Photon localisation in cold atomic gases

Cord Müller

UNIVERSITÄT BAYREUTH

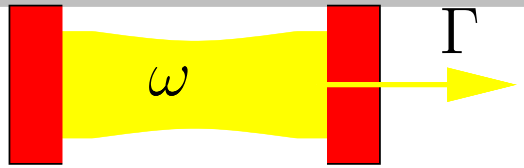
I. Cold atoms as light scatterers

II. Coherent photon transport theory

III. Experimental signatures

Invitation: five ways to trap a photon

- Cavity with quality factor $Q = \frac{\omega}{\Gamma} \gg 1$



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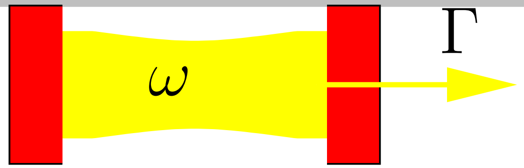

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$h_{\lambda\mu}$

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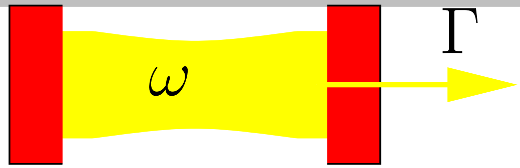

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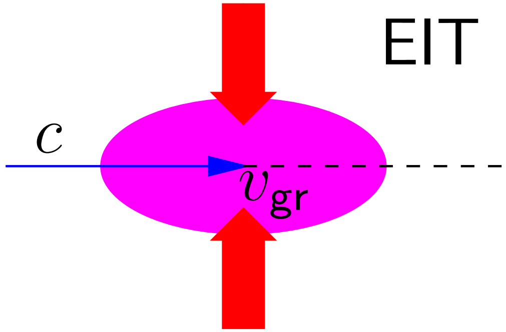

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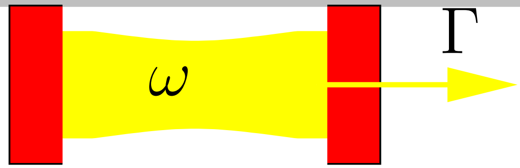

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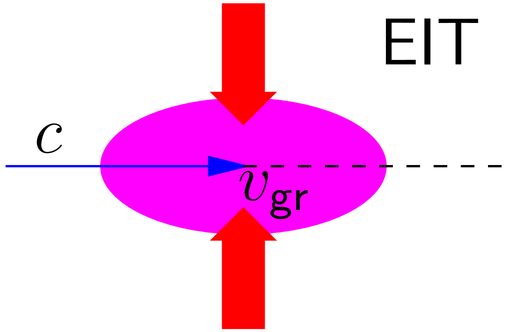

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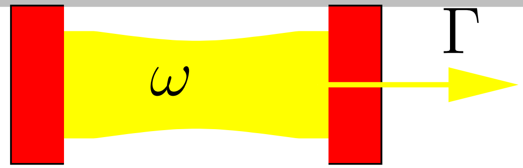

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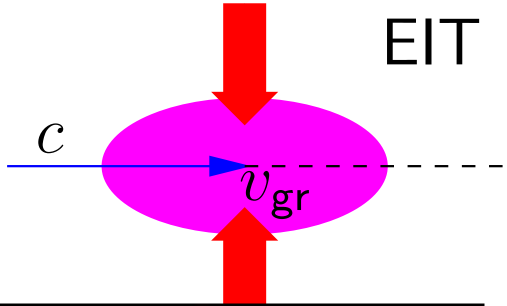

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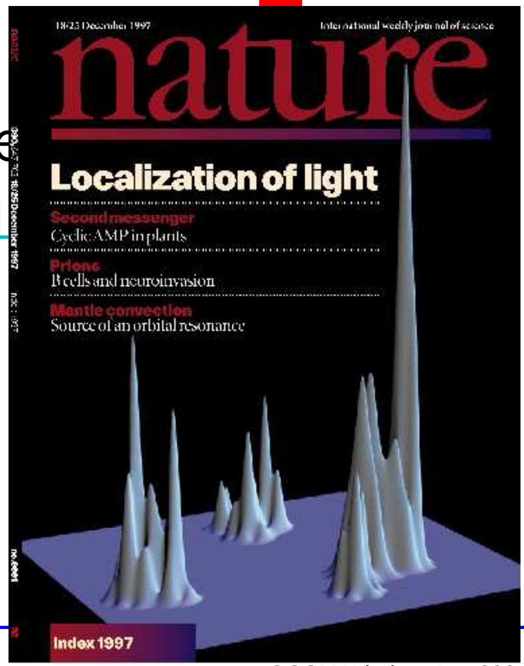
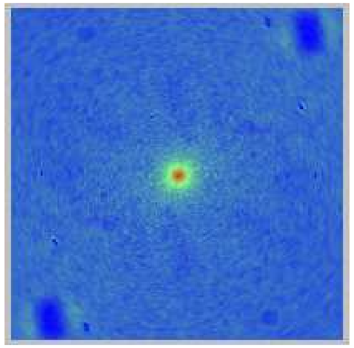
- Optics with colloidal suspensions [P.W. Anderson: "theory of white paint", 1985] or semiconductor powder [D. Wiersma et al., Nature (1997)]



Why light and cold atoms?

Propaganda response:

- Light scattering by cold atoms:
  - excellent laser coherence & polarisation control
  - atoms are identical resonant point scatterers
  - photons (almost) don't interact



Coherent backscattering (CBS) by Sr

[Bidel et al., PRL **88**, 203902 (2002)]

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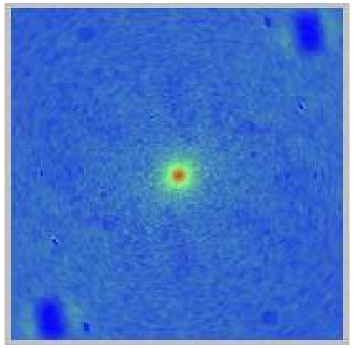

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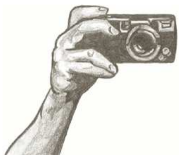
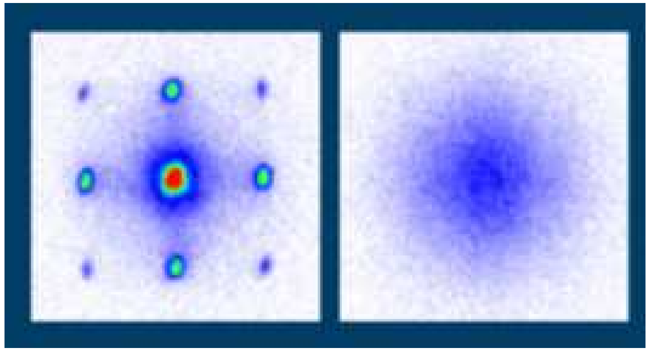
- Matter waves in a light potential:
  - tunable potential
  - controlled interaction
  - direct observation



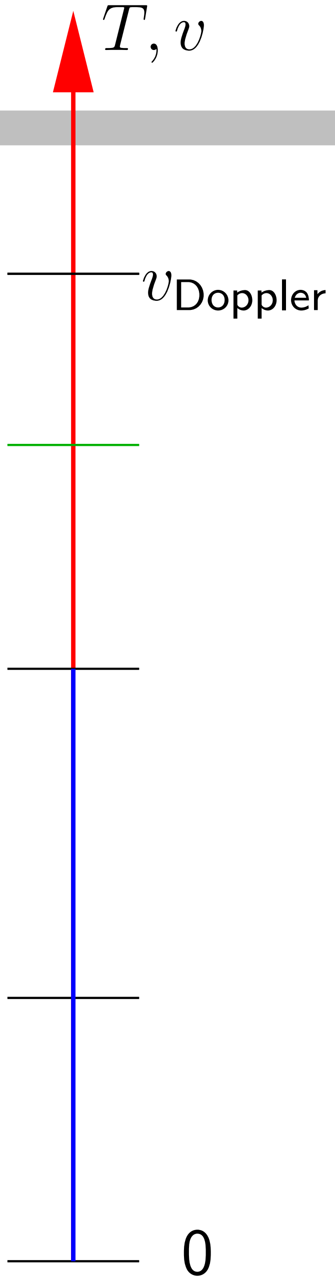


Mott-Hubbard transition with Rb BEC
[Greiner et al., Nature **415**, 39 (2002)]

I. Cold atoms as light scatterers

A. External degrees of freedom:

- Want quenched disorder — need slow atoms:
  Doppler effect negligible if
  $v_{\mathsf{rms}} < v_{\mathsf{Doppler}} = \Gamma/k \sim 10\,\mathrm{m/s}$



$h_{\lambda\mu}$

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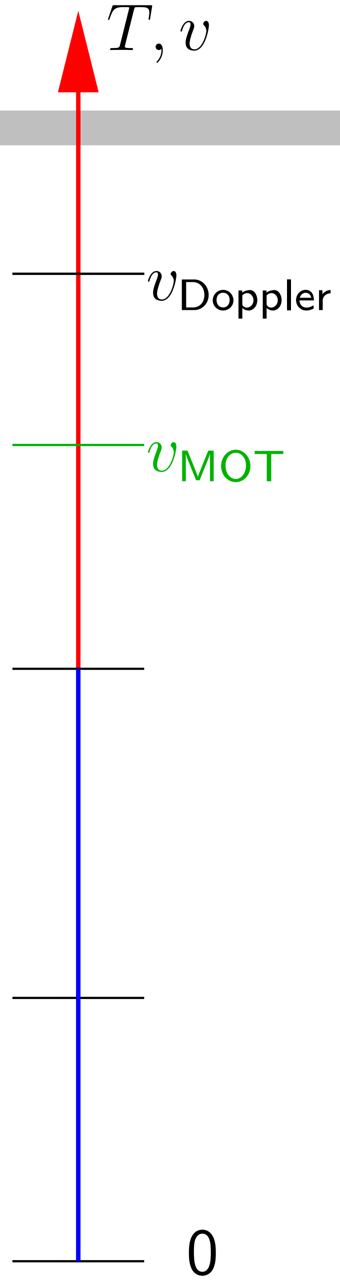

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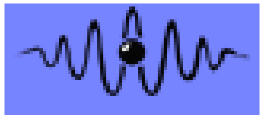

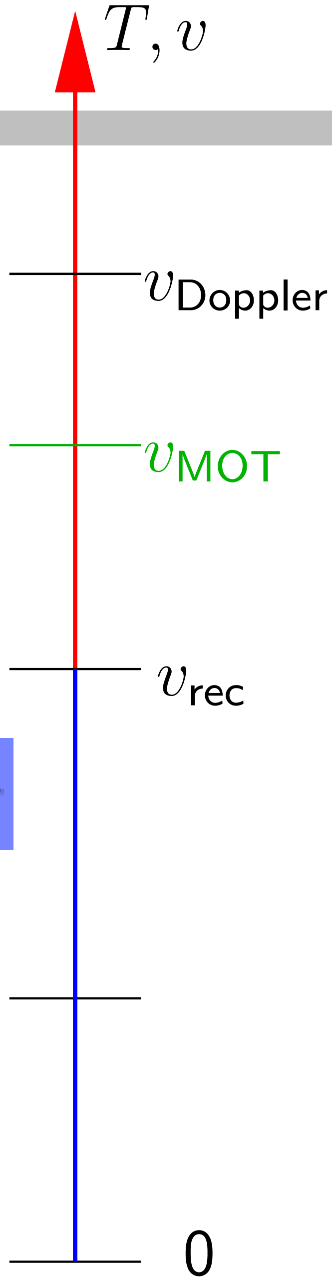

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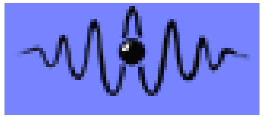

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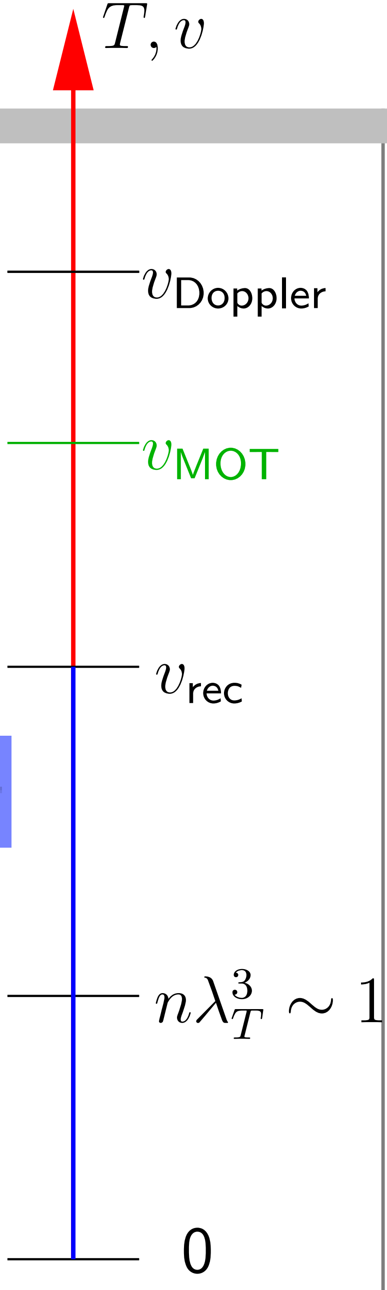

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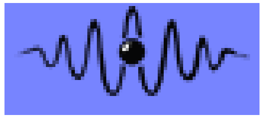

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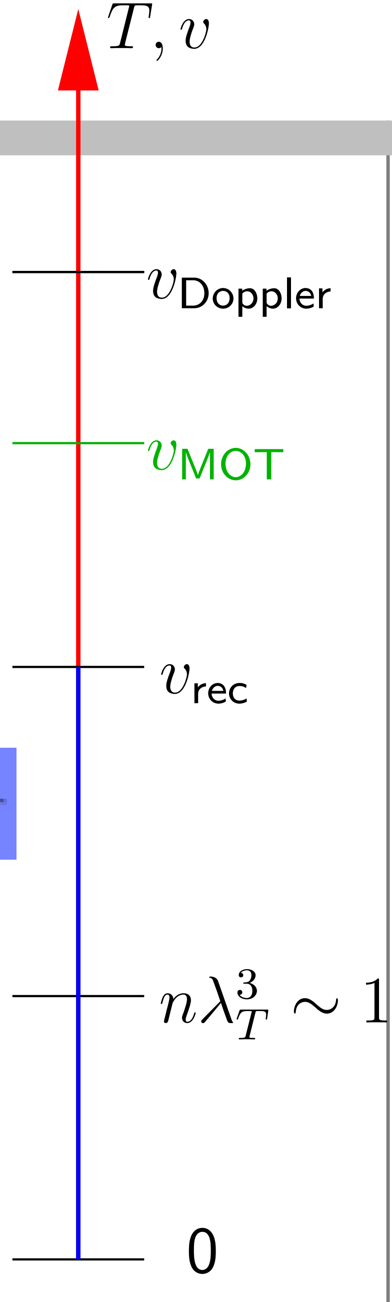
Around v_{MOT}: dilute sample of fixed classical point scatterers in $d = 3$.



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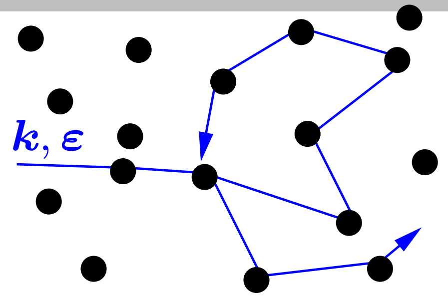
B. Internal degrees of freedom:

- Identical point scatterers with huge cross-section: $a_0 \ll \sqrt{\sigma_{\text{tot}}} \sim \lambda$



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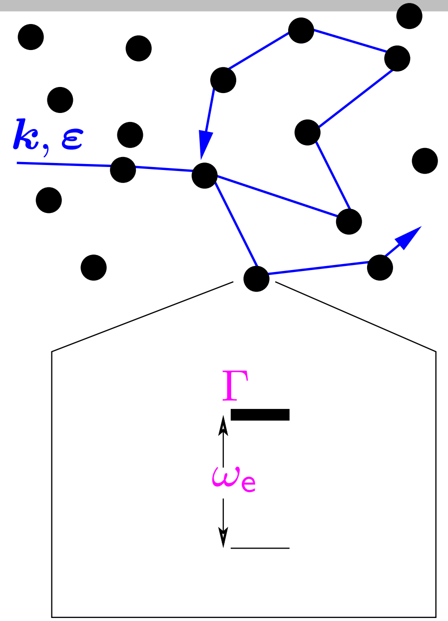

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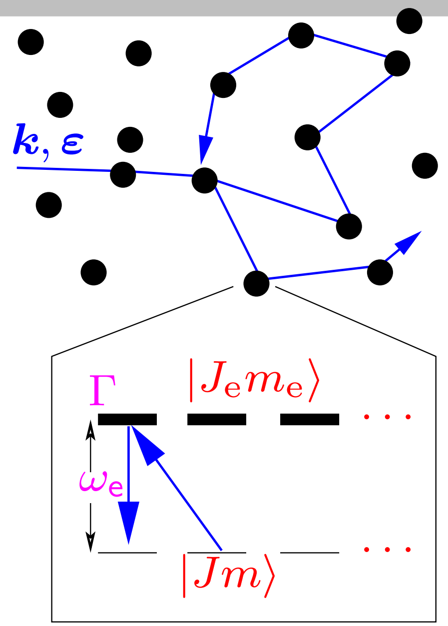

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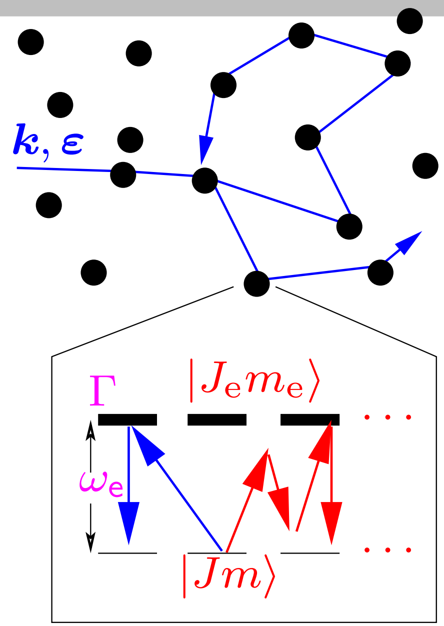

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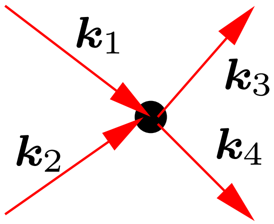

[2-photon scattering: T. Wellens et al., quant-ph/0403068]

[Master eq. approach: V. Shatokhin, CAM, A. Buchleitner]

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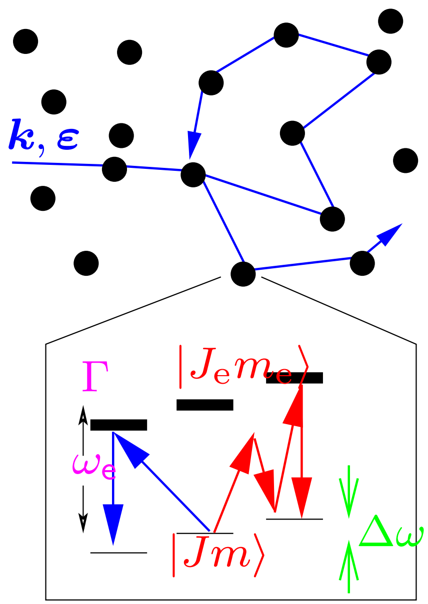

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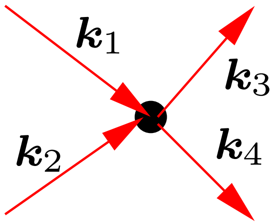

- 'Giant' magnetoactivity $B_\Gamma \sim 10^{-4}$ T
  [PhD by O. Sigwarth, poster 2.24]

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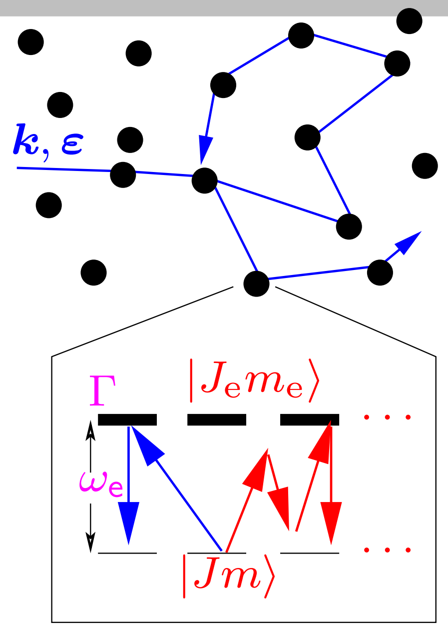

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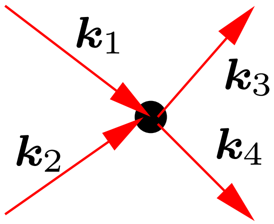

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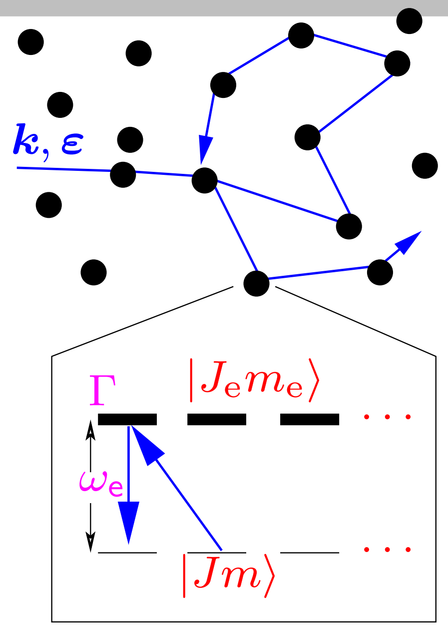

$$\omega_e/\Gamma \sim 10^8$$

$\longrightarrow$ affects transport time scale $\tau_{\rm tr}$

- Internal degeneracy $J > 0$
$\longrightarrow$ couples to photon polarisation $\boldsymbol{\varepsilon}$

$h_{\lambda\mu}$

II. Coherent photon transport theory

$h_{\lambda\mu}$

Theory of everything?

$h_{\lambda\mu}$

Theory of really everything

- Matter-light Hamiltonian with dipole interaction ($\hbar = c = 1$):

$$H_{\text{phot}} = \sum_{\boldsymbol{k},\boldsymbol{\varepsilon}\perp\boldsymbol{k}} k \; a^{\dagger}_{\boldsymbol{k}\boldsymbol{\varepsilon}} a_{\boldsymbol{k}\boldsymbol{\varepsilon}}, \qquad \boldsymbol{\varepsilon}\cdot\boldsymbol{k} = 0 \quad \text{transverse}$$

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- Fixed classical scatterers: focus onto internal quantum degrees of freedom

$h_{\lambda\mu}$

Microscopic photon scattering theory

- Diagrammatic single-particle transport theory:
  [Vollhardt & Wölfle, PRB (1980), v. Rossum & Nieuwenhuizen, RMP (1999)]

  Calculate $\langle G^{\mathsf{R}} \rangle$, $\langle G^{\mathsf{A}} G^{\mathsf{R}} \rangle$, ... for dilute medium $n\lambda^3 \ll 1$.

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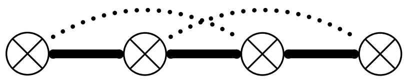
$\langle G \rangle$: Photon self-energy $\Sigma(\omega) = \otimes +$  $+ \dots$
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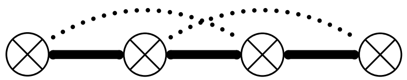
$\langle G \rangle$: Photon self-energy $\Sigma(\omega) = \otimes +$  $+ \dots$
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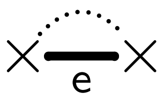
- Resonant scalar t-matrix of each atom

$$\otimes = \text{}$$  $$= \frac{g^2}{\omega - \omega_{\mathsf{e}} + \mathrm{i}\Gamma/2}\delta_{ij}$$

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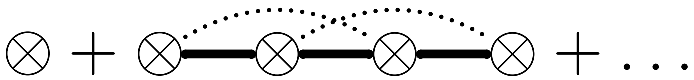
$\langle G \rangle$: Photon self-energy $\Sigma(\omega) =$ 
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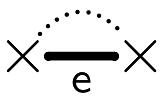
$$\otimes = $$  $$= \frac{g^2}{\omega - \omega_{\mathsf{e}} + \mathrm{i}\Gamma/2} \delta_{ij}$$

- Continuity equation for density $\Phi(\boldsymbol{q}, \Omega) = \sum_{\boldsymbol{k}\boldsymbol{k}'} \langle G^{\mathsf{A}} G^{\mathsf{R}} \rangle$

$$-\mathrm{i}\tau_{\mathsf{tr}} \Omega \Phi + \mathrm{i}\ell \boldsymbol{q} \cdot \boldsymbol{J} = 2\pi \ell \rho(\omega)$$

defines transport time $\tau_{\mathsf{tr}} = \ell/c + 1/\Gamma$

Microscopic photon scattering theory

- Diagrammatic single-particle transport theory:
  [Vollhardt & Wölfle, PRB (1980), v. Rossum & Nieuwenhuizen, RMP (1999)]

  Calculate $\langle G^{\mathsf{R}} \rangle$, $\langle G^{\mathsf{A}} G^{\mathsf{R}} \rangle$, ... for dilute medium $n\lambda^3 \ll 1$.

$\langle G \rangle$: Photon self-energy $\Sigma(\omega) = \otimes + \otimes\!-\!\otimes\!-\!\otimes\!-\!\otimes + \ldots$
defines elastic mean free path $\ell(\omega)$.

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$$\otimes = \times \overset{\cdots}{\underset{\mathsf{e}}{\textbf{—}}} \times = \frac{g^2}{\omega - \omega_{\mathsf{e}} + \mathrm{i}\Gamma/2} \delta_{ij}$$

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Ward identi-ology

- Resonance correction to transport time is $O(\Omega)$-residue in Ward identity [v. Rossum & Nieuwenhuizen, RMP (1999)]

$h_{\lambda\mu}$

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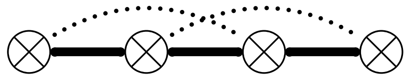
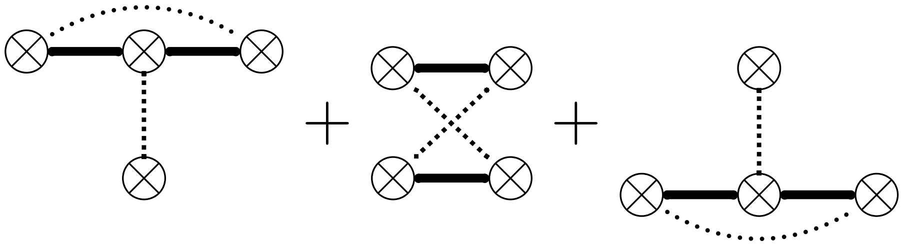


$$\mapsto U^{(2,\mathsf{I})} = \quad + \quad +$$



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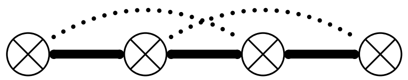
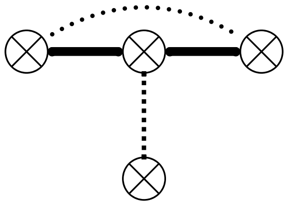
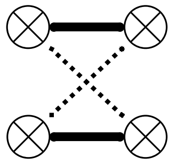
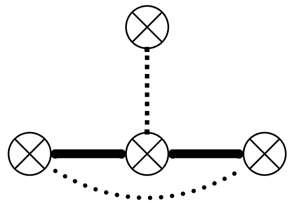
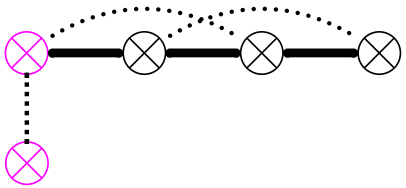
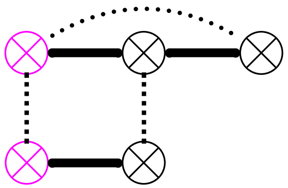
$\mathrm{Im}\,\Sigma^{(2)} = \mathrm{Im}\,($  $)$

$\mapsto U^{(2,\mathrm{I})} =$  $+$  $+$ 

- Resonance generates additional vertices:

$U^{(2,\mathrm{II})} =$  $+$  $+\, c.c.$

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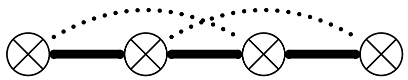
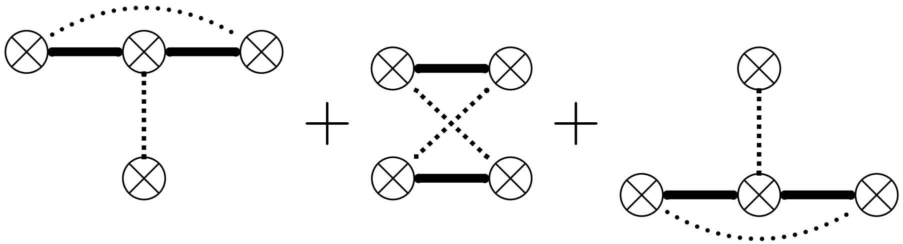


$$\mapsto U^{(2,\mathrm{I})} =$$



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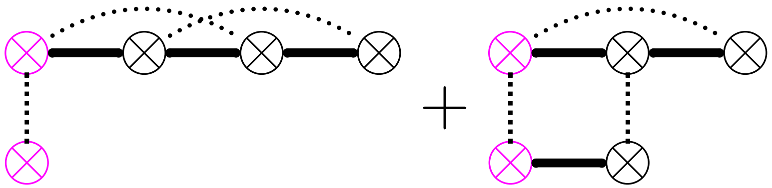

- "Optical theorem": $\mathrm{Im}\,\Sigma^{(1)} = \mathrm{Im}\,\otimes \quad \mapsto \quad U^{(1)} =$

$h_{\lambda\mu}$

III. Experimental signatures

Part 1: **resonant radiation trapping**

$h_{\lambda\mu}$

Radiation trapping

- Resonance-dominated transport: $\tau_{\mathsf{tr}} \approx \tau_{\mathsf{nat}} = \Gamma^{-1} \gg \ell/c$

typical values: $\ell \approx 10^{-4}\mathrm{m}$, $\tau_{\mathsf{nat}} \approx 30\,\mathrm{ns}$

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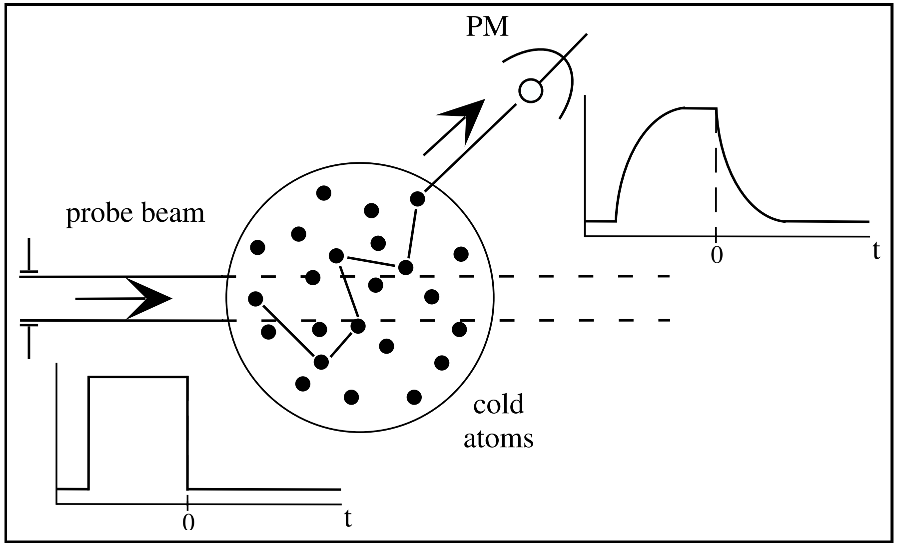
- Experiment @ Nice:

[Labeyrie et al. PRL (2003)]



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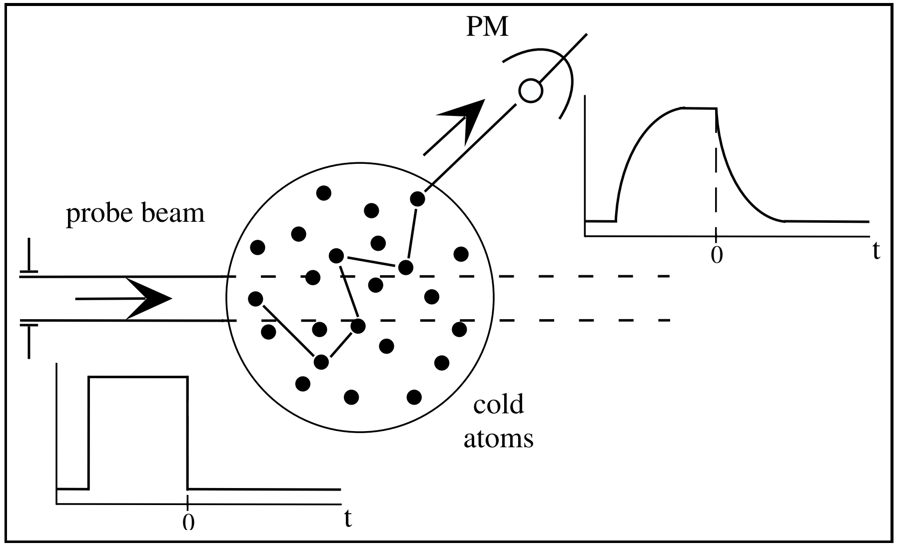
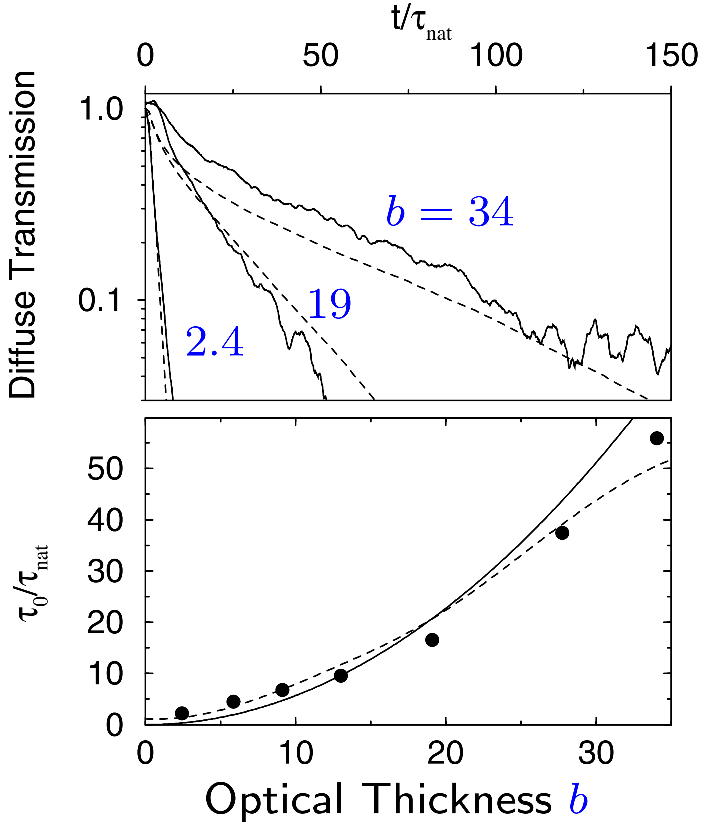


Diffusion $D = \frac{\ell^2}{3\tau_{\mathrm{tr}}} \sim \frac{b^2}{\tau_0}$



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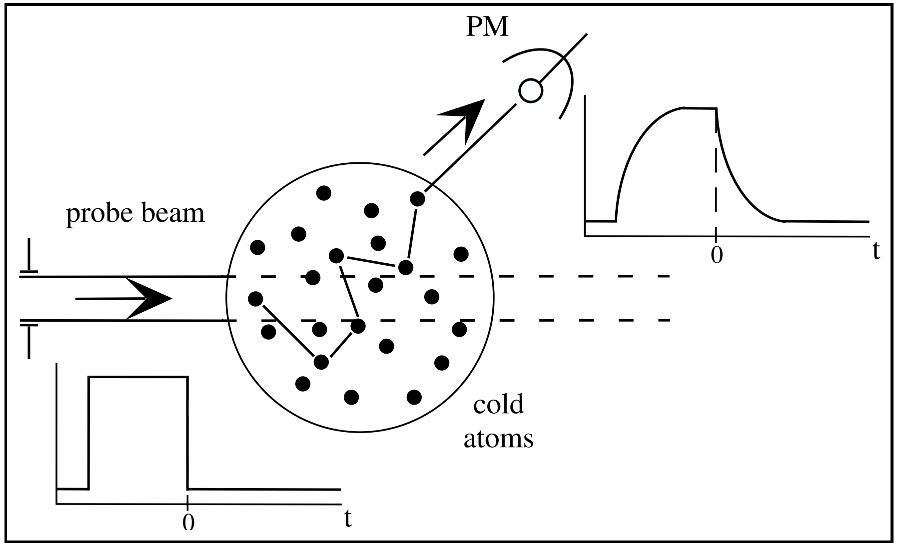

Diffusion $D = \frac{\ell^2}{3\tau_{\text{tr}}} \sim \frac{b^2}{\tau_0} \approx 1\,\text{m}^2/\text{s}$

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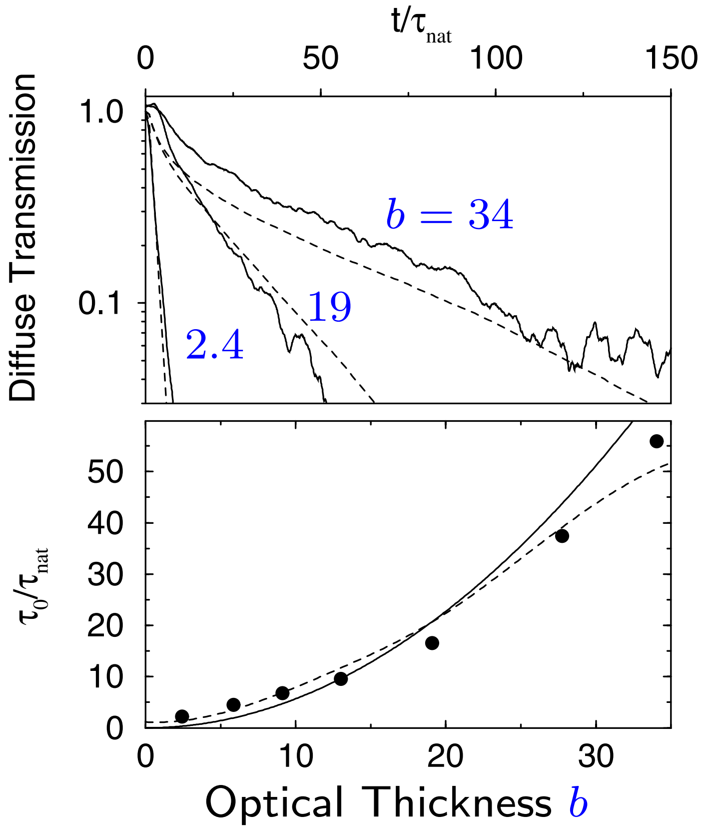

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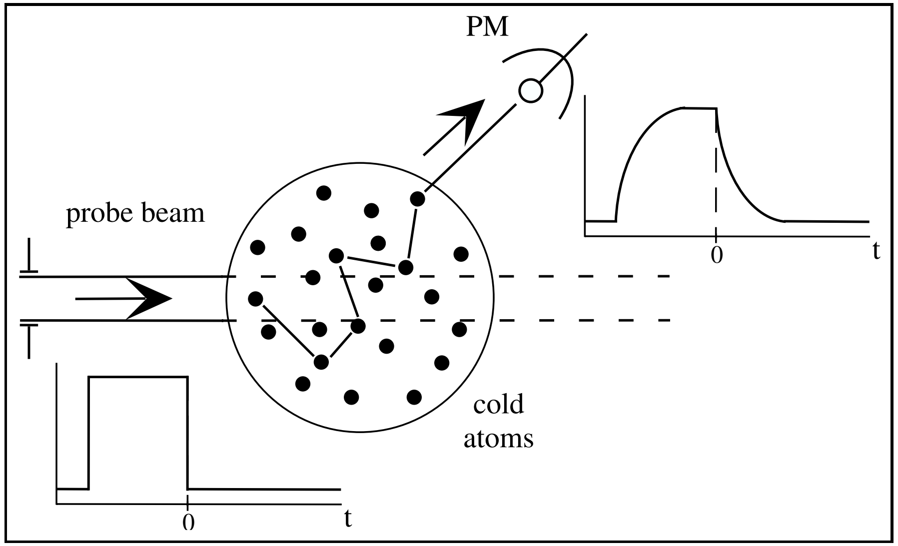

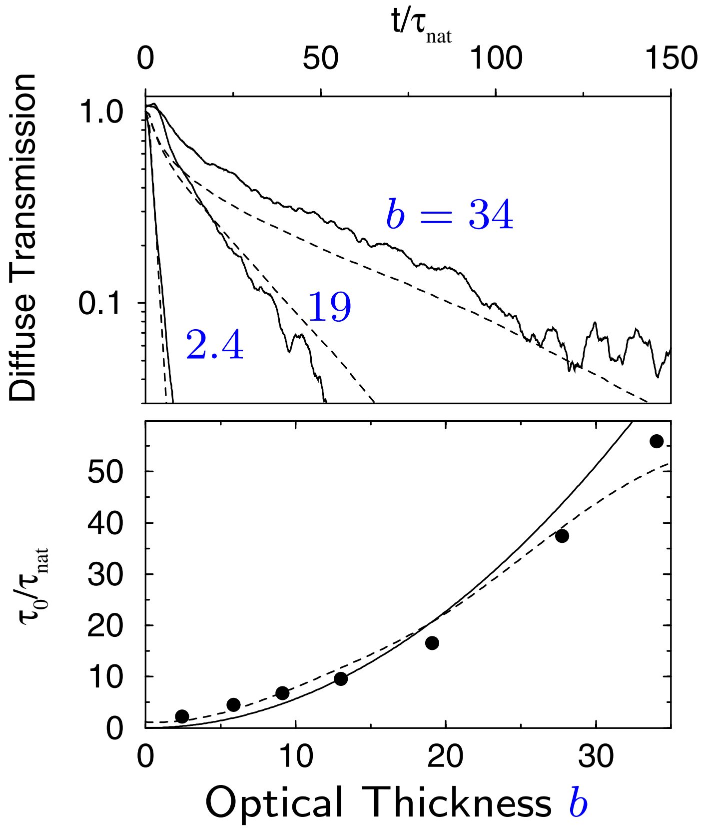

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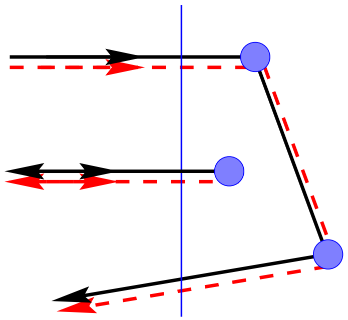
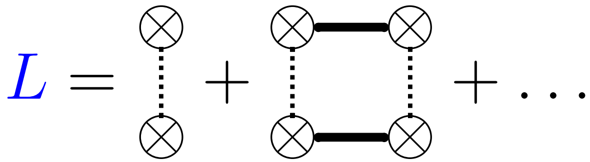
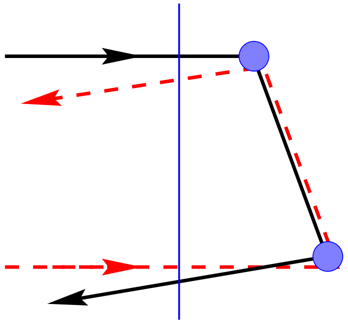
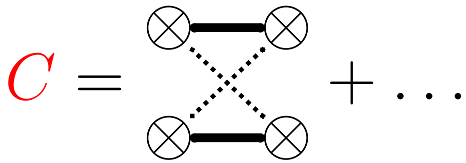
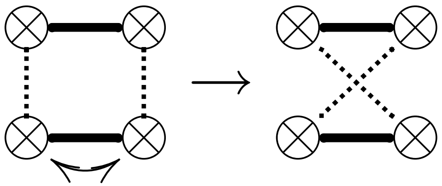
- Time scale is set. What about interference?

Weak localisation: strategy of calculation

- Interpretation of diagrams:

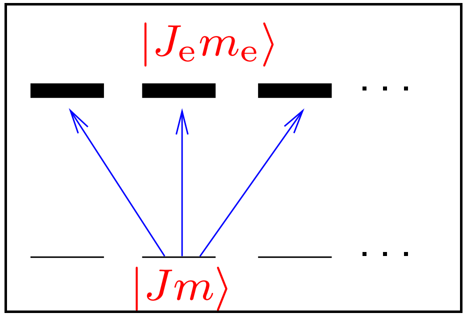
Real space	Reciprocal space
	Ladder diagrams 
	Crossed diagrams 

- 1. sum geometrical series for L
  2. use reciprocity trick to obtain C:



Impact of internal degeneracy

- A *degenerate* dipole transition ($J > 0$) has an internal *spin* degree of freedom coupling to the photon polarisation ε.



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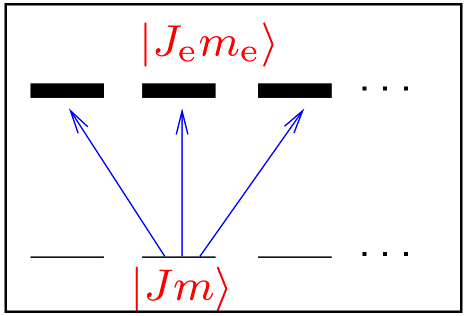
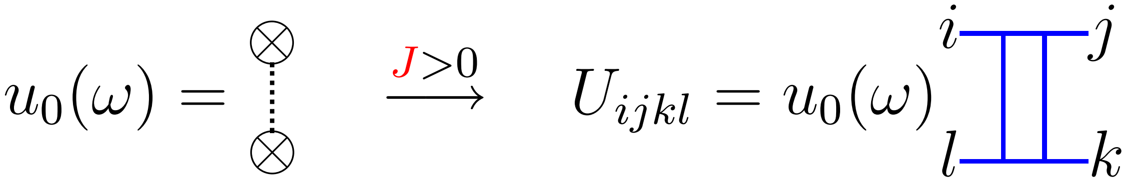


- Photon scattering vertex acquires topology of a ribbon:



$h_{\lambda\mu}$

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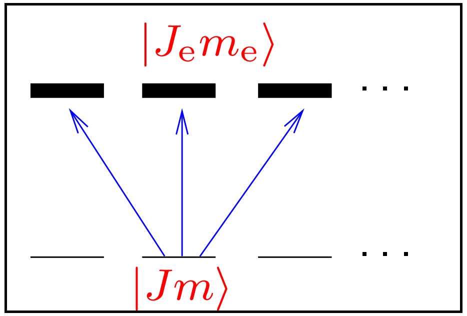

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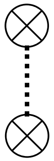
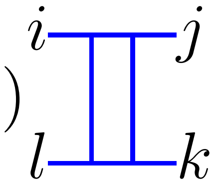
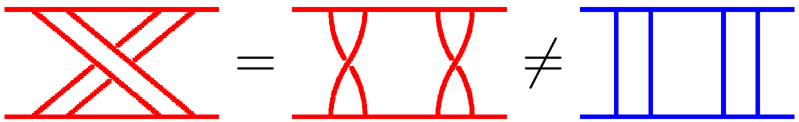
$u_0(\omega) =$  $\xrightarrow{J>0}$ $U_{ijkl} = u_0(\omega)$ 

- Equivalence between crossed and ladder diagrams lost:



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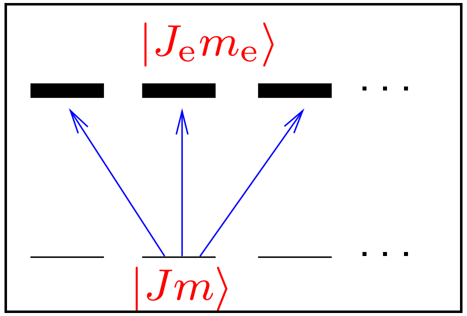

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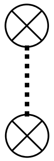
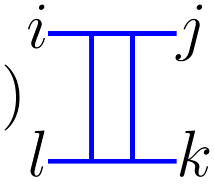
$u_0(\omega) =$  $\xrightarrow{J>0}$ $U_{ijkl} = u_0(\omega)$ 

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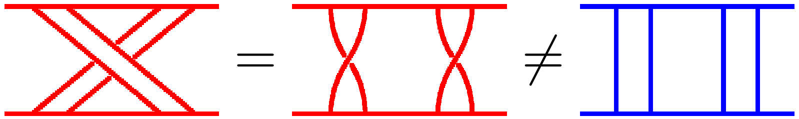
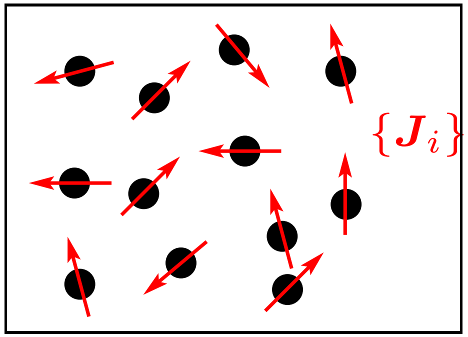


- Absence of time-reversal symmetry:

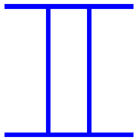
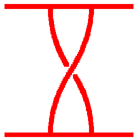
[Jonckheere et al., PRL (2000)]



Vertex eigenvalues

- Diagonalisation into irreducible components $K = 0, 1, 2$:

[CAM & C. Miniatura, J.Phys.A (2002)]

 $= \sum_K \lambda_K \mathsf{T}_K,$  $= \sum_K \chi_K \mathsf{T}_K$

$$\lambda_K = 3(2J_\mathrm{e}+1)\begin{Bmatrix} 1 & 1 & K \\ J_\mathrm{e} & J_\mathrm{e} & J \end{Bmatrix}^2, \quad \chi_K = 3(2J_\mathrm{e}+1)\begin{Bmatrix} 1 & J_\mathrm{e} & J \\ 1 & J & J_\mathrm{e} \\ K & 1 & 1 \end{Bmatrix}$$

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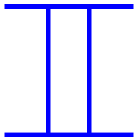
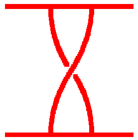
 $= \sum_K \lambda_K \mathsf{T}_K,$  $= \sum_K \chi_K \mathsf{T}_K$

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- Selection rules:

(i) $\lambda_0 = 1$ for all J, J_e (energy conservation)

(ii) $\chi_K = \lambda_K = 1$ for $J = 0$ (isotropic dipole).

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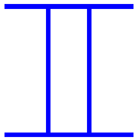
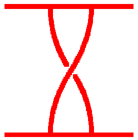
 $= \sum_K \lambda_K \mathsf{T}_K ,$  $= \sum_K \chi_K \mathsf{T}_K$

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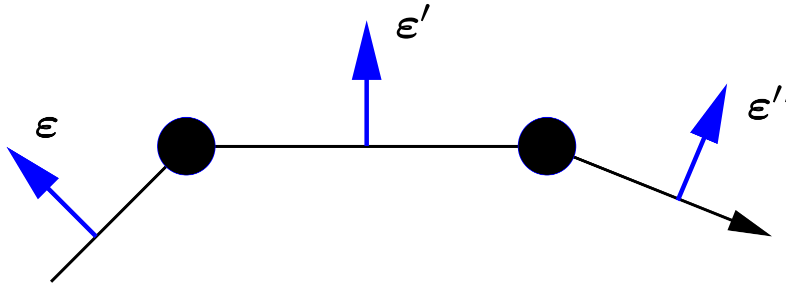
- Natural generalization to arbitrary spin and interaction

[with G. Montambaux]

Bulk transport: Diffusion and depolarization

- Exact diagonalisation of transverse propagator $\forall \Omega, \boldsymbol{q}$: eigenvalues $b_0 = 1$, $b_1 = \frac{1}{2}$, $b_2 = \frac{7}{10}$ at $q \to 0$.



  - field transversality $b_{1,2} < 1$: "spin-orbit"

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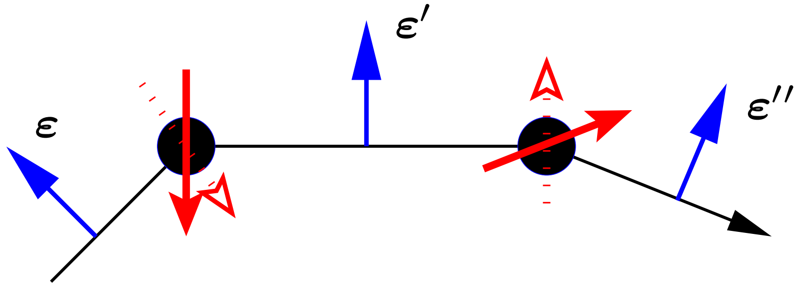

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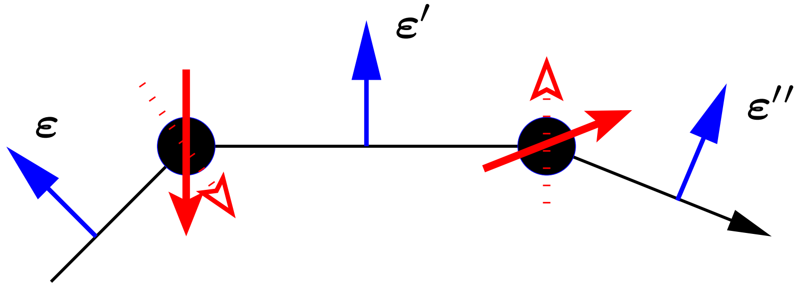

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$h_{\lambda\mu}$

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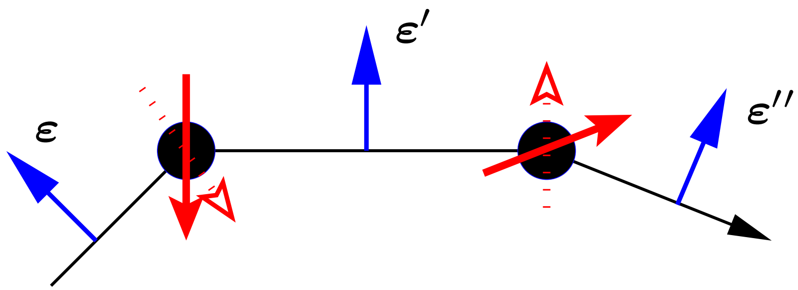

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- Diffusion approximation: $L(q) \approx \sum_K \frac{l_K}{Dq^2 + 1/\tau_d(K)}$

- Polarization relaxation times $\tau_d(K) = \frac{\tau_{tr}}{1/(b_K \lambda_K) - 1}$

$h_{\lambda\mu}$

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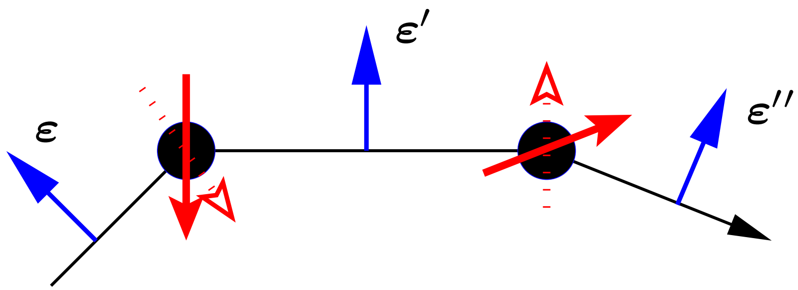
- Exact diagonalisation of transverse propagator $\forall \Omega, \boldsymbol{q}$:
eigenvalues $b_0 = 1$, $b_1 = \frac{1}{2}$, $b_2 = \frac{7}{10}$ at $q \to 0$.



  - field transversality $b_{1,2} < 1$: "spin-orbit"
  - atomic degeneracy $\lambda_{1,2} \leq 1$ "spin-flip"

- Diffusion approximation: $L(q) \approx \sum_K \frac{l_K}{Dq^2 + 1/\tau_d(K)}$

- Polarization relaxation times $\tau_d(K) = \frac{\tau_{tr}}{1/(b_K \lambda_K) - 1}$

- Conserved intensity: diffusive mode with $1/\tau_d(0) = 0$.

Cooperon: Dephasing of weak localization

- Weak localization contribution:

$$C(q) \approx \sum_K \frac{c_K}{Dq^2 + 1/\tau_c(K)}$$

$h_{\lambda\mu}$

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[Akkermans, Miniatura, & Müller, cond-mat/0206298]

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- Anomalous (non-thermal) photon dephasing due to partial trace over large ground-state degeneracy $(2J+1)^N$ of the atomic medium.

"uncompensated magnetic impurities at zero magnetic field"

[Y. Imry, cond-mat/0202044 + refs]

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III. Experimental signatures

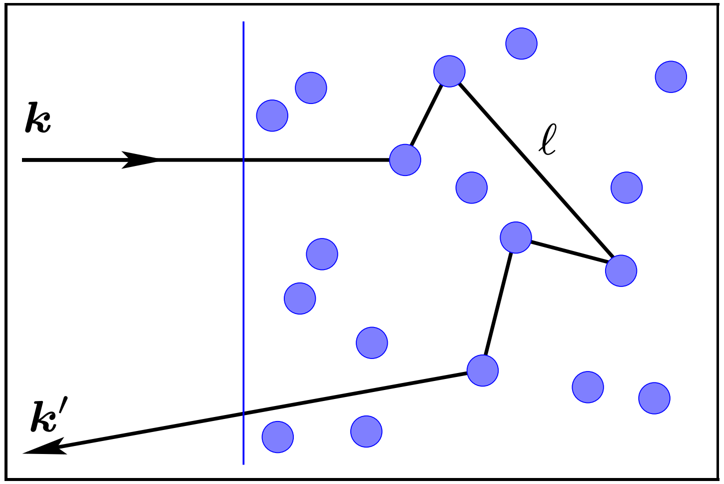
Part 2: **Coherent Backscatteriing**

$h_{\lambda\mu}$

Coherent Backscattering (CBS)

- Scattering by random sample:

$$\langle I \rangle = \sum_p |a_p|^2$$



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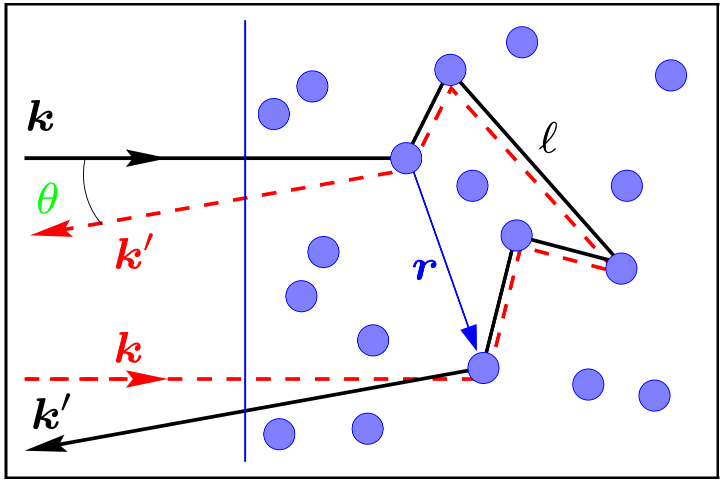
$$\langle I \rangle = \sum_p |a_p + a_{\tilde{p}}|^2$$

Counterpropagating amplitudes!

$$\phi(\theta) = (\boldsymbol{k} + \boldsymbol{k}') \cdot \boldsymbol{r} \approx k\ell\theta$$



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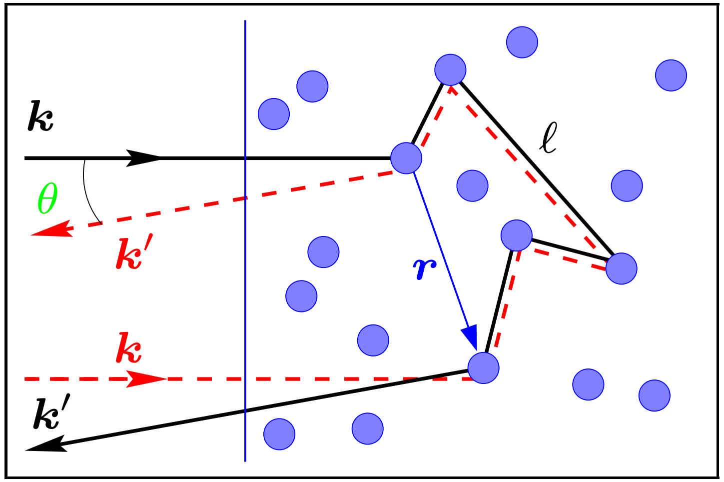
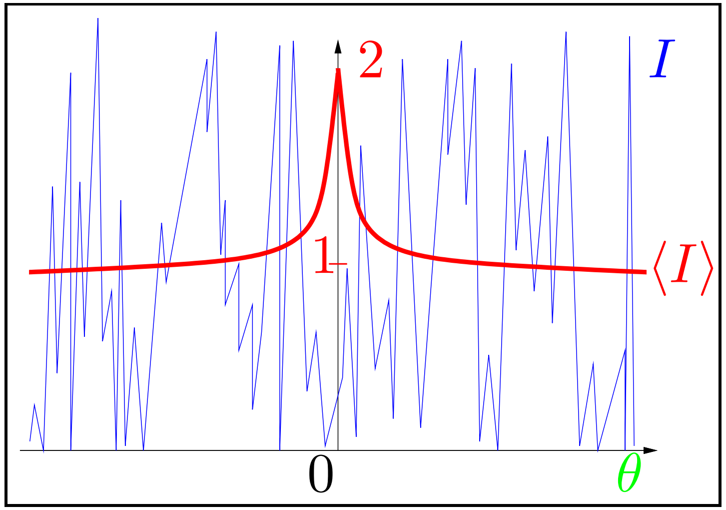




CBS: Pairwise constructive interference survives the ensemble average for

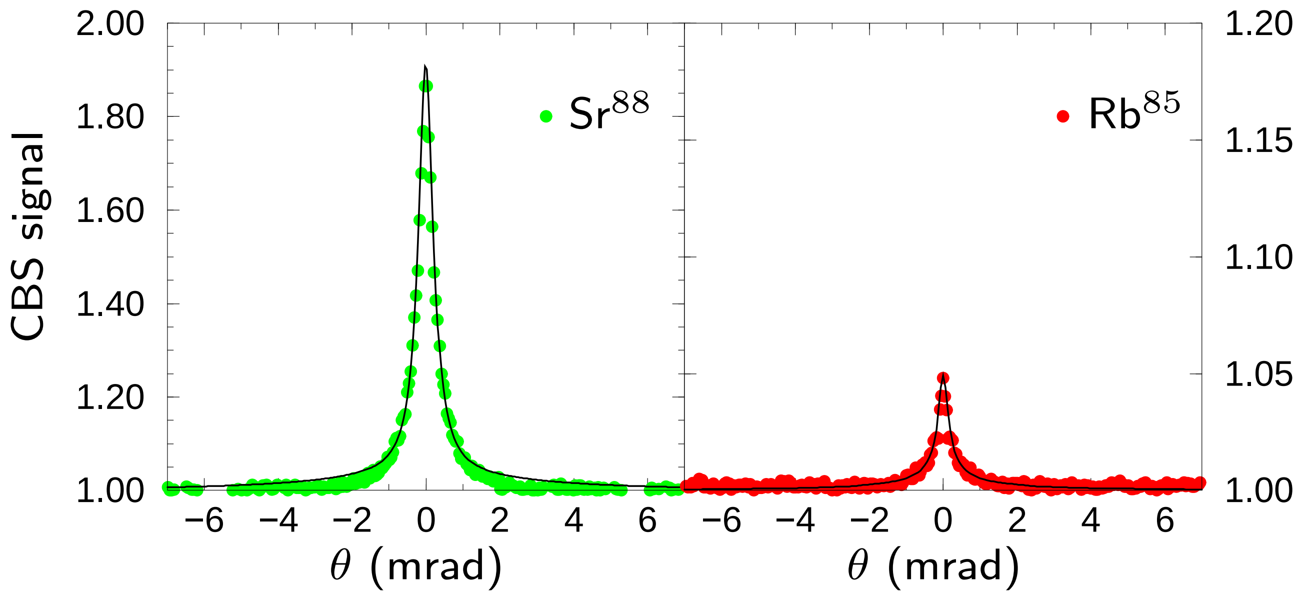
$$|\theta| < 1/k\ell$$

"random collection of Young slits"

Experimental signature

- CBS by atoms without and with internal degeneracy:

$J = 0, J_e = 1$ $J = 3, J_e = 4$



[Bidel et al., PRL **88**, 203902 (2002)] [Labeyrie et al., EPL **61**, 327 (2003)]

- Theory: analytic internal degeneracy

[Müller & Miniatura, J. Phys. A (2002)]

$+$ MC simulation of photon trajectories

[Labeyrie, Delande et al., PRA (2003)]

Summary

- "Photons do it like electrons, but all at once and resonantly"

$$\tau_d \sim \tau_c \sim \tau_{\mathsf{tr}}$$

$h_{\lambda\mu}$

Summary

- "Photons do it like electrons, but all at once and resonantly"

$$\tau_d \sim \tau_c \sim \tau_{\mathrm{tr}}$$

- Atoms: exact microscopic theory for τ_d, τ_c as function of experimental parameters J, J_{e}.

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Open questions

- Towards strong localisation of photons in a gas of 'immobile' atoms: Self-consistent perturbation (Ward identities, etc.) in strongly disordered limit $n\lambda^3 \to 1$

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- ...ask me again at Windsor 2007 ...

$\overleftrightarrow{h}_{\lambda\mu}$ *Quantum Transport of Light and Matter*

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Thanks to colleagues

$h_{\lambda\mu}$

QCCM 2004's motto:

"If you wanted to draw this as a diagram, how would it look like?"

Thanks to colleagues