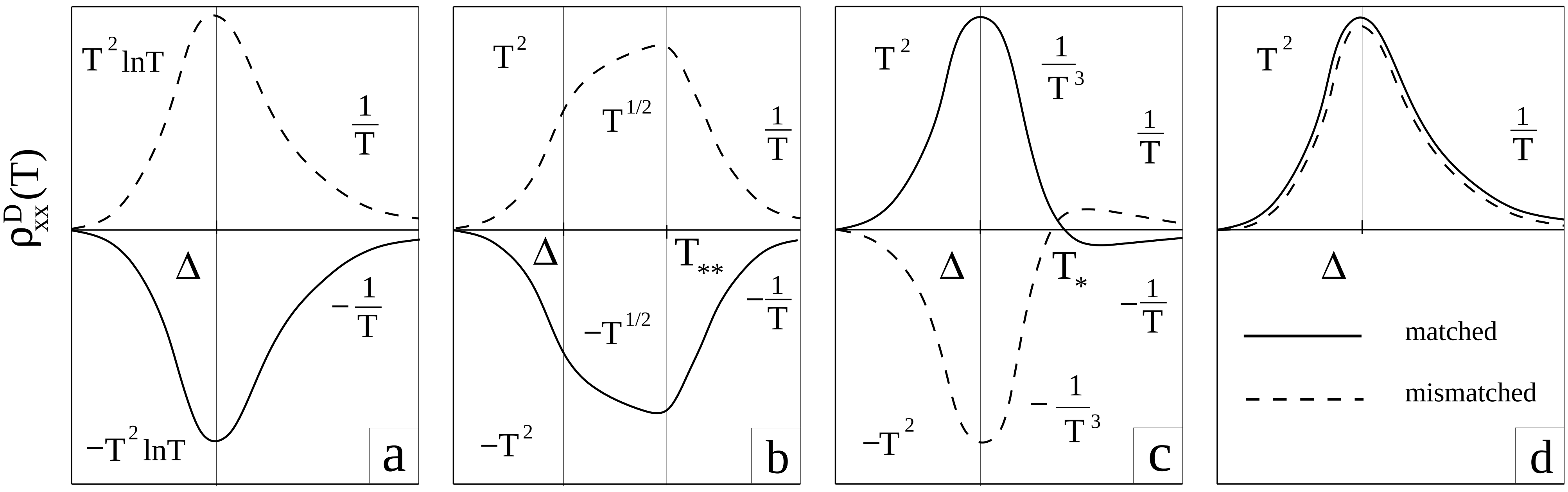
Low T: "anomalous drag":

Peak at $T \sim \Delta$ (Landau level width), **oscillatory sign**,

amplitude $\sim 1\,\Omega/\square$ for exper. parameters

double peak structure in each Landau level: $\rho_D \propto \frac{(\mu - E_N)^2}{\Delta^2}\left[1 - \frac{(\mu - E_N)^2}{\Delta^2}\right]^2$

Low-temperature drag for different inter-layer distances



Schematic T–dependence of low-temperature drag in different regimes:

a) diffusive, $R_c/a \ll 1$

b) weakly ballistic, $1 \ll R_c/a \ll \omega_c/\Delta$

c) ballistic, $\omega_c/\Delta \ll R_c/a \ll N\Delta/\omega_c$

d) ultra-ballistic, $N\Delta/\omega_c \ll R_c/a$

Outlook

- **quantum kinetic equation for magnetodrag:**
  particle-hole symmetry should be taken into account!
- **Drag in non-equilibrium (large bias, microwaves,...)**
- **Composite-fermion-drag near filling factor $\nu = 1/2$:**
  "normal" vs. "anomalous" drag, pairing instability,
  effect of disorder (effective random magnetic field)
- **Phonon drag in strong B**
- **Drag in the Quantum Hall regime:**
  quantum interference, localization, criticality

Summary

- **Low temperature, $T \ll \Delta$:**
  - $\longrightarrow$ **particle–hole asymmetry due to Landau quantization**
  - $\longrightarrow$ **oscillatory Coulomb drag.**
- **Higher temperatures:**
  - $\longrightarrow$ **particle–hole asymmetry due to the spectrum curvature**
  - $\longrightarrow$ **non-monotonic T-dependence.**