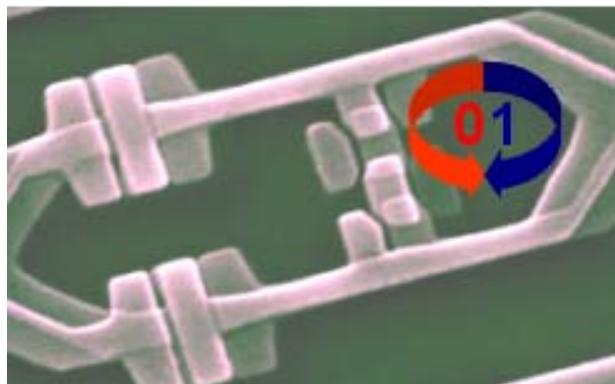


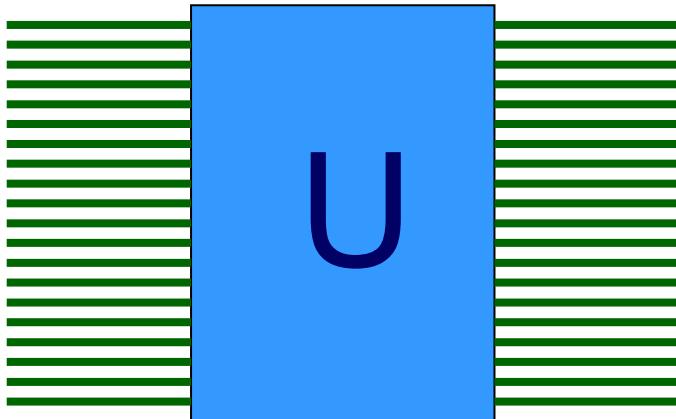
# SUPERCONDUCTING QUANTUM BITS

Hans Mooij

*Summer School on  
Condensed Matter Theory  
Windsor, August 18, 2004*



# quantum computer



input - unitary transformations - output

N qubits,  $2^N$  states

quantum algorithms  
factorization (Shor)  
searching (Grover)

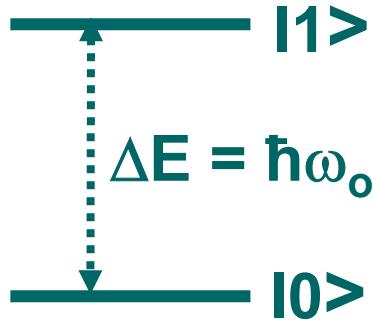
error correction

quantum bits  
states  $|0\rangle, |1\rangle$   
 $\Psi = \alpha|0\rangle + \beta|1\rangle$

initialization  
controlled operations  
interaction  
large number  
individual readout

non-trivial two-bit gates  
controlled-not, swap

# quantum bit: two level quantum system

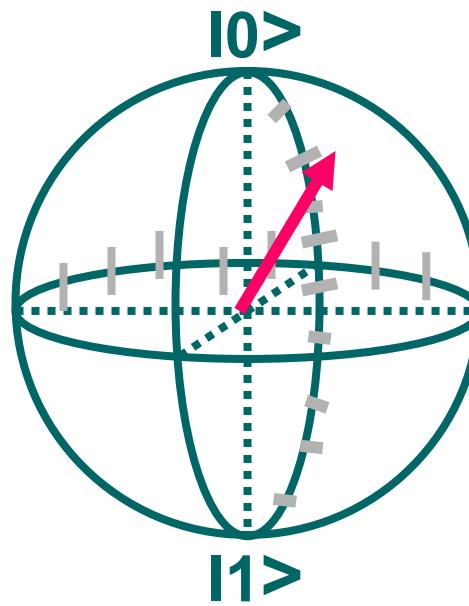
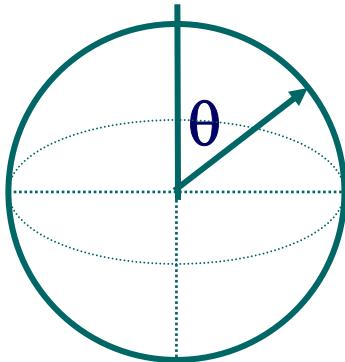


$$\Psi = \alpha |0\rangle + \beta |1\rangle$$

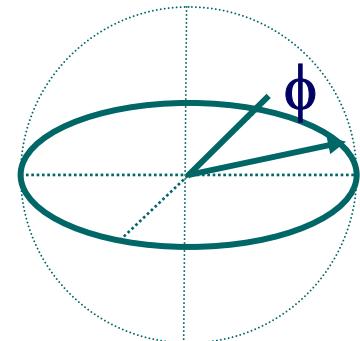
$$|\alpha|^2 + |\beta|^2 = 1$$

$$\alpha = \cos \theta \quad \beta = e^{i\phi} \sin \theta$$

*latitude:*  
angle  $\theta$   
content  $|0\rangle$  and  $|1\rangle$   
measured in readout



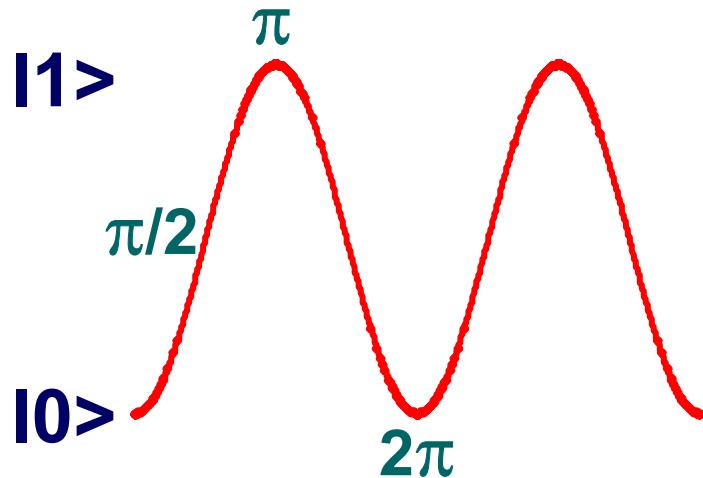
*longitude:*  
phase  $\phi$   
irrelevant in readout  
 $d\phi/dt = \Delta E/\hbar$



# quantum manipulations

resonant electromagnetic wave  $\nu = \Delta E/h$

Rabi oscillation



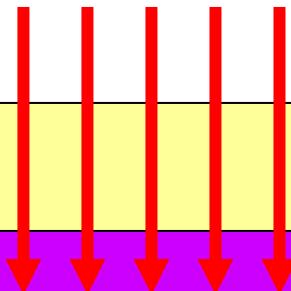
$$\Psi = \alpha|0\rangle + \beta|1\rangle$$

$$\alpha(t) = \cos(\theta_0 + \omega_R t)$$

$$\beta(t) = \sin(\theta_0 + \omega_R t)$$

length of microwave pulse determines final state

**measurement**



**quantum system**

**environment**

## **well-established quantum technologies**

- nuclear magnetic resonance**
- photons**
- ions / atoms**

**basic manipulations well-controlled  
in present form not scalable**

# NMR

**$10^{18}$  identical molecules in liquid**

**each molecule has N identifiable spins**

**$10^{18}$  parallel N-qubit computers**

**ensemble-properties represent qubits**

**readout: measurement of spin magnetic moments**

nuclear spins

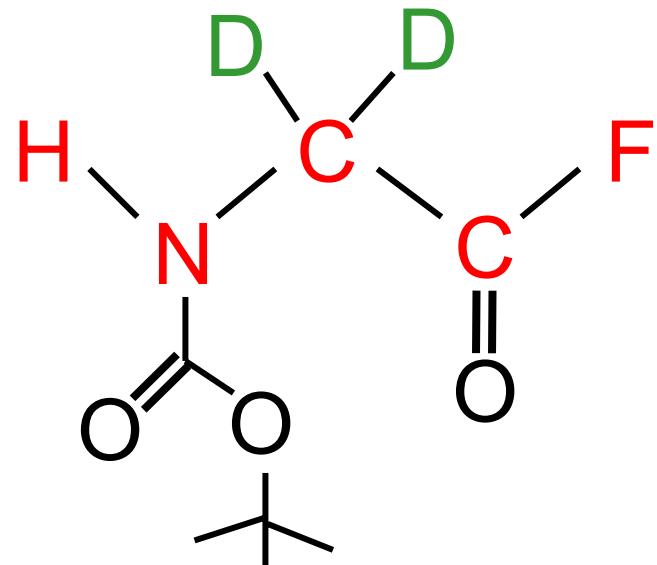


E<sub>2</sub>-E<sub>1</sub> proportional to magnetic field strength

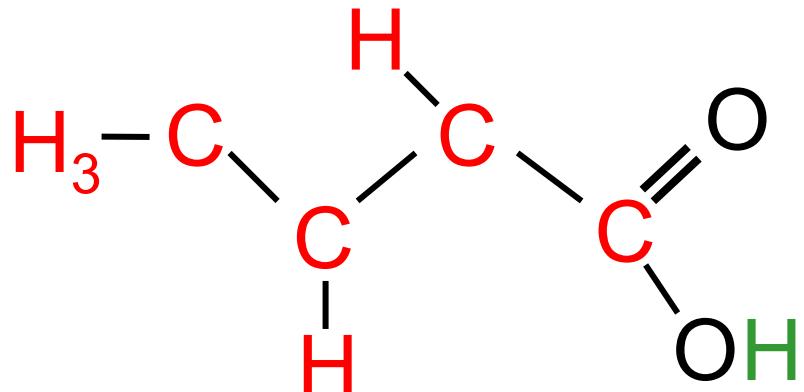
weak interaction between the spins in the same molecule

no interaction between molecules

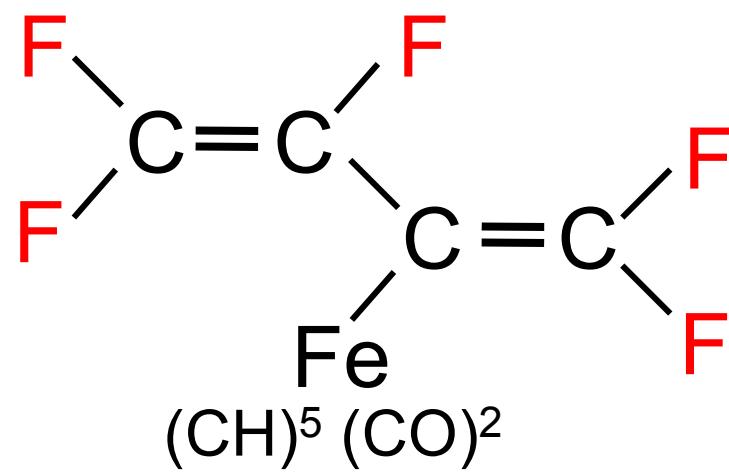
Deutsch-Jozsa



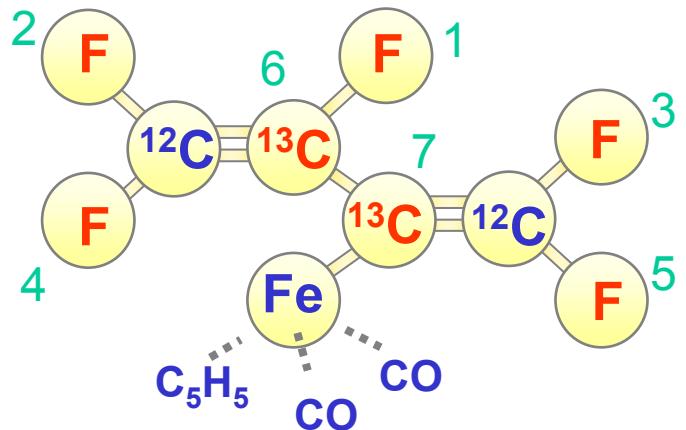
7-spin coherence



Order-finding

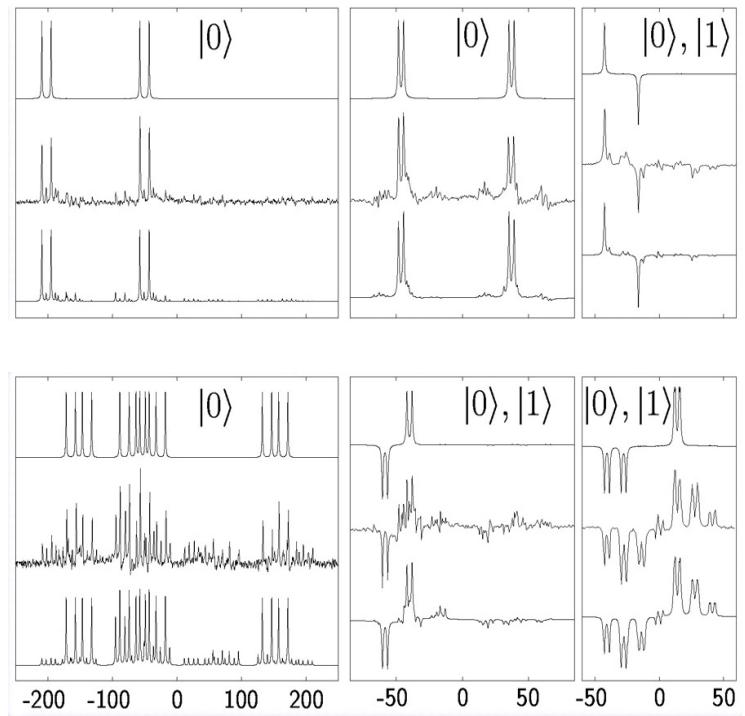
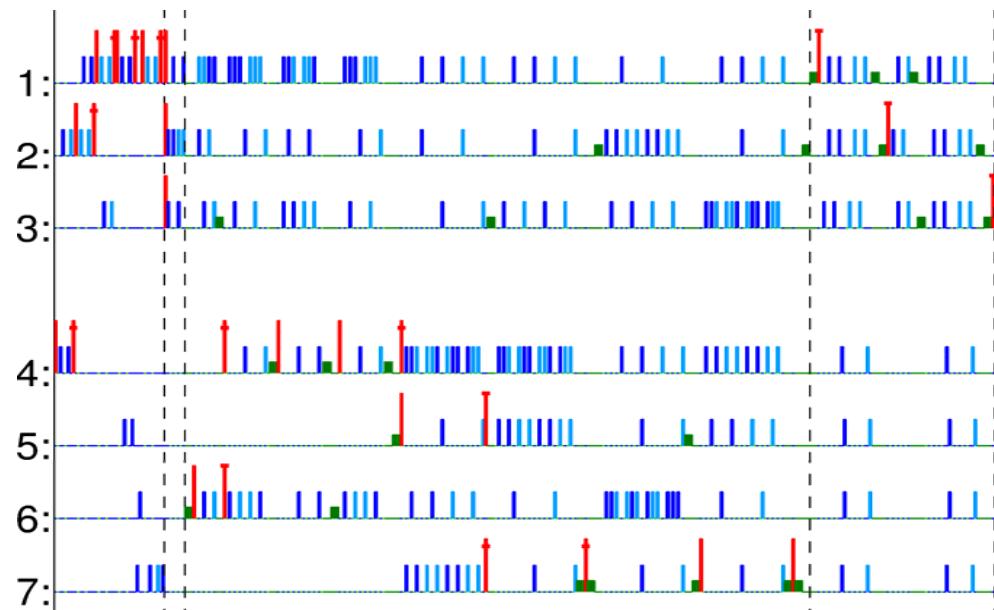


# factoring 15 with nuclear spins



Vandersypen et al., *Nature* **414**, 883 (2001)

$$15 \approx 3 \times 5$$

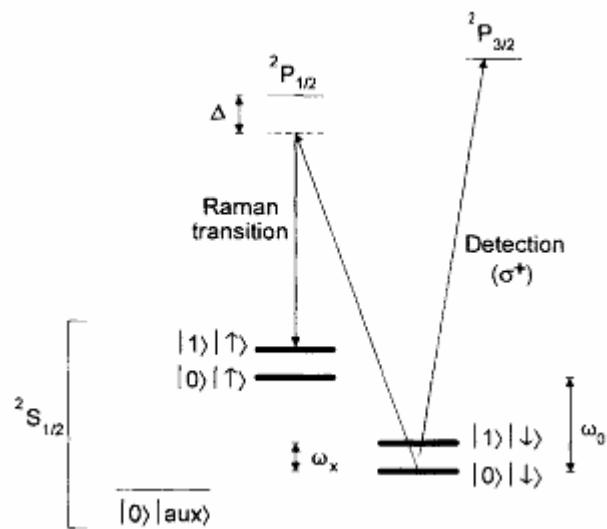


# **photons**

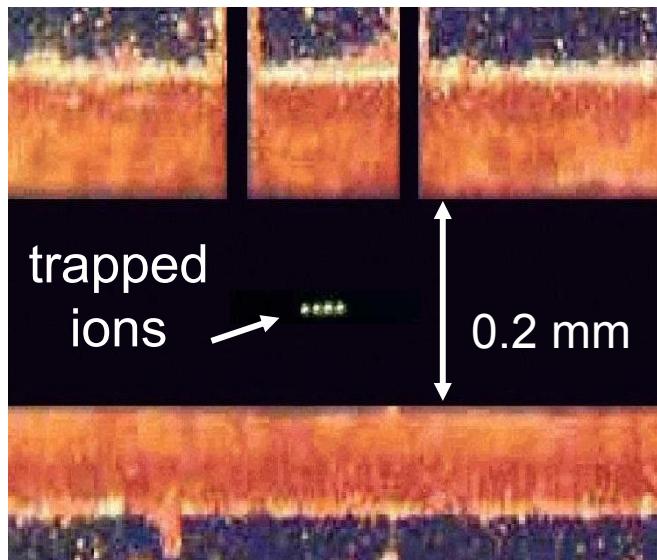
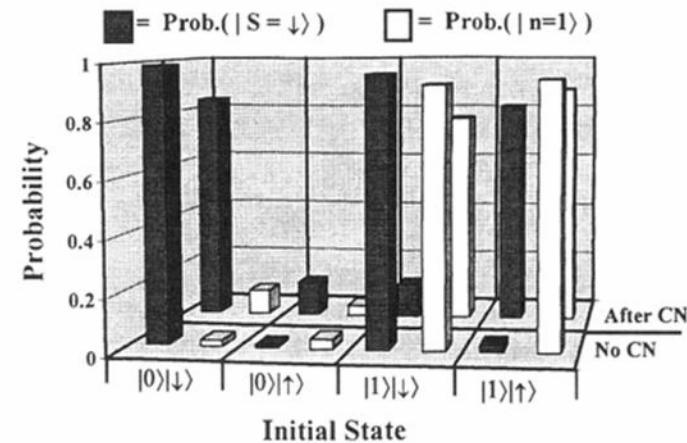
**'free' single photons  
beam splitters  
polarizers  
non-linear Kerr cells  
single photon detectors**

**photons in a cavity  
interaction through an atom**

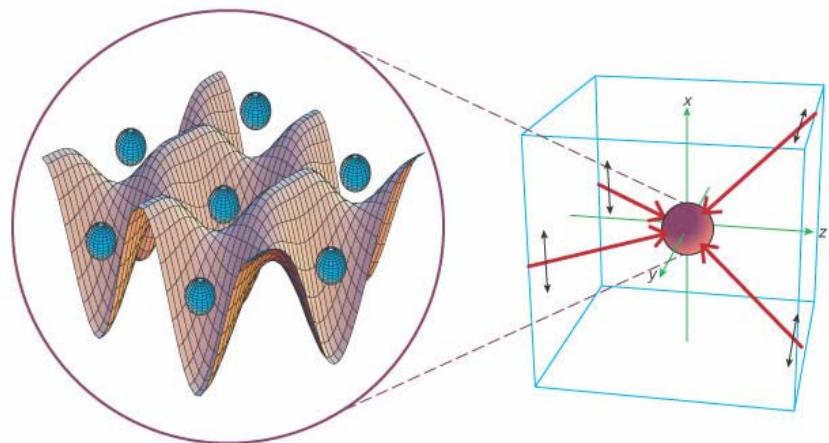
# ions in an ion trap



c. monroe et al. PRL 75 4714 1995

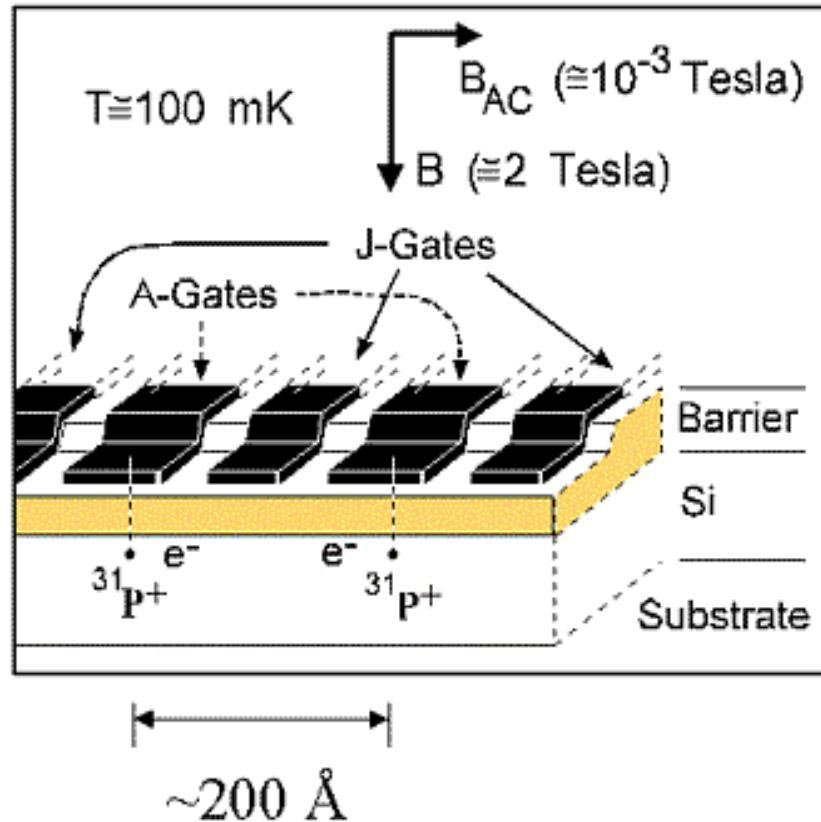


Courtesy D. Wineland, NIST



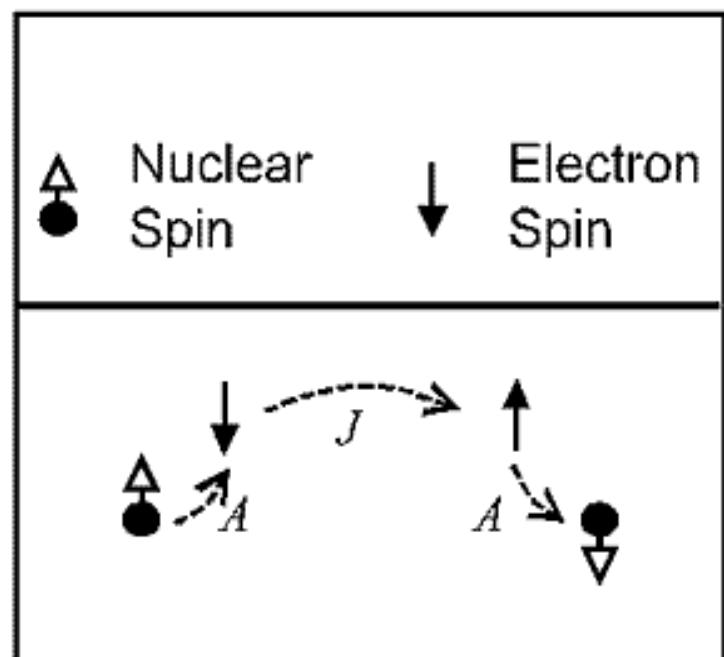
## **new scalable quantum technologies**

- ion traps with micro-control electrodes 'atom chip'**
- superconducting systems**
- single spins in semiconductors**
- x x x x**

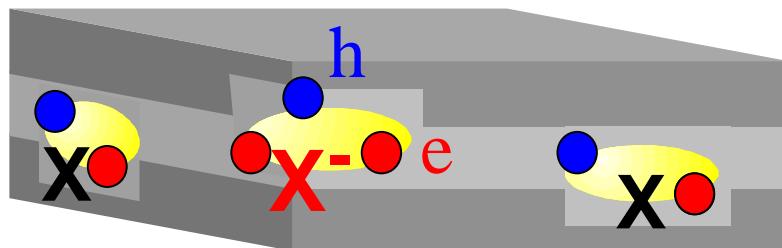


## “A Silicon-based nuclear spin quantum computer”

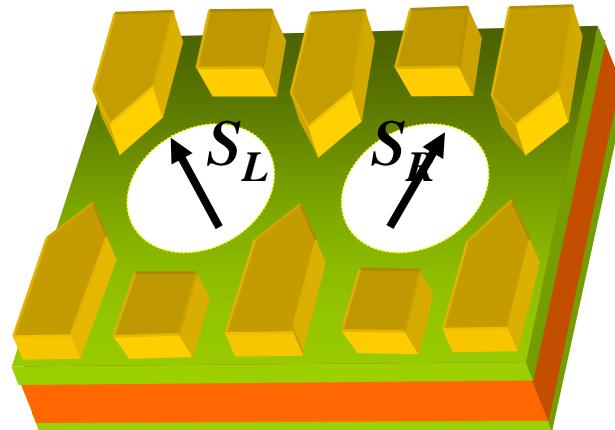
B. E. Kane, *Nature*, May 14, 1998  
also, quant-ph/0003031



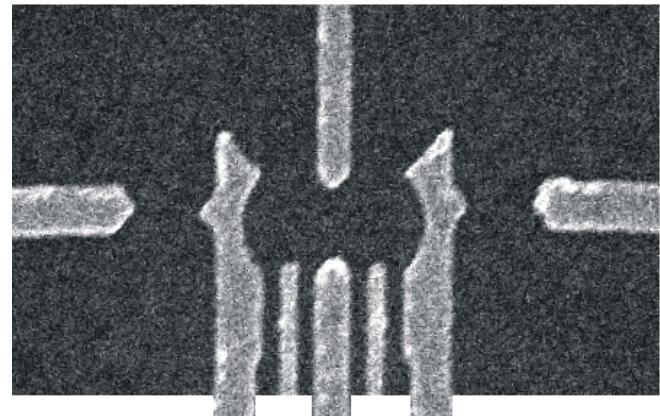
# single spins in a semiconductor



Gammon et al  
Imamoglu et al, PRL 1999



Loss & DiVincenzo, PRA 1998



200 nm

# mesoscopic Josephson junction circuits

*Josephson coupling energy*

$$U_J = E_J (1 - \cos\phi)$$

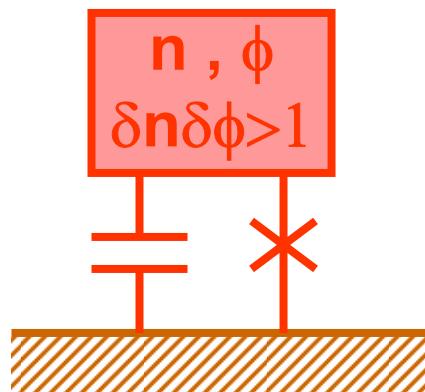
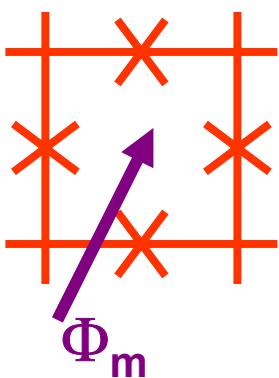
$$E_J = 2\Delta (h/e^2) / (8R_n)$$

*Coulomb charging energy*

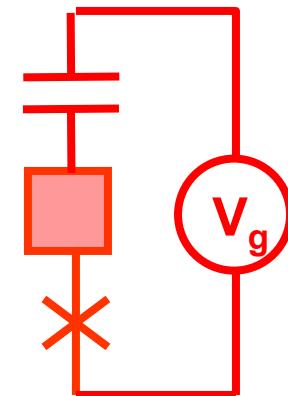
$$U_C = E_C 4n^2$$

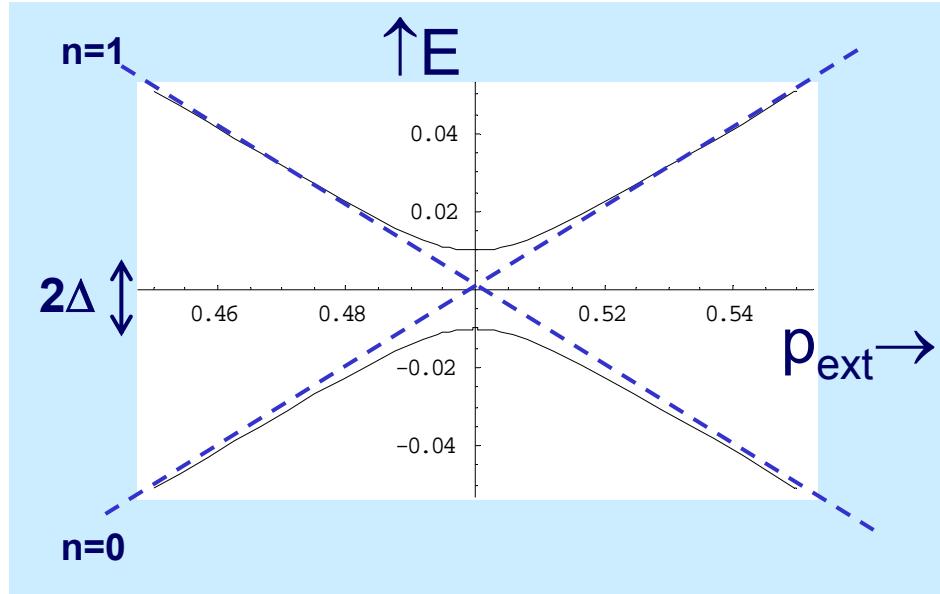
$$E_C = e^2 / 2C$$

$E_J/E_C \gg 1$   
phase excitations  
fluxons



$E_C/E_J \gg 1$   
charge excitations





**charge qubit**

$$p_{\text{ext}} = V_g C_g / 2e$$

$$\Delta = E_J$$

**n : Cooper pairs**

**flux qubit**

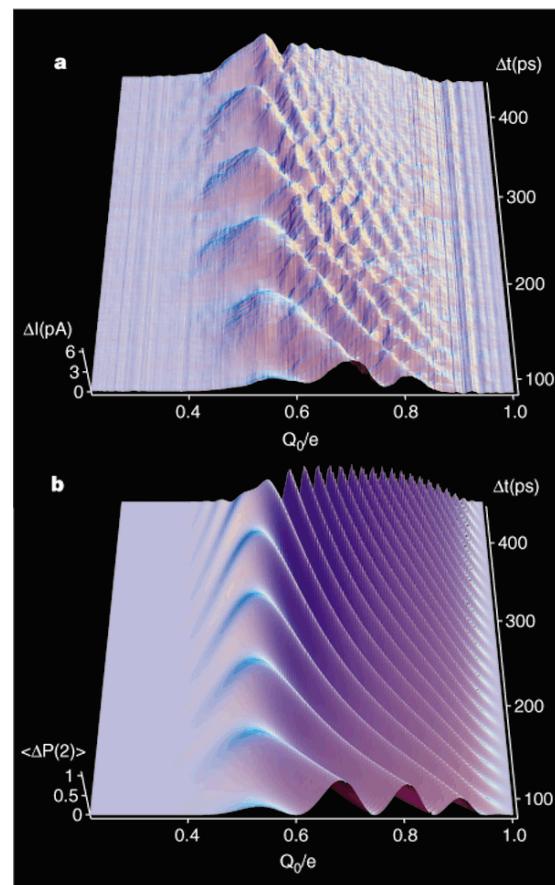
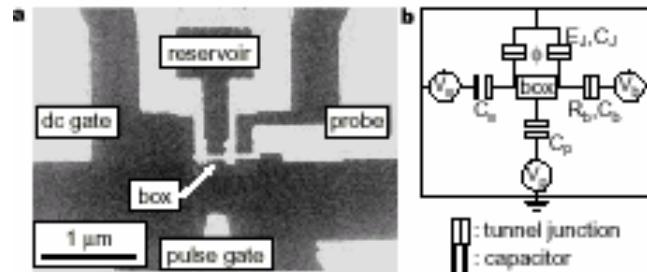
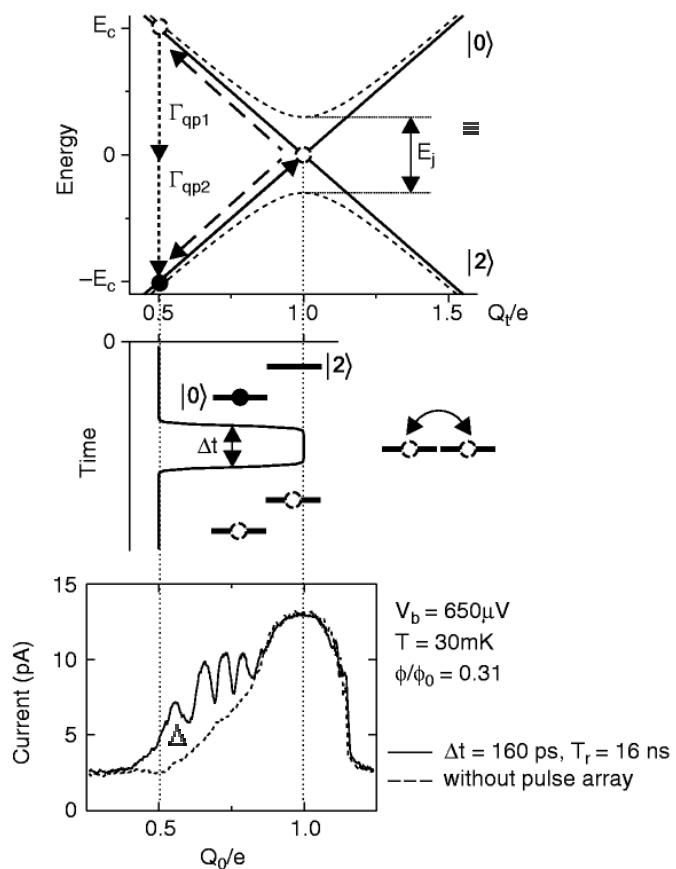
$$p_{\text{ext}} = \Phi / \Phi_0$$

$$\Delta : \exp\{ -(E_J/E_c)^{1/2} \}$$

**n : fluxons**

# Nakamura, Pashkin, Tsai

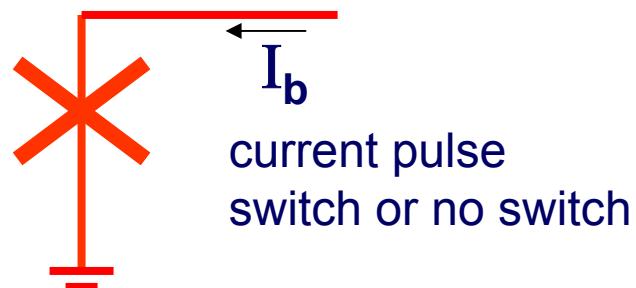
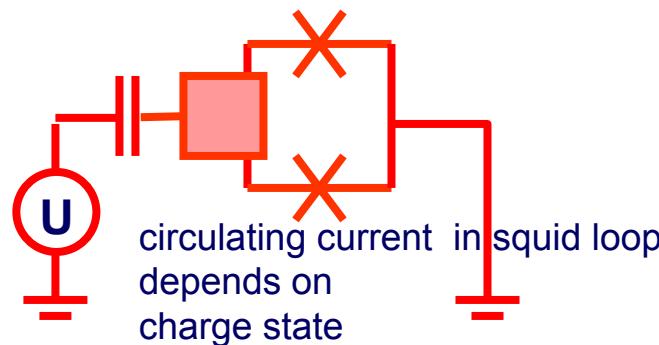
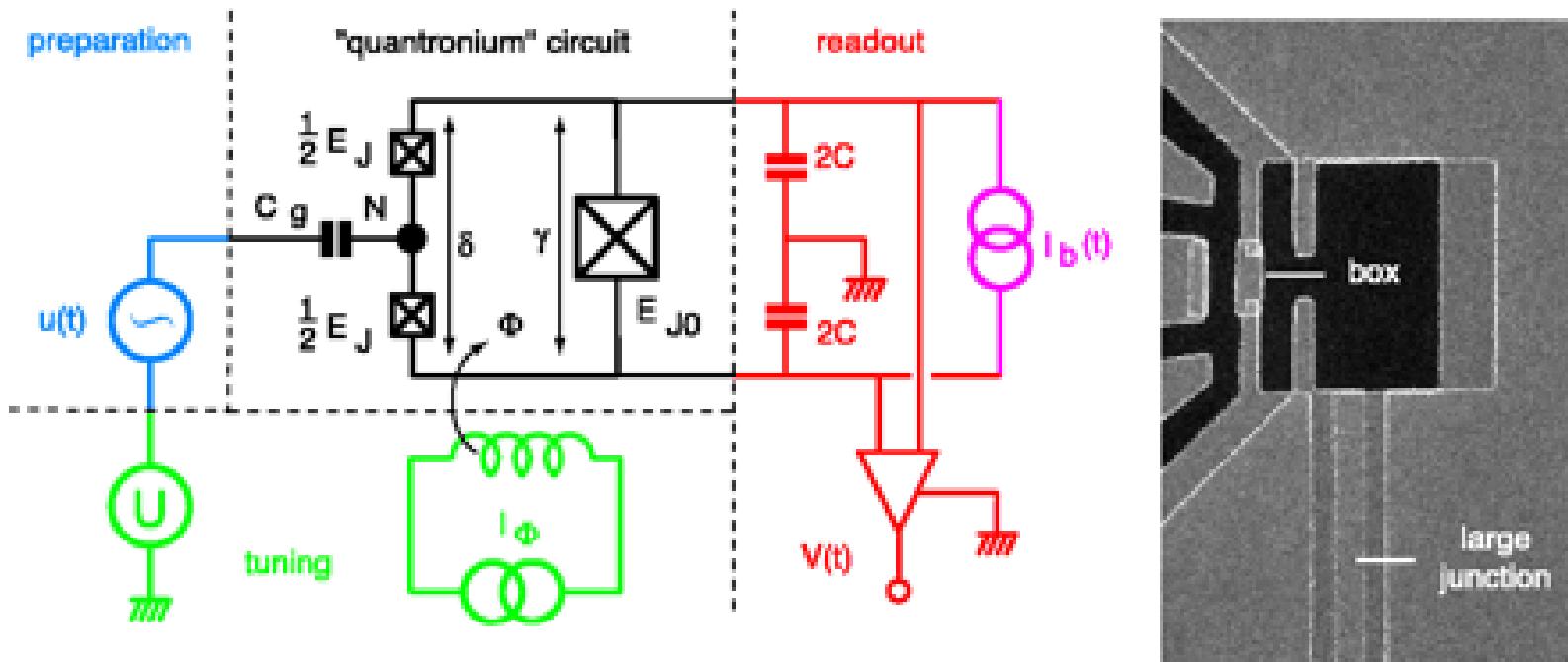
Nature 398, 786 (1999)

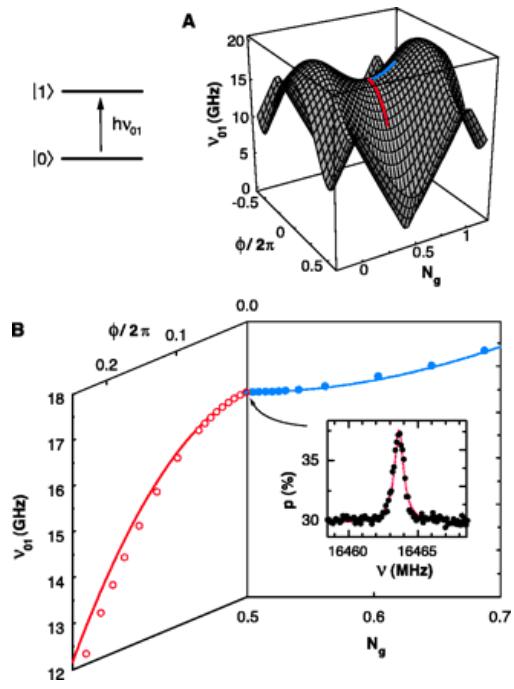




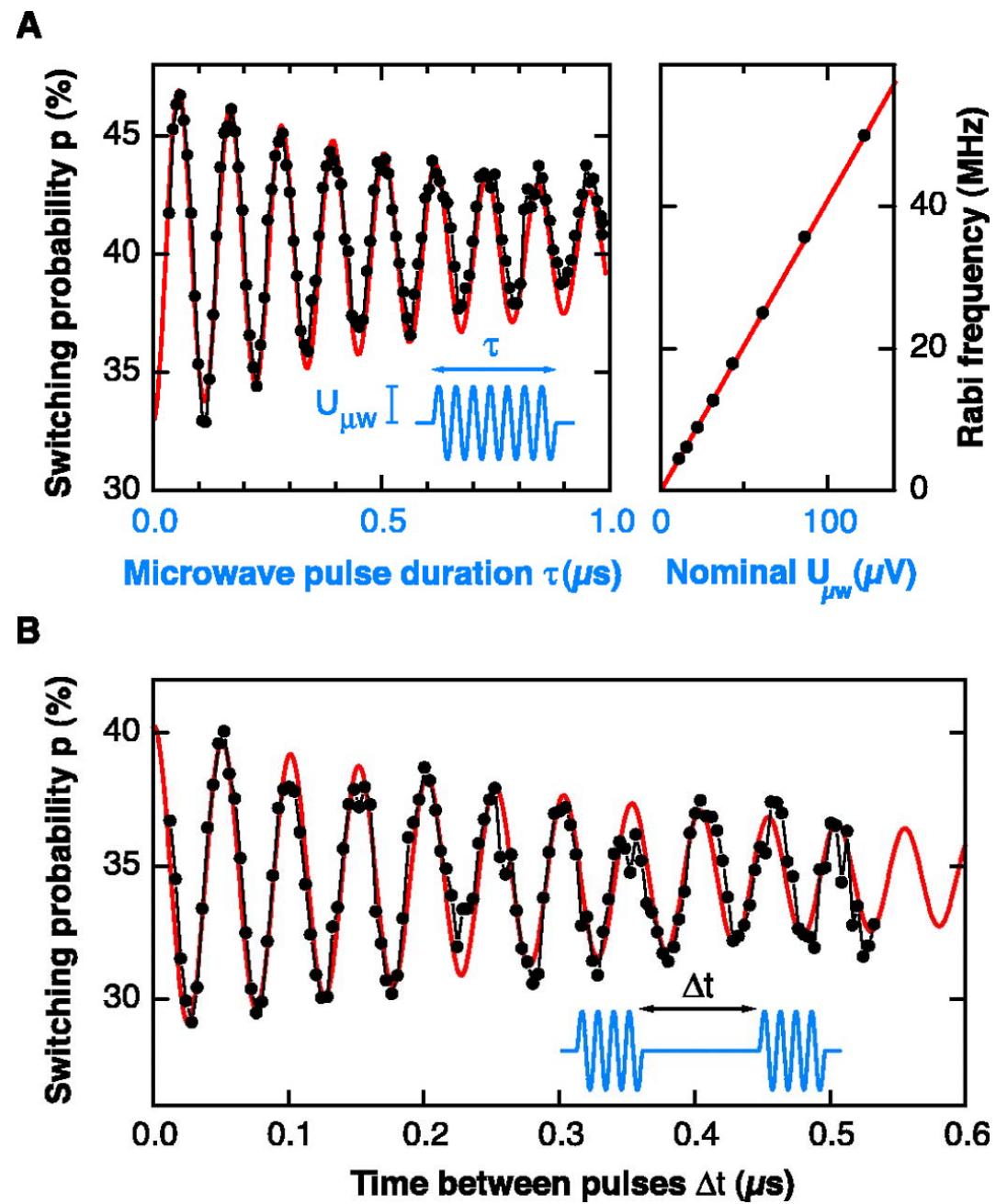
# charge qubit with phase read-out

D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, M.H. Devoret, Science 296, 886 (2002)



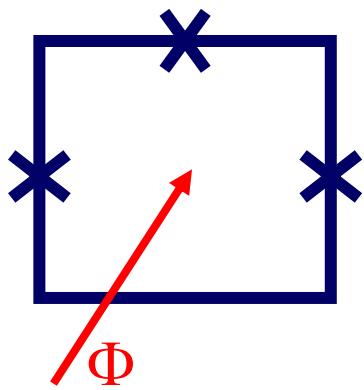


'magic point'



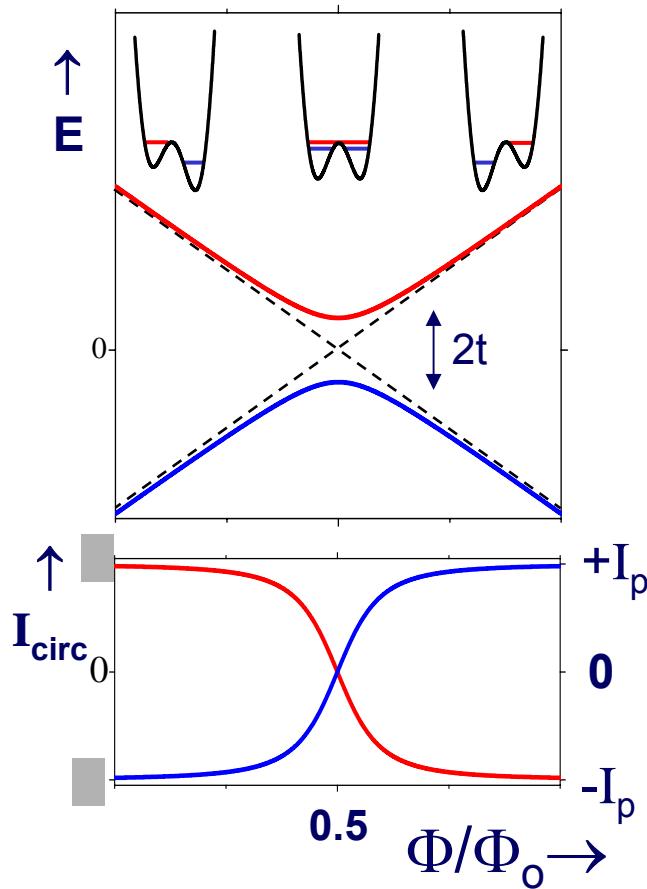
# persistent-current quantum bit

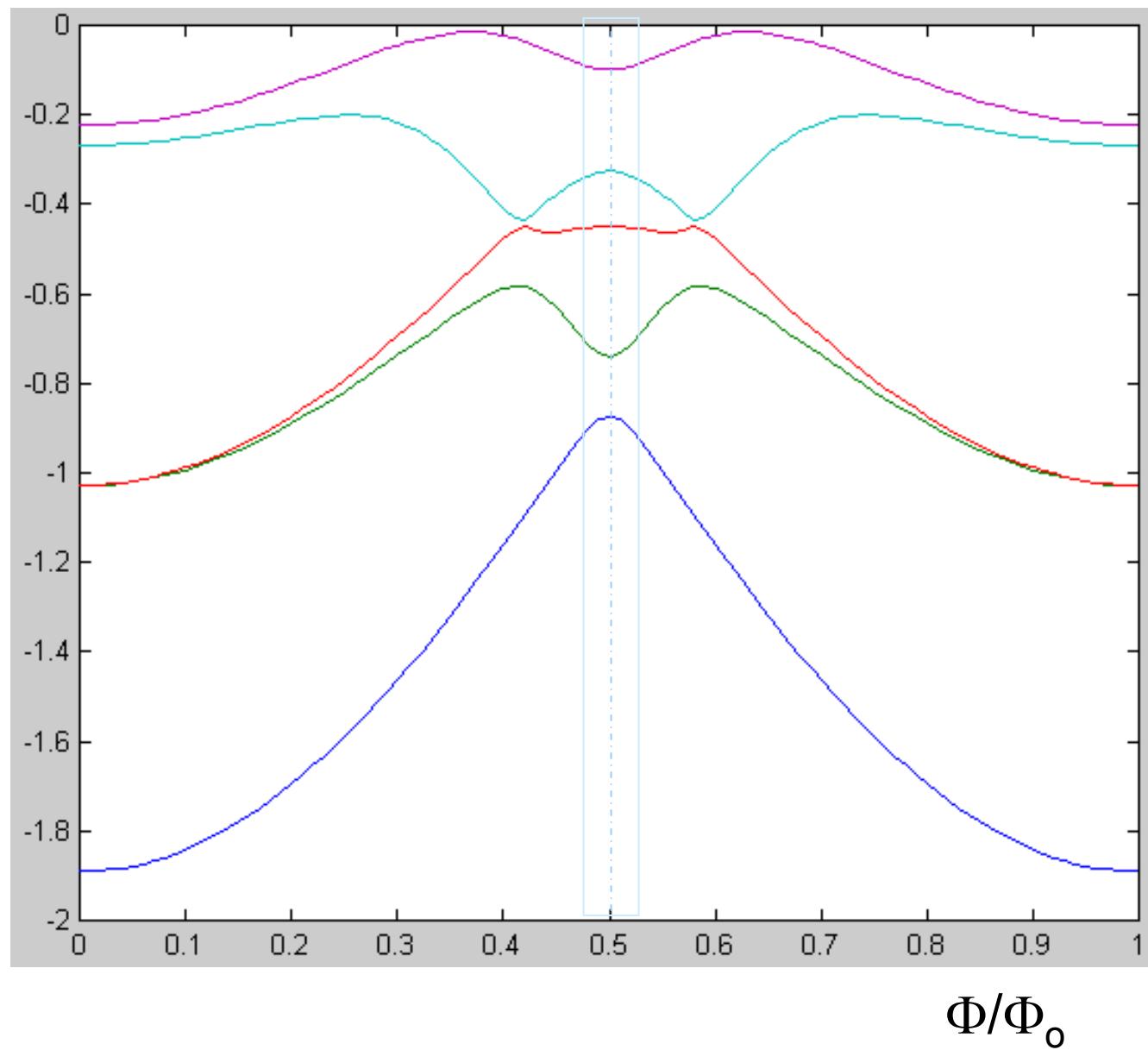
flux qubit with three junctions, small geometric loop inductance



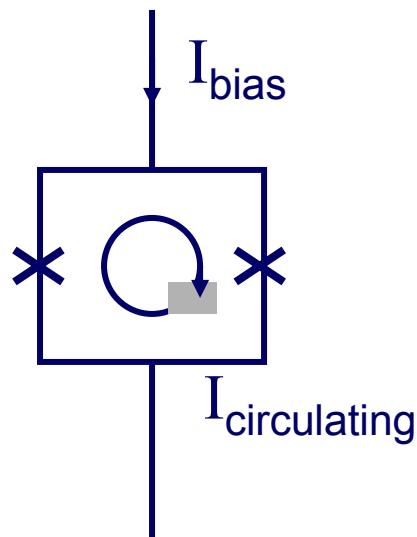
$$H = h\sigma_z + t\sigma_x$$

with  $h = (\Phi/\Phi_0 - 0.5) \Phi_0 I_p$



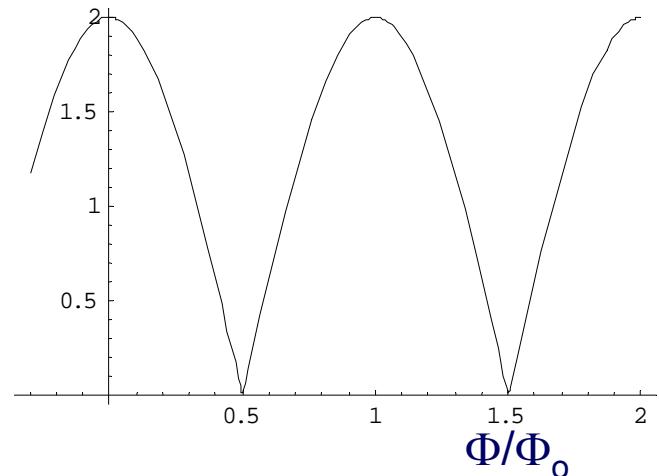


# SQUID: $I_s \max(\Phi)$



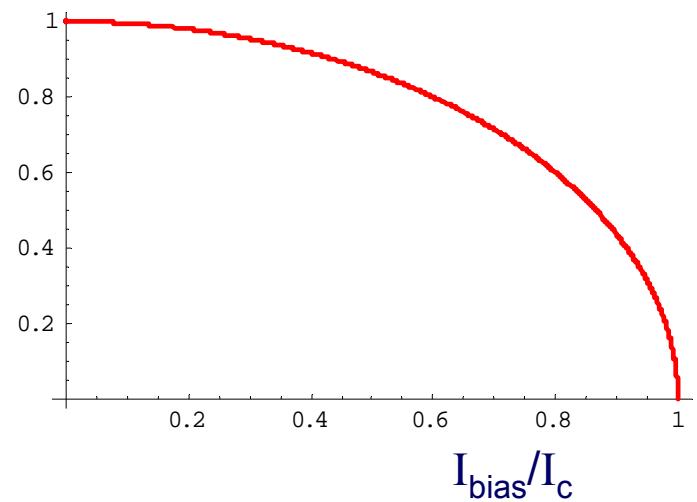
maximum supercurrent

$$I_{s \max} / I_c$$

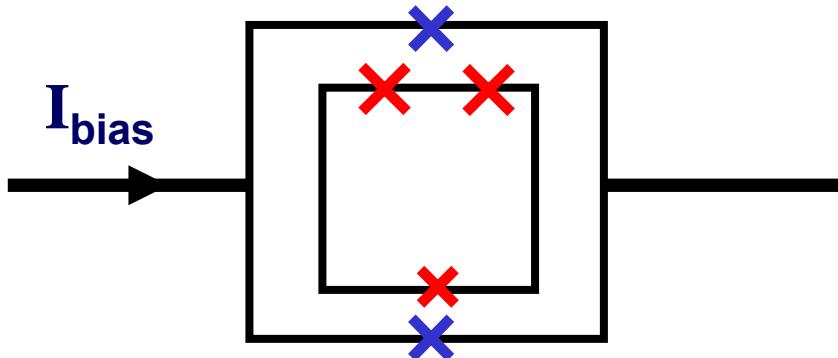


circulating current at  $f=0.5$

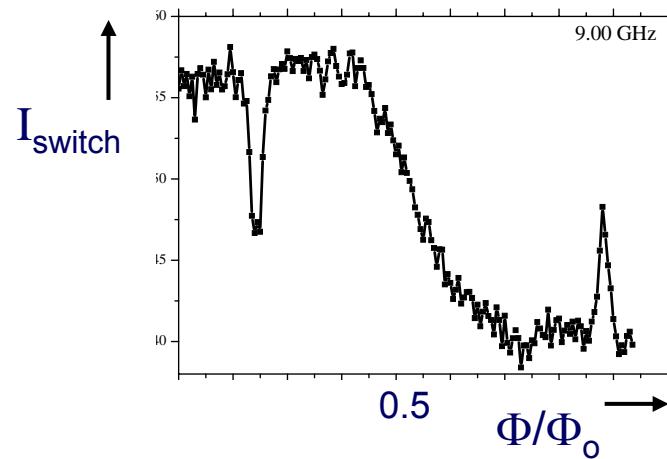
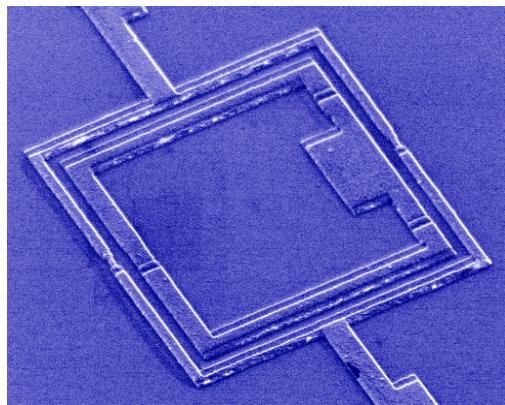
$$I_{\text{circ}} / I_c$$



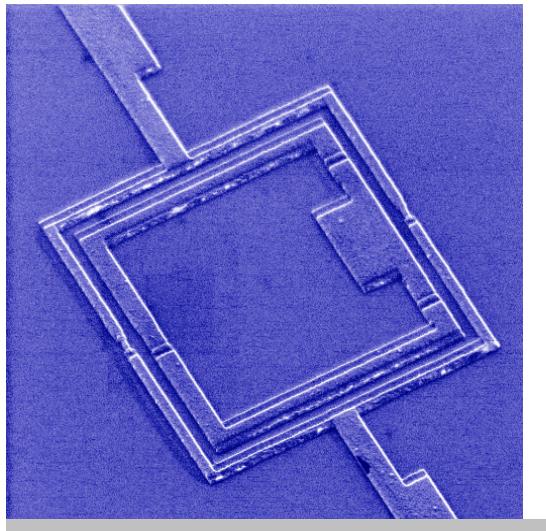
# SQUID measurement



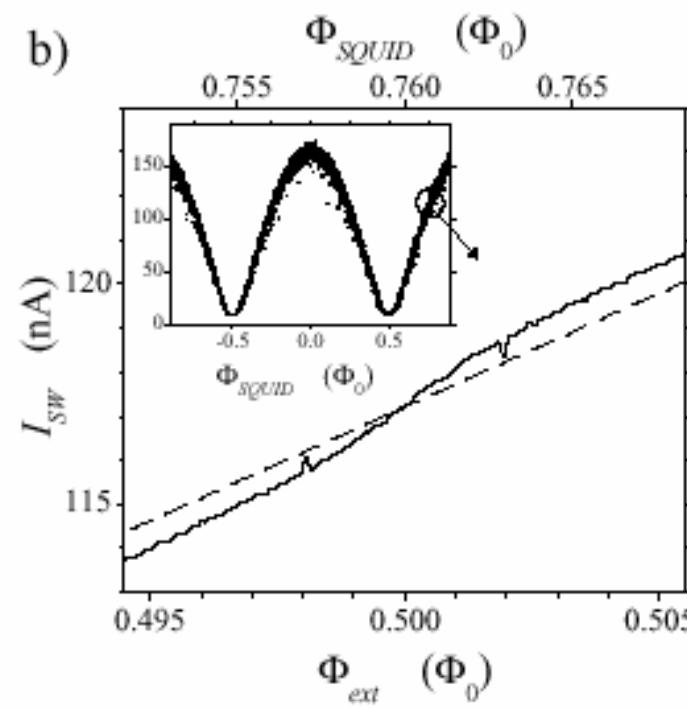
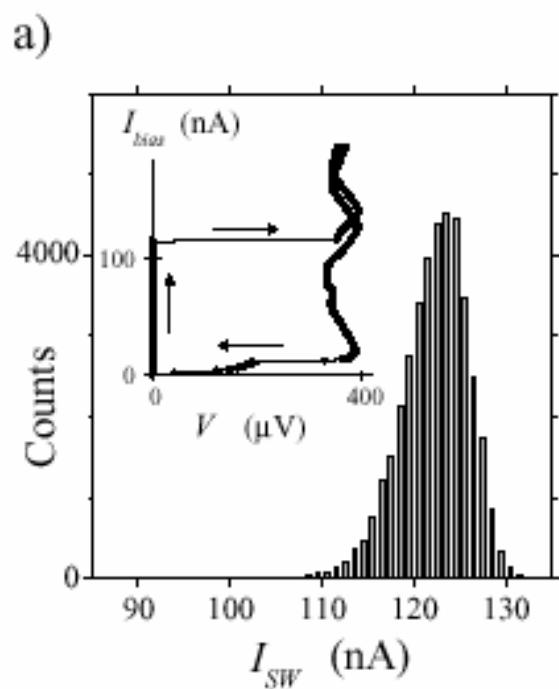
qubit generates flux  $\pm LI_p \approx 10^{-3} \Phi_0$   
measured with  
hysteretic (unshunted) SQUID  
maximum supercurrent depends on  
flux in the SQUID loop



measurement switching current:  
(RF 9 GHz applied)  
5000-10000 ramps

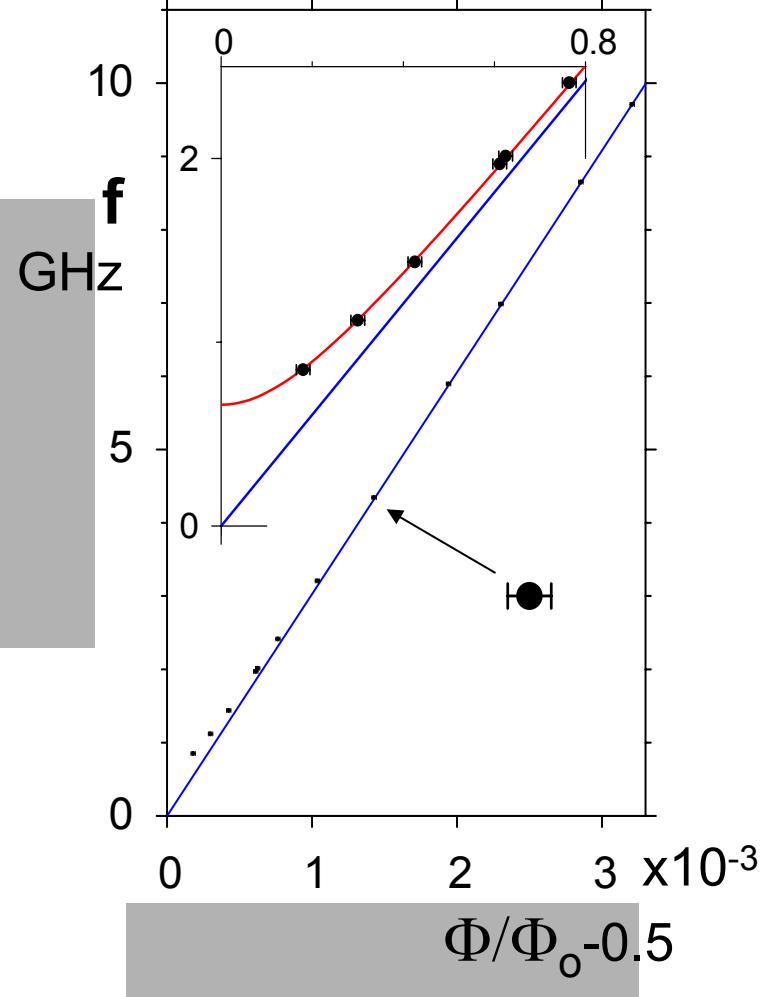
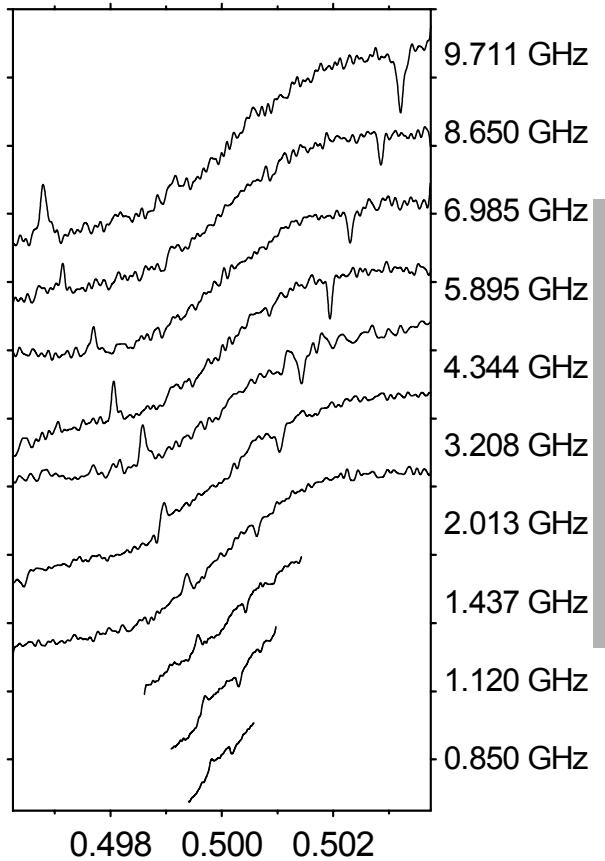


**SQUID**  $I_c$  200 nA  
underdamped, no ohmic shunt  
measurement of  $I_{\text{switch}}$   
by repeated ramping  
histogram of 5,000-10,000 points  
average value recorded



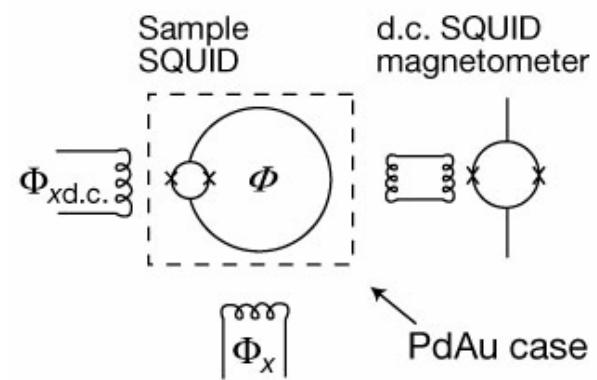
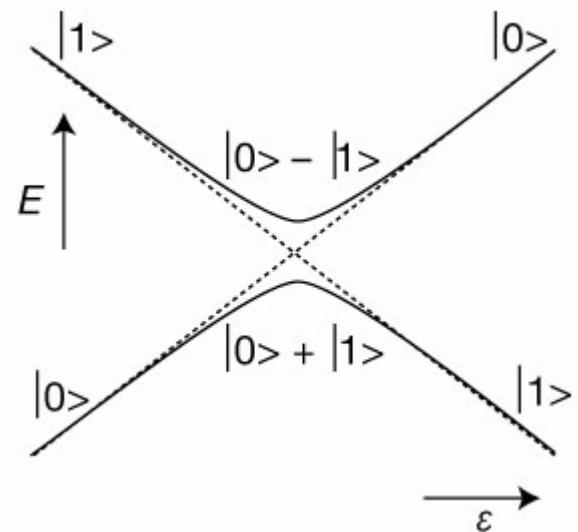
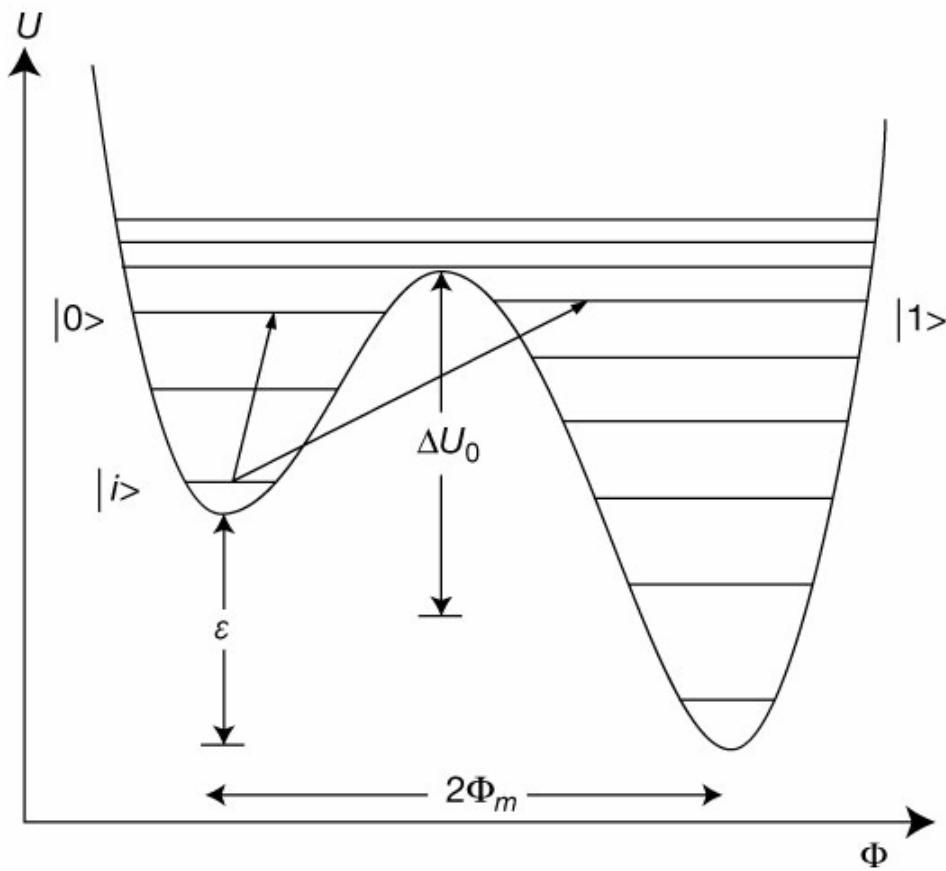
# superposition of states with clockwise and anti-clockwise-current

switching current



Van der Wal et al., Science 290, 773 (2000)

also SUNY (Friedman, Lukens et al.)



$\Delta E$  and  $E_B$  manipulated with flux  
 $E_J = 76 \text{ K} (\cos \Phi_{\text{d.c.}} / \Phi_0)$ ,  $E_C = 9 \text{ mK}$   
 $\Delta U_0 = 9 \text{ K}$

**quantum system**

**environment**

# Decoherence

relaxation  $T_1 \tau_{\text{relax}}$

dephasing  $T_2 \tau_\phi$

## - pseudospin - boson bath (harmonic oscillators)

Milena Grifoni, Frank Wilhelm, Gerd Schon et al

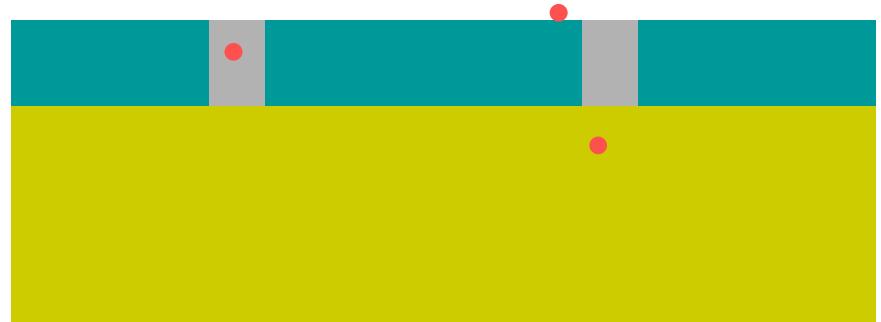
$\tau_{\text{relax}}$  determined by spectral density at  $\Delta E$

$\tau_\phi$  determined by spectral density at low  $\omega$   
measurement circuit designed to optimize

## - pseudospin - spin bath ? → Phil Stamp

## - 1/f type noise: flux, charge, critical current

## 1/f noise



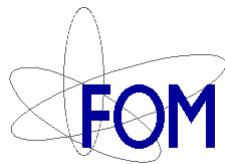
- charge noise: charged defects in barrier, substrate or surface lead to non-integer induced charge. Static offset, 1/f noise.
- critical current noise: neutral defects in barrier.
- flux noise: trapped vortices, magnetic domains, magnetic impurities, nuclear spins

# Delft flux qubits

Kees Harmans  
Alexander ter Haar  
Irinel Chiorescu  
Adrian Lupascu  
Floor Pauw  
Patrice Bertet  
Jelle Plantenga  
Jonathan Eroms  
Yasunobu Nakamura (NEC)  
Kouichi Semba (NTT)



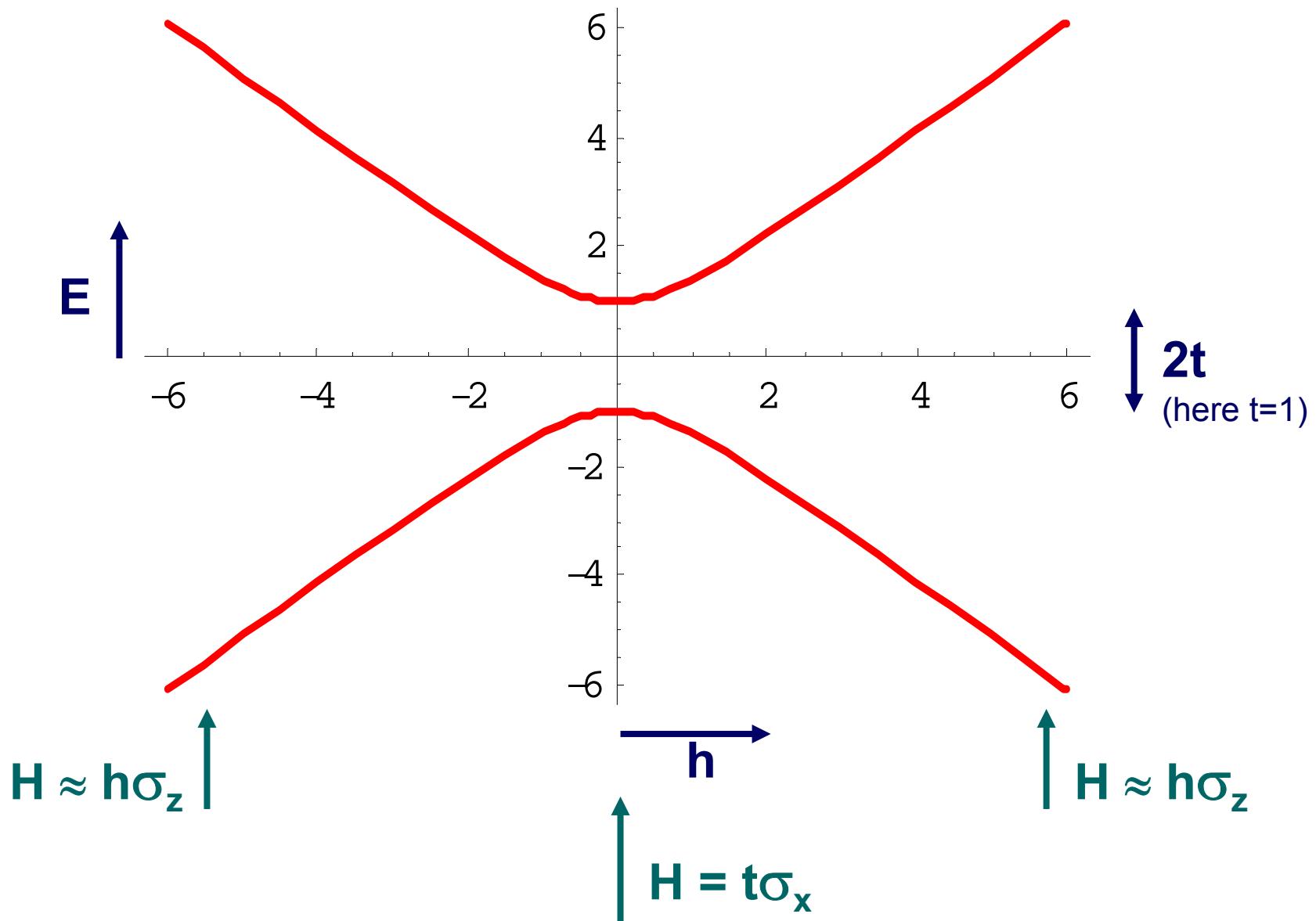
funding: FOM, EU  
ARO, Kavli

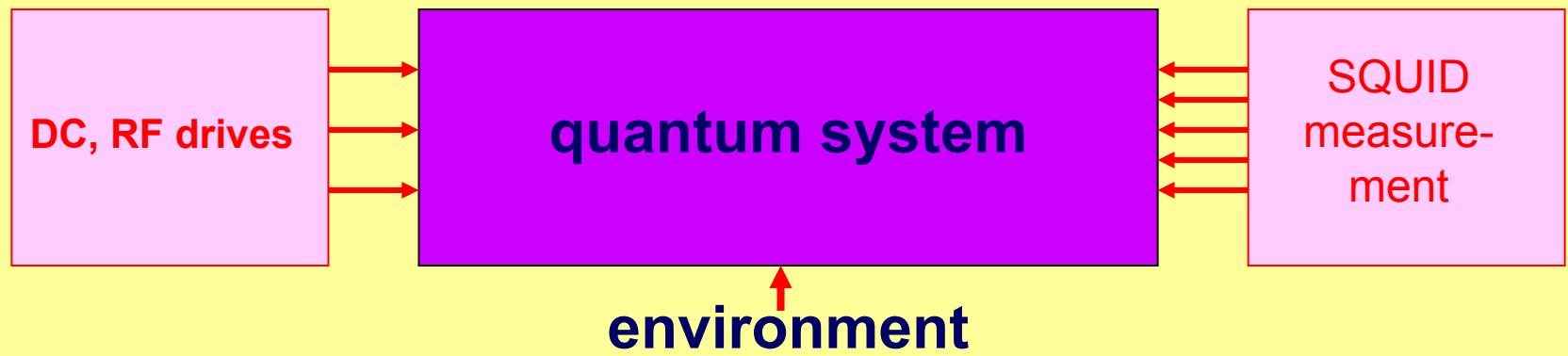


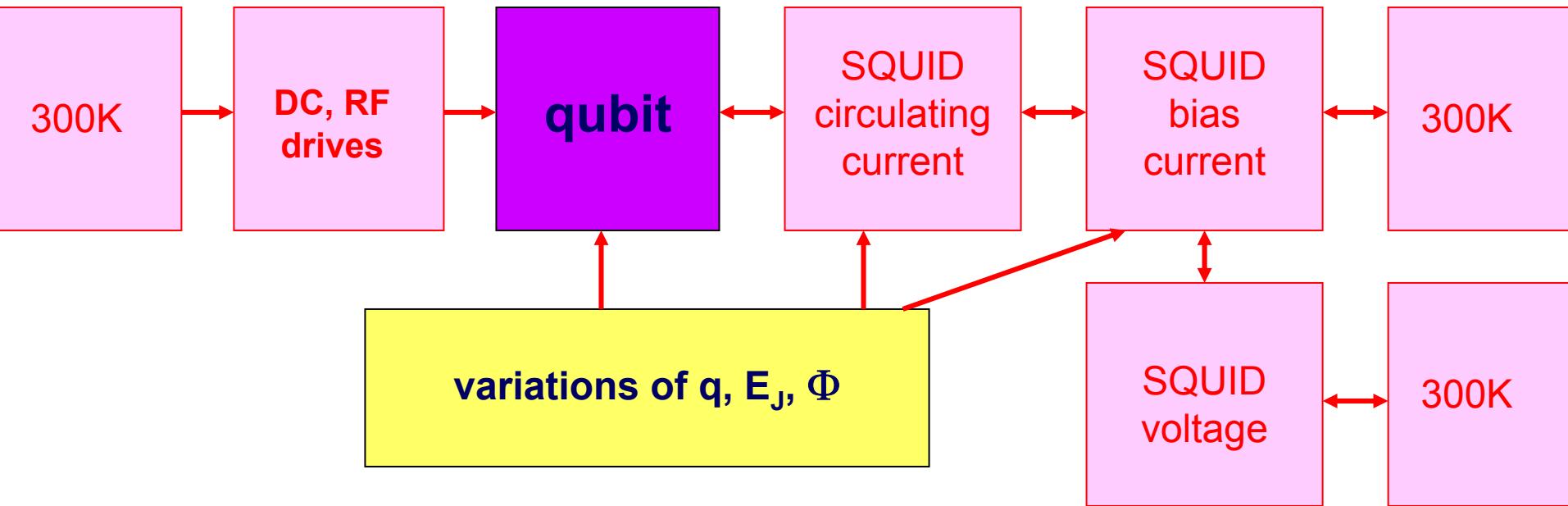
$h$  set by external flux  
 $t$  set by fabrication

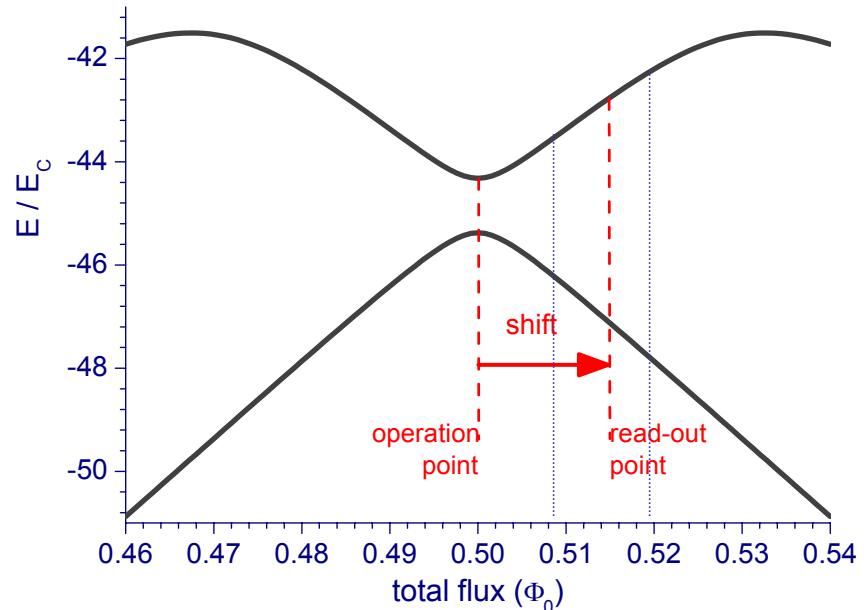
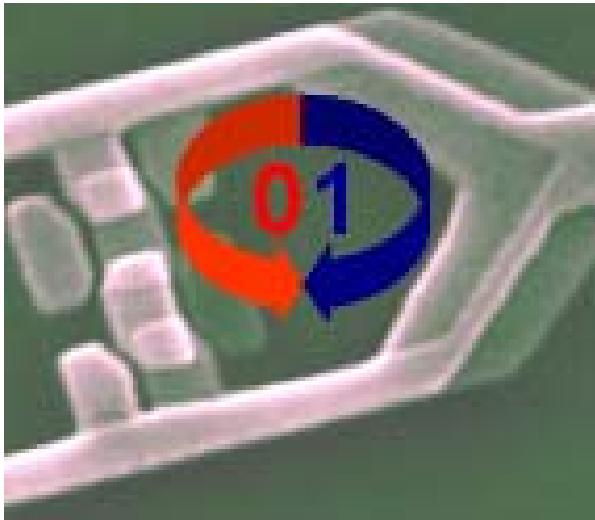
excitation, measurement  $\sigma_z$

$$H = h\sigma_z + t\sigma_x$$



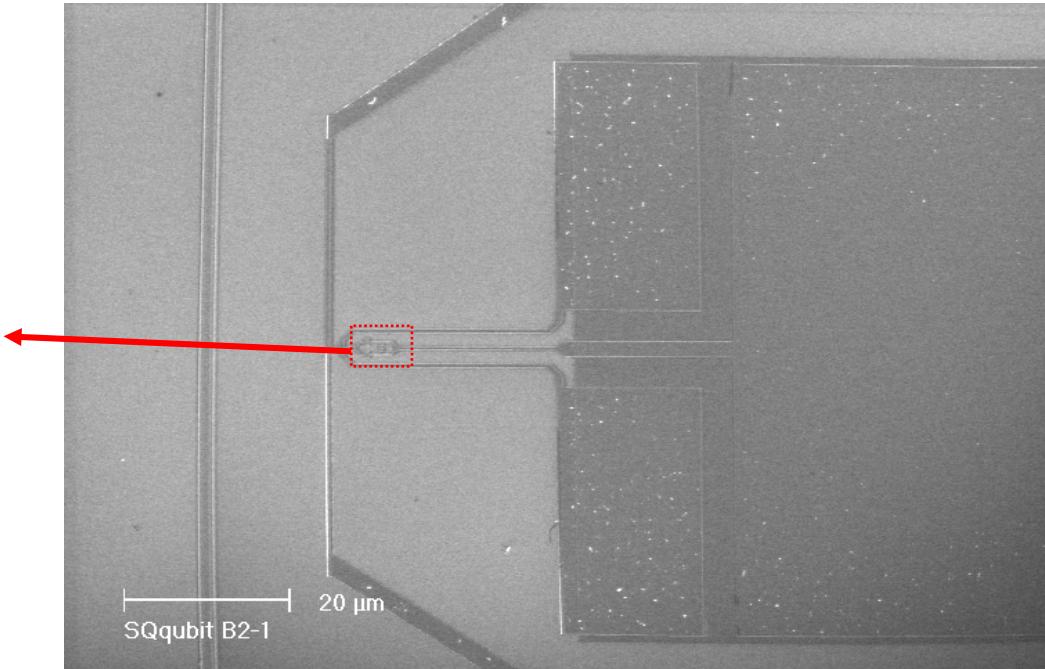
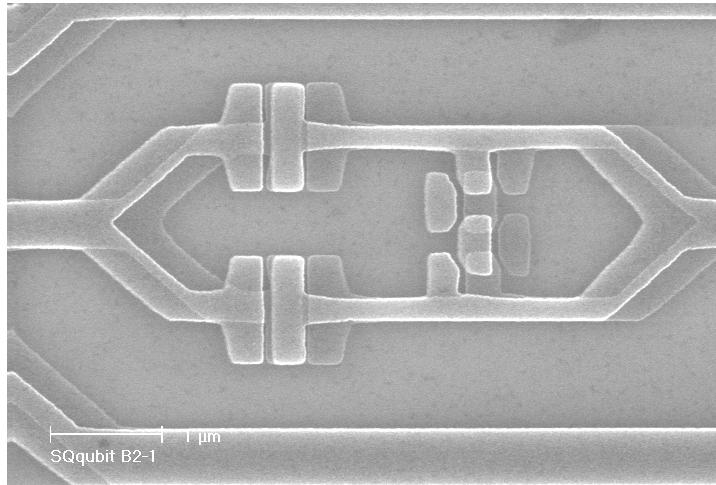






intended principle:

- quantum operations at symmetry point where  $dE/d\Phi=0$  (but also qubit circulating currents are zero)
- use changing SQUID circulating current for automatic shift to a read-out bias where there is qubit current to measure



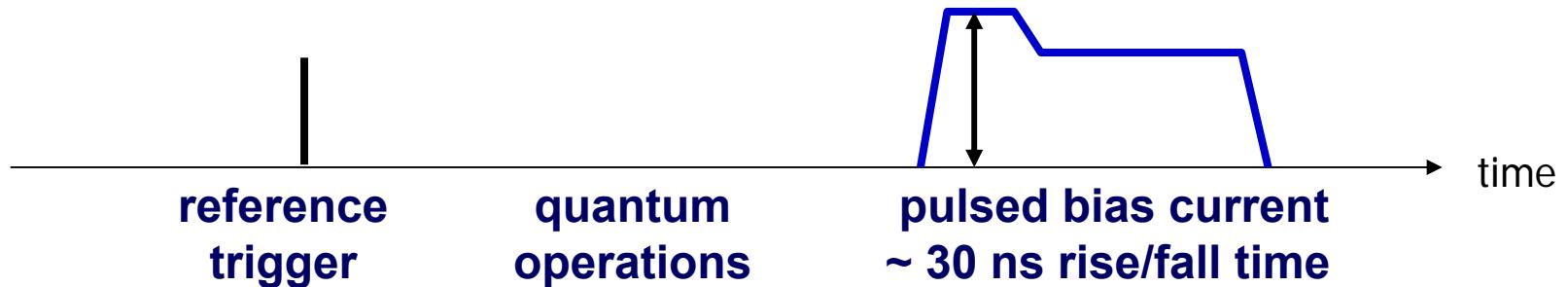
**well-designed electromagnetic environment**

**calculated**       $T_1 = 100 \mu\text{s}$   
 $T_2 = 20 \mu\text{s}$

**operation time** 1 ns

**SQUID shunted by large capacitance on chip**

# SQUID readout:

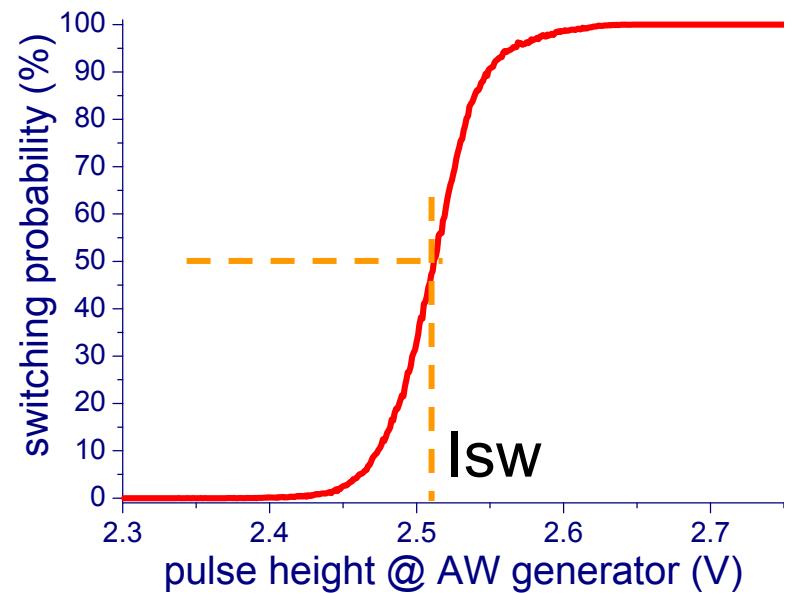


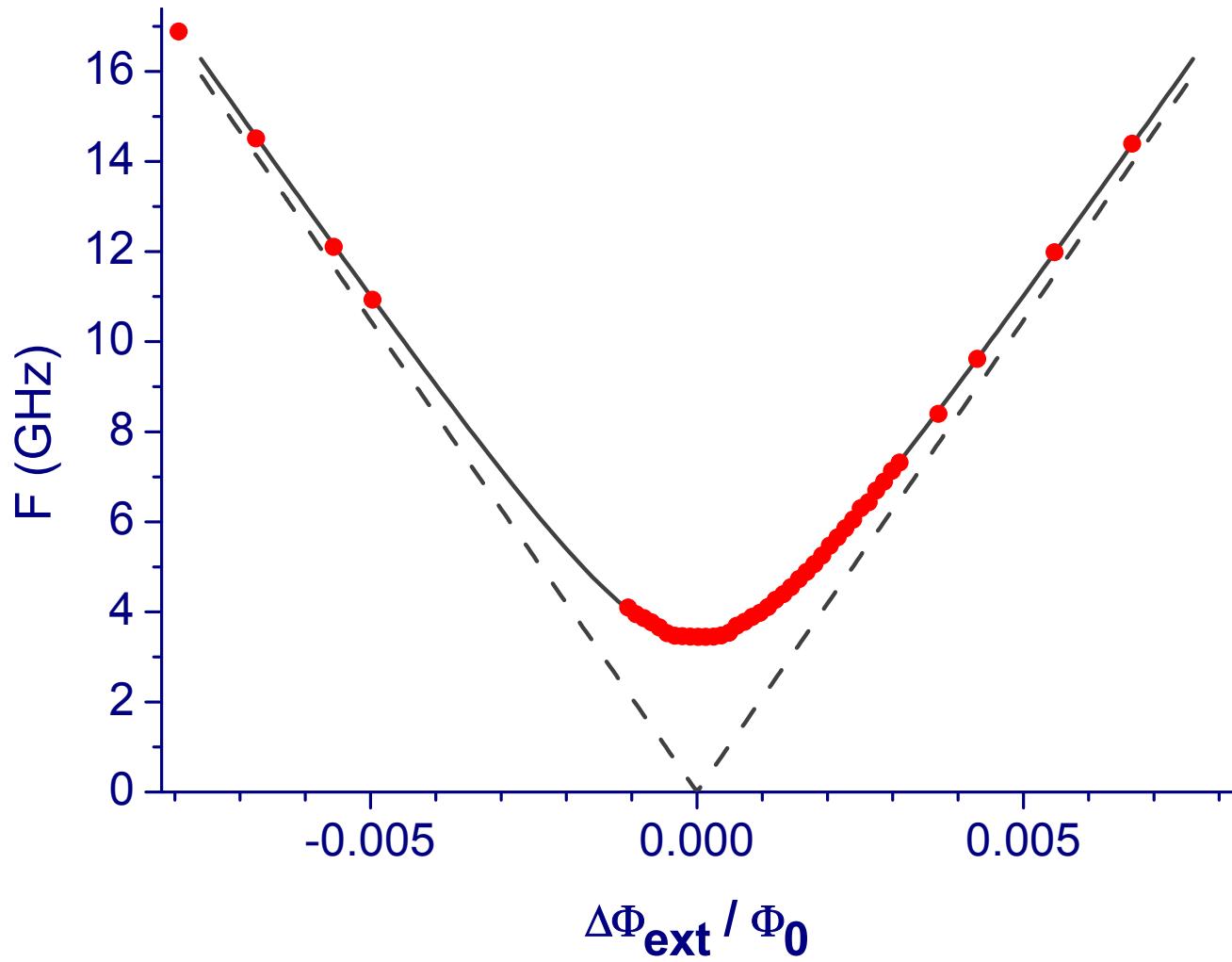
**only two possible outputs:**

**SQUID switched to gap voltage**

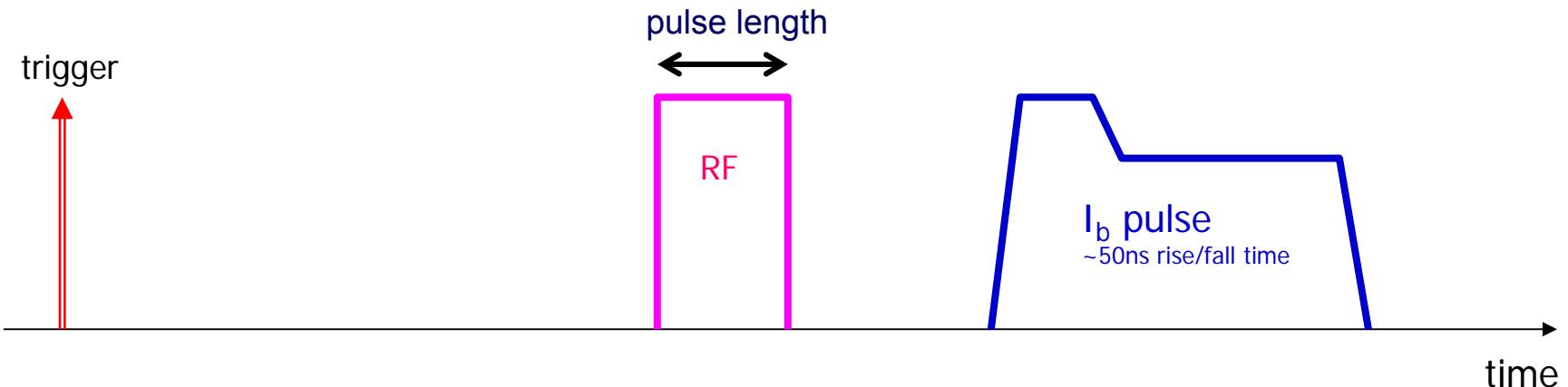
**SQUID still at  $V=0$**

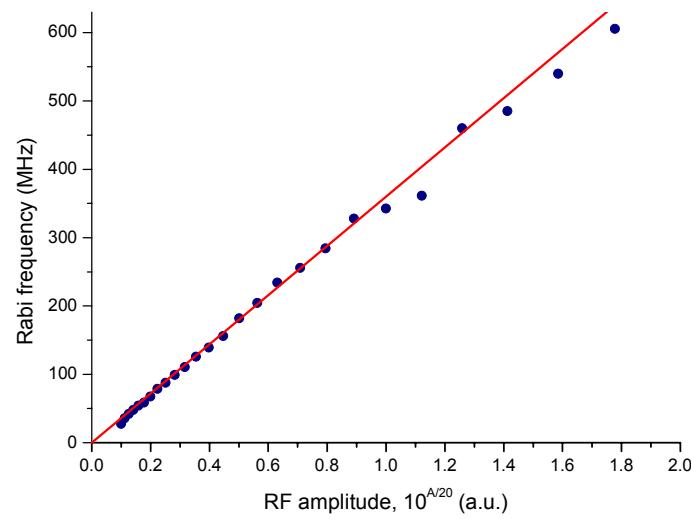
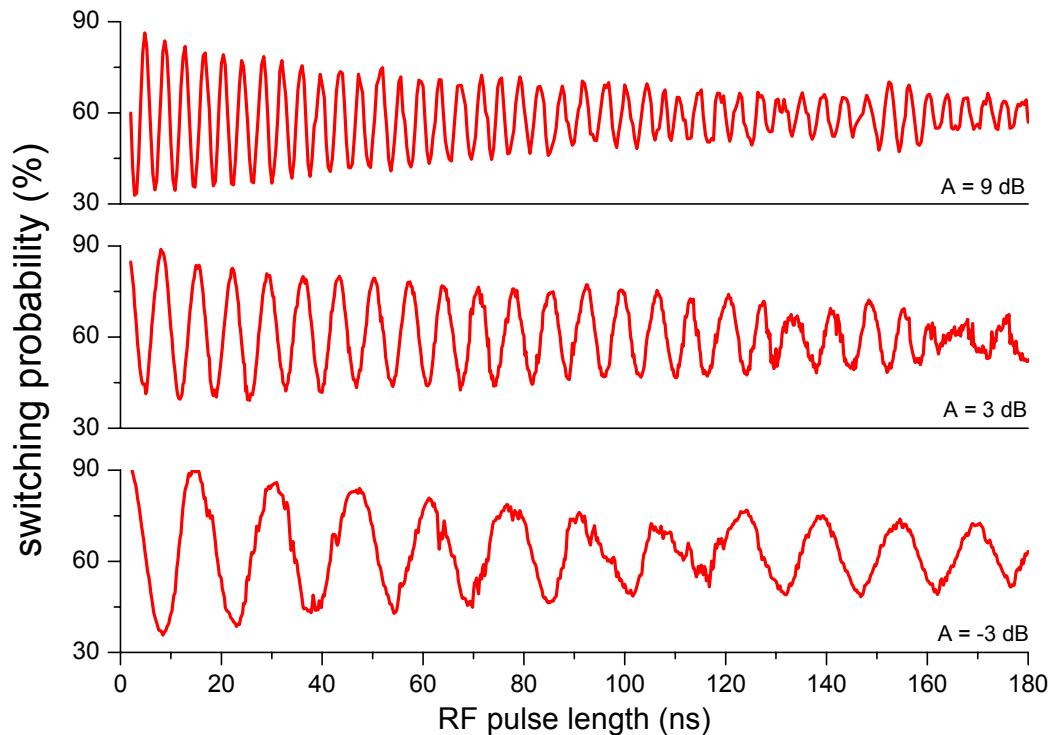
**pulse height adjusted  
to give  $\sim 50\%$  switching**

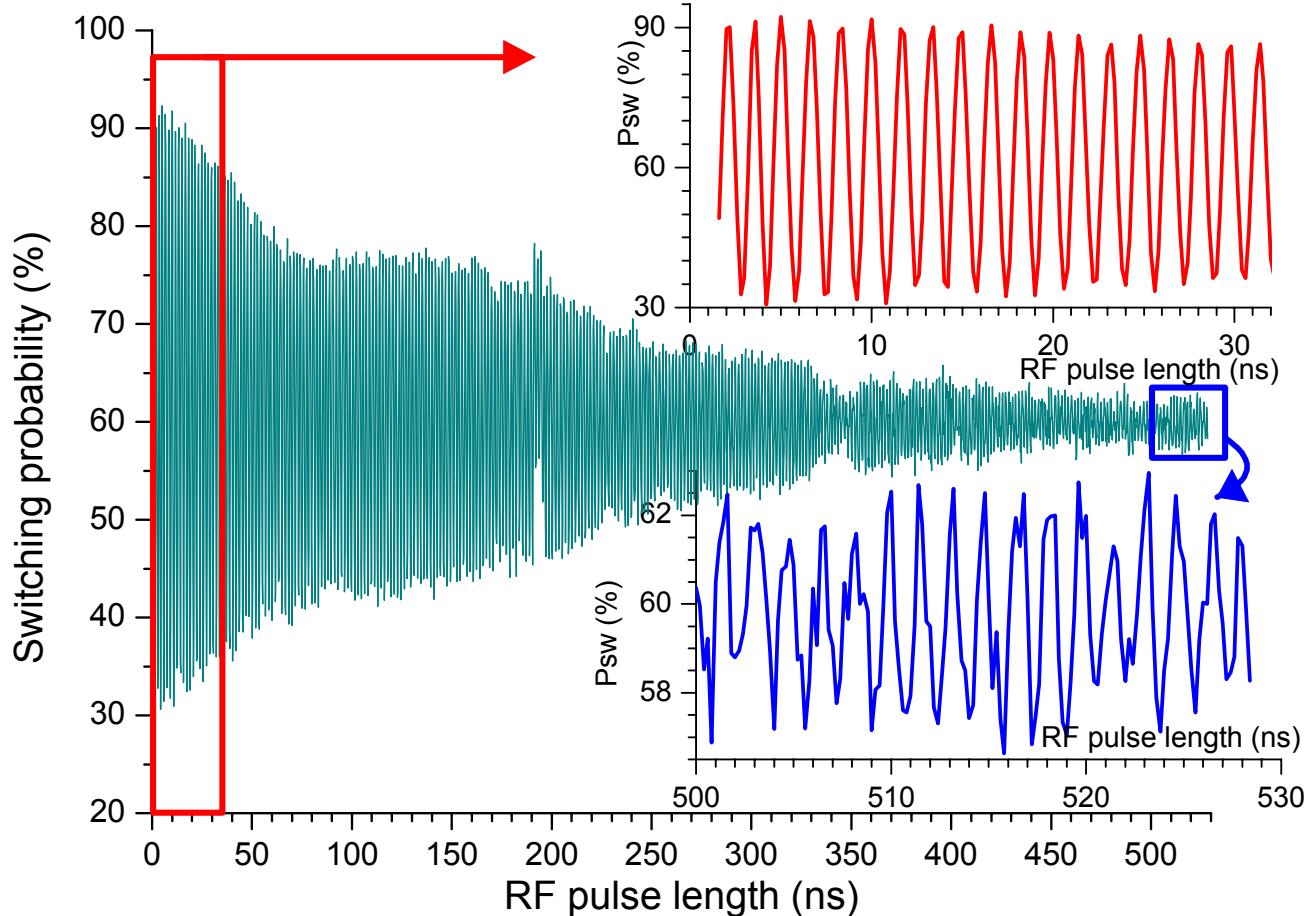




## Rabi: microwave pulse with varying length



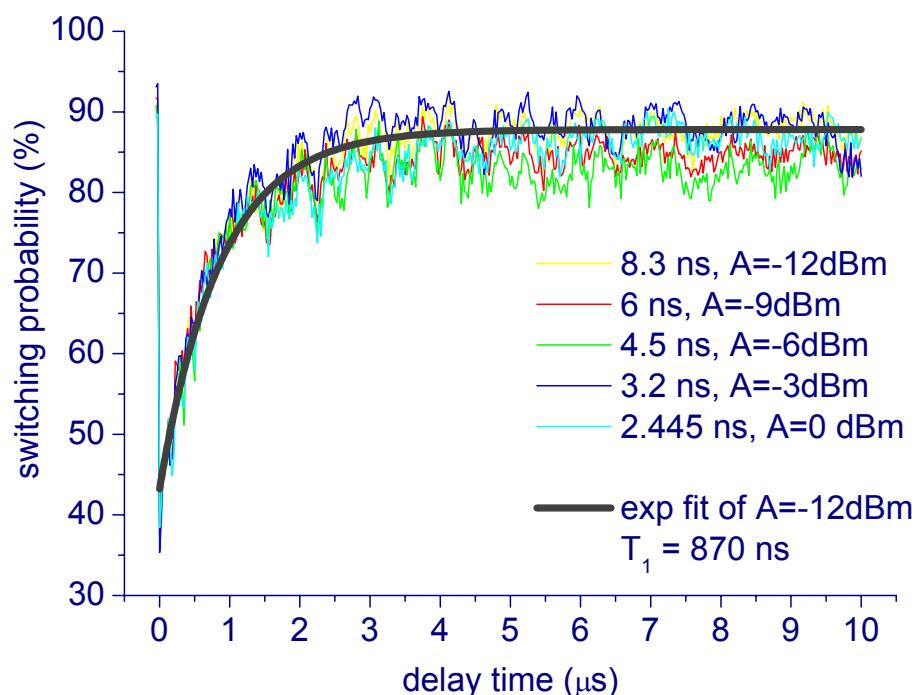
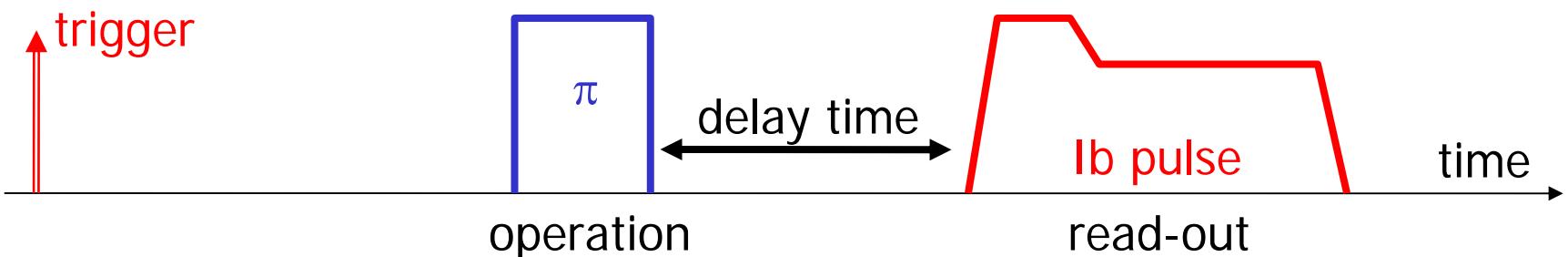




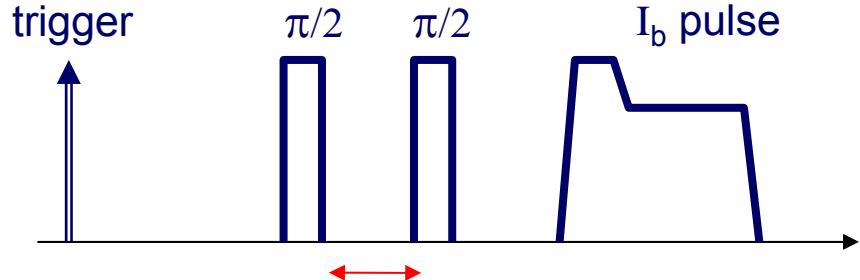
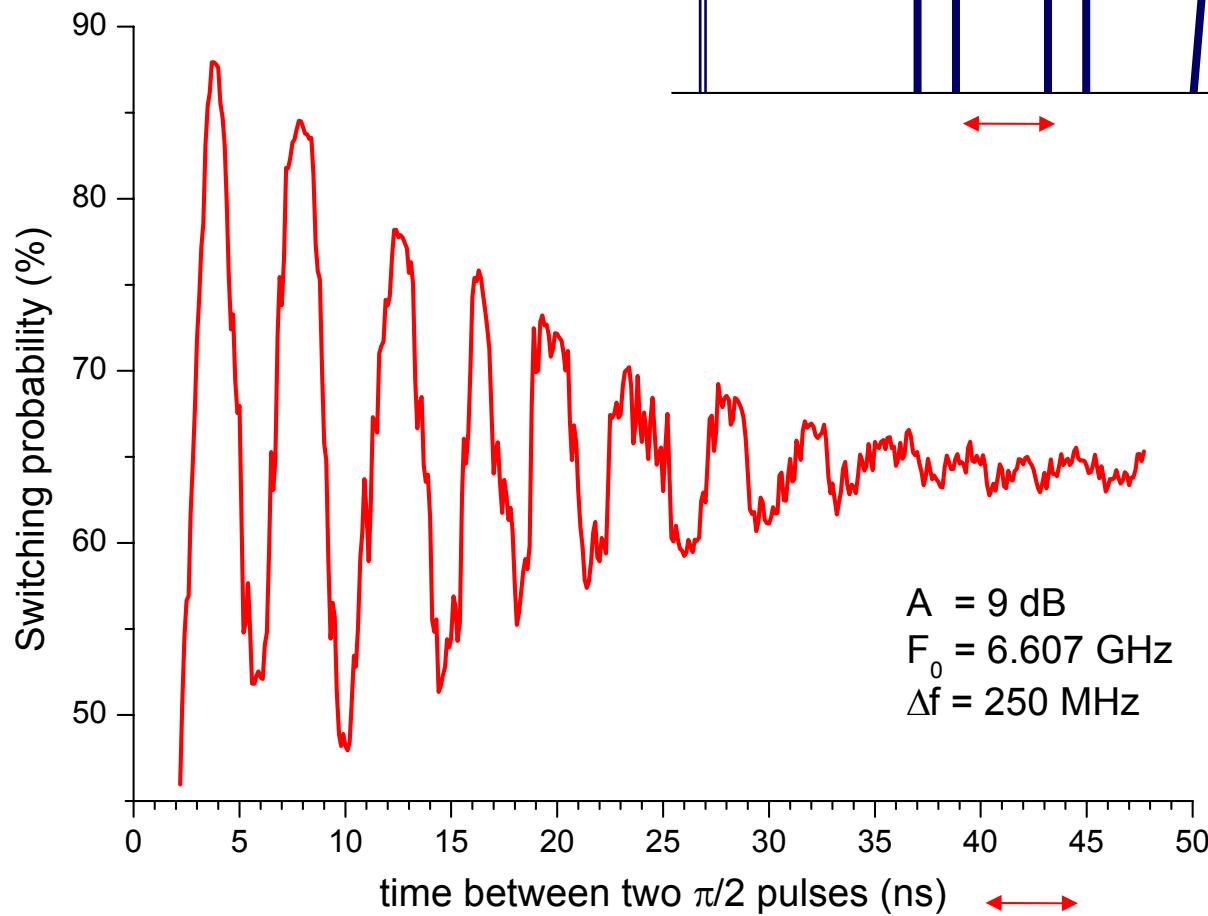
# measurement of relaxation time

observed values: 40 ns to 300  $\mu$ s

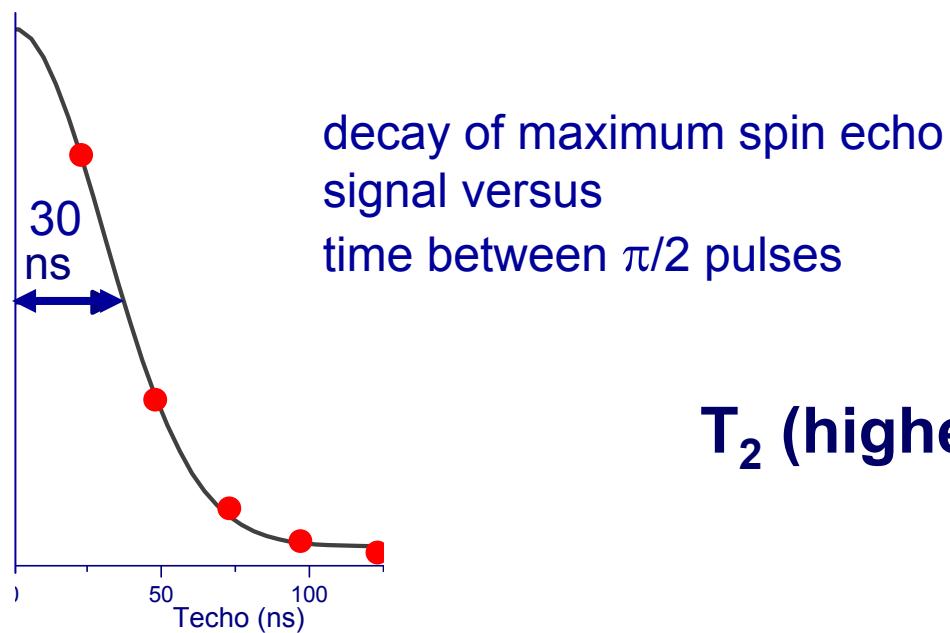
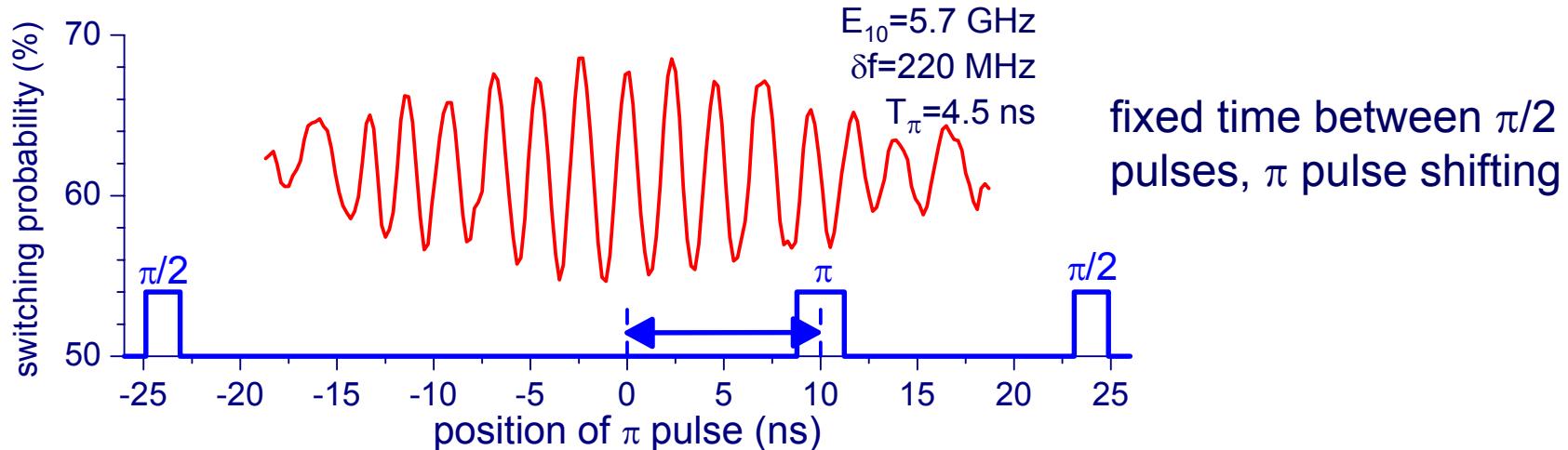
one  $\pi$  pulse and read-out pulse delayed



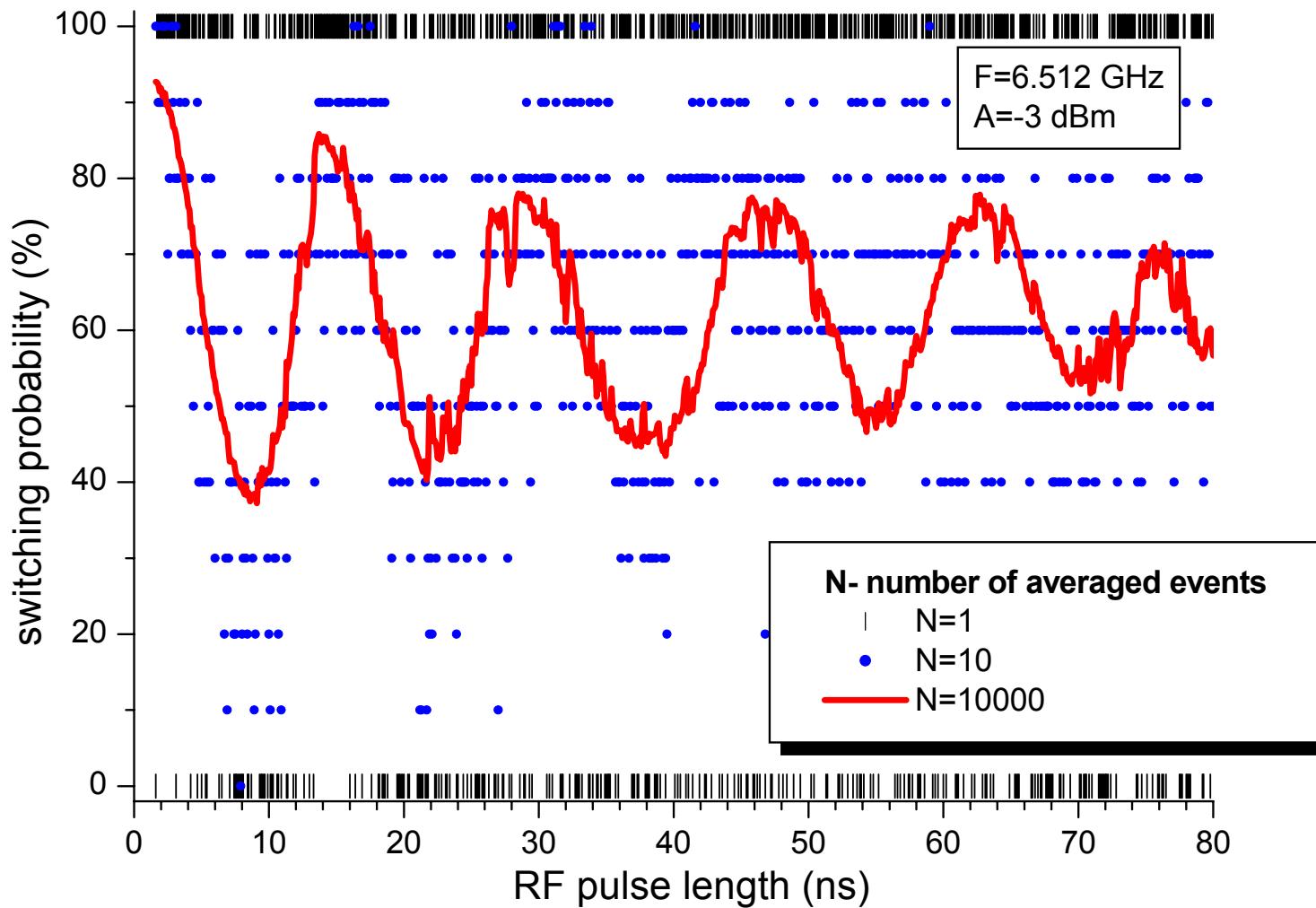
# measurement $T_2$ time Ramsey

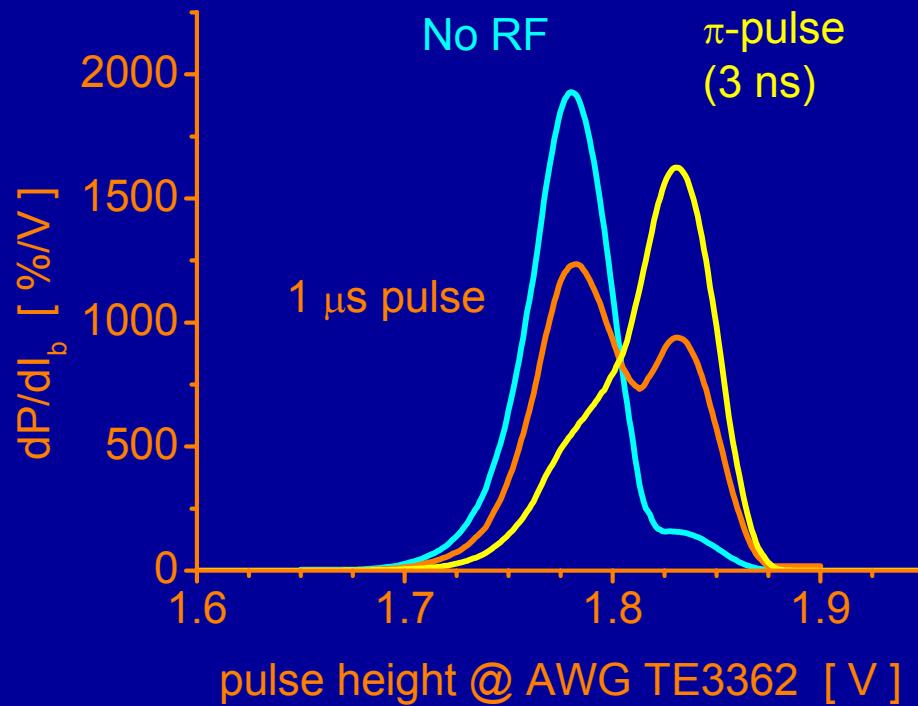
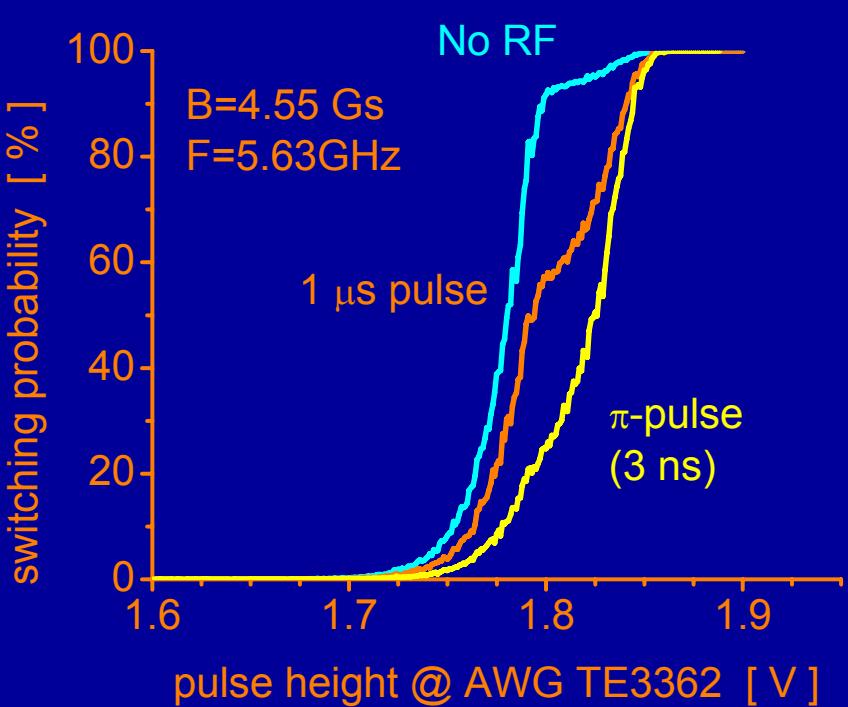


# spin echo



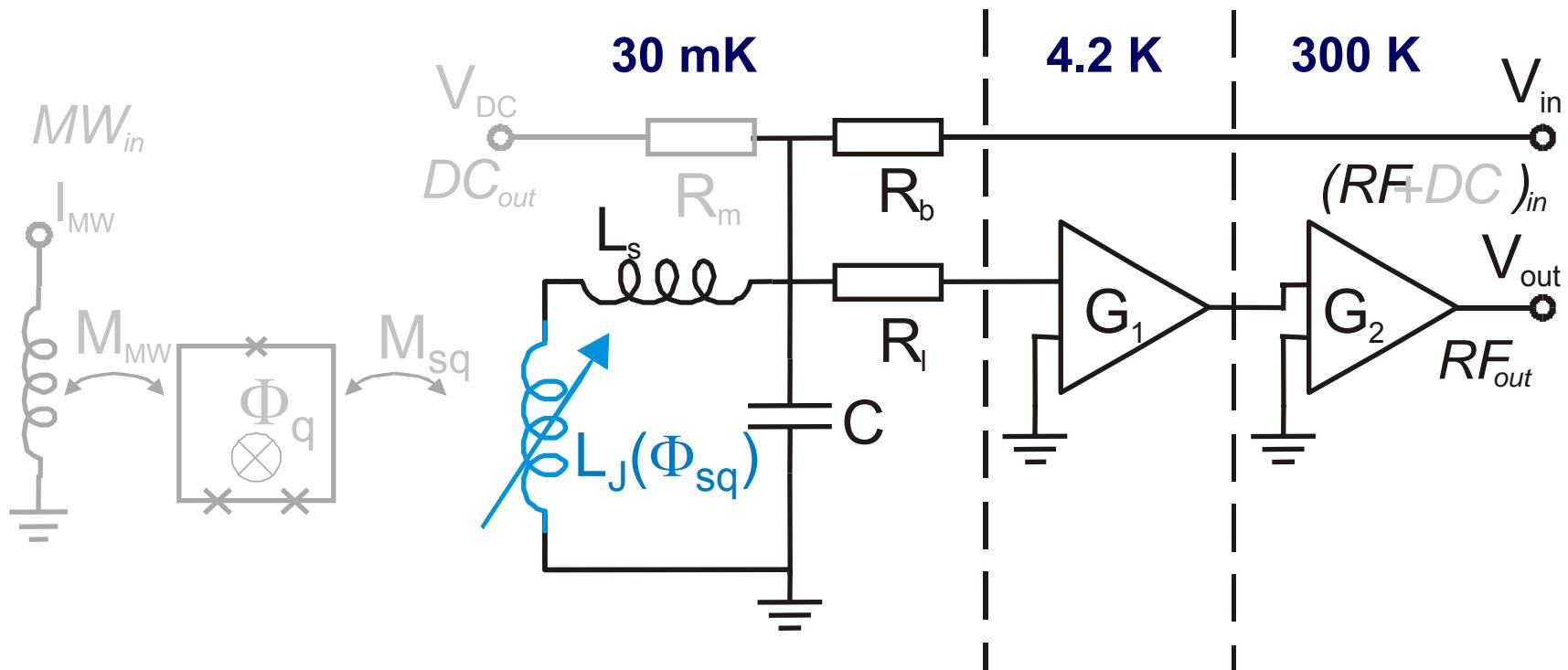
# Rabi with low number of measurements, single shot contains some information





Kouichi Semba, NTT, Delft  
TE3362 AWG

# inductive readout, circuit



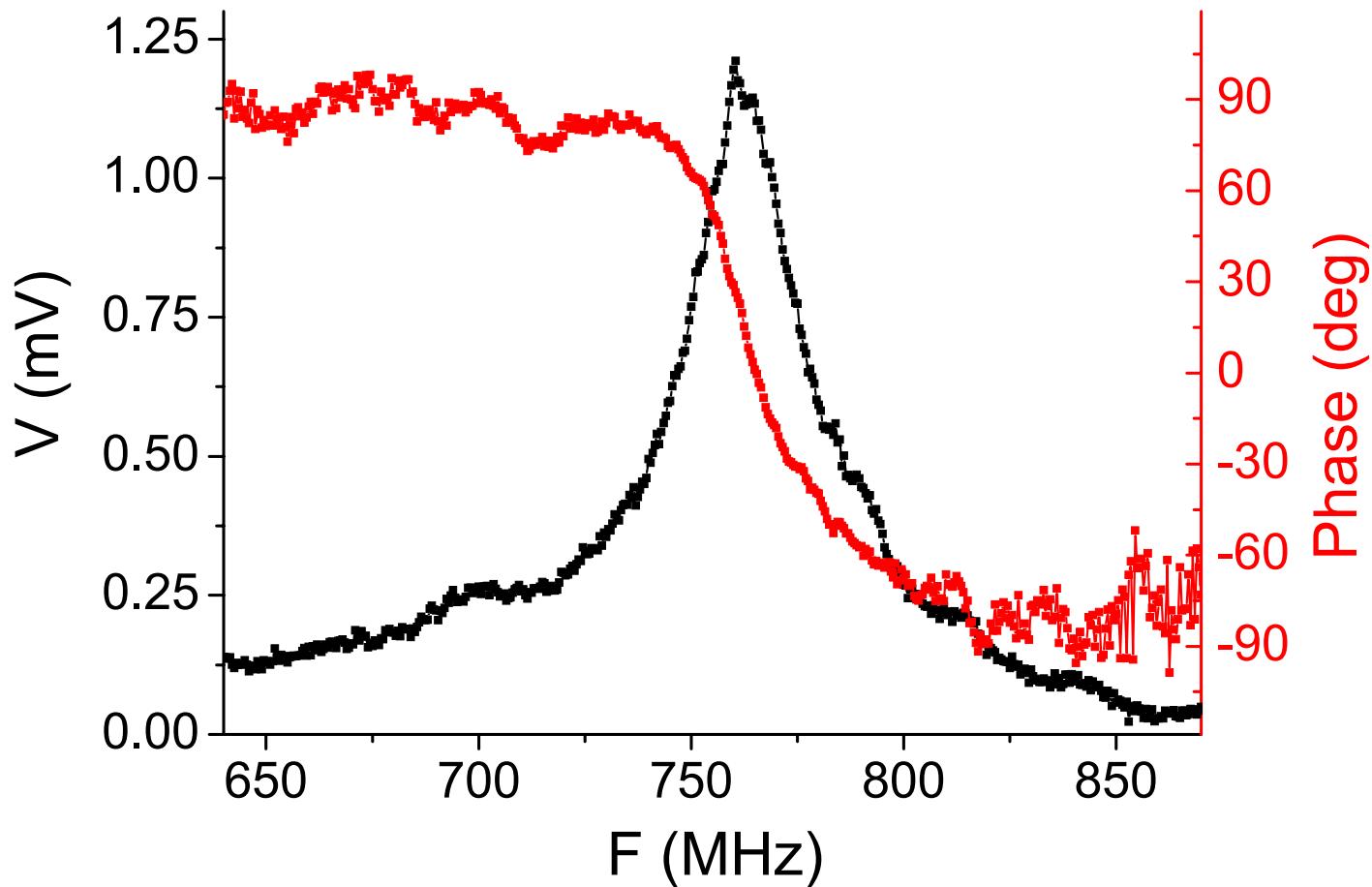
circuit  
parameters

$$\begin{aligned} C &= 12 \text{ pF} \\ L_s &\sim 2.5 \text{ nH} \\ R_L &= 820 \text{ W} \\ R_b &= 5.6 \text{ kW} \\ R_m &= 11 \text{ kW} \\ Q &\sim 40 \end{aligned}$$

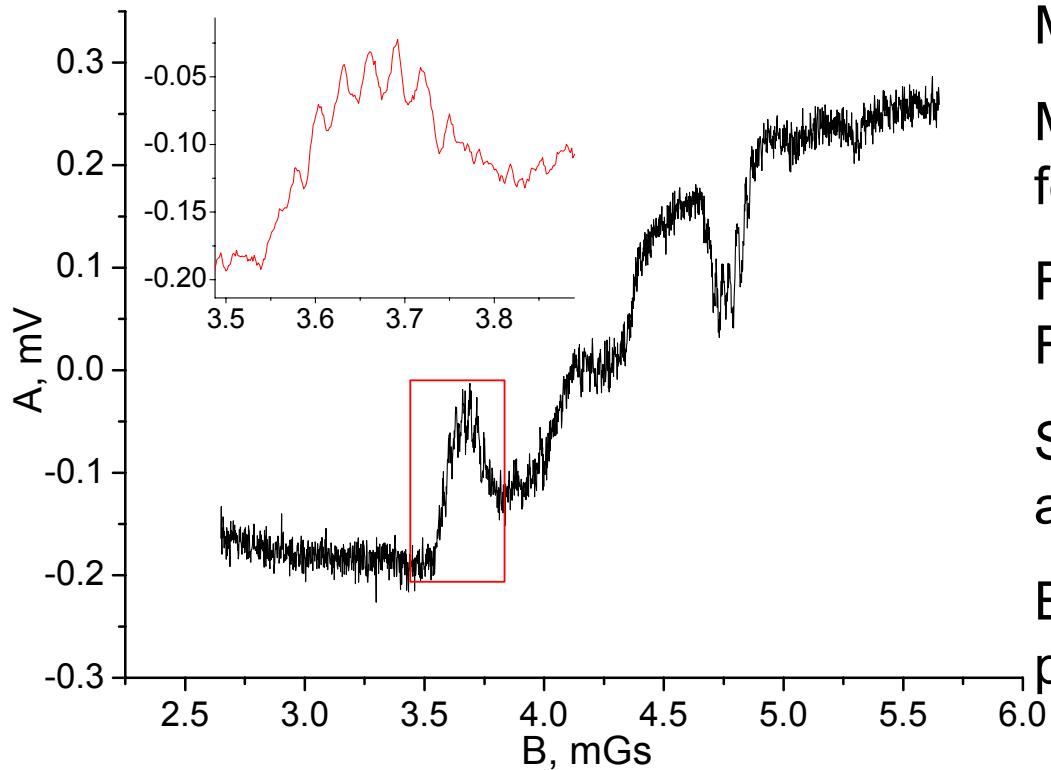
Adrian Lupascu, Kees Harmans,  
Raymond Schouten

# resonance curve

at  $B=0$ :  $F_{\text{res}} = 762.2 \text{ MHz}$ ,  $Q = 33$



# Typical spectroscopy curve



Measurements at step s1

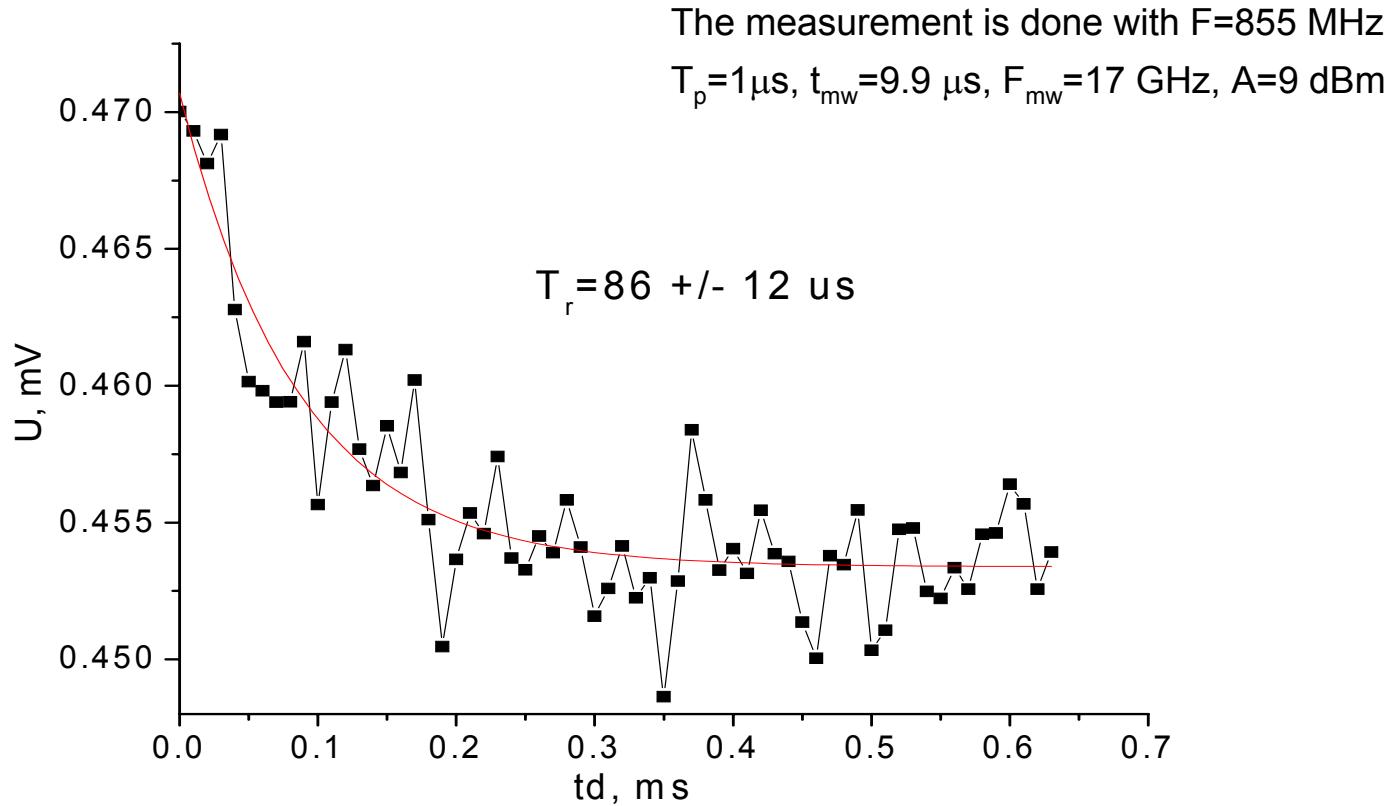
Measurement scheme:  $1\mu\text{s}$  MW followed by  $2\mu\text{s}$  readout pulse

$F_{\text{MW}}=17\text{ GHz}$ ,  $F_{\text{IM}}=855\text{ MHz}$ ,  
 $F_{\text{res}}=838\text{ MHz}$

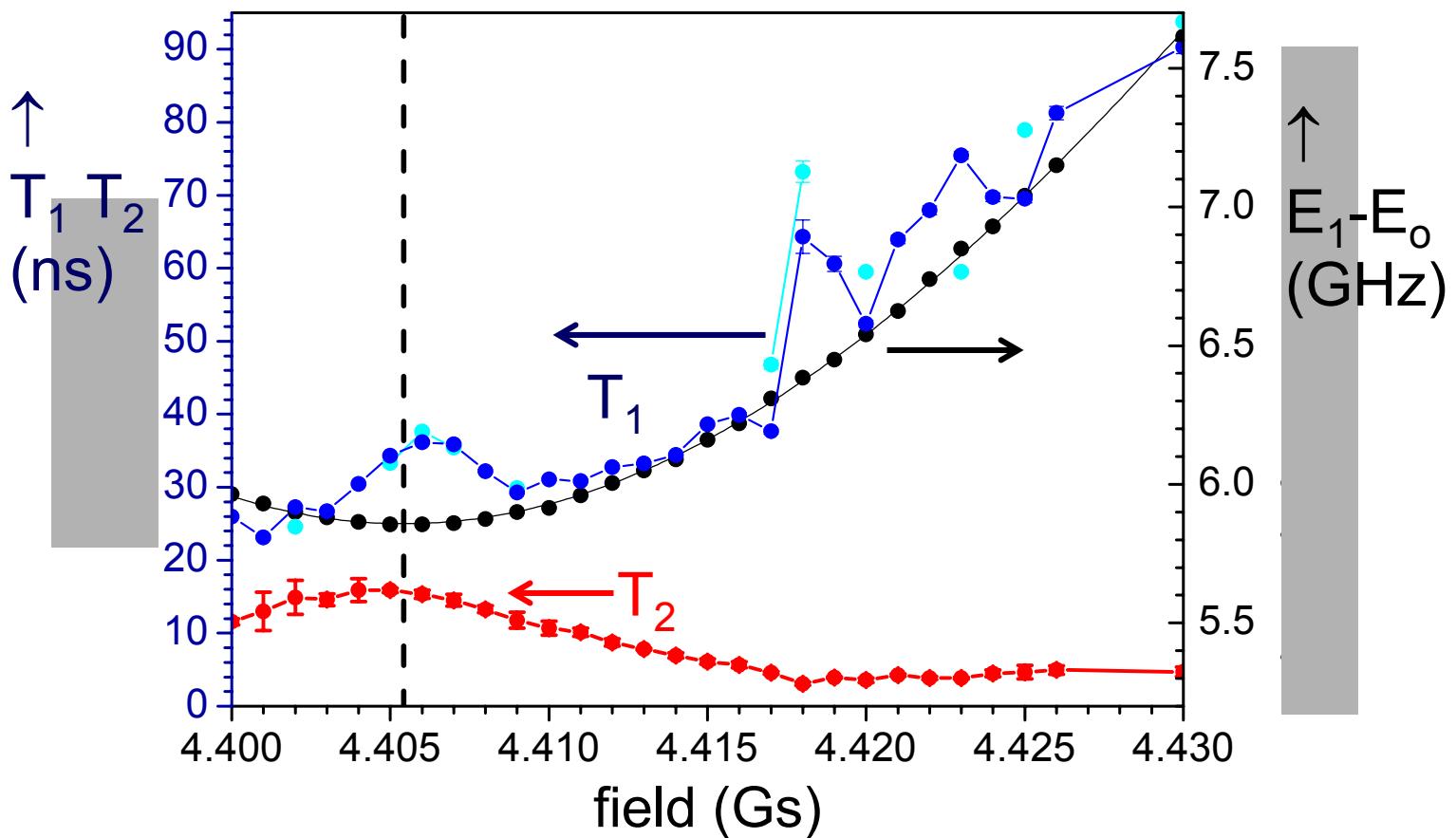
SQUID excitation: current amplitude less than  $1/2I_c$

Estimated sub-peaks period:  $750\text{ MHz}$

# relaxation

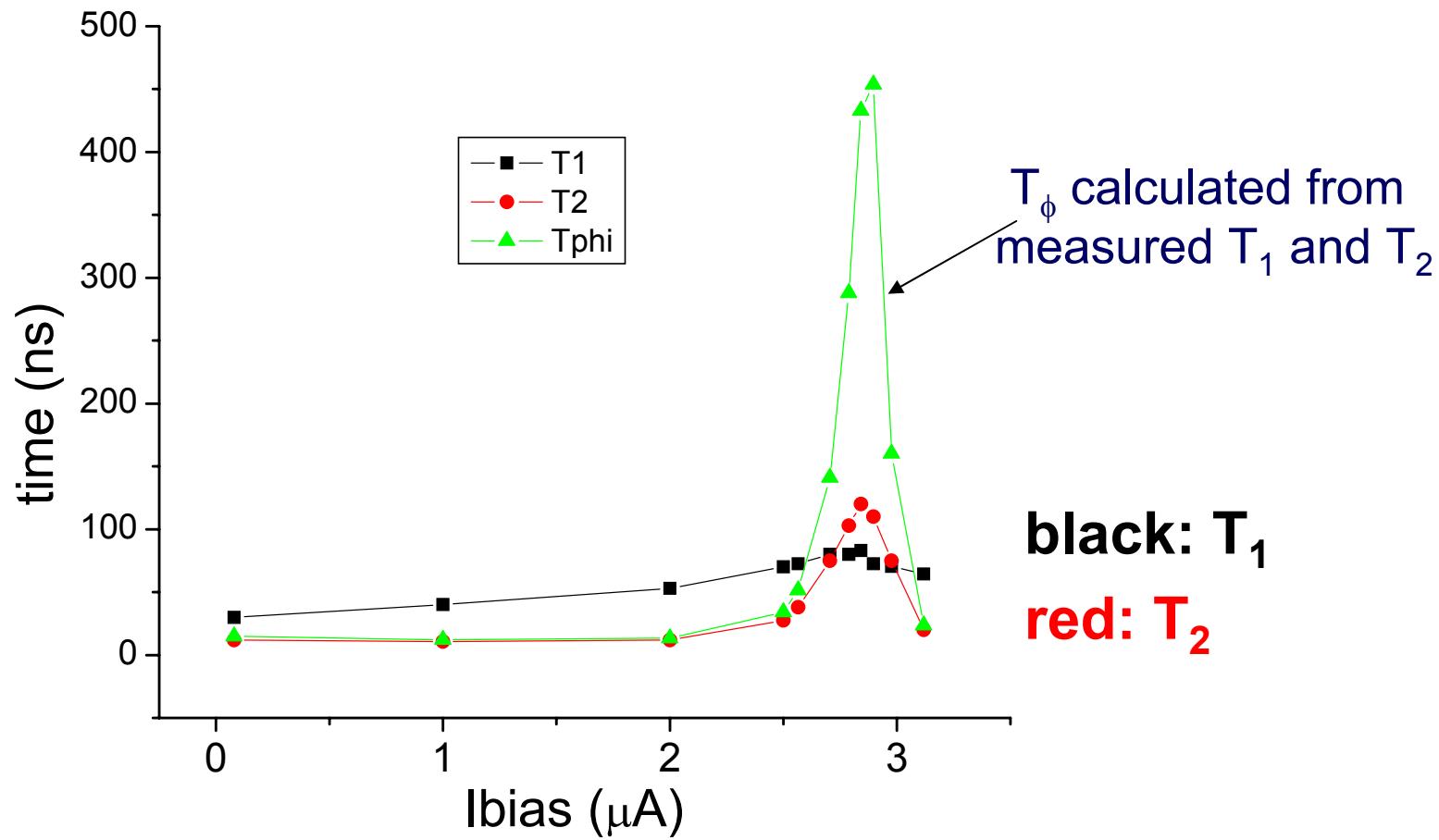


(preliminary) conclusion: relaxation time about  $60\mu s$

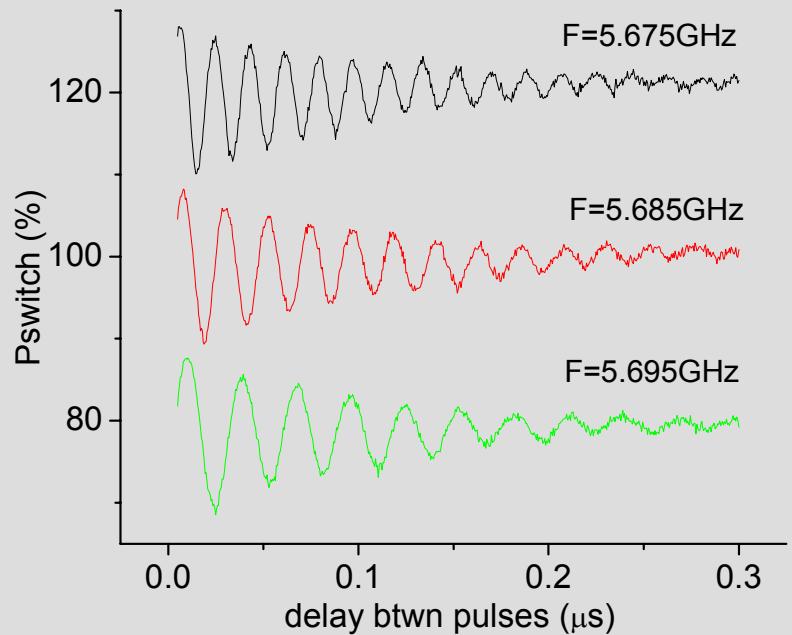
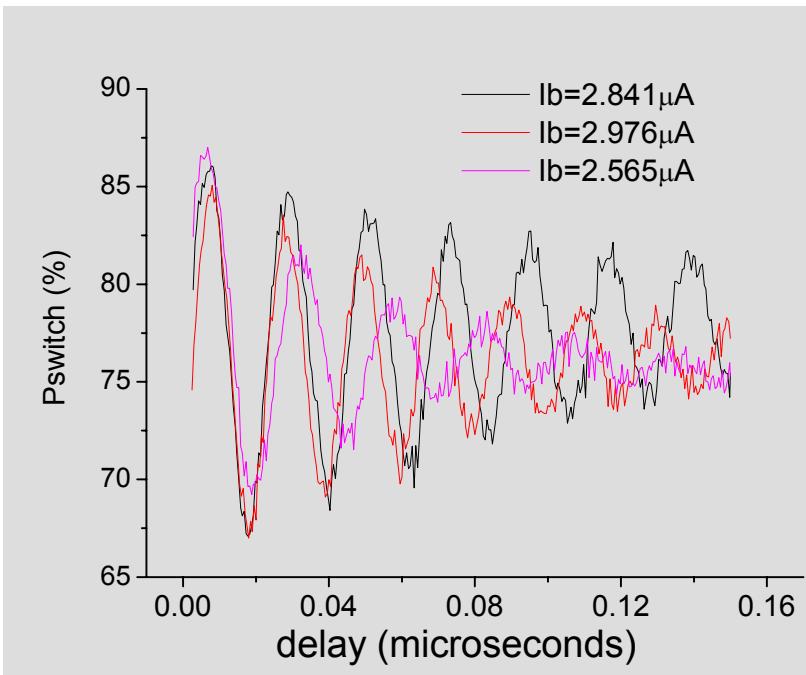


## influence of bias current on $T_1$ and $T_2$

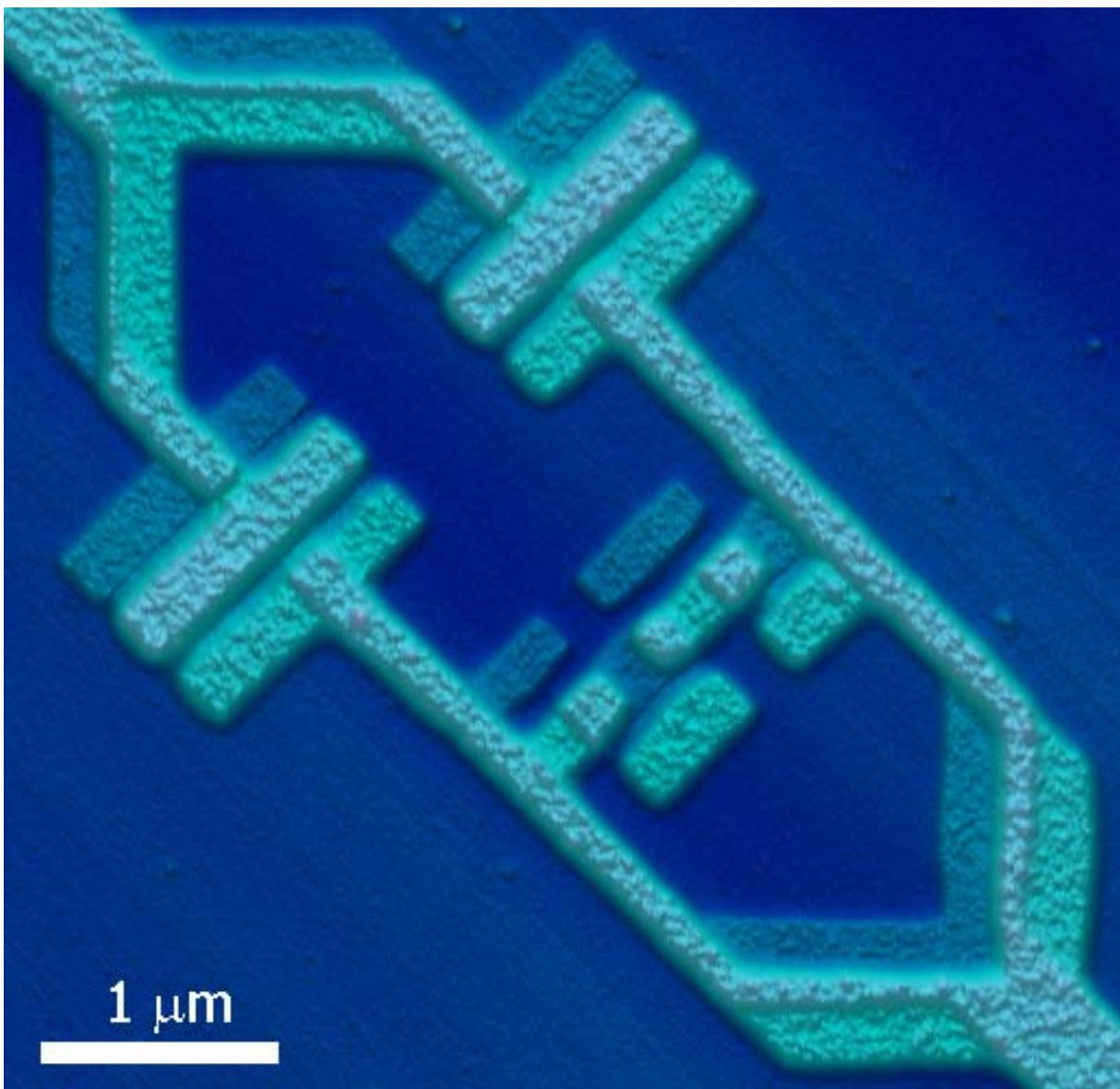
$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}$$



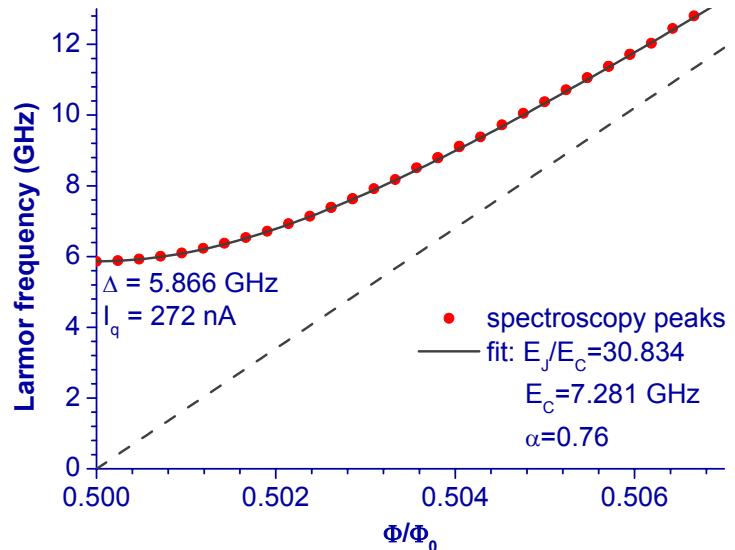
# Ramsey in presence of transport current



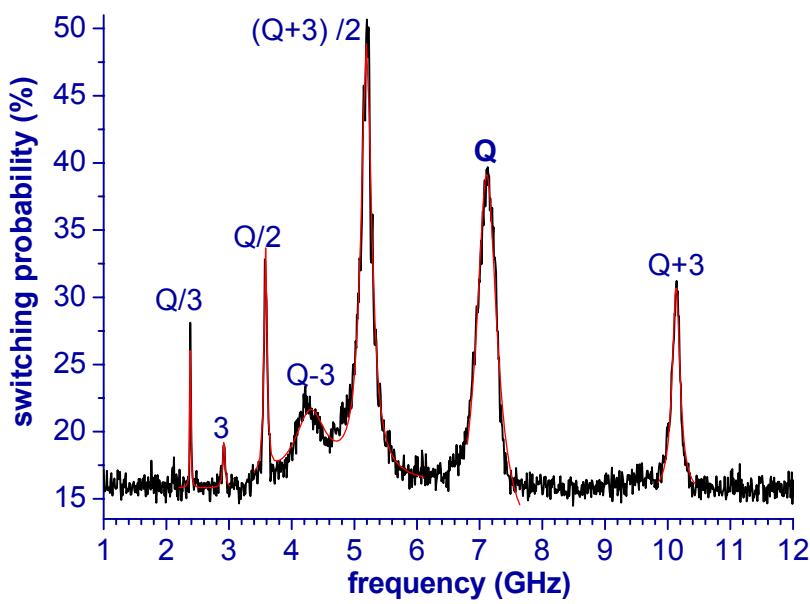
$$T_1 = 70 \text{ ns}$$
$$T_2 = 120 \text{ ns}$$



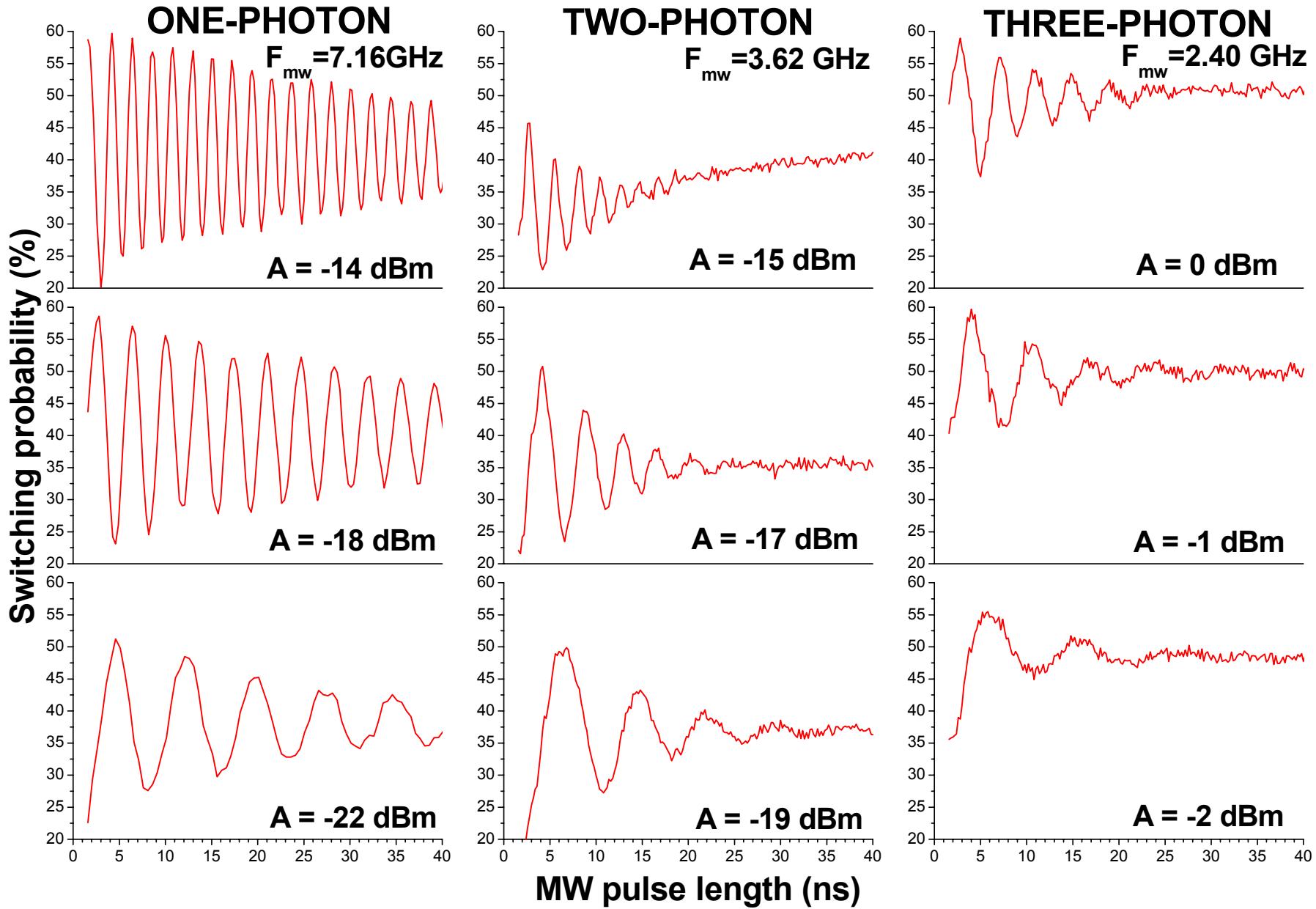
**Patrice Bertet, Irinel Chiorescu**

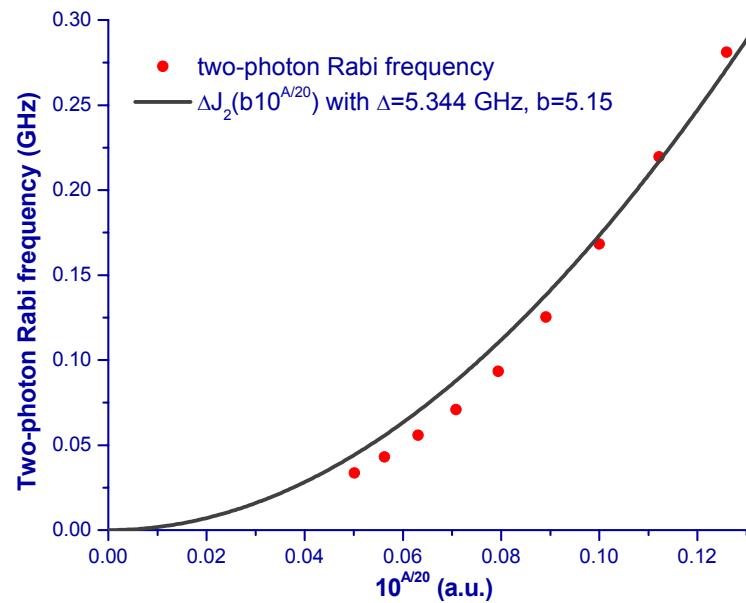
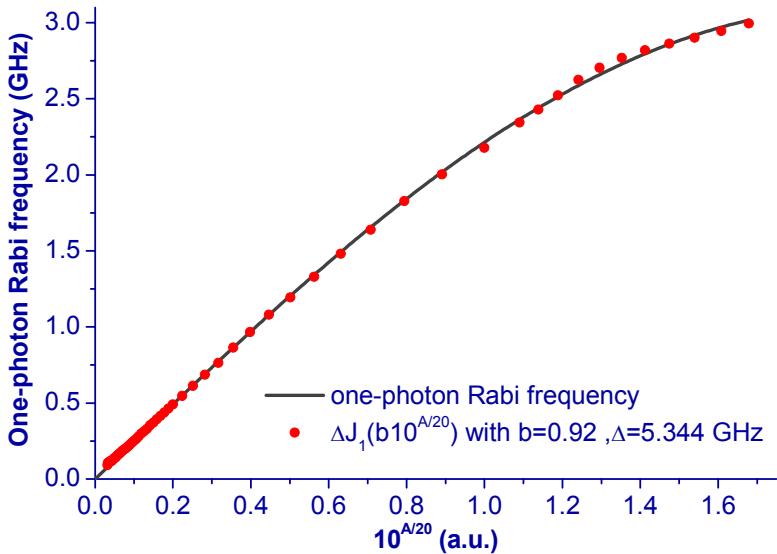


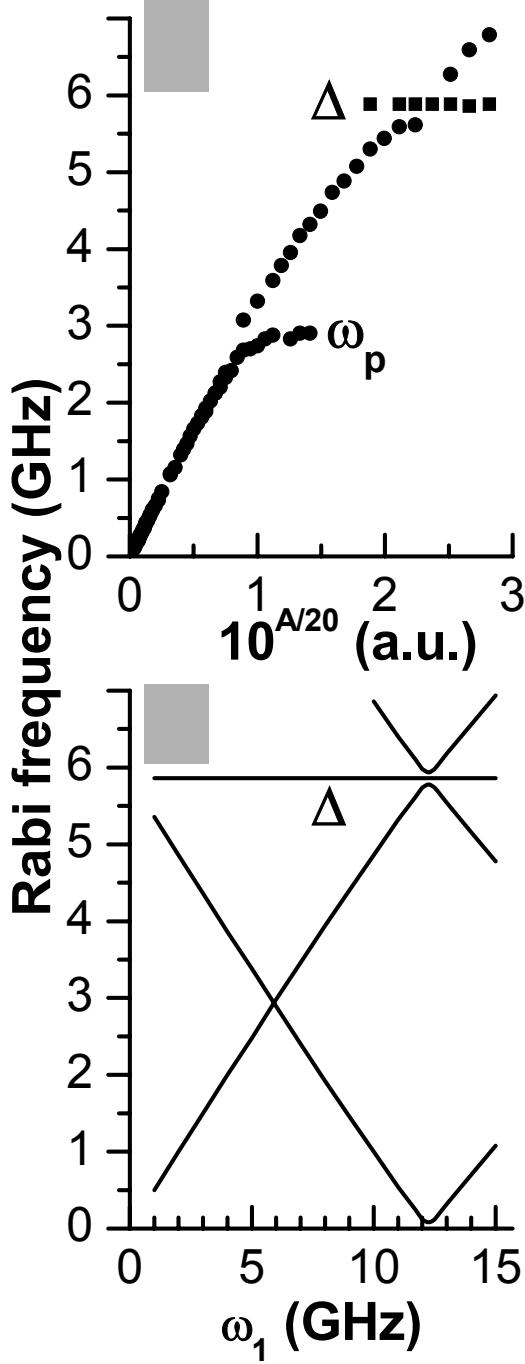
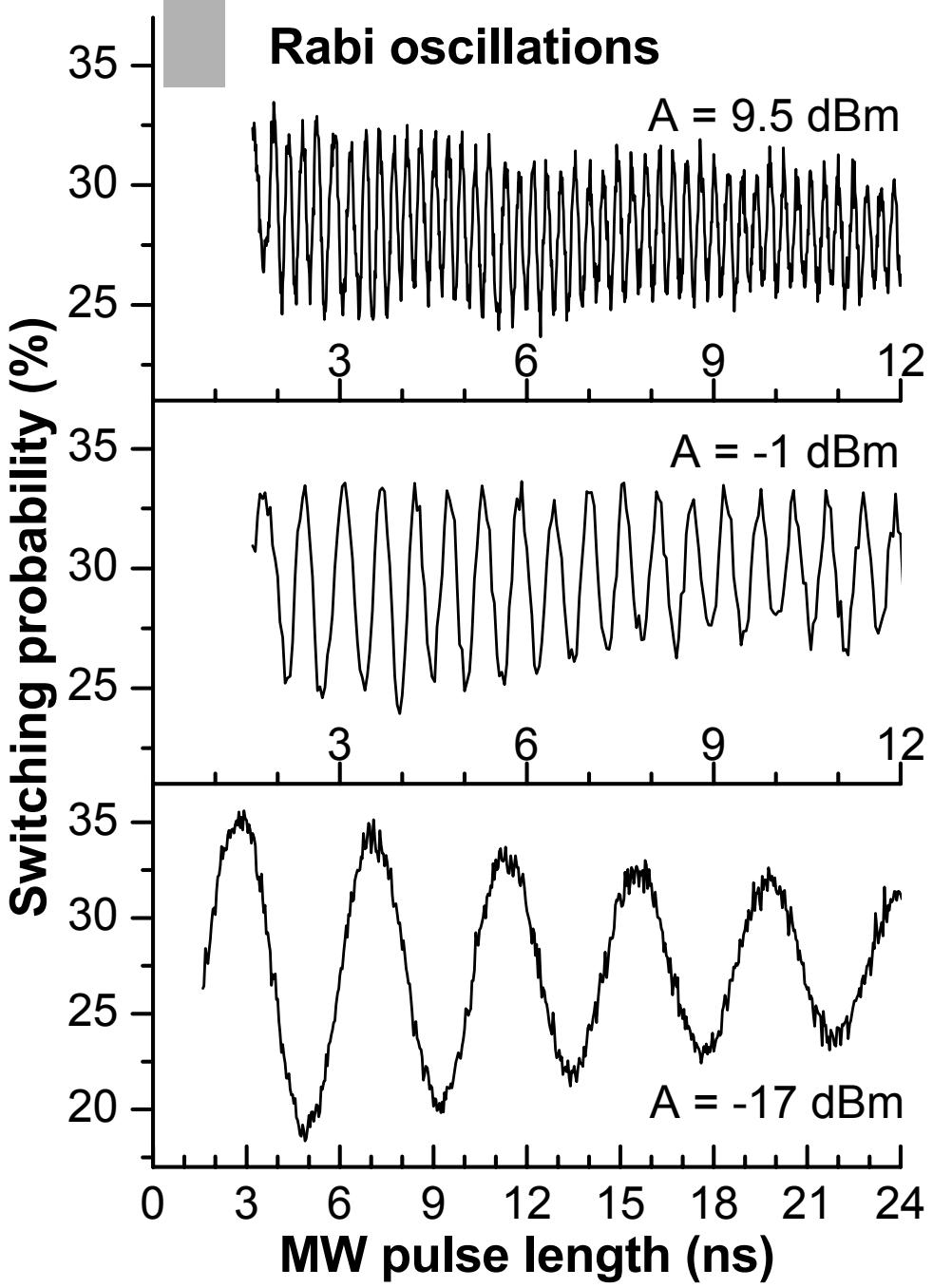
gap 5.866 GHz  
 persistent current 272 nA



spectroscopy peaks  
 $Q$ : qubit 1,2,3 fotons  
 $3$ :  $\omega_p$  squid 2.91 GHz  
 $Q\pm 3$ : sidebands



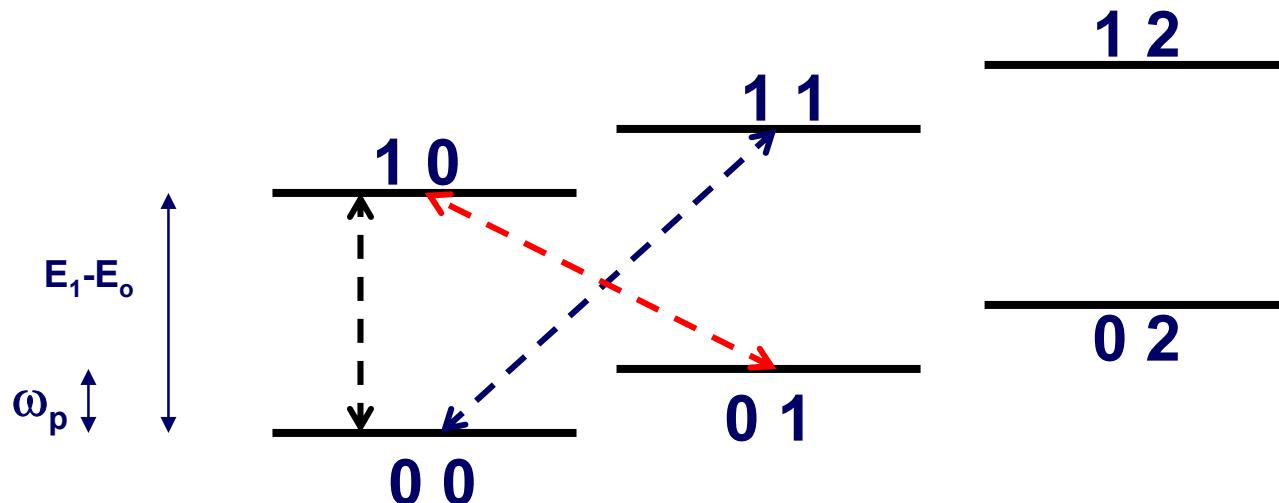


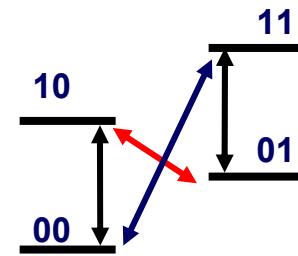
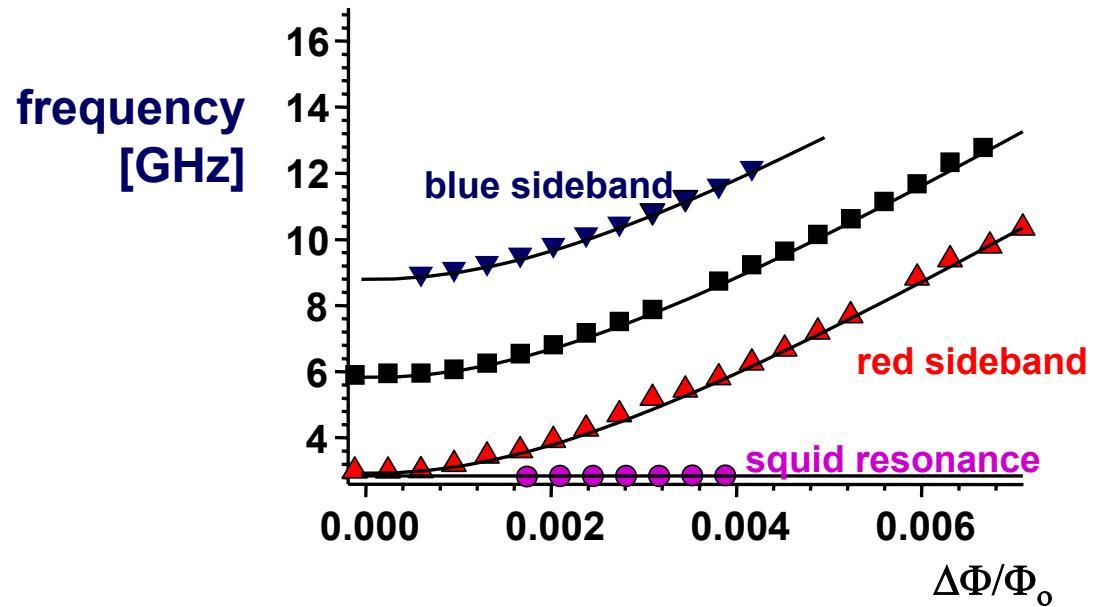


# flux qubit coupled to harmonic oscillator

harmonic oscillator: measurement SQUID shunted by large external capacitance; Q about 150

strong coupling accidental

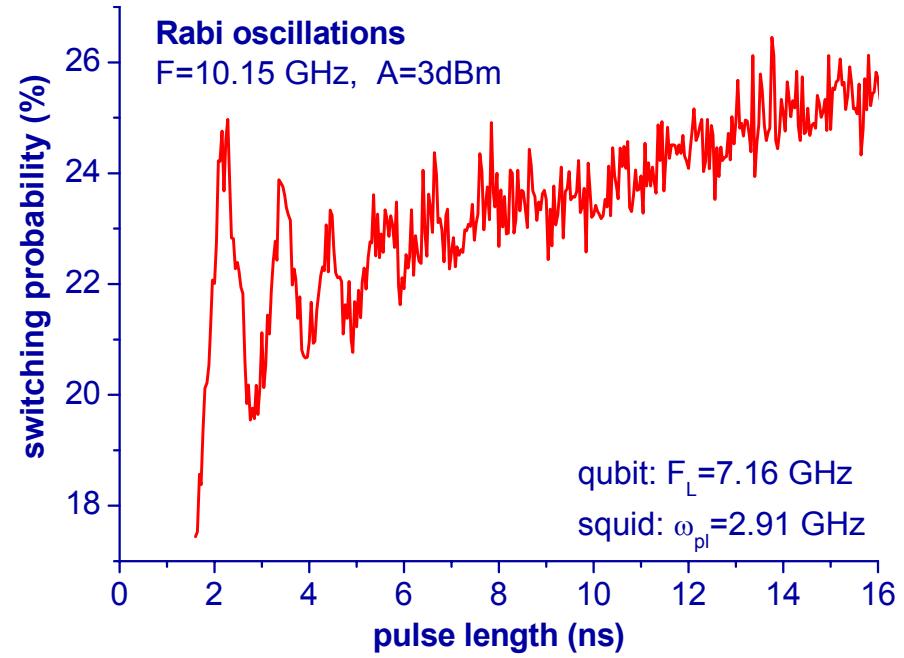
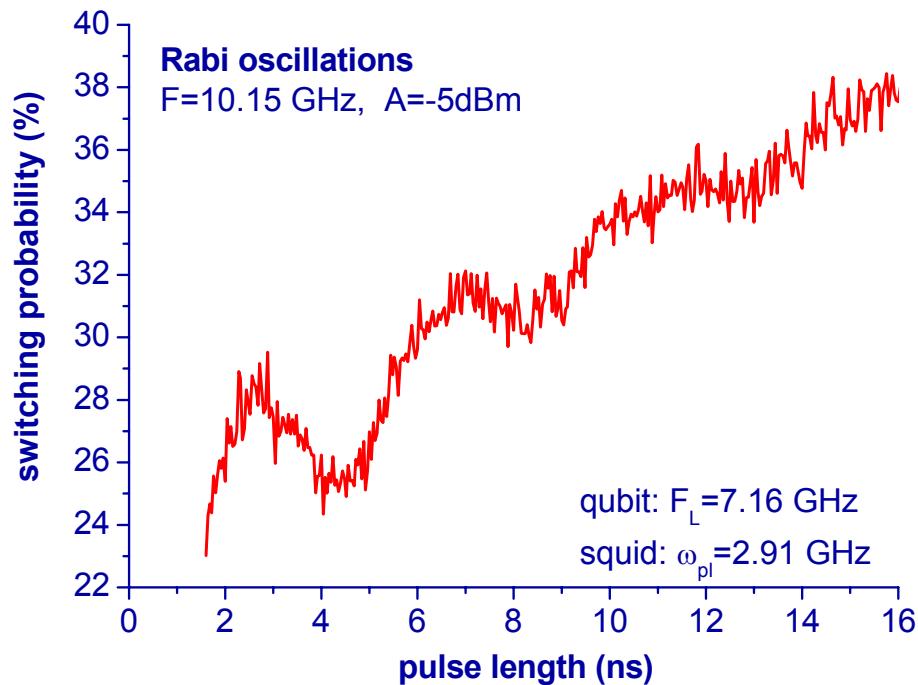


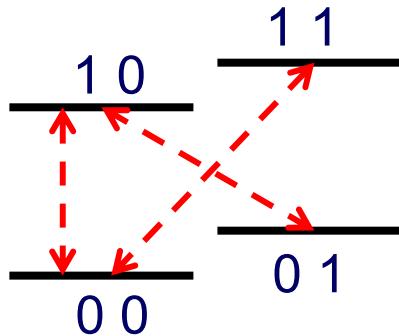
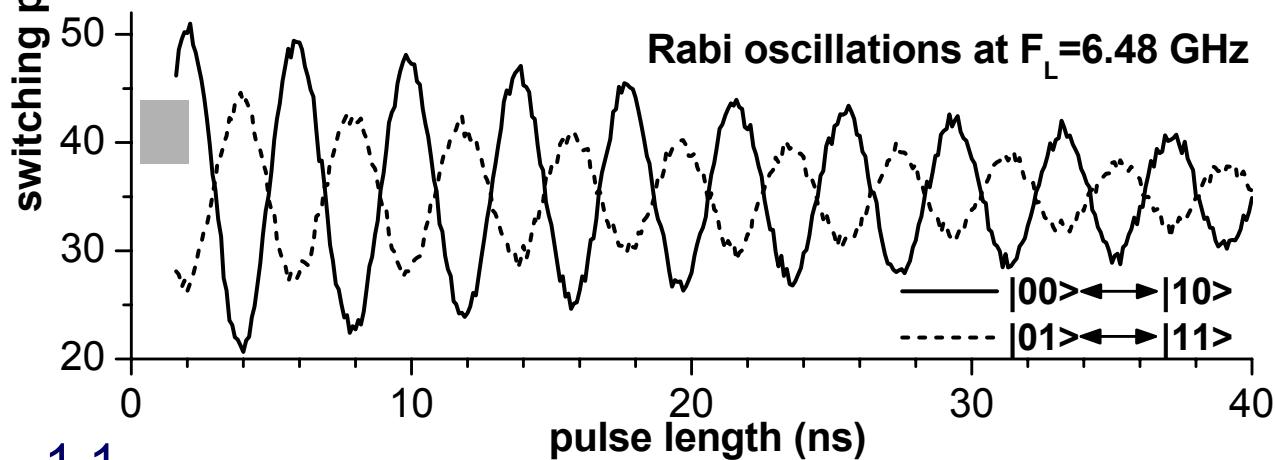
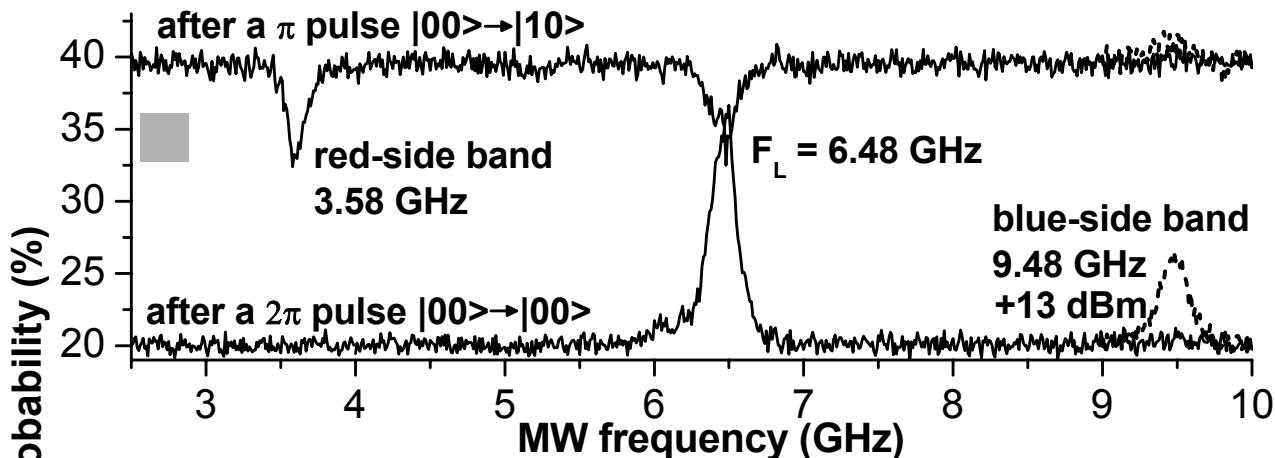


# coherent oscillations of the coupled system

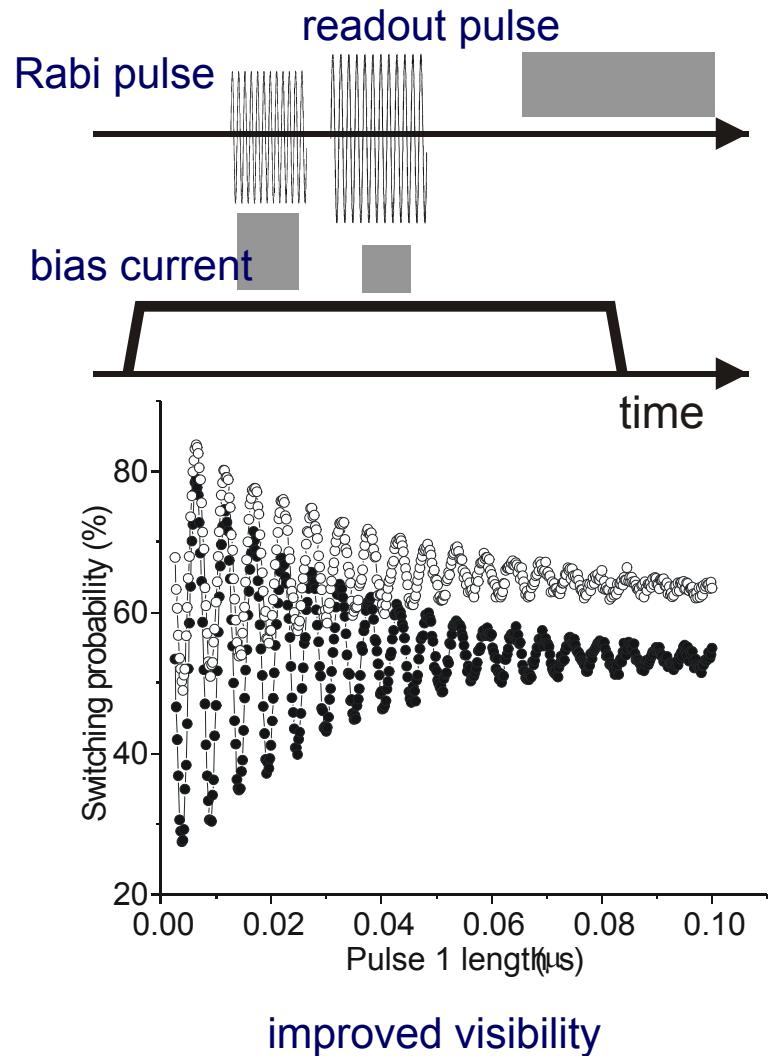
qubit Larmor frequency 7.16 GHz  
plasma frequency : 2.91 GHz

coupled system at 10.15 GHz (blue sideband)

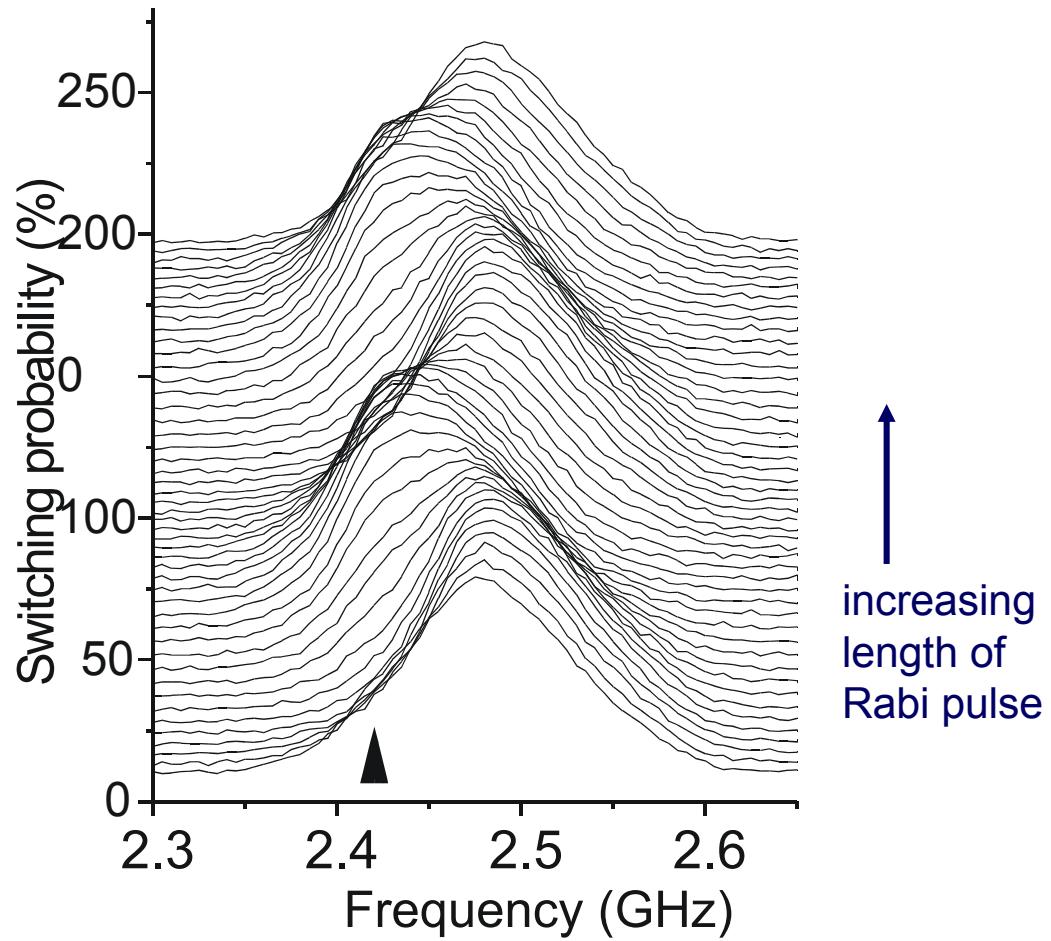




# SQUID readout with qubit-dependent resonant pulse

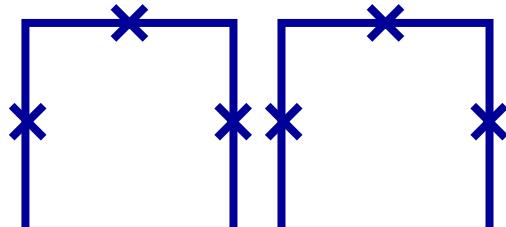


improved visibility

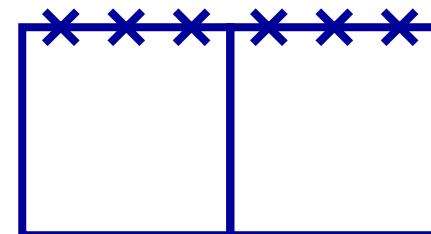


increasing  
length of  
Rabi pulse

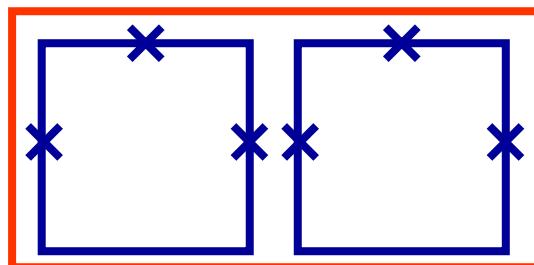
## coupling of qubits



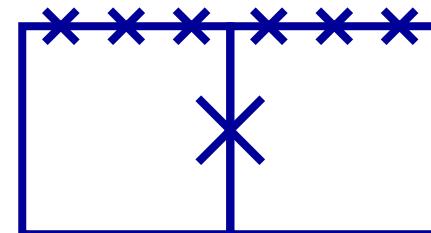
0.01 pH    4 MHz



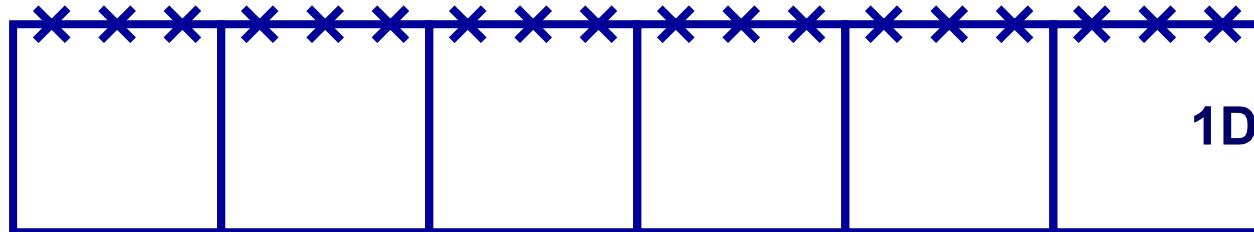
2 pH    0.8 GHz



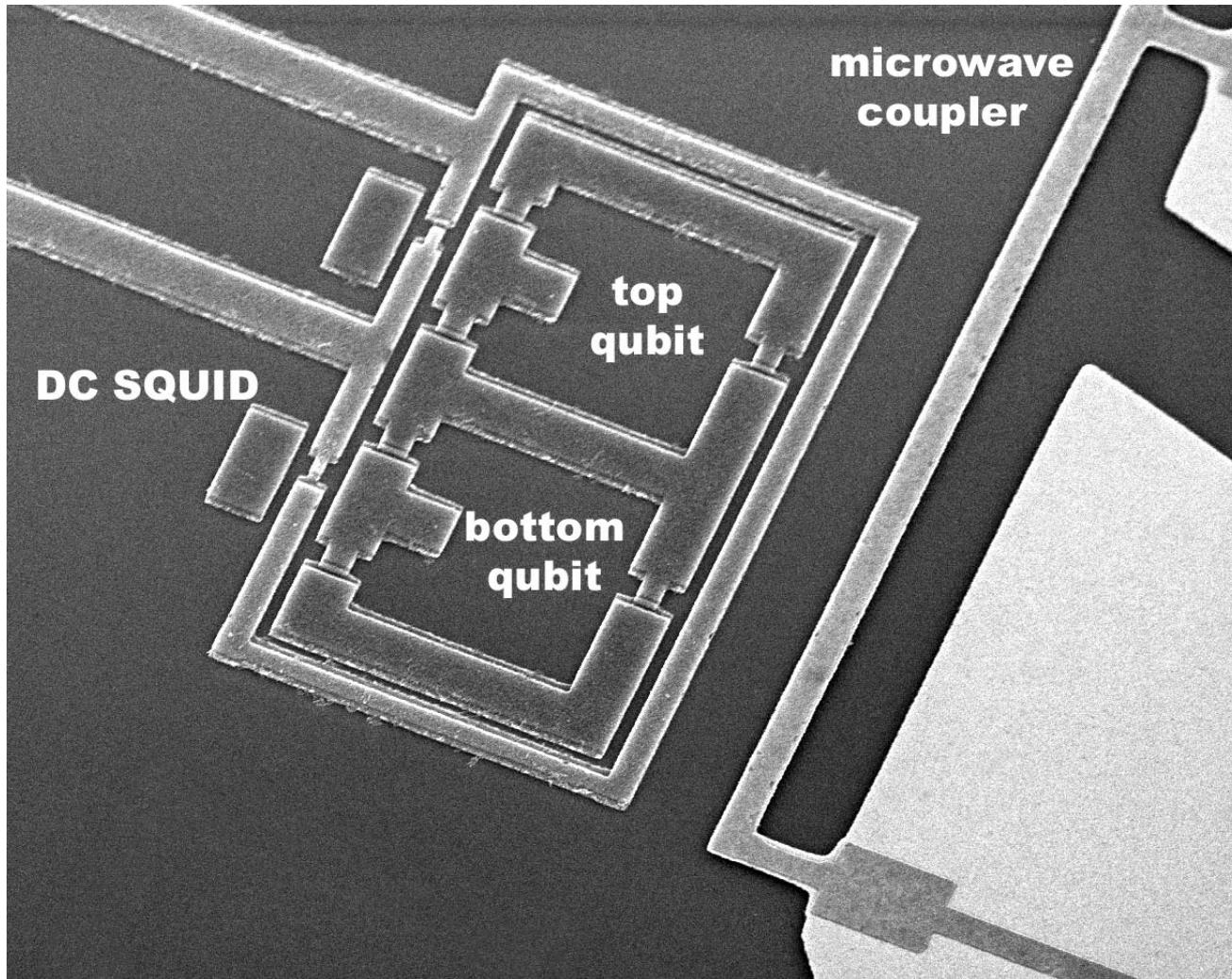
0.3 pH    0.1 GHz



10 pH    4 GHz



1D qubit chain

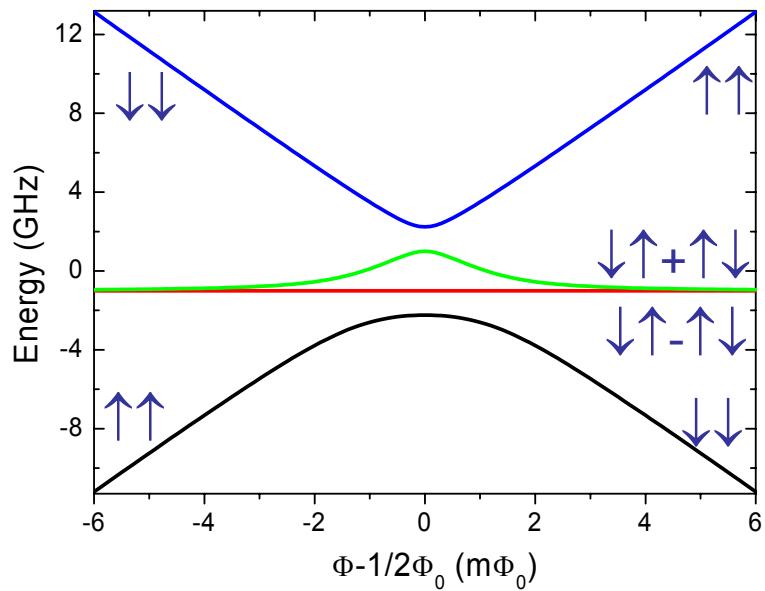


Hannes Majer  
Floor Pauw  
Alexander ter Haar

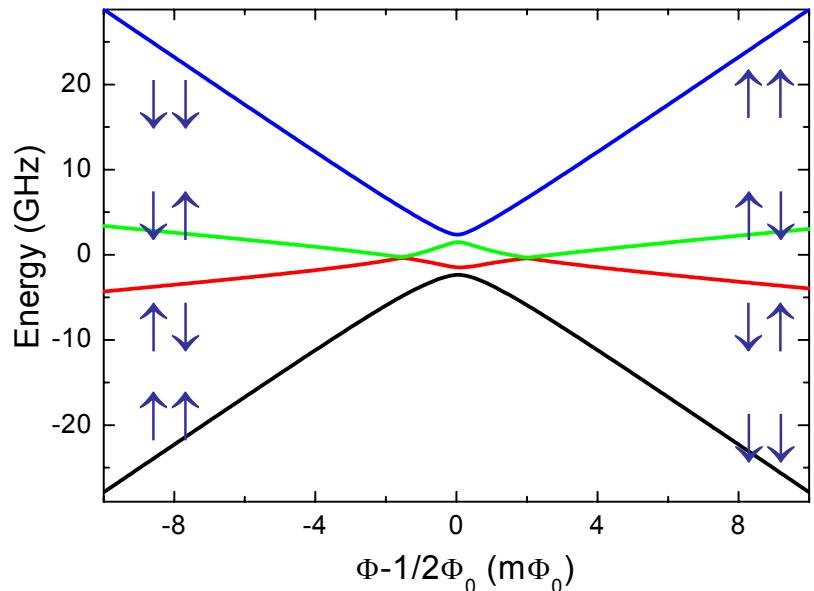
$$H = h_1\sigma_z^1 + t_1\sigma_x^1 + h_2\sigma_z^2 + t_2\sigma_x^2 + j\sigma_z^1\sigma_z^1$$

asymmetry in  $h$ :  $E_J$  and area (shift of  $f=0.5$ )

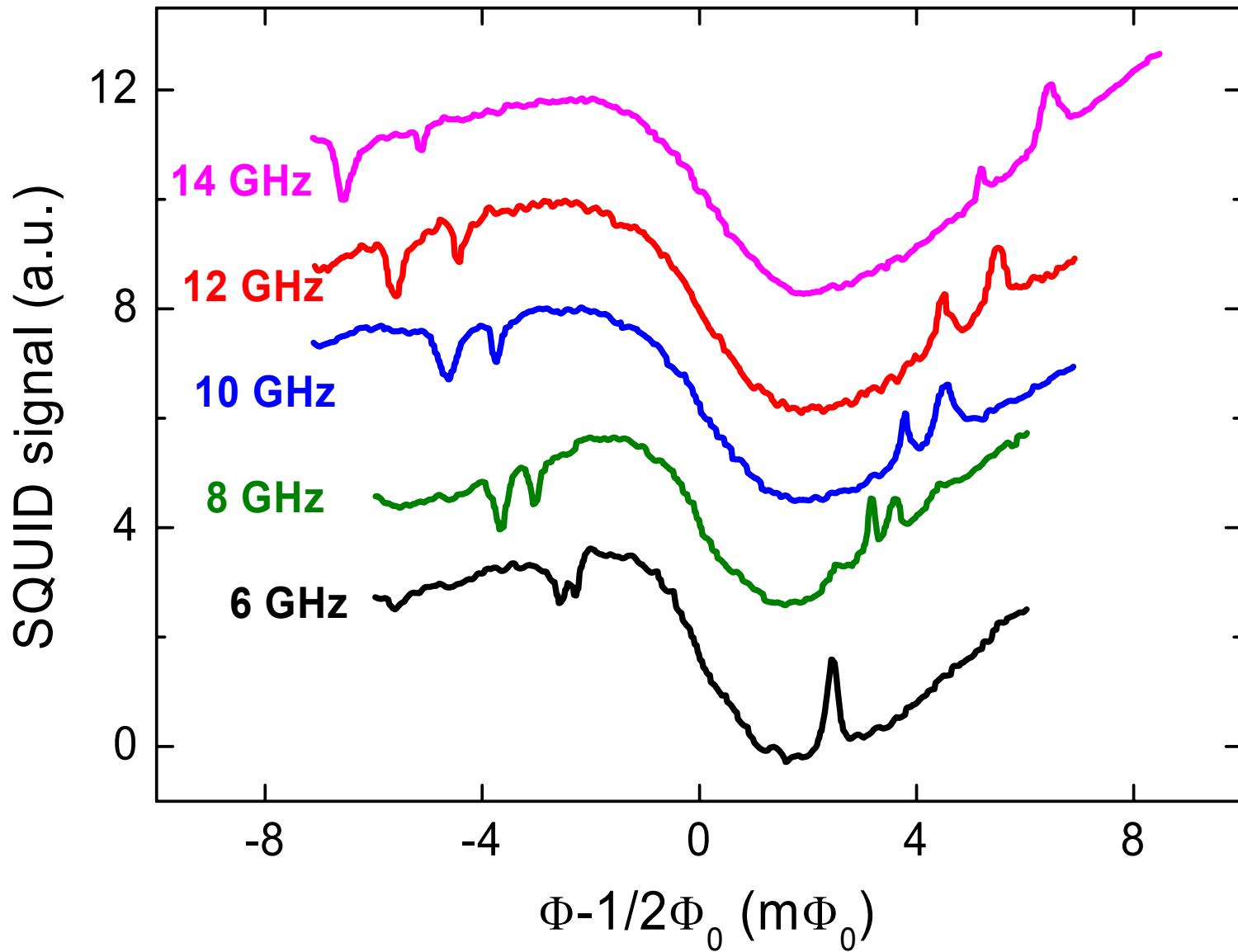
asymmetry  $t$ :  $E_J$ , junction ratio  $\alpha$

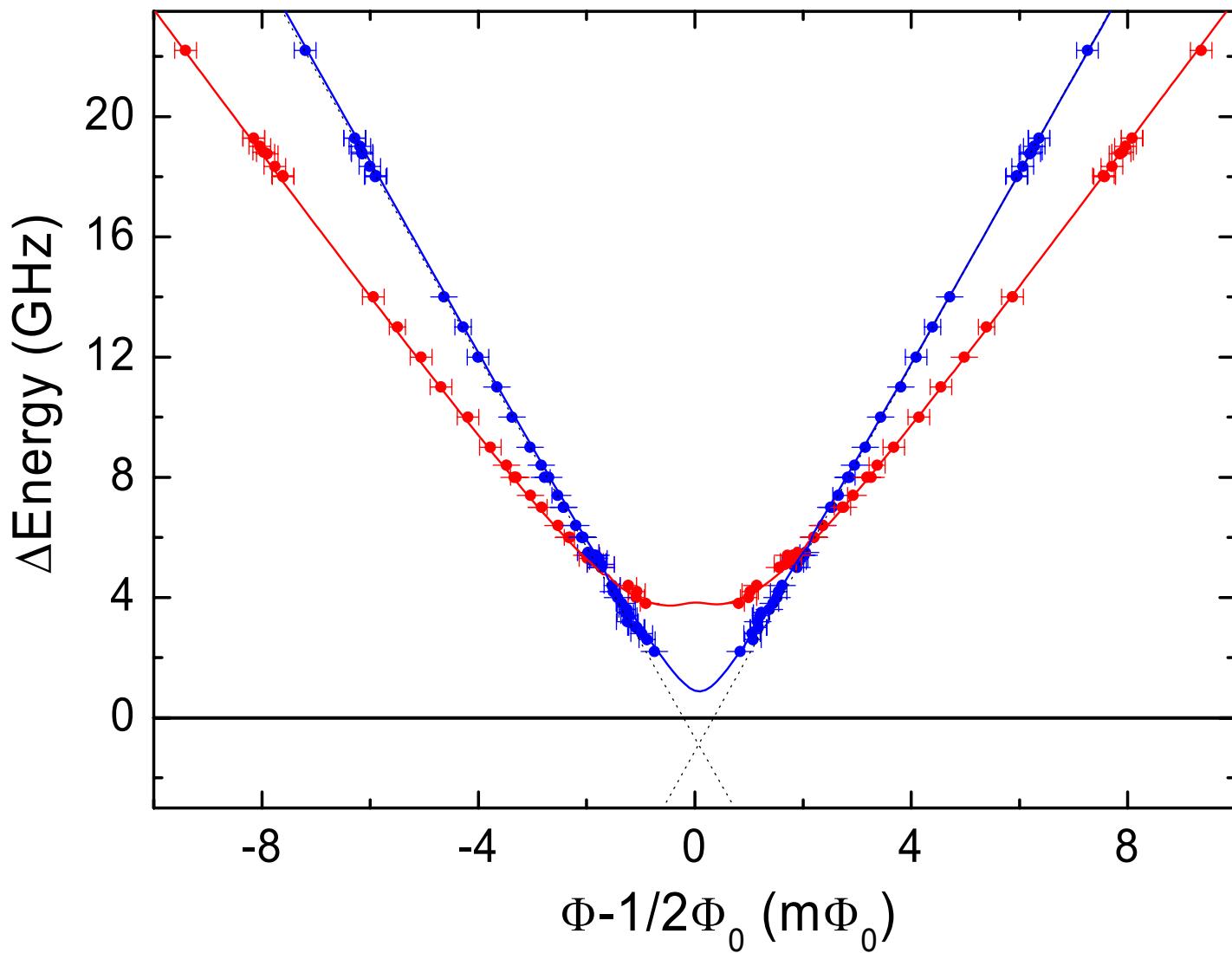


symmetric qubits



asymmetric qubits





## results of fit

$$I_{p1} = 512 \text{ nA} \pm 6 \text{nA}$$

$$t_1 = 0.45 \text{ GHz} \pm 0.2 \text{ GHz}$$

$$I_{p2} = 392 \text{nA} \pm 5 \text{nA}$$

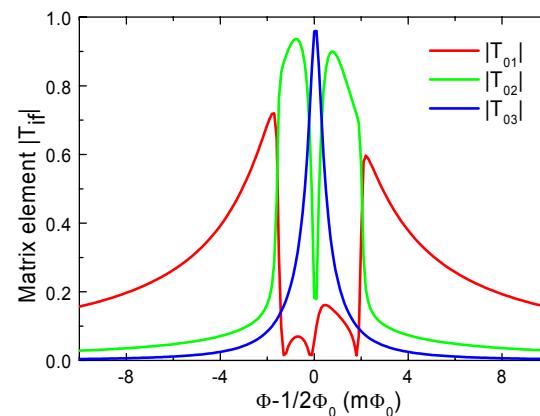
$$t_2 = 1.9 \text{ GHz} \pm 0.1 \text{ GHz}$$

**sigma = -0.027% ± 0.004% (difference areas)**

**j = 0.50 GHz ± 0.03 GHz (coupling strength)**

no transitions to 3rd excited state:

low transition probability



# conclusions

- flux qubit behaves quantum mechanically
- coherent driving possible (to a certain extent)
- decoherence partly due to circuit, partly to defects
- coherent driving observed of 2-level system
  - +harmonic oscillator

# Delft flux qubits

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Irinel Chiorescu  
Adrian Lupascu  
Floor Pauw  
Patrice Bertet  
Jelle Plantenga  
Jonathan Eroms  
Yasunobu Nakamura (NEC)  
Kouichi Semba (NTT)



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ARO, Kavli

