Electron-electron interactions in metallic diffusive wires

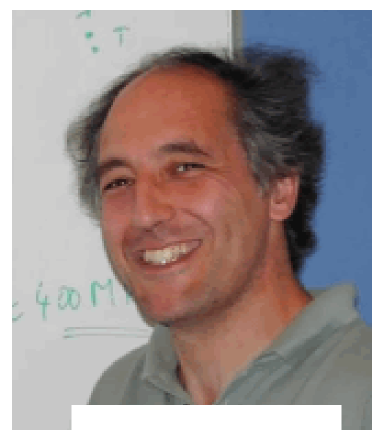
Quantronics Group
CEA Saclay, France


B. Huard


A. Anthore


F. Pierre


N. O. Birge


D. Esteve


H. Pothier

MSU
East Lansing, USA

Ingredients

Altshuler

NS junction

Phase coherence

Mirlin

Weak localisation

1ppm

Coulomb interaction

Lerner

von Delft

Diffusion

Energy exchange

Tunnel spectroscopy

Kondo effect

ZBA

Distribution function

Boltzmann equation

Landauer

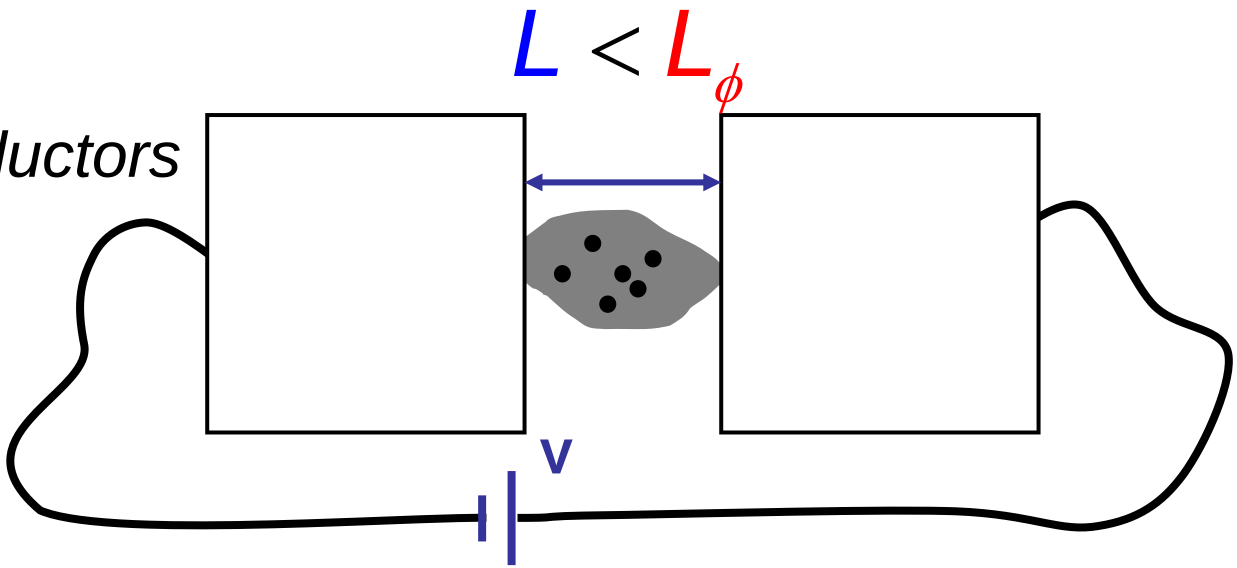
Coulomb blockade

Outline

- phase coherence and electrical transport
- phase coherence in wires and interactions
- interactions and energy exchange
- effect of magnetic impurities

Electrical transport and coherence

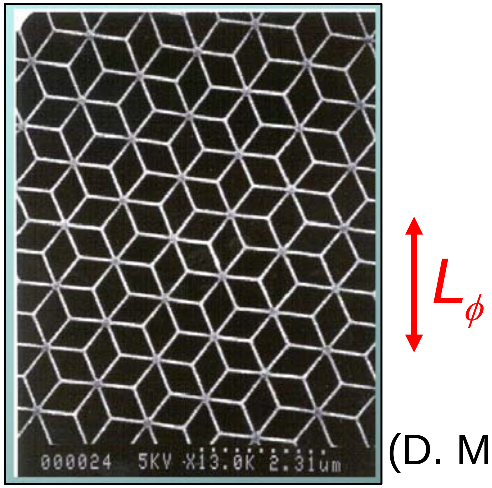
Fully coherent conductors



Transport=scattering problem → Landauer formalism

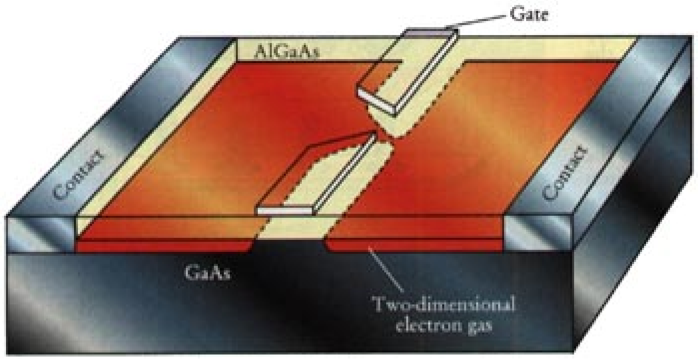
Larger conductors:

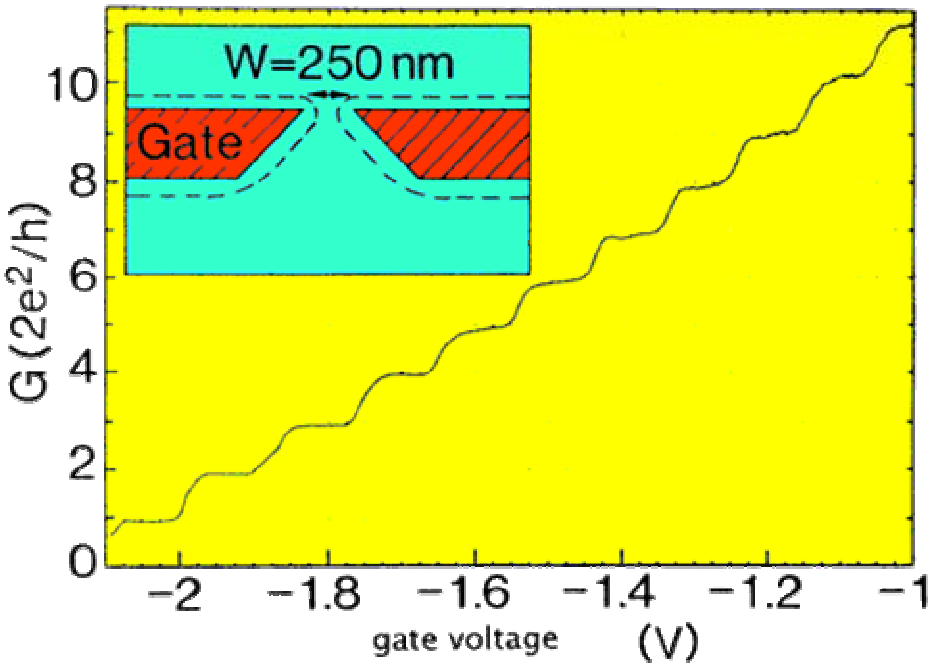
size of mesoscopic effects depends on L_ϕ



(D. Mailly)

Fully coherent conductors
1. Quantum point contacts





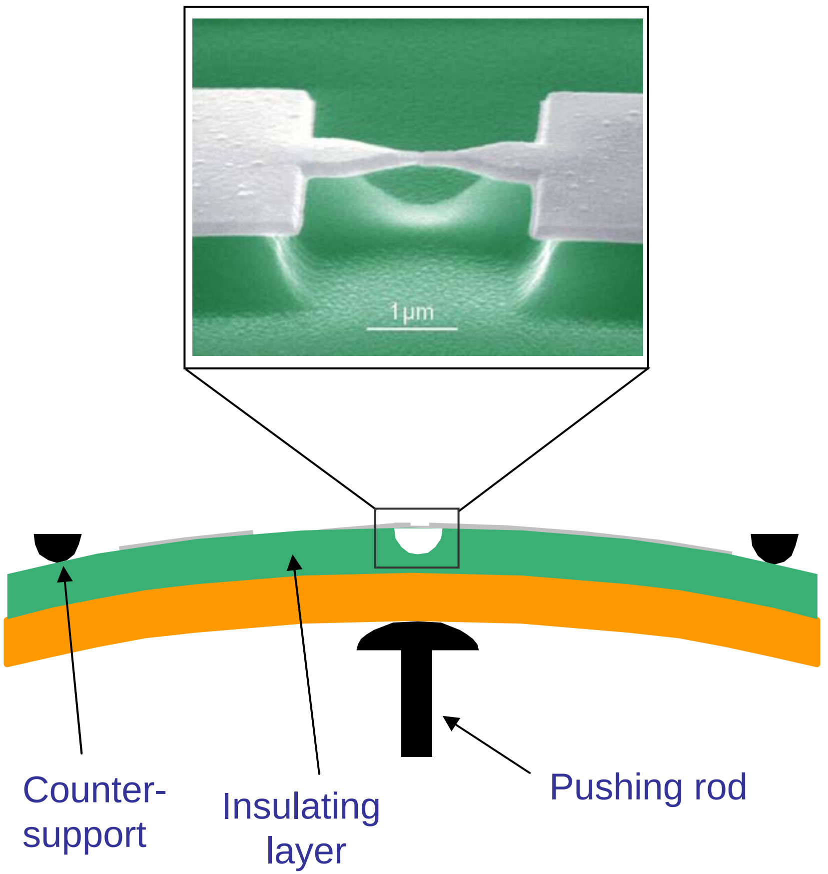
van Wees; Wharam (1988)

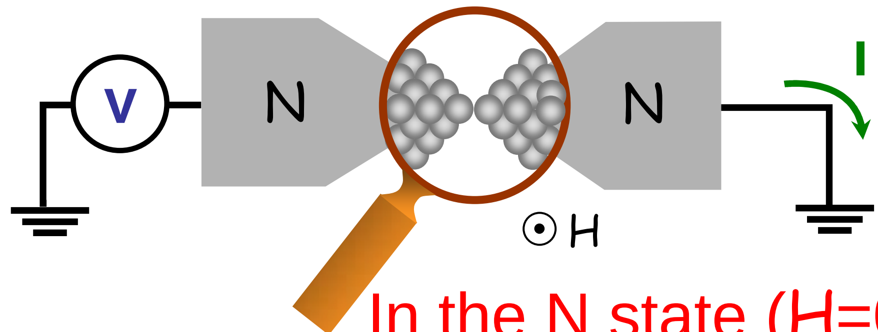
τ_i=1 for all open channels

0<τ_N<1 for the last channel

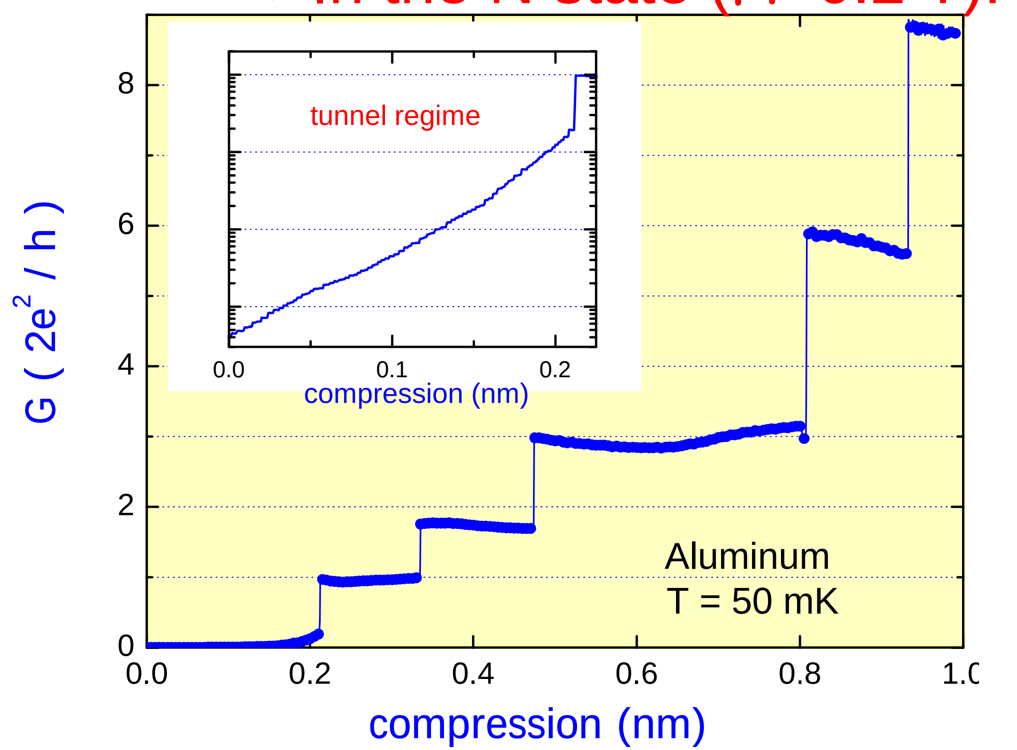
Landauer (1957) $$G = \frac{2e^2}{h}\sum \tau_i = \frac{2e^2}{h}\left(N - 1 + \tau_N\right)$$

Fully coherent conductors
2. Atomic contacts

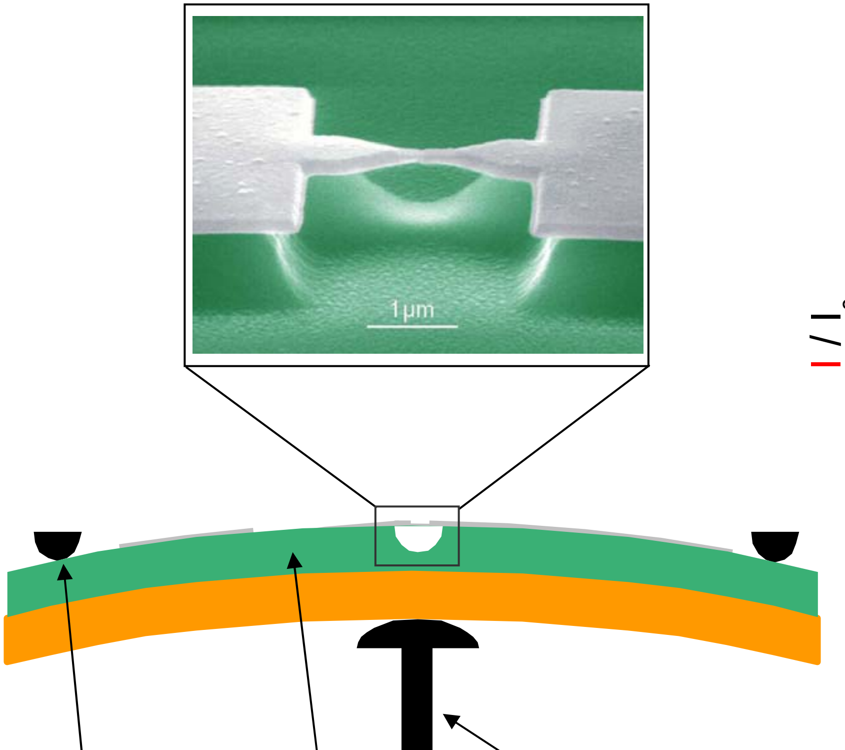




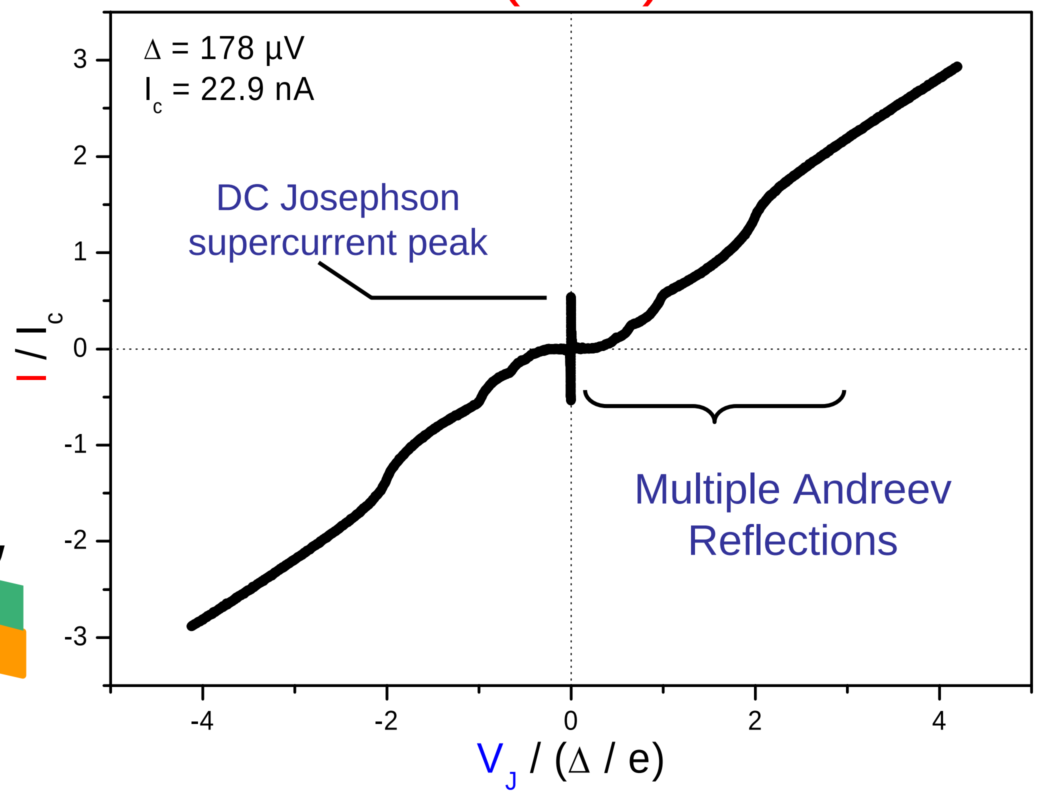
In the N state (H=0.2 T):



Fully coherent conductors
2. Atomic contacts



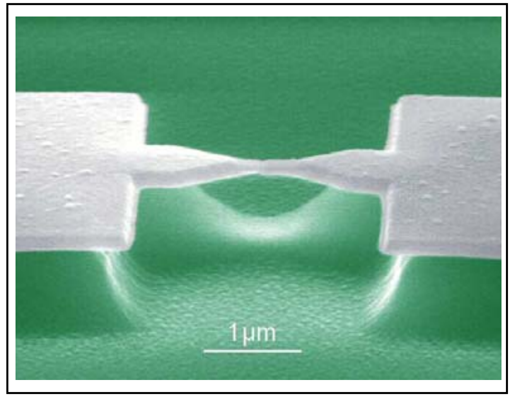
In the S state (H=0):



$\{\tau_1 \ldots \tau_N\}$ determined from the IV of the contact in the S state

Scheer *et al.* (Nature, 1998)

Fully coherent conductors
2. Atomic contacts



N. Agrait, A. Levy-Yeyati,
J.M. van Ruitenbeek
Phys. Reps. **377**, 81-380 (2003)

$\{\tau_1 \dots \tau_N\}$

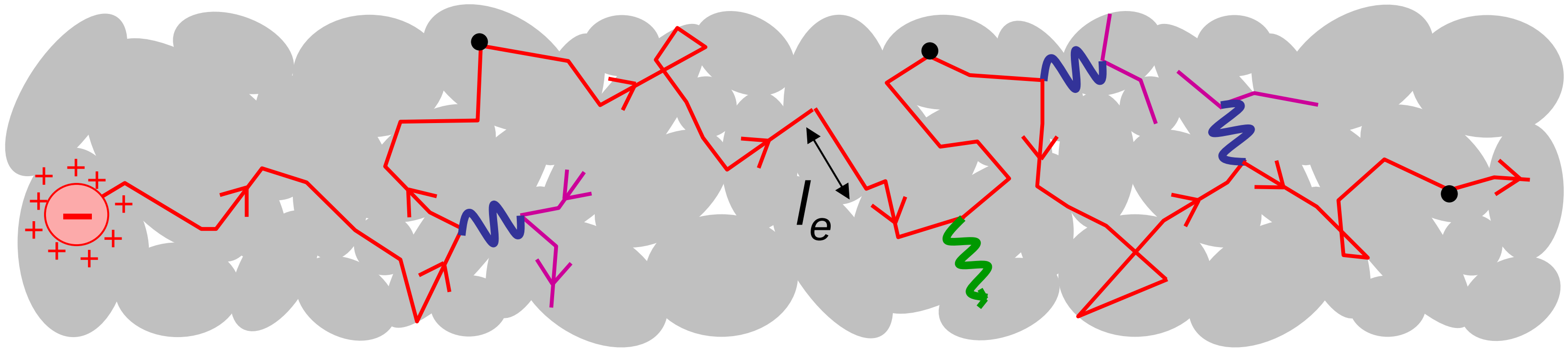
Normal state

- …
- Coulomb Blockade
- Shot Noise
- Thermopower
- Conductance

Superconducting state

- …
- Shapiro Steps
- Shot Noise
- Current-Voltage characteristic
- Josephson supercurrent

Transport in metallic thin films



Elastic scattering

Grain boundaries
Film edges •
Impurities

→

Diffusive states
$l_e \approx 40$ nm
$D = v_F l_e / 3$

Inelastic scattering

Coulomb interactions
Phonons

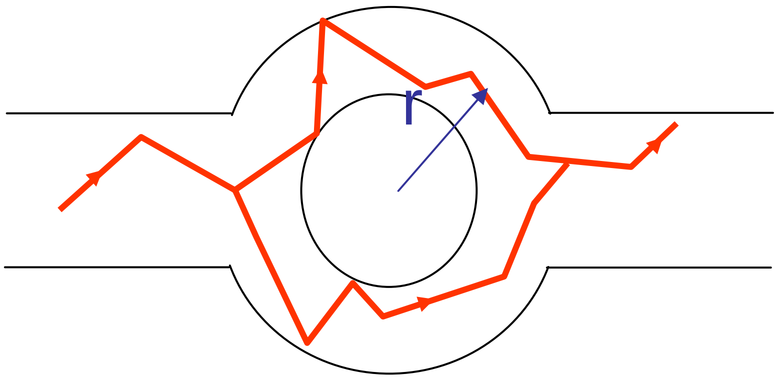
→

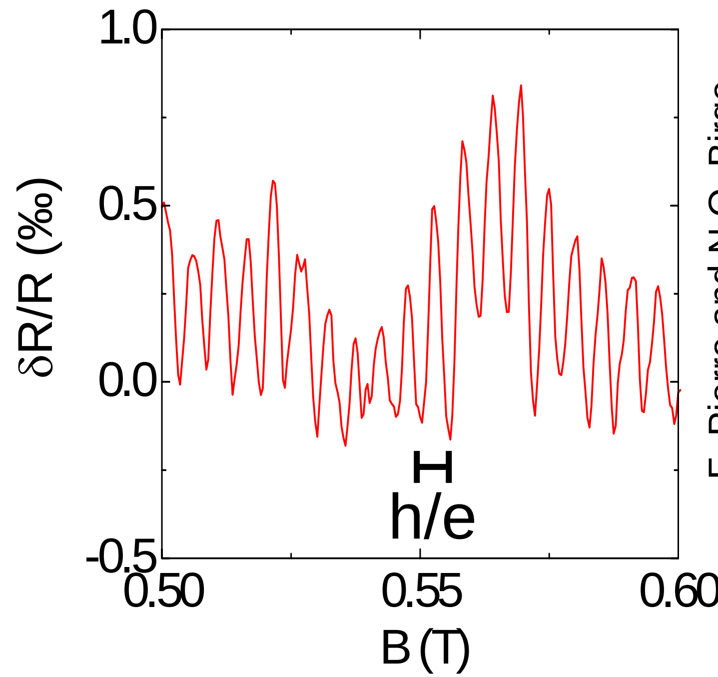
- Limit coherence (L_ϕ)
- Redistribute energy

Typically, $\lambda_F \ll l_e \ll L_\phi \leq L$

Interference effects and L_ϕ

-Aharonov-Bohm effect





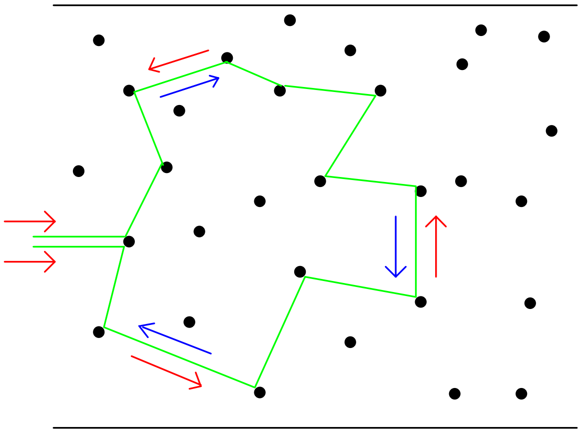
F. Pierre and N.O. Birge, PRL **86**, 206804 (2002)

$$P_{transm.} = \left| A_{up} + A_{down} \right|^2 = P_{up} + P_{down} + 2\mathrm{Re}\left(A_{up} A_{down} \right)$$

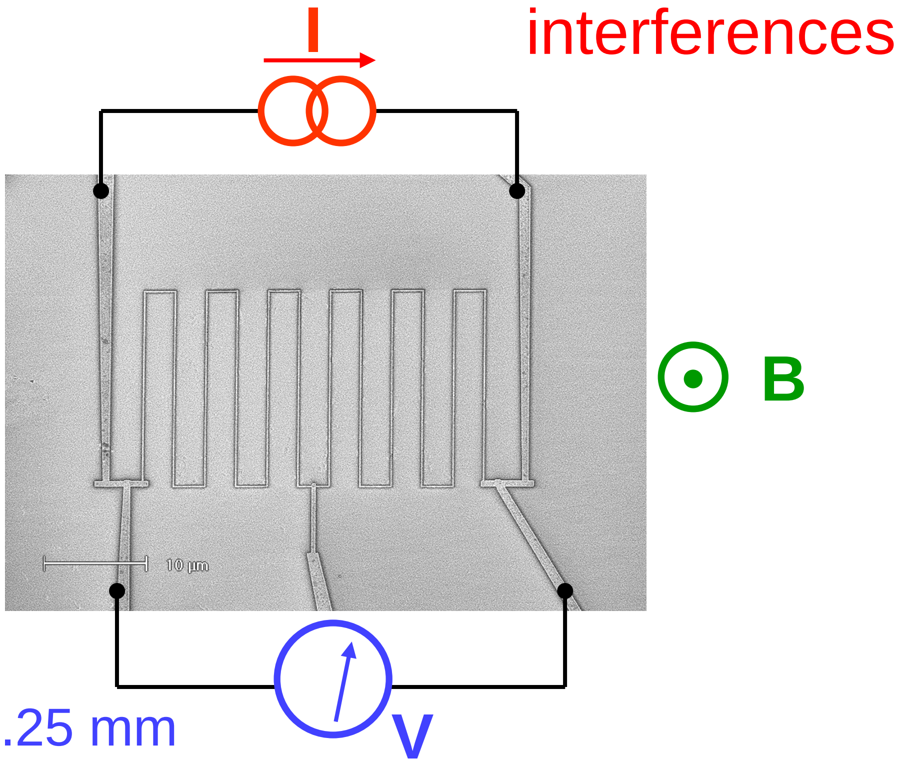
$$\frac{\Delta G}{e^2/h} = C \frac{L_T}{\pi r} \sqrt{\frac{L_\phi}{\pi r}} \exp\left(-\frac{\pi r}{L_\phi} \right)$$

Interference effects and L_ϕ

- *Aharonov-Bohm effect*
- *Weak localization*

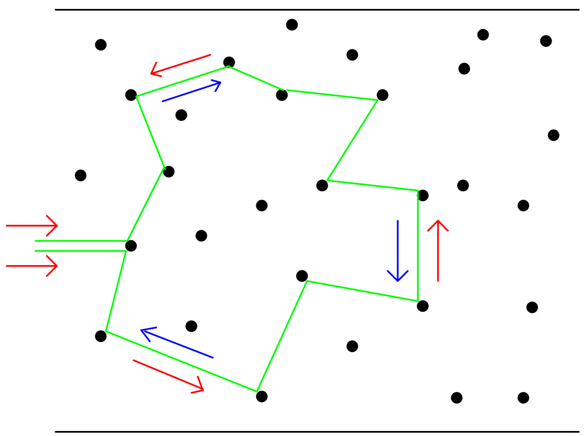


$$P_{return} = \left|A_\rightarrow + A_\leftarrow\right|^2 = \underbrace{P_\rightarrow + P_\leftarrow}_{\text{classic}} + \underbrace{2\mathrm{Re}\left(A_\rightarrow A_\leftarrow\right)}_{\text{quantum interferences}}$$

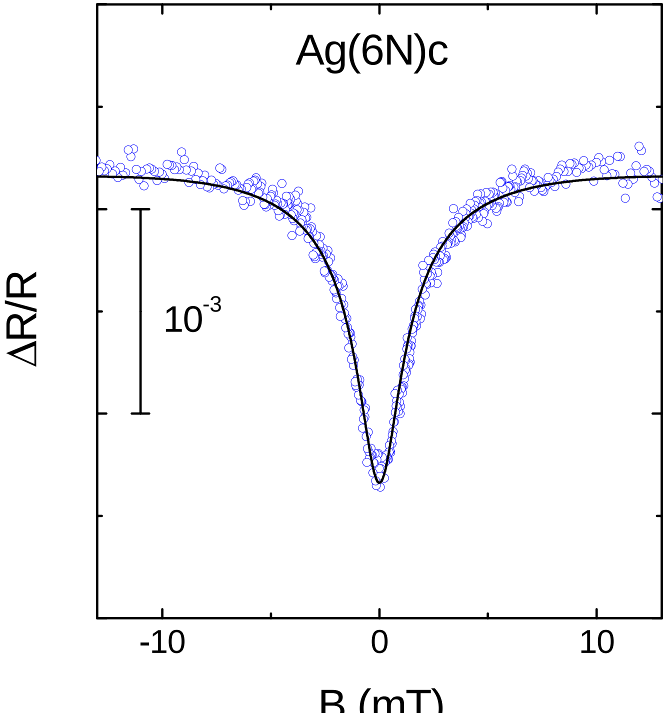


Interference effects and L_ϕ

- *Aharonov-Bohm effect*
- *Weak localization*



$$P_{return} = \left|A_{\rightarrow} + A_{\leftarrow}\right|^2 = \underbrace{P_{\rightarrow} + P_{\leftarrow}}_{\text{classic}} + \underbrace{2\mathrm{Re}\left(A_{\rightarrow}A_{\leftarrow}\right)}_{\text{quantum interferences}}$$



In the case of strong spin-orbit coupling (Mirlin tonight):

$$\frac{R(B=0) - R_\infty}{R_\infty} = -\frac{R}{R_K}\frac{L_\phi}{L}$$

$$\left(R_K = \frac{h}{e^2} \approx 25\,812\ \Omega\right)$$

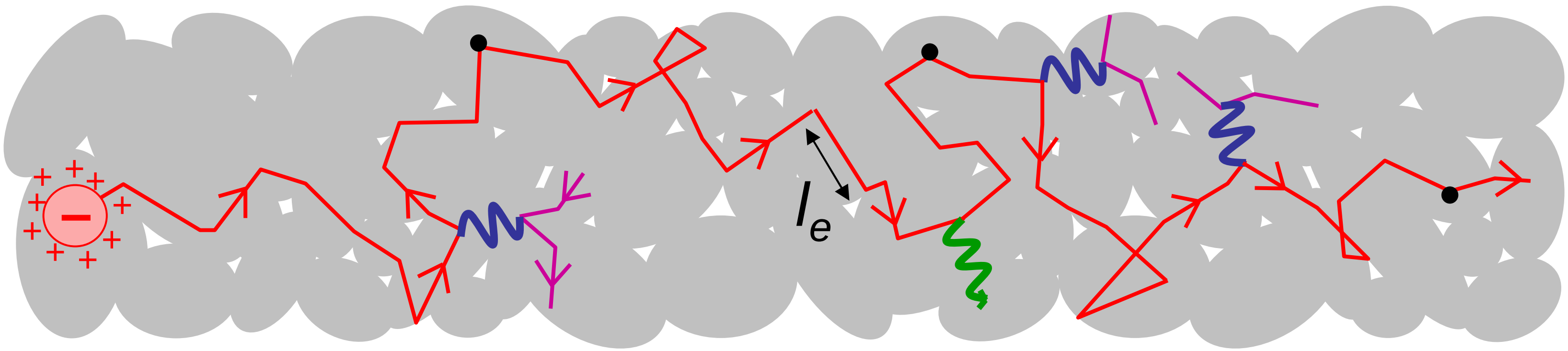
Interference effects and L_ϕ

- *Aharonov-Bohm effect*
- *Weak localization*
- *Conductance Fluctuations*
- *Persistent currents*
- *Superconducting proximity effect*
- ...

Size of the effects depends on $L_\phi = \sqrt{D\tau_\phi}$

- phase coherence and electrical transport
- phase coherence in wires and interactions
- interactions and energy exchange
- effect of residual impurities

Extension of τ_ϕ



determined by the dominant inelastic process

e-ph

e-e

τ_ϕ reflects interactions

Predictions for τ_ϕ at low T

(Altshuler, Aronov, 1979)

At low T, τ_ϕ limited by e-e interactions

Screening depends on dimensionality

(At energy E, compare $\sqrt{\hbar D/E}$ with transverse dimensions)

$$\left(\sum_{q_x, q_y, q_z} \frac{\cdots}{\left(Dq^2 + i\omega\right)^{\cdots}} \right)$$

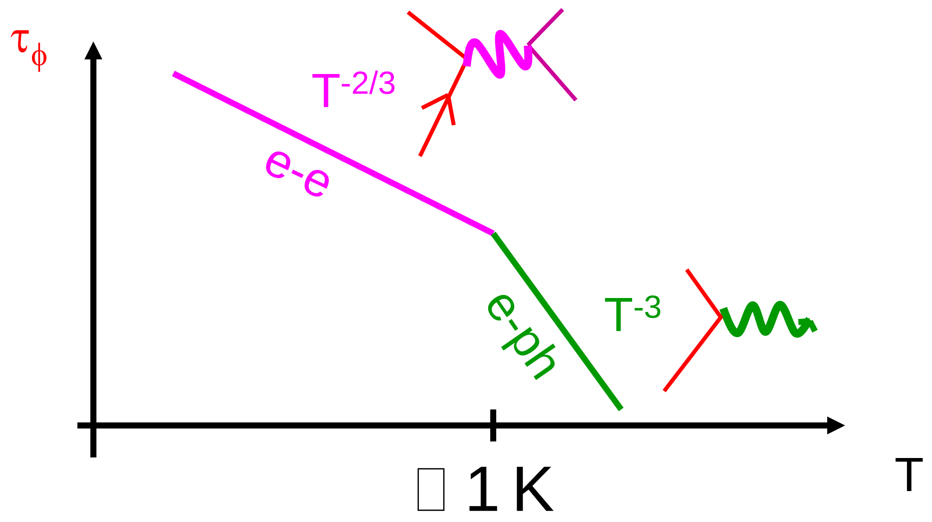
τ_ϕ depends on dimensionality

« wires » (1d regime) : $L_\phi = \sqrt{D\tau_\phi} >$ transverse dimensions

($E \sim \hbar / \tau_\phi$ rule the game)

$\tau_\phi(T)$ in wires

(Altshuler, Aronov, Khmelnitskii, 1982)



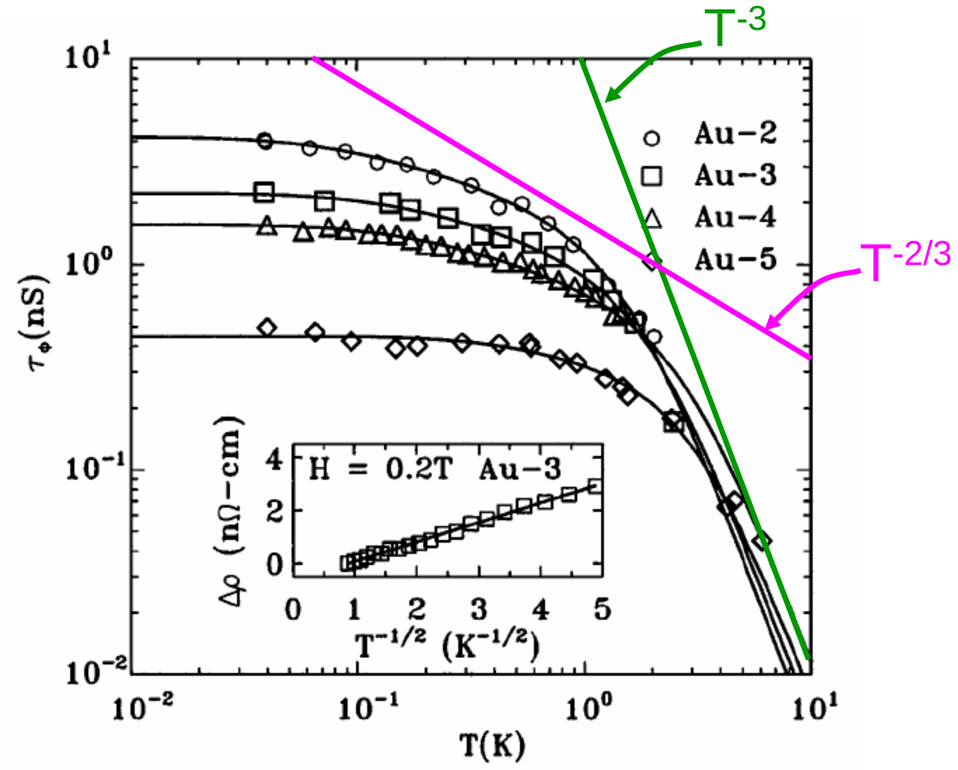
$$\tau_\phi = (A\,T^{2/3} + B\,T^3)^{-1}$$

$$A = \frac{1}{\hbar}\left(\frac{\pi k_B^2}{4\nu_F L w t}\frac{R}{R_K}\right)^{1/3}$$

Screened Coulomb interaction at d=1

$\tau_\phi(T)$ measurements at low T

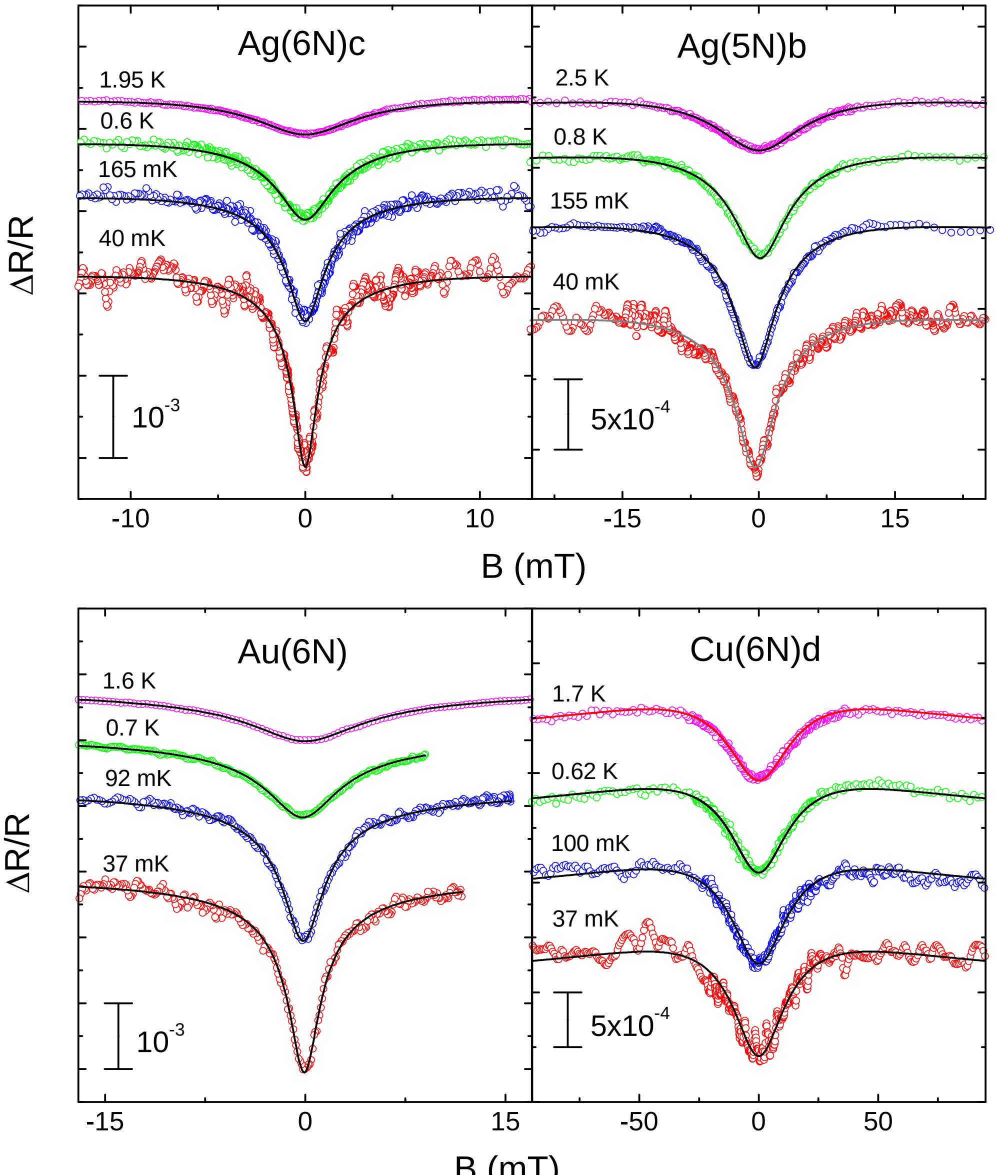
(Mohanty, Jariwala and Webb, PRL **78**, 3366 (1997))



"Saturation" of τ_ϕ:

e-e interaction badly understood ?
another process dominates ?

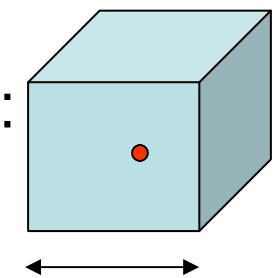
Measuring $\tau_\phi(T)$: raw data



5N = 99.999 % source purity

6N = 99.9999 % " "

1 ppm of *impurities* :



100 atoms ~ 25 nm

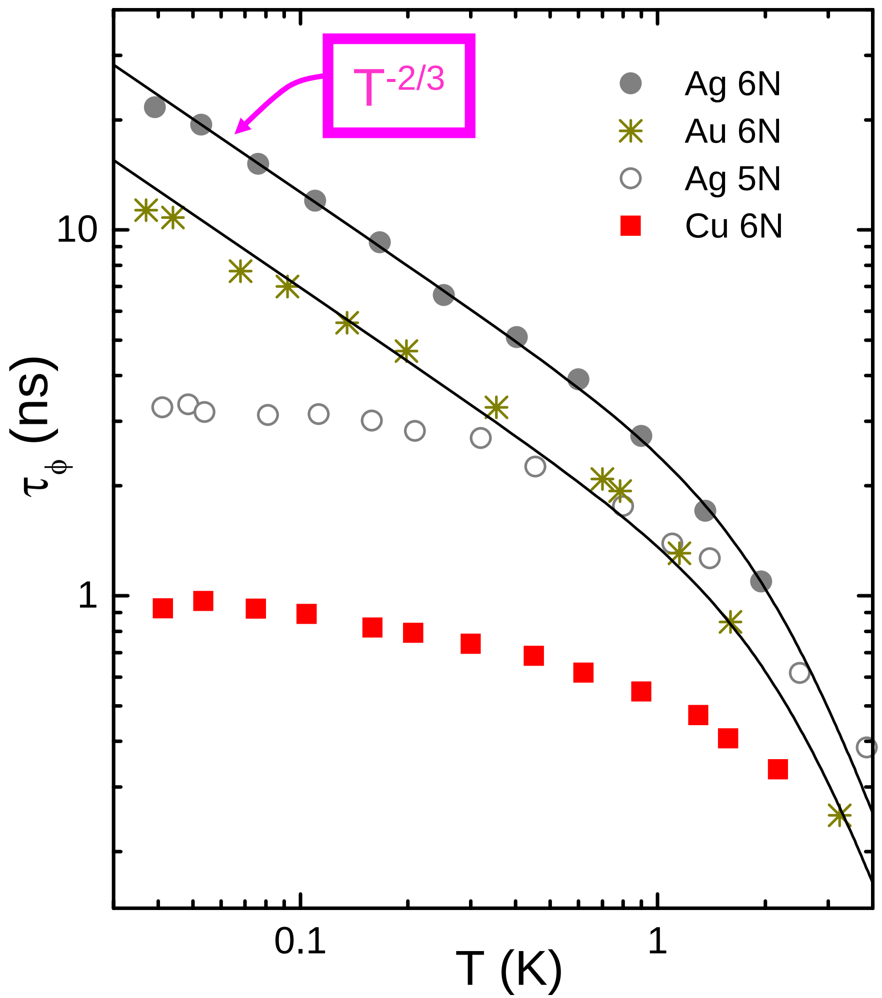
Ag(6N) & Au(6N):

ΔR grows as T decreases

Ag(5N) & Cu(6N):

ΔR saturates below ~ 100mK

$\tau_\phi(T)$ in Ag, Au & Cu wires



F. Pierre *et al.*,
PRB **68**, 0854213 (2003)

5N = 99.999 % source material purity
6N = 99.9999 % " " "

Low T behavior vs. Purity:

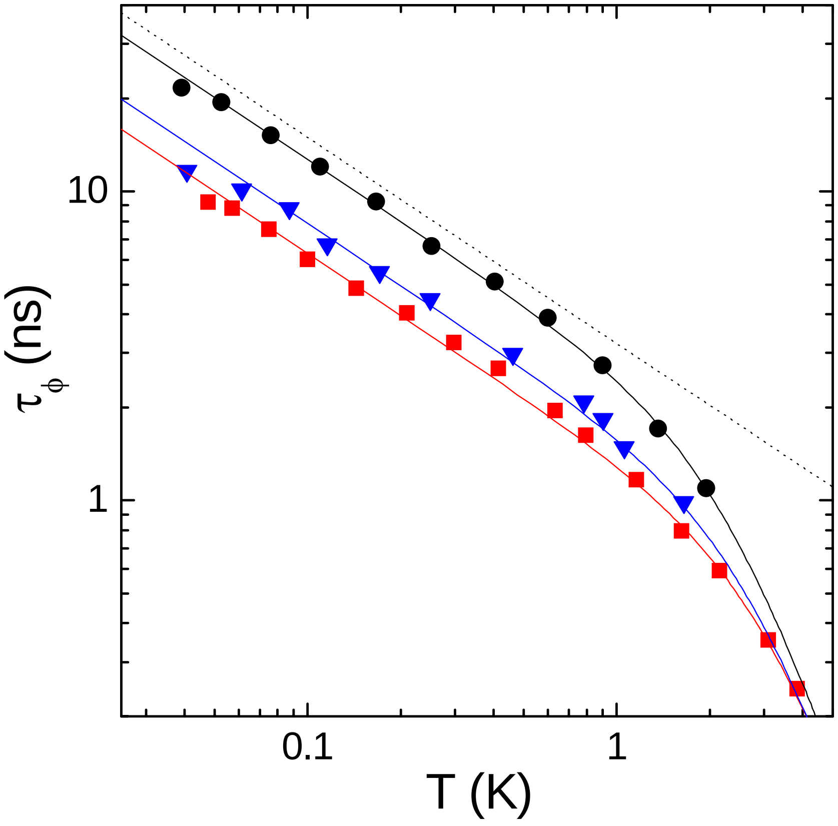
- **Ag 6N**, **Au 6N**
  → agreement with AAK theory

- **Ag 5N**, **Cu 6N**
  → saturation of $\tau_\phi(T)$

Saturation of τ_ϕ is sample dependent

Quantitative comparison with theory for clean samples

$$\tau_\phi = (A\,T^{2/3} + B\,T^3)^{-1}$$



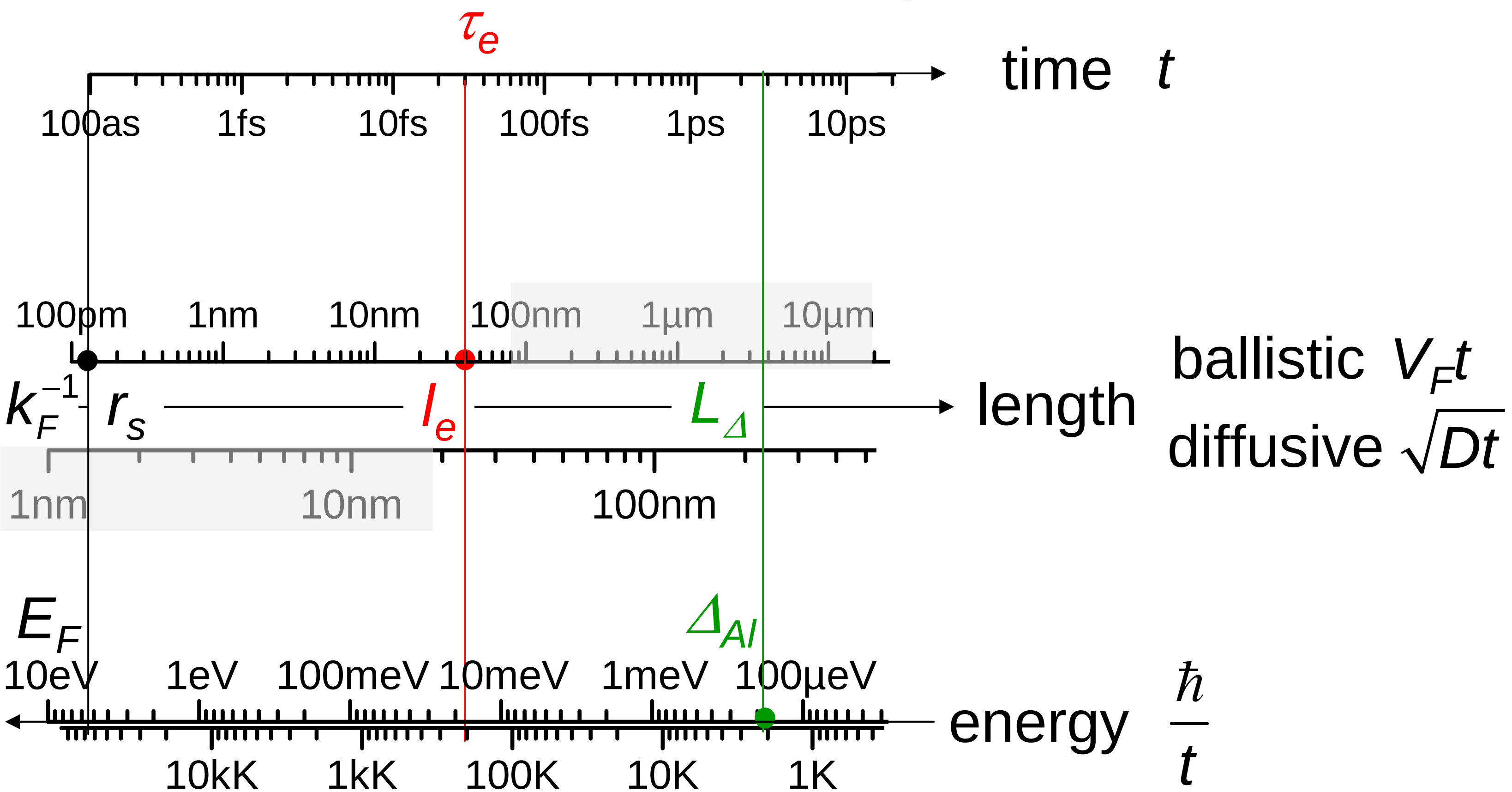
Sample	A_{thy} ($\text{ns}^{-1}\,\text{K}^{-2/3}$)	A ($\text{ns}^{-1}\,\text{K}^{-2/3}$)
■ Ag(6N)a	0.55	0.73
▼ Ag(6N)b	0.51	0.59
● Ag(6N)c	0.31	0.37
Ag(6N)d	0.47	0.56
Au(6N)	0.40	0.67

F. Pierre *et al.*,
PRB **68**, 0854213 (2003)

$$A_{thy} = \frac{1}{\hbar}\left(\frac{\pi k_B^2}{4\nu_F L w t}\,\frac{R}{R_K}\right)^{1/3}$$

Orders of magnitude in diffusive wires

1. intrinsic parameters

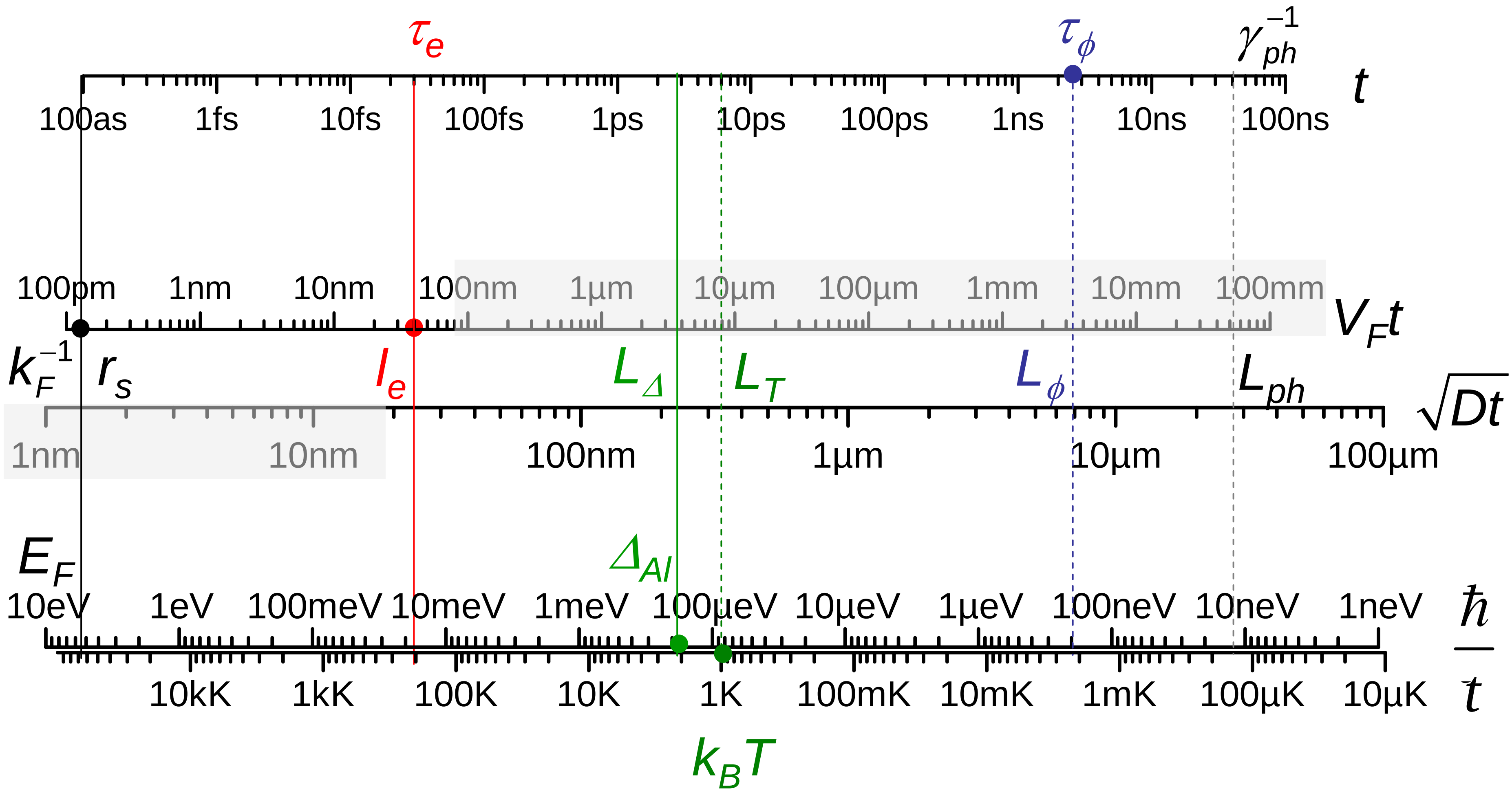


$k_F^{-1} \ll l_e$: good metal

D=185 cm^2/s
V_F=1.39 10^6 m/s (Ag)
ν_F=1.03 10^{47} $J^{-1}m^{-3}$

Orders of magnitude in diffusive wires

2. at T=1 K

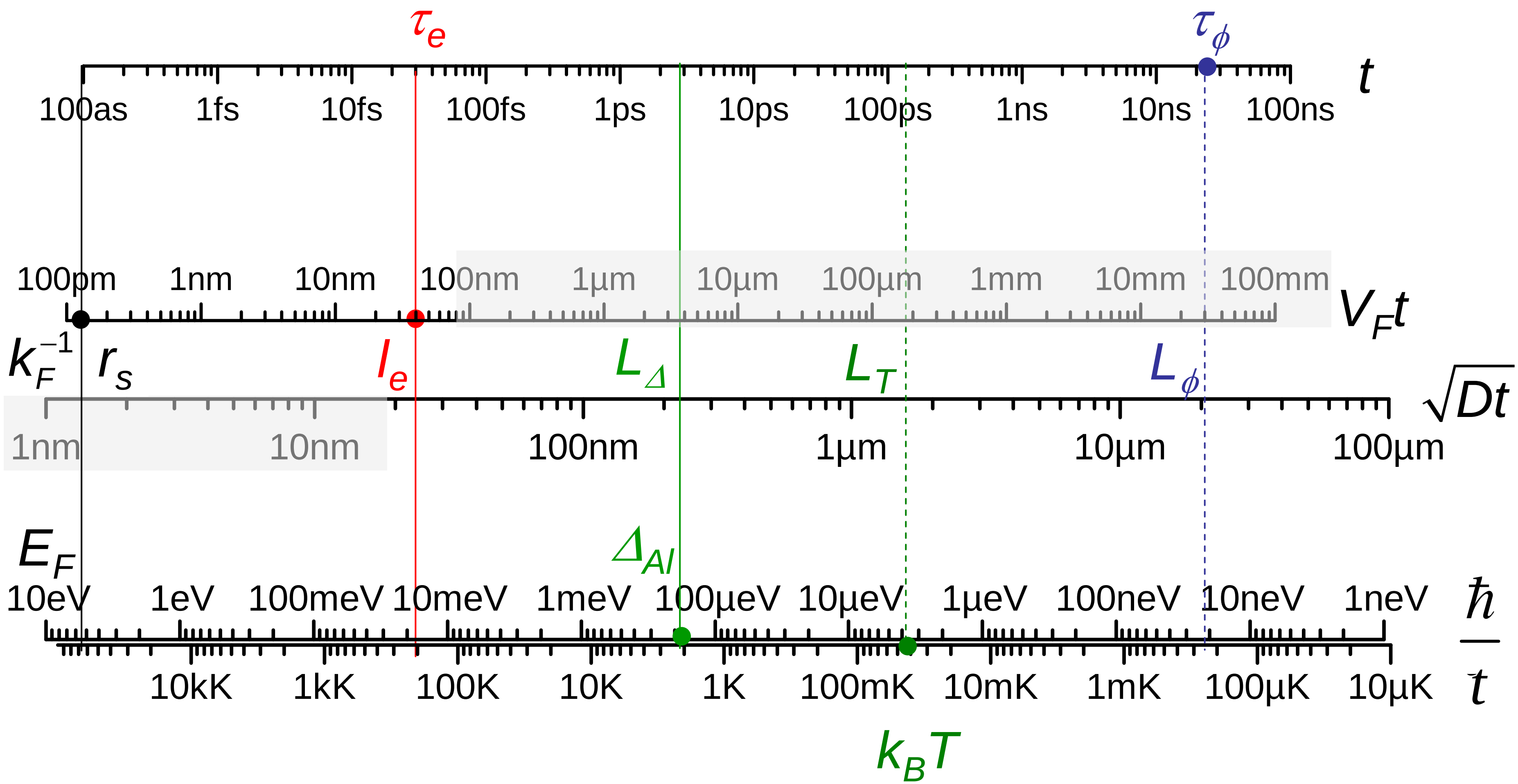


$$L_T = \sqrt{\frac{\hbar D}{k_B T}},\; L_\phi \propto T^{-1/3},\; L_{ph} \propto T^{-3/2}$$

$D=185\ cm^2/s$
$V_F=1.39\ 10^6$ m/s (Ag)
$\nu_F=1.03\ 10^{47}\ J^{-1}m^{-3}$

Orders of magnitude in diffusive wires

3. at T=40 mK



L_{ph} □ 3.5 mm out of this scale

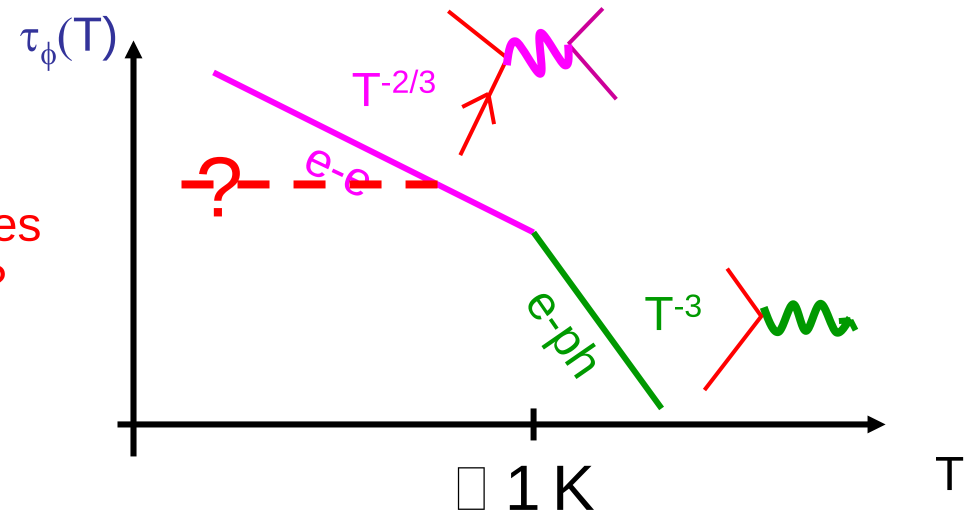
D=185 cm^2/s
$V_F=1.39\ 10^6$ m/s (Ag)
$\nu_F=1.03\ 10^{47}\ J^{-1}m^{-3}$

- phase coherence and electrical transport
- phase coherence in wires and interactions
- interactions and energy exchange
- effect of residual impurities

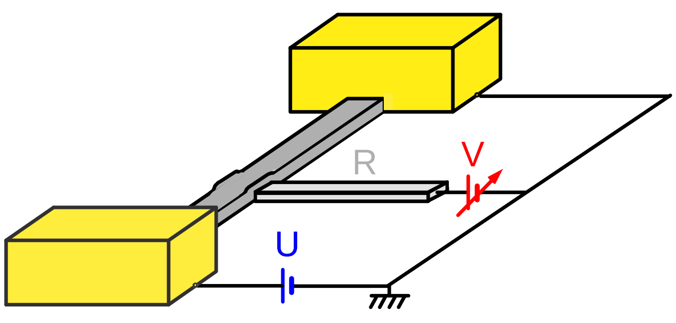
Investigation of inelastic processes

1st method : τ_ϕ

Another process dominates
in not-so-pure samples?

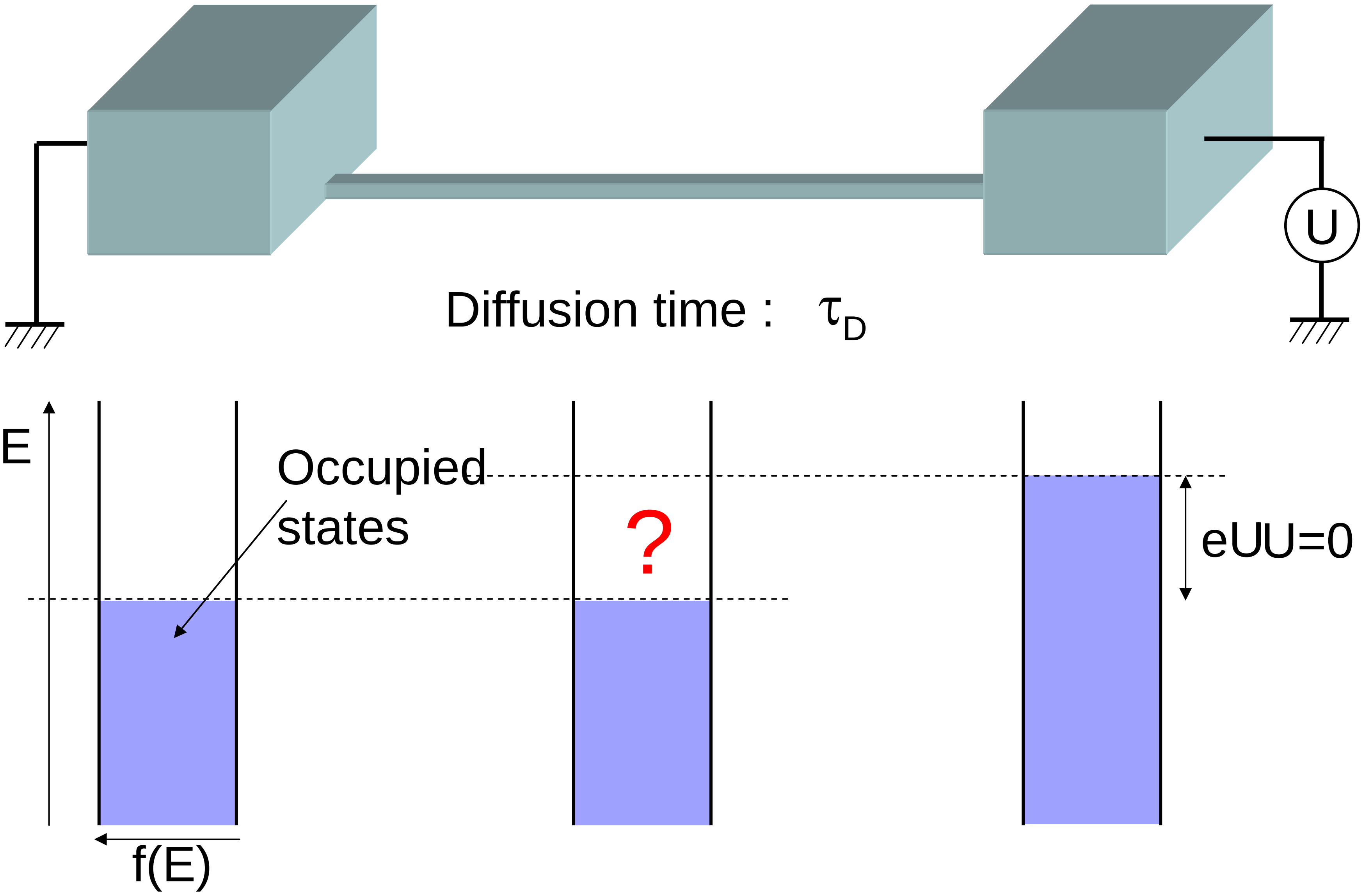


2nd method : measure energy exchange rates



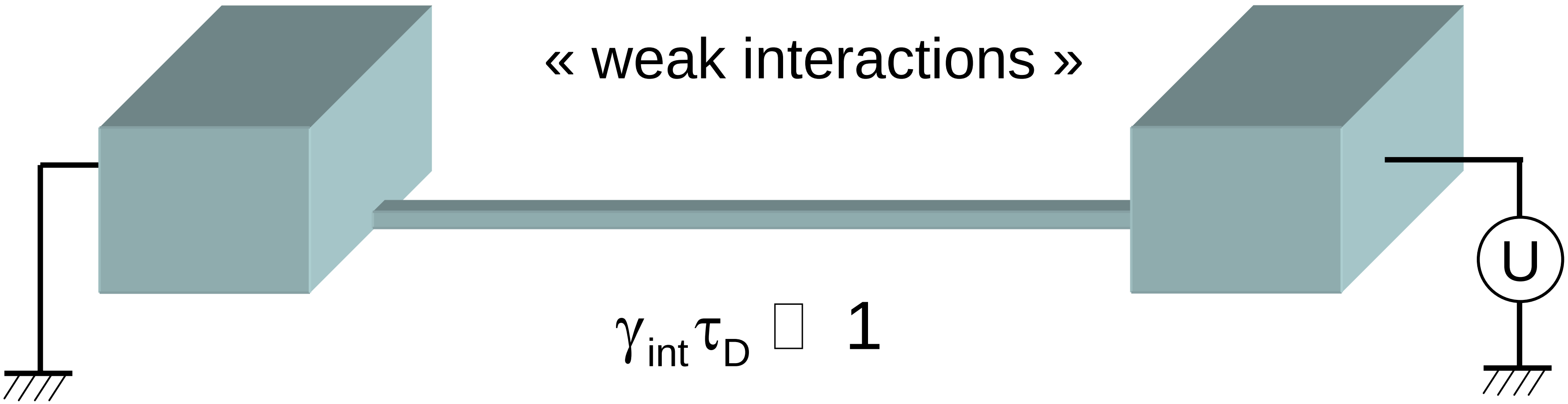
Distribution f(E)
reflects the
exchange rates

Distribution function and energy exchange rates

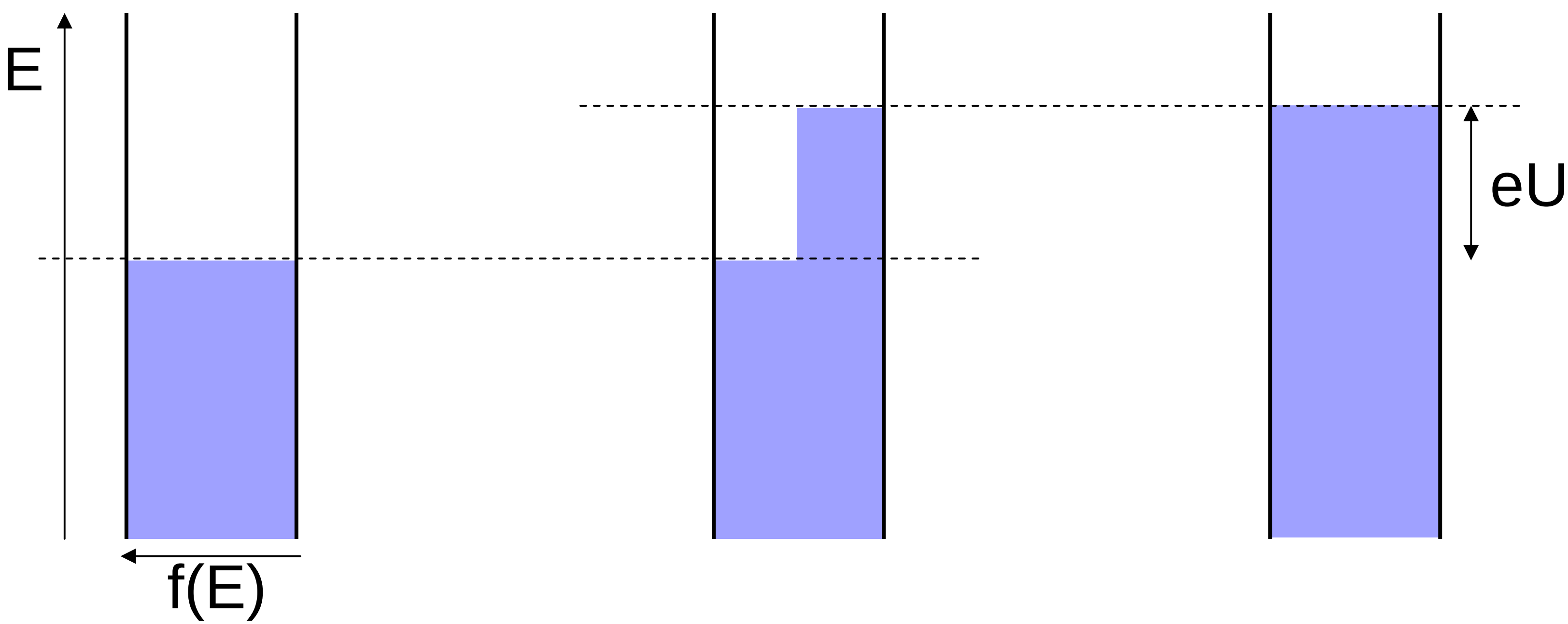


Distribution function and energy exchange rates

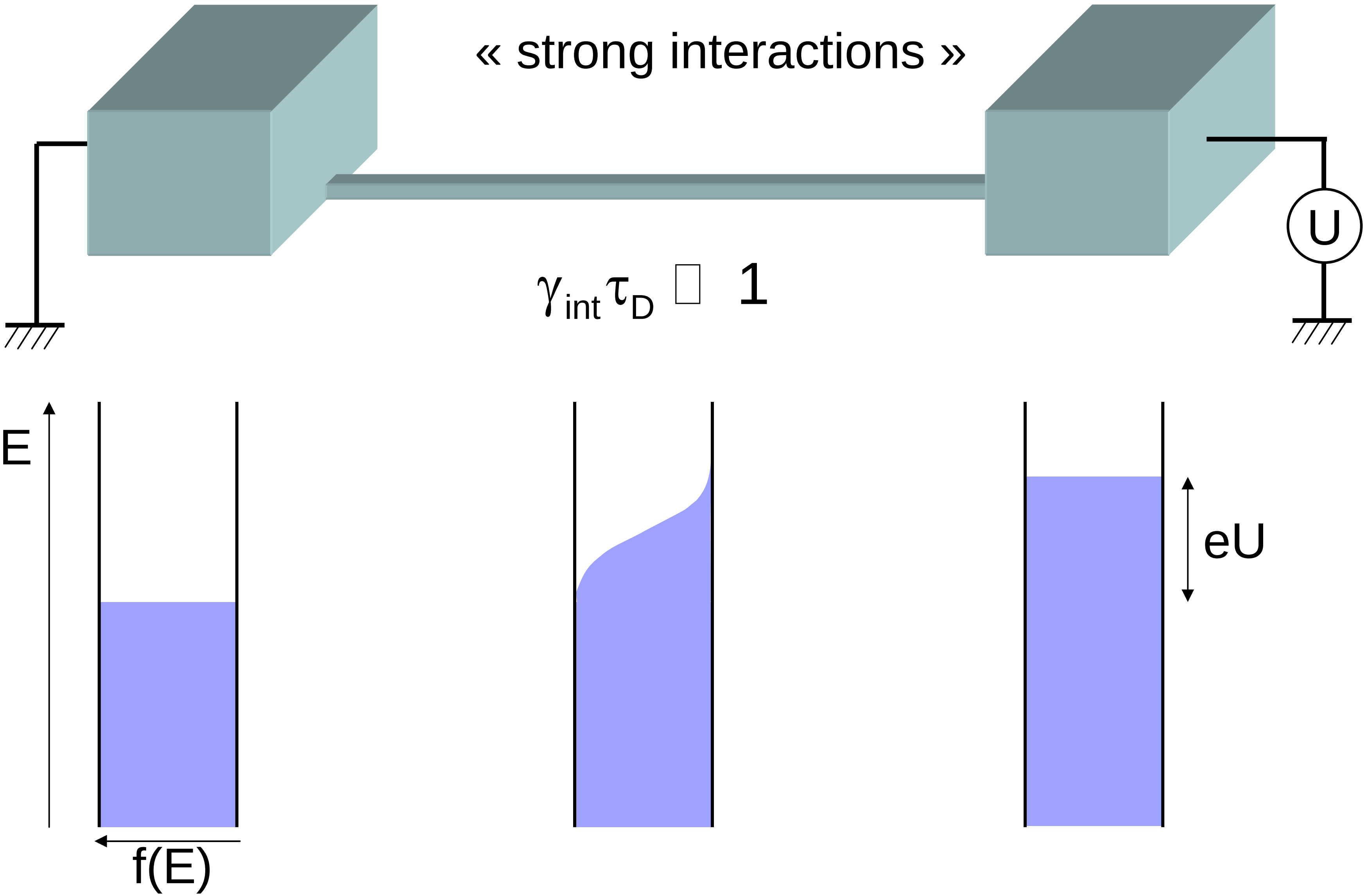
« weak interactions »



$$\gamma_{int}\tau_D \ll 1$$



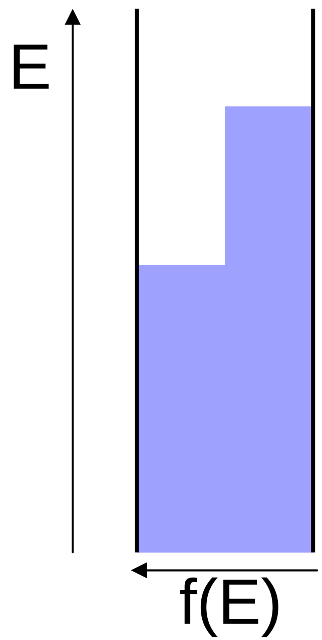
Distribution function and energy exchange rates



Distribution function and energy exchange rates

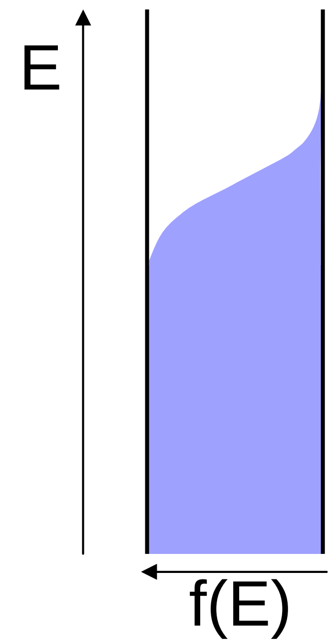
« weak interactions »

$$\gamma_{int}\tau_D \ll 1$$



« strong interactions »

$$\gamma_{int}\tau_D \gg 1$$

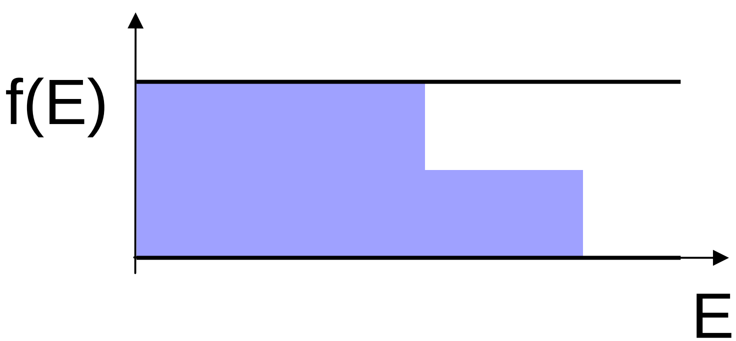


f(E) $\Longleftrightarrow$ interactions

Distribution function and energy exchange rates

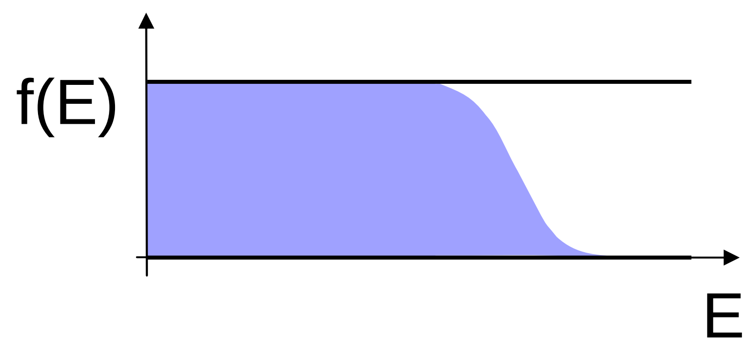
« weak interactions »

$$\gamma_{int}\tau_D \ll 1$$



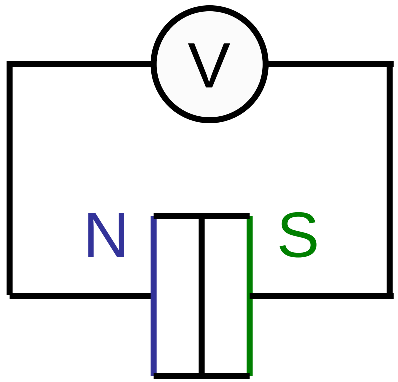
« strong interactions »

$$\gamma_{int}\tau_D \gg 1$$



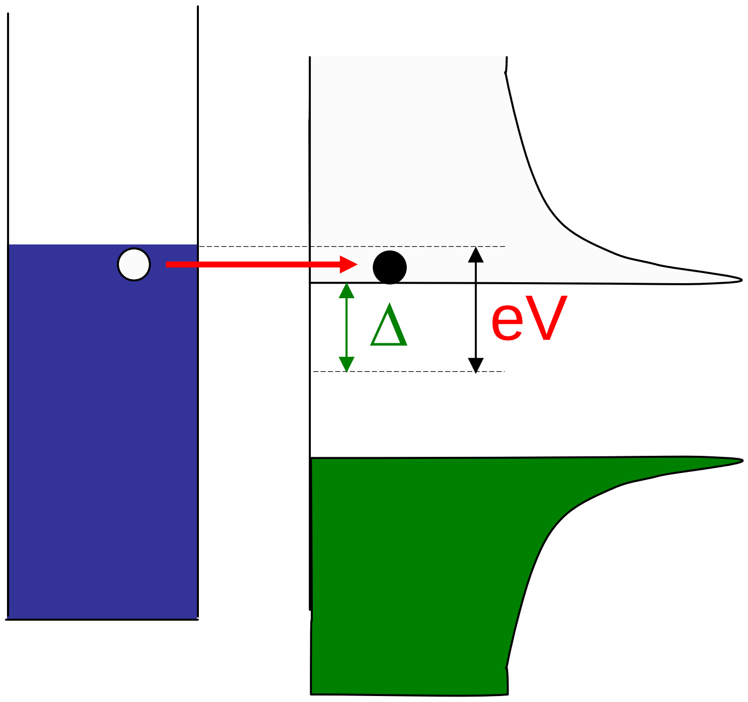
f(E) ⟺ interactions

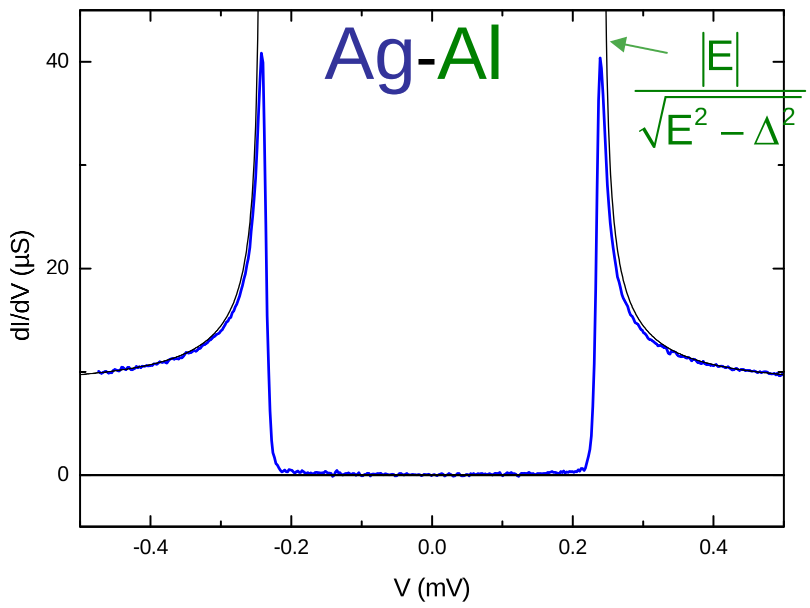
Accessing f(E) : conductance of an N-S tunnel junction



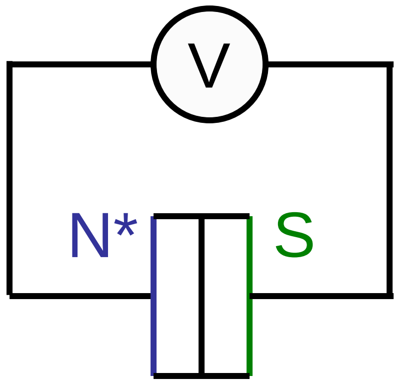
$$I = \frac{1}{eR_T}\int dE\, n_S(E)\,\big(f_N(E-eV) - f_S(E)\big)$$

$$\frac{dI}{dV} = -\frac{1}{R_T}\int dE\, n_S(E)\, f_N'(E-eV)$$



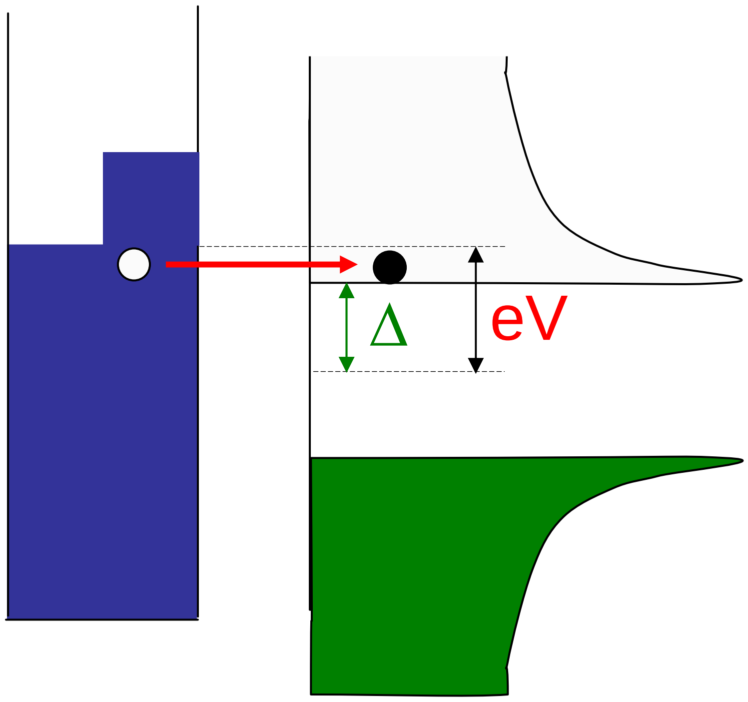


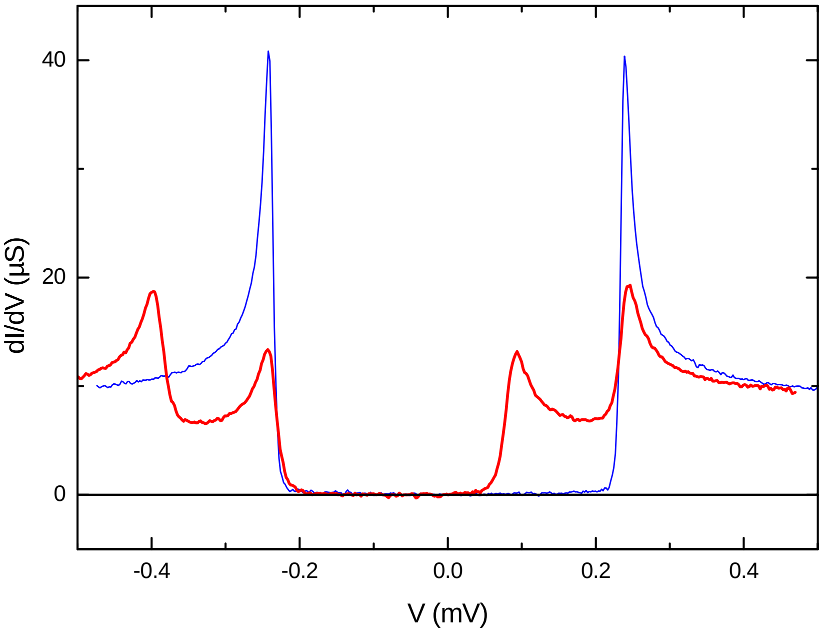
Accessing f(E) : conductance of an N-S tunnel junction



$$I = \frac{1}{eR_T} \int dE \, n_S(E) \left(f_N(E - eV) - f_S(E) \right)$$

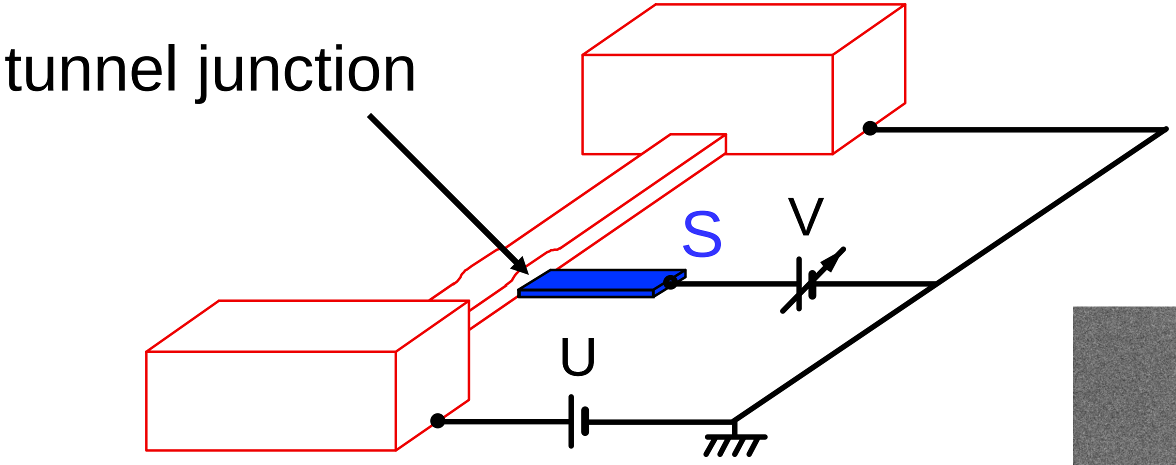
$$\frac{dI}{dV} = -\frac{1}{R_T} \int dE \, n_S(E) \, f_N'(E - eV)$$

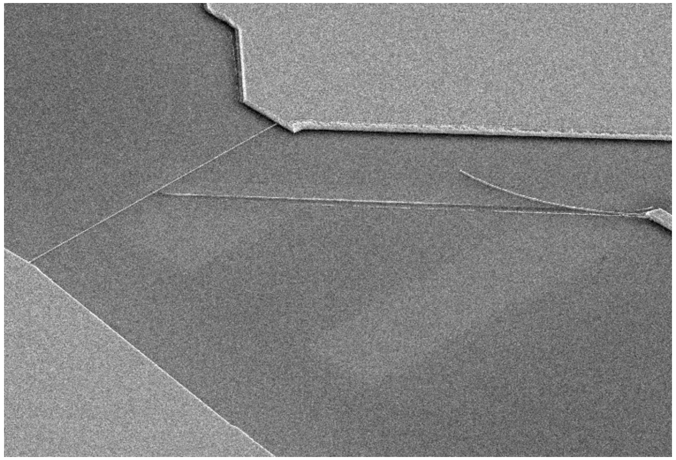




Spectroscopy of f_N

Experimental setup





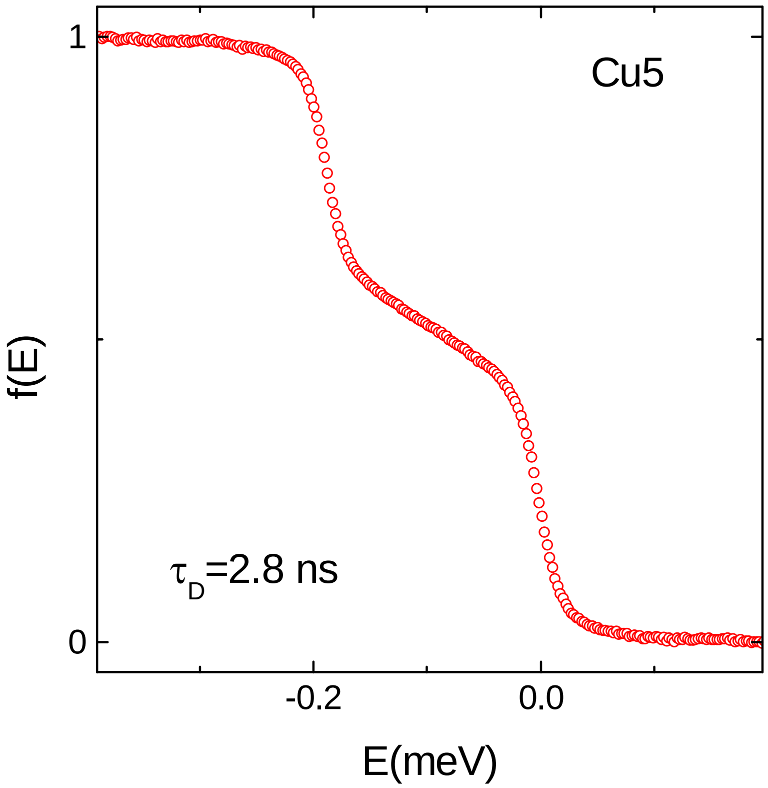
L=5 to 40 µm

Diffusion time: $\tau_D = \frac{L^2}{D} = 1$ to 60 ns

$$\frac{dI}{dV}(V) \xrightarrow[\textit{deconvolution}]{\textit{numerical}} f(E)$$

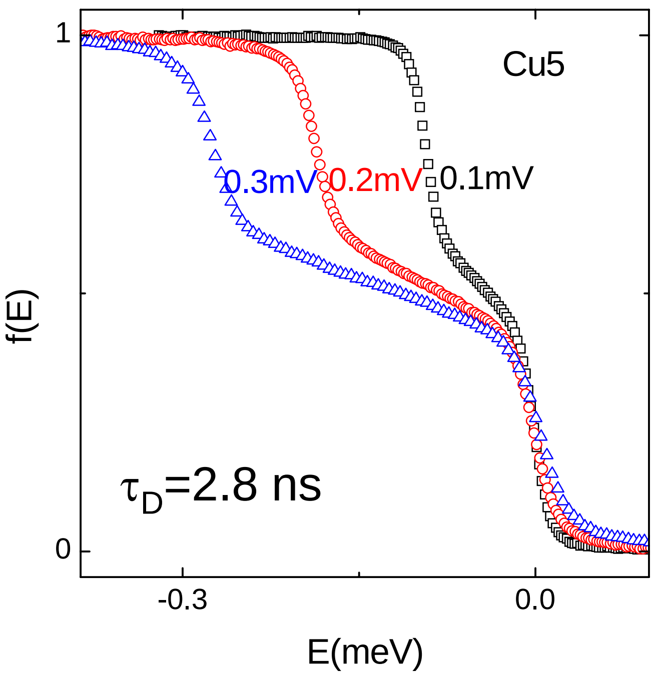
f(E) measurement

U=0.2 mV



Problems raised by f(E) measurements

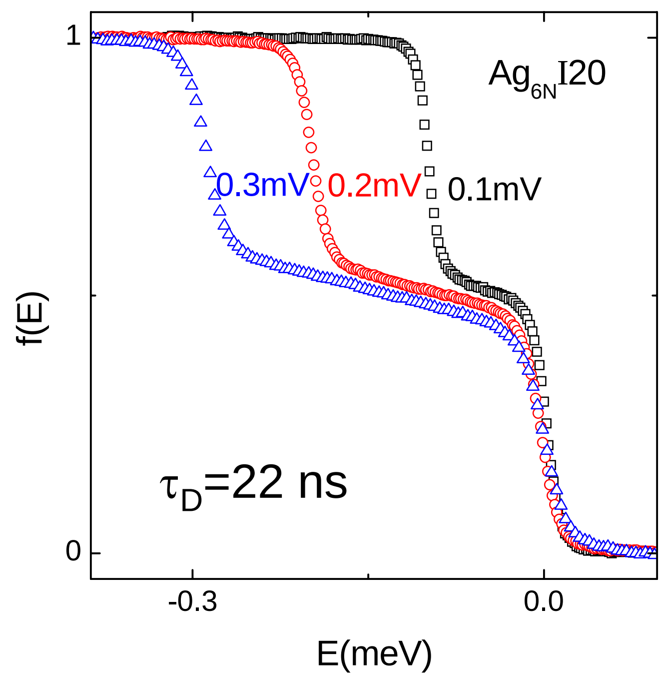
Ag_{5N} - $Cu_{5N,4N}$ - Au_{4N}



- large rates
- slope $\propto 1/U$ (scaling)

H. Pothier *et al.*,
PRL **79**, 3490 (1997)

Ag_{6N} (99.9999%)



- small rates
- similar slopes at $\neq$ U's

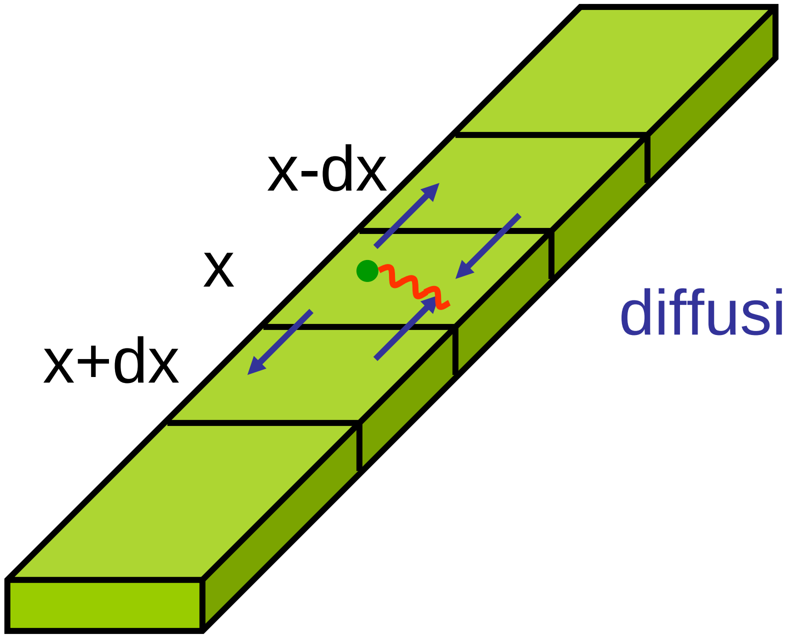
F. Pierre *et al.*,
J. Low Temp Phys. 118, 437 (2000)

Stronger interactions ?!

Calculation of f(x,E)

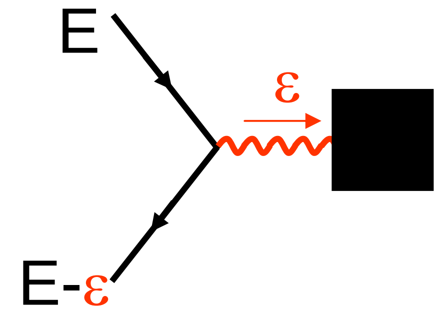
Boltzmann equation in the diffusive regime (Nagaev, Phys. Lett. A, 1992):

$$\cancel{\frac{\partial f(E)}{\partial t}} + D\frac{\partial^2 f(E)}{\partial x^2} = I_{in}\left(x,E,\{f\}\right) - I_{out}\left(x,E,\{f\}\right)$$



diffusion in real space

transfers of population in the energy space



Boundary conditions :

$$f_{x=0}(E) = f_{x=L}(E) = \text{Fermi function}$$

Calculation of f(x,E)

<u>Boltzmann equation in the diffusive regime (Nagaev, Phys. Lett. A, 1992):</u>

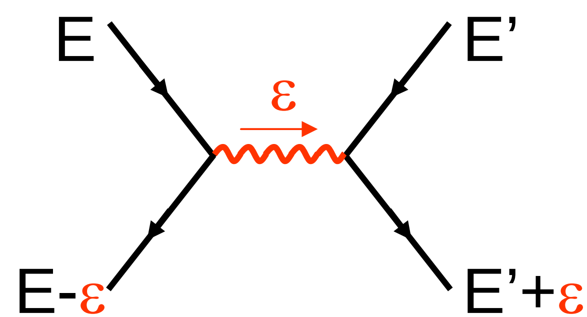
$$D \frac{\partial^2 f(E)}{\partial x^2} = I_{in}\left(x,E,\{f\}\right) - I_{out}\left(x,E,\{f\}\right)$$

e-e interactions :

$$\frac{\kappa}{\varepsilon^{3/2}}$$

(Altshuler, Aronov, Khmelnitskii, 1982)

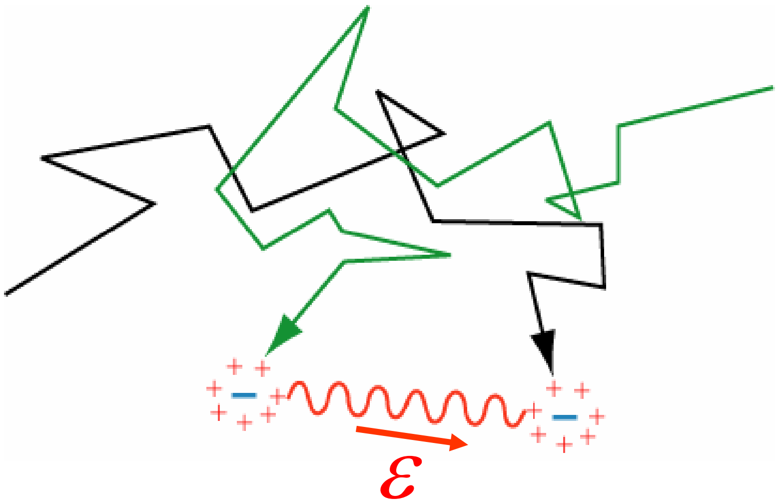
$$I_{out}\left(x,E,\{f\}\right) = \int dE' d\varepsilon \, K(\varepsilon) \, f(E) \left[1 - f(E-\varepsilon)\right] f(E') \left[1 - f(E'+\varepsilon)\right]$$



fn3D

Theory of screened Coulomb interaction in the diffusive regime

(Altshuler, Aronov, Khmelnitskii, 1982)



ingredients:
- polarisability ↘
- overlap ↗

Prediction for 1D wire :

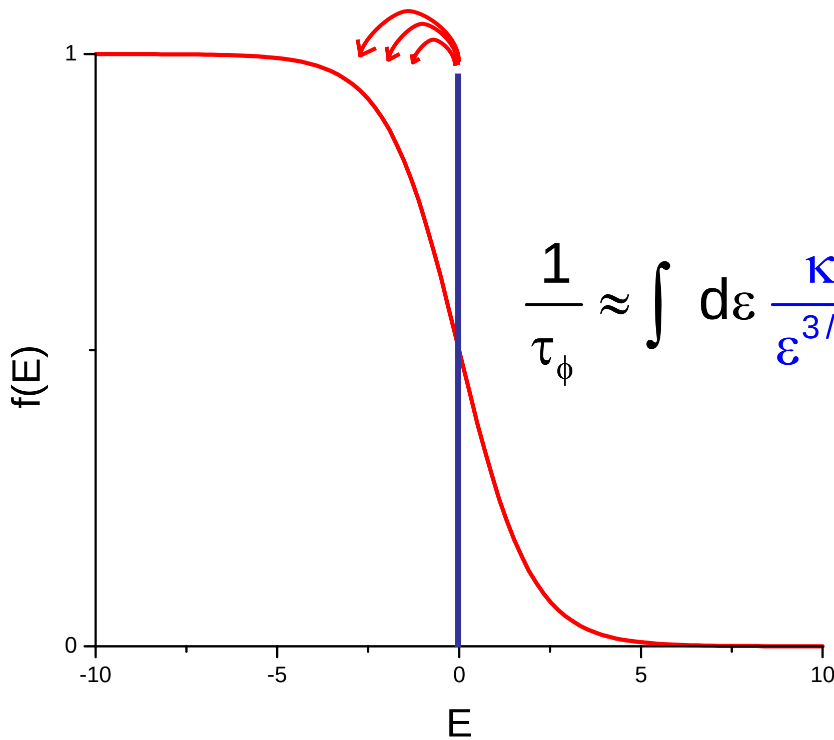
$$K(\varepsilon) = \frac{\kappa}{\varepsilon^{3/2}}$$

$$\left(\propto \int \frac{\mathrm{d}q}{D^2 q^4 + \omega^2} \right)$$

$$\kappa = \left(\sqrt{2D}\ \pi\ \hbar^{3/2}\ \nu_F\ \mathrm{S_e} \right)^{-1}$$

Collision int. for Coulomb interactions and $\tau_\phi(T)$

$$I_{out}\left(x,E,\{f\}\right) = \int dE'\, d\varepsilon\, \frac{\kappa}{\varepsilon^{3/2}}\, f(E)\left[1-f(E-\varepsilon)\right] f(E')\left[1-f(E'+\varepsilon)\right]$$



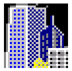
$$\frac{\partial f}{\partial t} = I\left(x,0,f_{Fermi}\right) = -\frac{f}{\tau_\phi}$$

$$\frac{1}{\tau_\phi} \approx \int d\varepsilon\, \frac{\kappa}{\varepsilon^{3/2}}\left[1-f(E-\varepsilon)\right]\int dE'\; f(E')\left[1-f(E'+\varepsilon)\right]$$

$$= \int_{\hbar/\tau_\phi}^{kT} d\varepsilon\, \frac{\kappa}{\varepsilon^{3/2}}\; kT$$

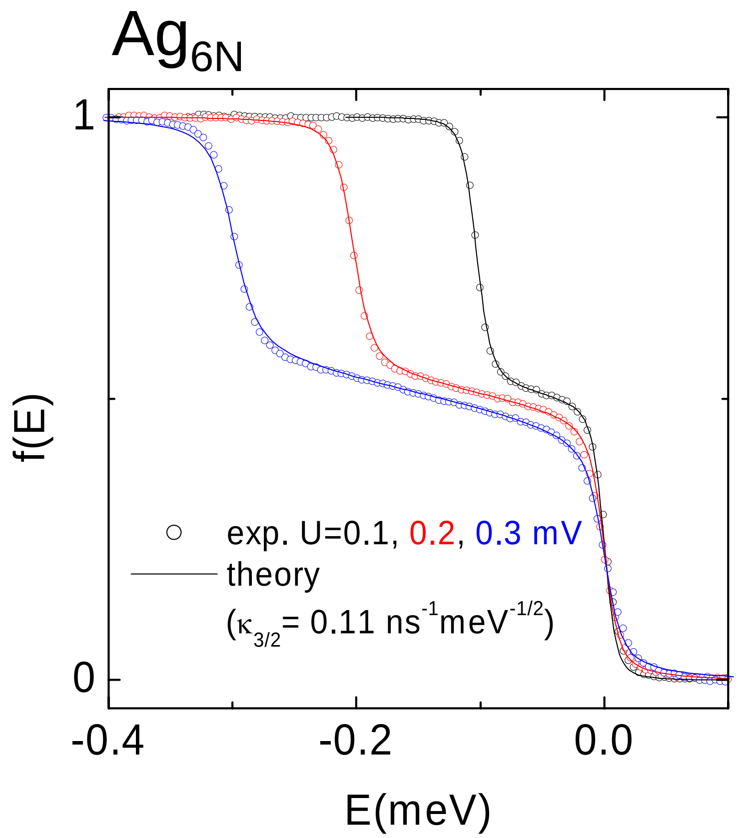
$$\approx \frac{2\kappa}{\left(\hbar/\tau_\phi\right)^{1/2}}\; kT$$

$$\frac{1}{\tau_\phi} \approx \frac{\left(2\kappa kT\right)^{2/3}}{\hbar^{1/3}} \propto T^{2/3}$$



fn3D

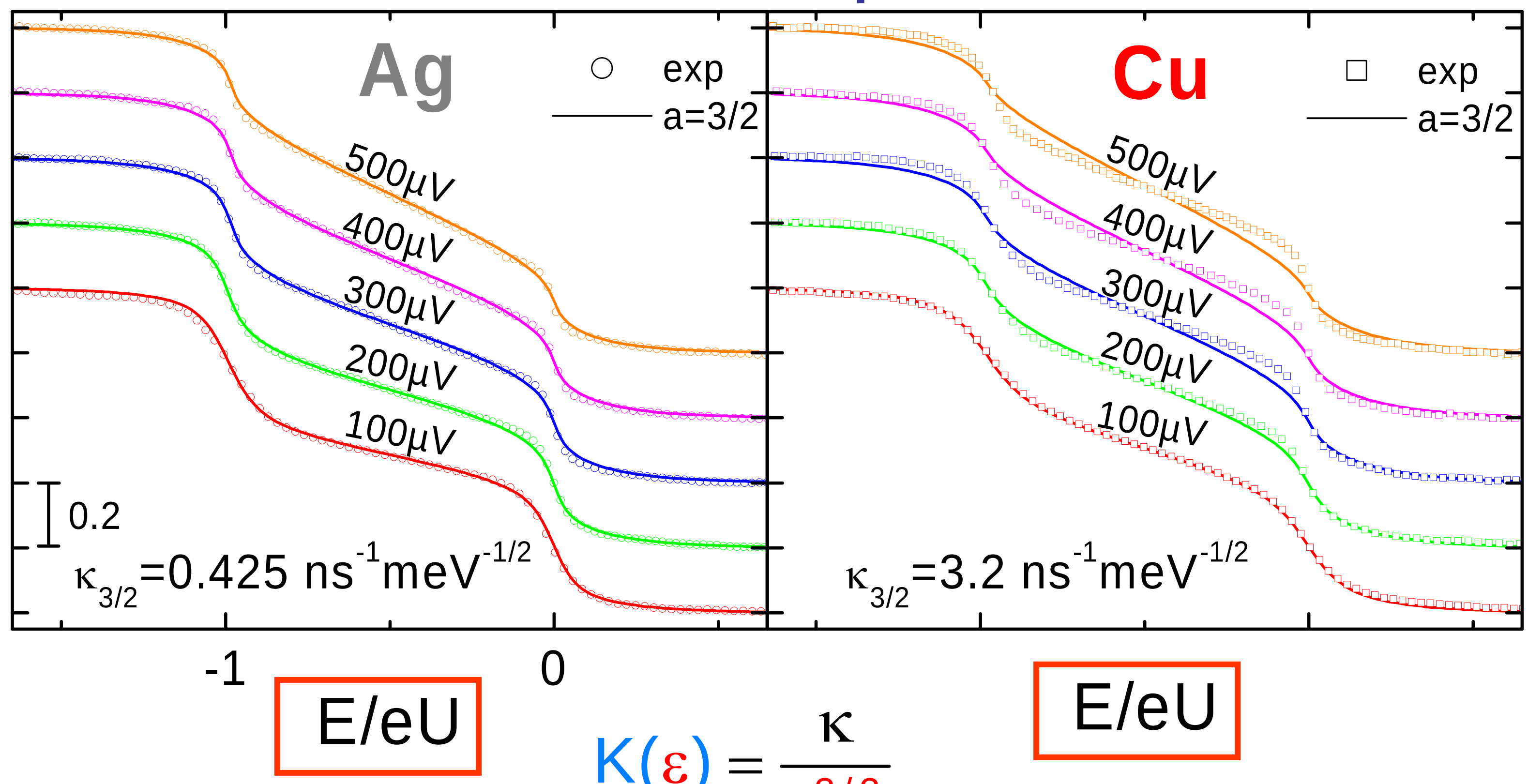
f(E) in clean samples: Coulomb interaction only



$$K(\varepsilon) = \frac{\kappa}{\varepsilon^{3/2}}$$

—— Fits

Fitting f(E) with Coulomb interactions: other examples

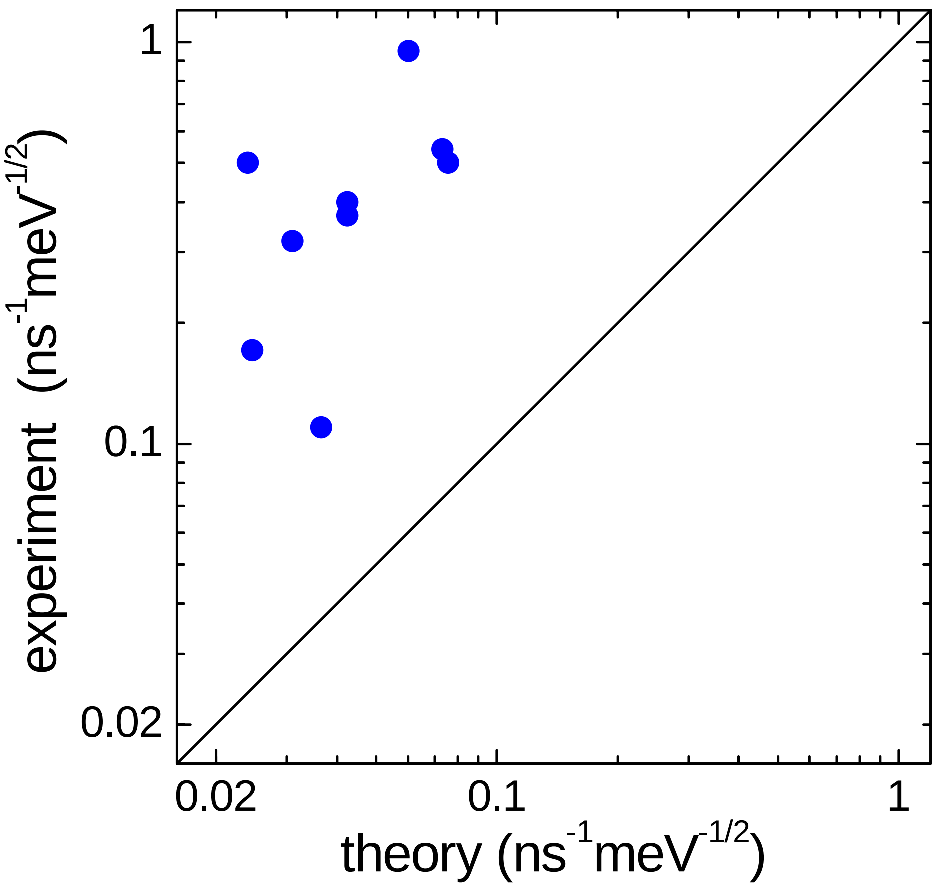


$$K(\varepsilon) = \frac{\kappa}{\varepsilon^{3/2}}$$

OK

U-dependence not reproduced

f(E) in clean samples: Coulomb interaction intensity



$$K(\varepsilon) = \frac{\kappa}{\varepsilon^{3/2}}$$

$$\kappa^{th} = \left(\sqrt{2D}\ \pi\ \hbar^{3/2}\ \nu_F\ S_e \right)^{-1}$$

(Huard *et al.*,
Solid State Commun. (2004))

Good fits, but « wrong » intensities !

Comparison of the results of the two methods

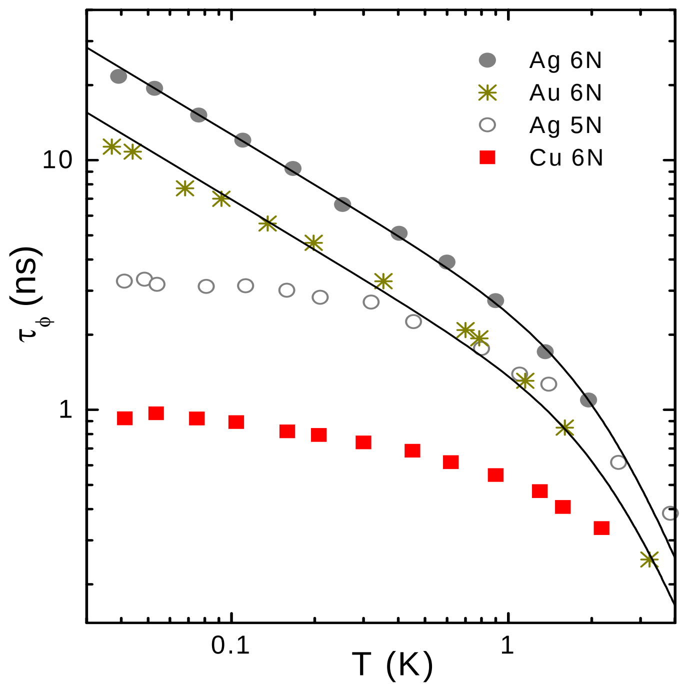
	$\tau_\phi(T)$		f(E)
Ag_{6N}	$\propto T^{-2/3}$	*Coulomb interactions*	$K(\varepsilon) = \frac{\kappa}{\varepsilon^{3/2}}$ (intensity?)
Ag_{5N} $Cu_{5N,4N}$ Au_{4N}	saturation	*Other mechanism*	fast relaxation rates $K(\varepsilon) \not\propto \frac{1}{\varepsilon^{3/2}}$

ROLE OF RESIDUAL IMPURITIES ?

- phase coherence and electrical transport
- phase coherence in wires and interactions
- interactions and energy exchange
- effect of residual impurities

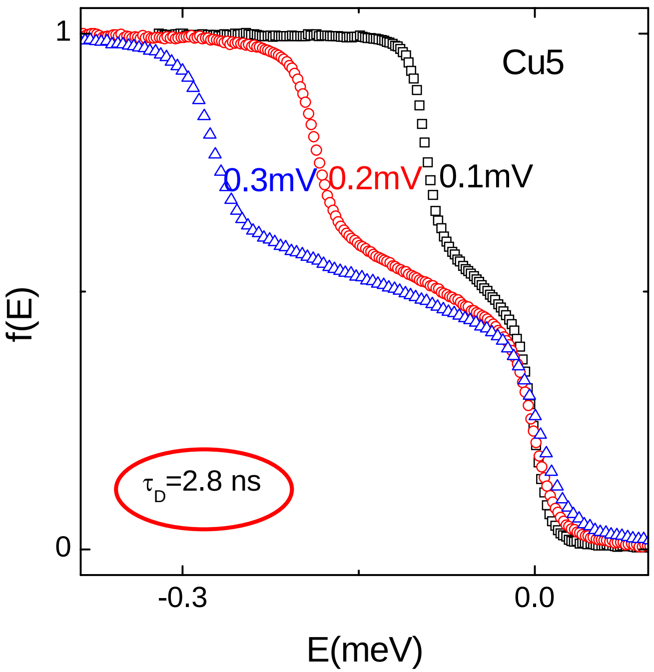
The two puzzles

$\tau_\phi(T)$ measurements



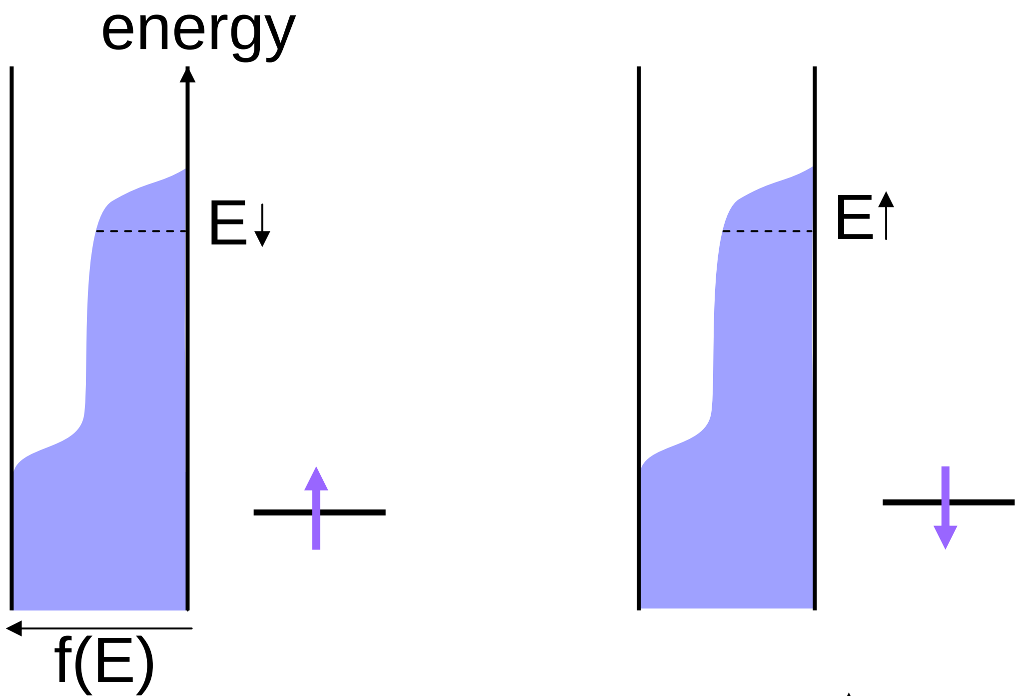
f(E) measurements

Ag_{5N} - $Cu_{5N,4N}$ - Au_{4N}

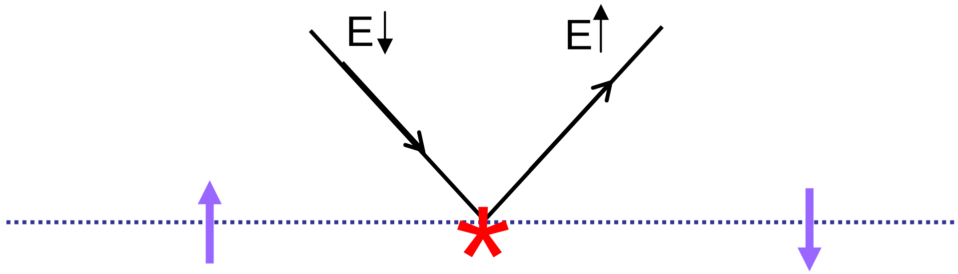


Anomalous interactions in the less pure samples

Spin-flip scattering on a ***magnetic*** impurity

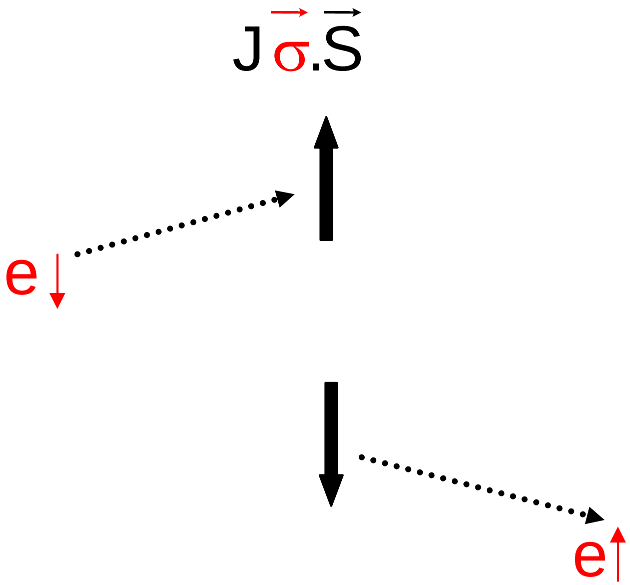


- dephasing
(Hikami, Larkin and Nagaoka, Prog. Theor. Phys. 1980)
- no change of energy



rate maximal at Kondo temperature

Kondo effect

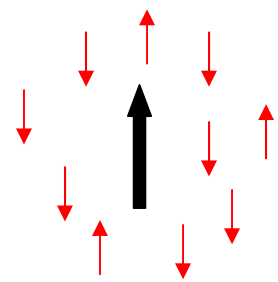


Spin-flip scattering

⇨ increased resistivity

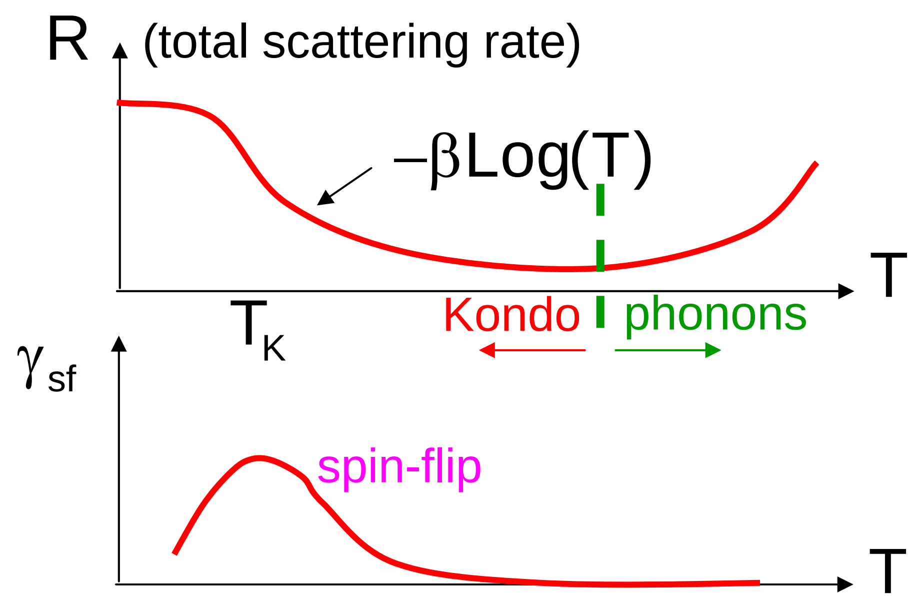
⇨ reduction of τ_ϕ

Collective effect:

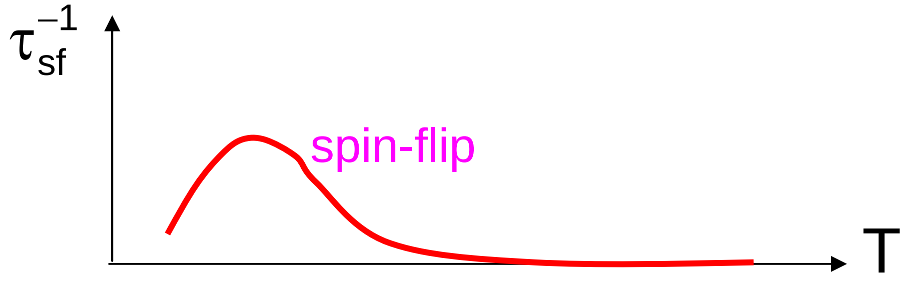


Formation of a singlet spin state

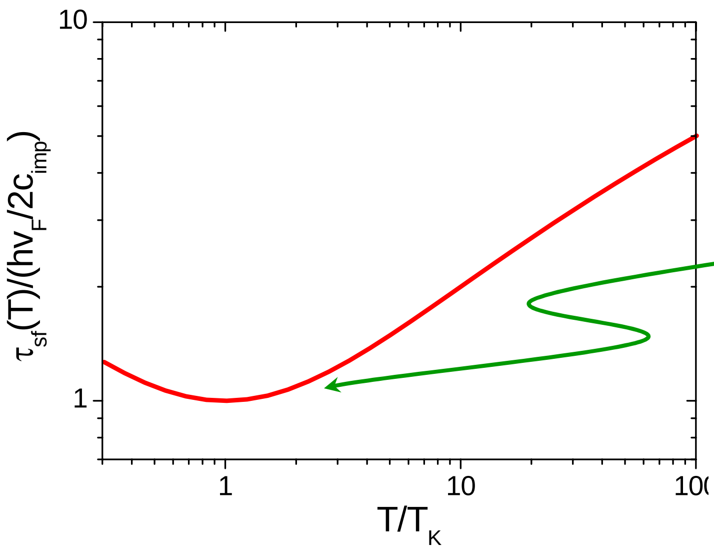
$k_B T_K \approx E_F e^{-1/\nu J}$



Nagaoka-Suhl expression of the spin-flip scattering rate near T_K



$$\frac{1}{\tau_{sf}} = \frac{c_{mag}}{\pi \hbar \nu_F} \frac{\pi^2 S(S+1)}{\pi^2 S(S+1) + \ln^2(T/T_K)}$$



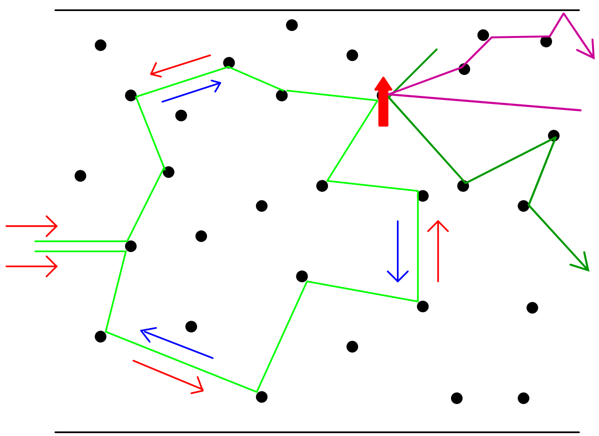
Weak temperature dependance near T_K !!

Link to τ_ϕ(T) saturation?

From τ_{sf} to τ_ϕ

Another important timescale: τ_K

Lifetime of the spin state
of a magn. imp.



(V.I. Fal'ko, JETP Lett. **53**, 340 (1991))

If $\tau_K < \tau_{sf}$ *Other electrons matter*

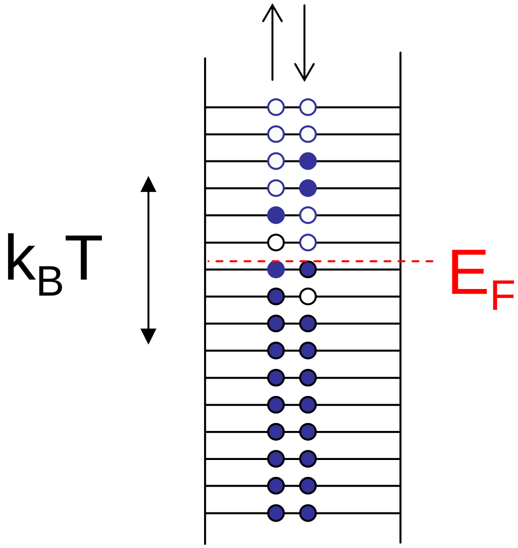
⇨ Randomising effect

$$\frac{1}{\tau_\phi} = \frac{1}{\tau_{ee}} + \frac{1}{\tau_{e-ph}} + \frac{1}{\tau_{sf}}$$

If $\tau_K > \tau_{sf}$ *The spin states of the mag. imp. seen by time-reversed electrons are correlated*

$$\frac{1}{\tau_\phi} = \frac{1}{\tau_{ee}} + \frac{1}{\tau_{e-ph}} + \frac{2}{\tau_{sf}}$$

Comparison of τ_{sf} and τ_K



$\nu_F k_B T$ concentration of electrons that can spin-flip

c_{imp} concentration of magnetic impurities

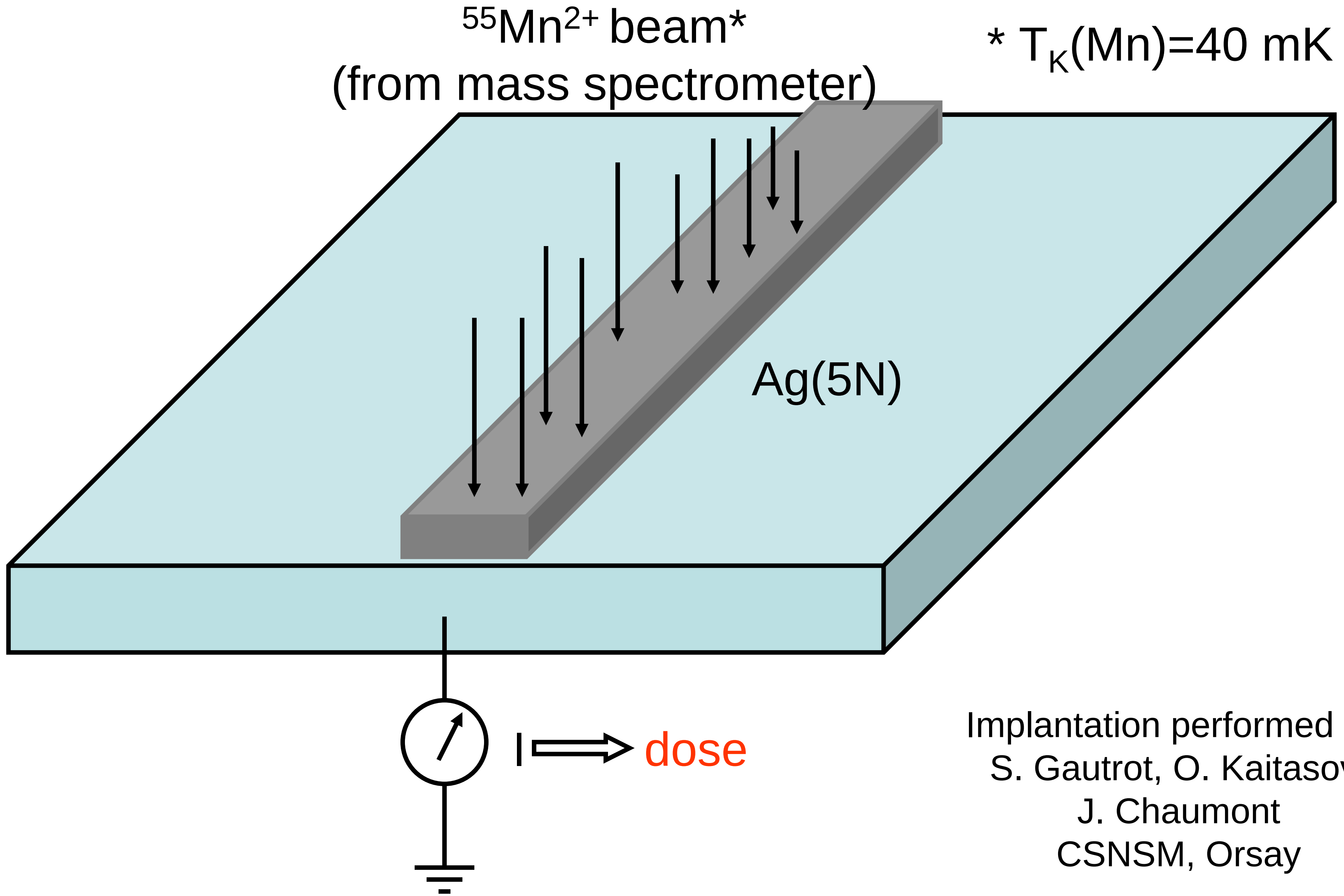
If $\nu_F k_B T > c_{imp}$: $\tau_K < \tau_{sf}$

$$\frac{1}{\tau_\phi} = \frac{1}{\tau_{ee}} + \frac{1}{\tau_{e-ph}} + \frac{1}{\tau_{sf}}$$

Numerically,
for Au, Ag, Cu, …

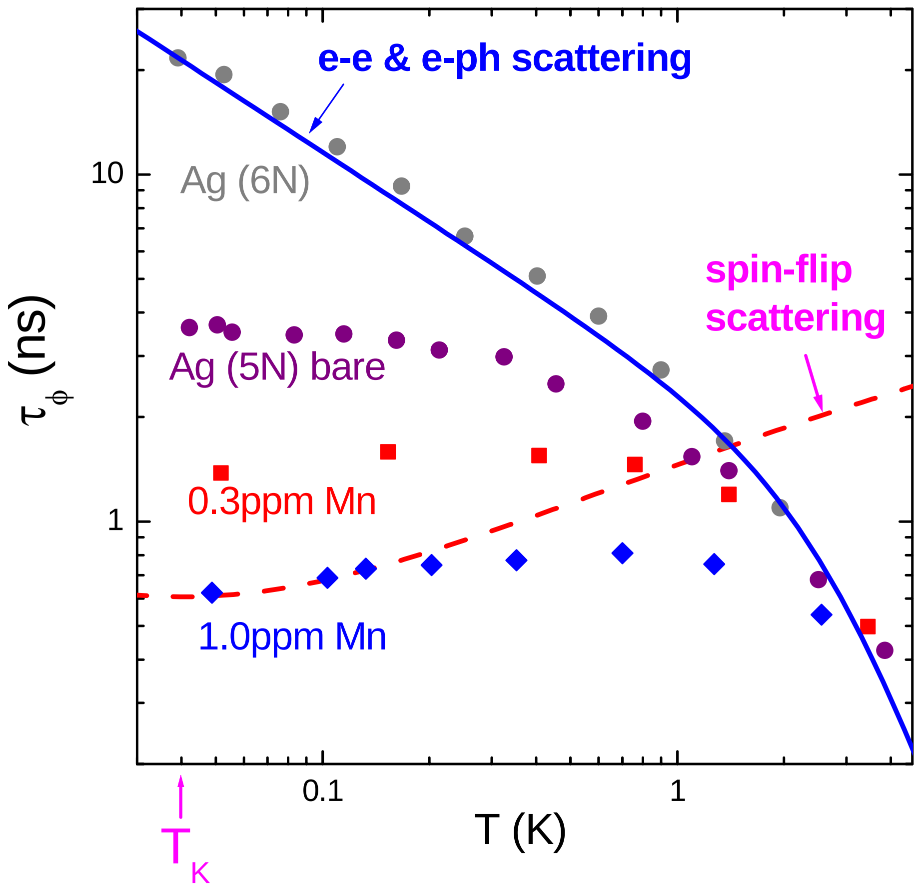
$$T > 40\ \text{mK} \times c_{imp}(\text{ppm})$$

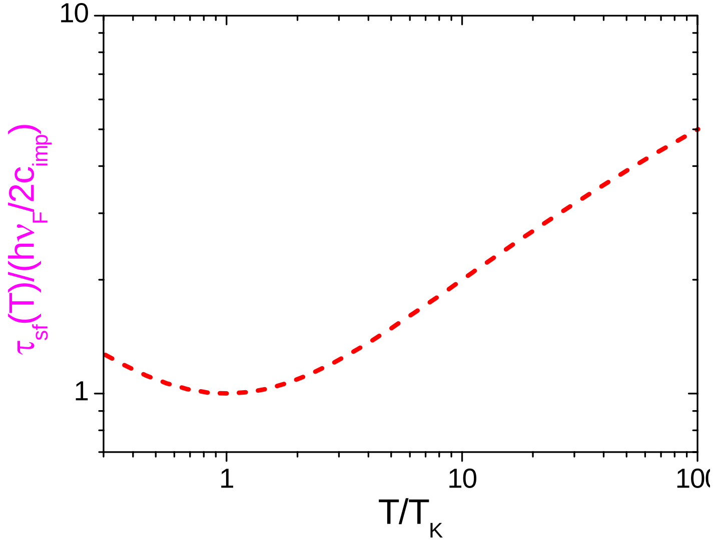
Experimental investigation of the effect of magnetic impurities



Implantation performed by:
S. Gautrot, O. Kaitasov,
J. Chaumont
CSNSM, Orsay

Effect of magnetic impurities on τ_ϕ



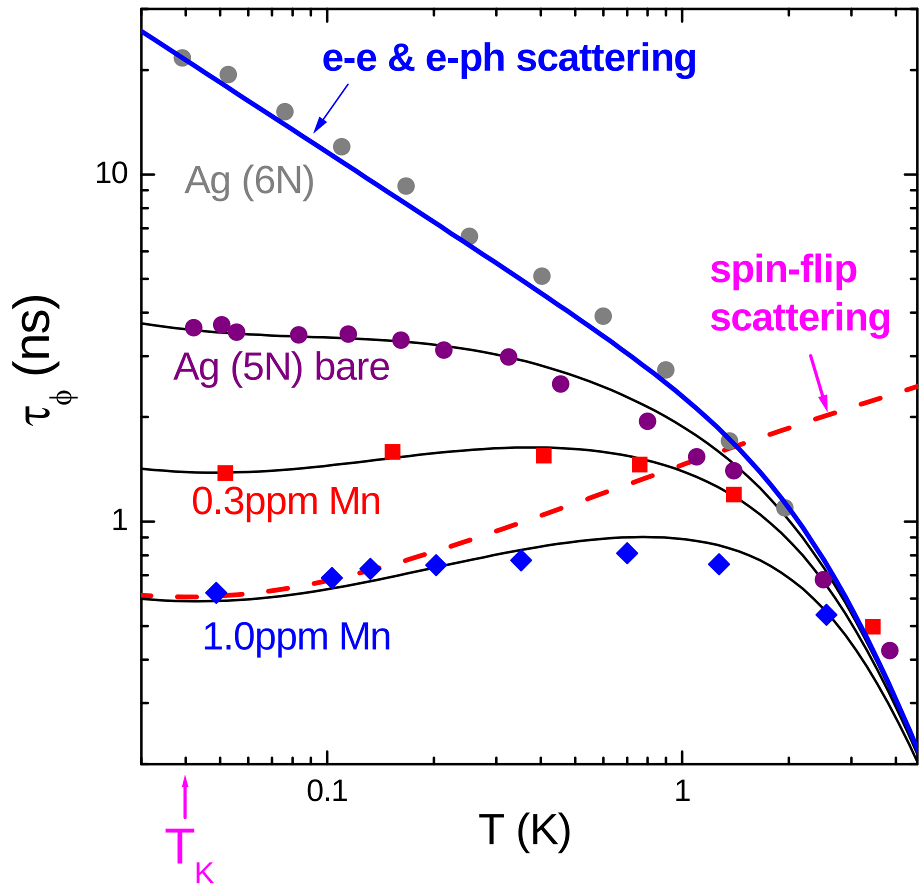


Spin-flip rate peaks at T_K:

$$\tau_\phi(T_K) = \frac{0.6 \text{ ns}}{c_{imp}\,(\text{ppm})}$$

$$\frac{1}{\tau_\phi} = \frac{1}{\tau_{ee}} + \frac{1}{\tau_{e-ph}} + \frac{1}{\tau_{sf}}$$

Effect of magnetic impurities on τ_ϕ



F. Pierre *et al.*,
PRB **68**, 0854213 (2003)

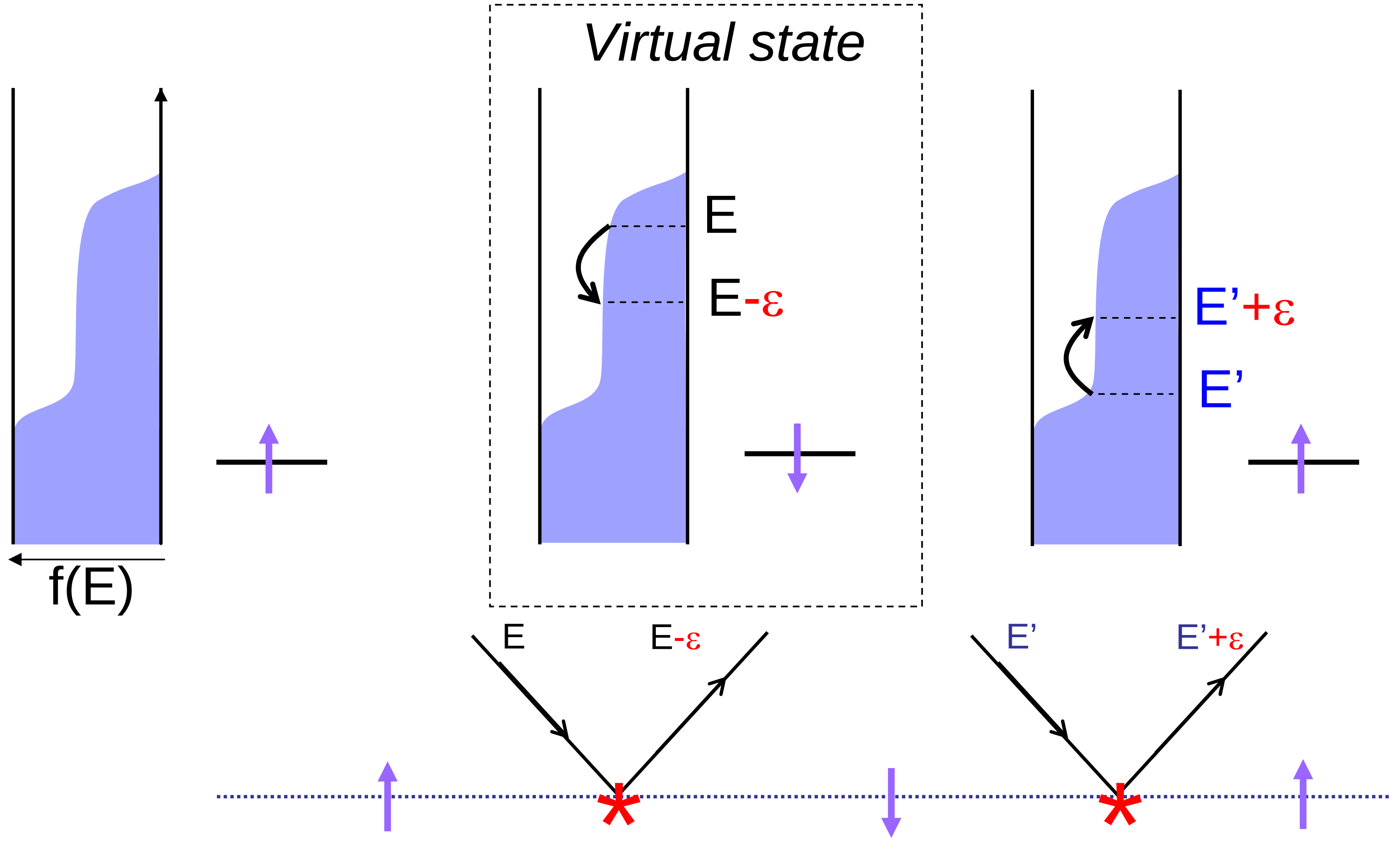
<u>Fit parameters:</u>
Ag(5N) bare: *0.13 ppm*
+ 0.3 ppm : *0.40 ppm*
+ 1 ppm : *0.96 ppm*

Above T_K : partial compensation of e-e and s-f

→ apparent saturation

What about energy exchange ?

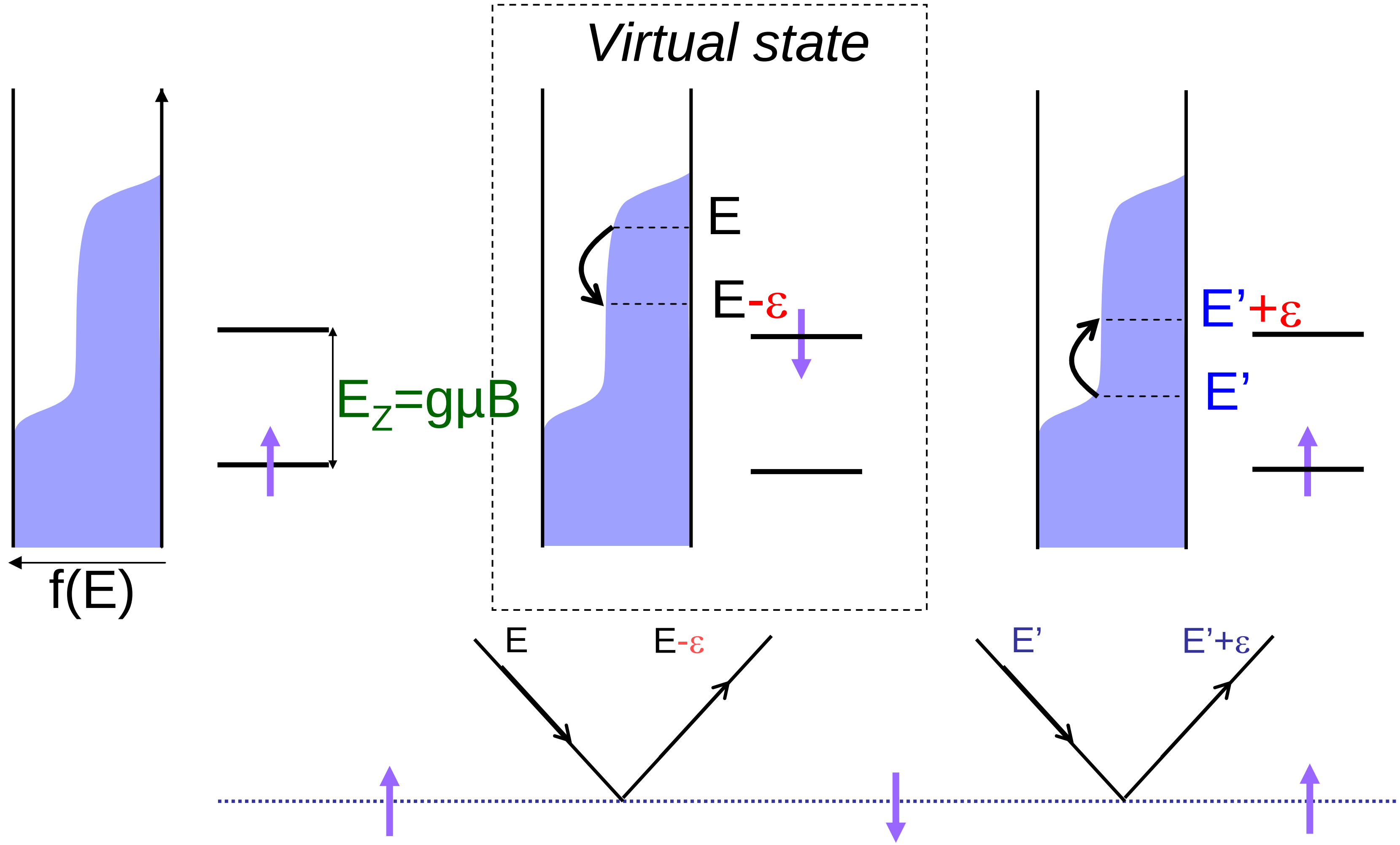
Interaction between electrons mediated by a magnetic impurity



Reinforced by Kondo effect

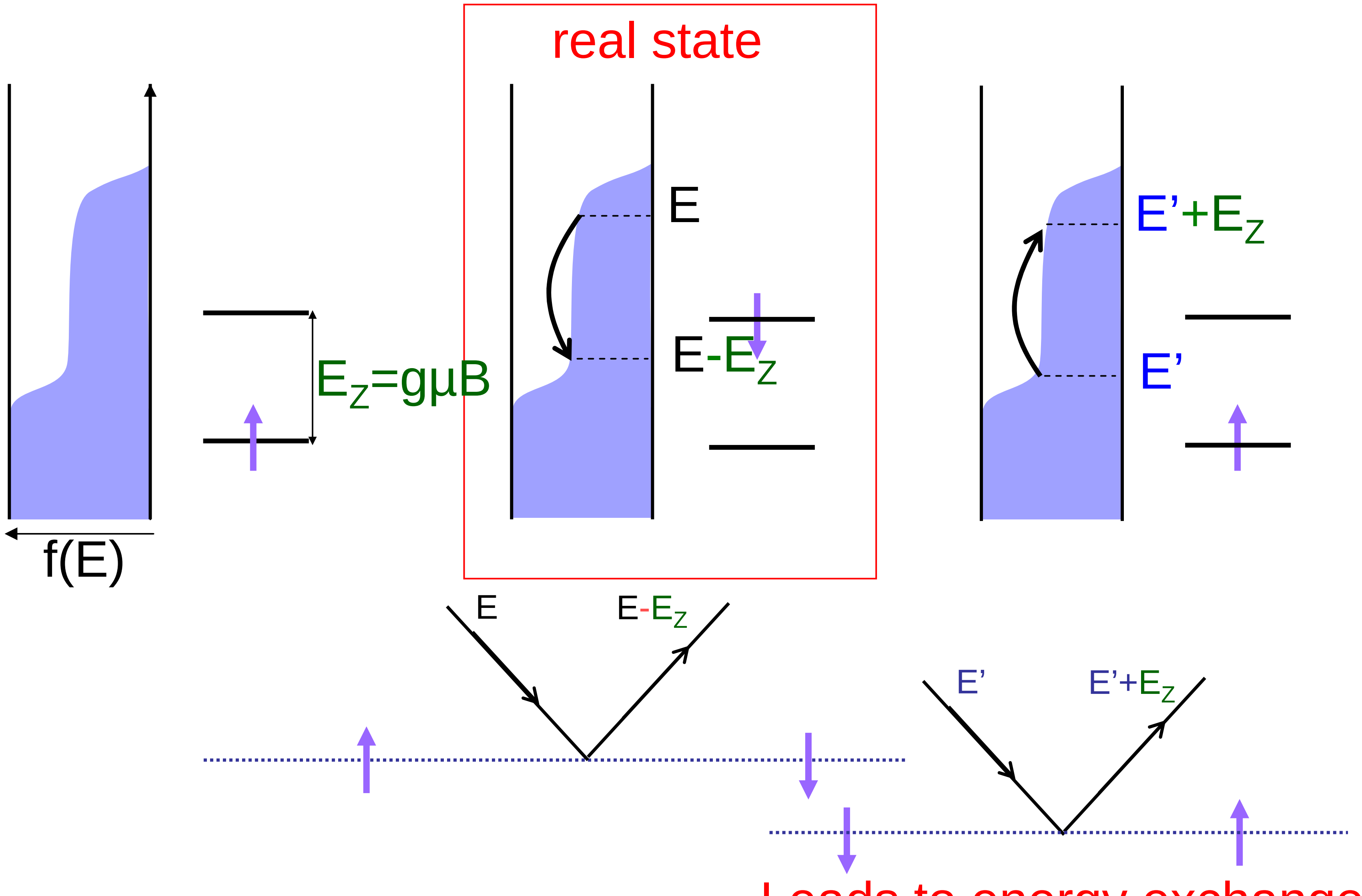
Kaminski and Glazman, PRL **86**, 2400 (2001)

Interaction mediated by a magnetic impurity : *effect of a small magnetic field*



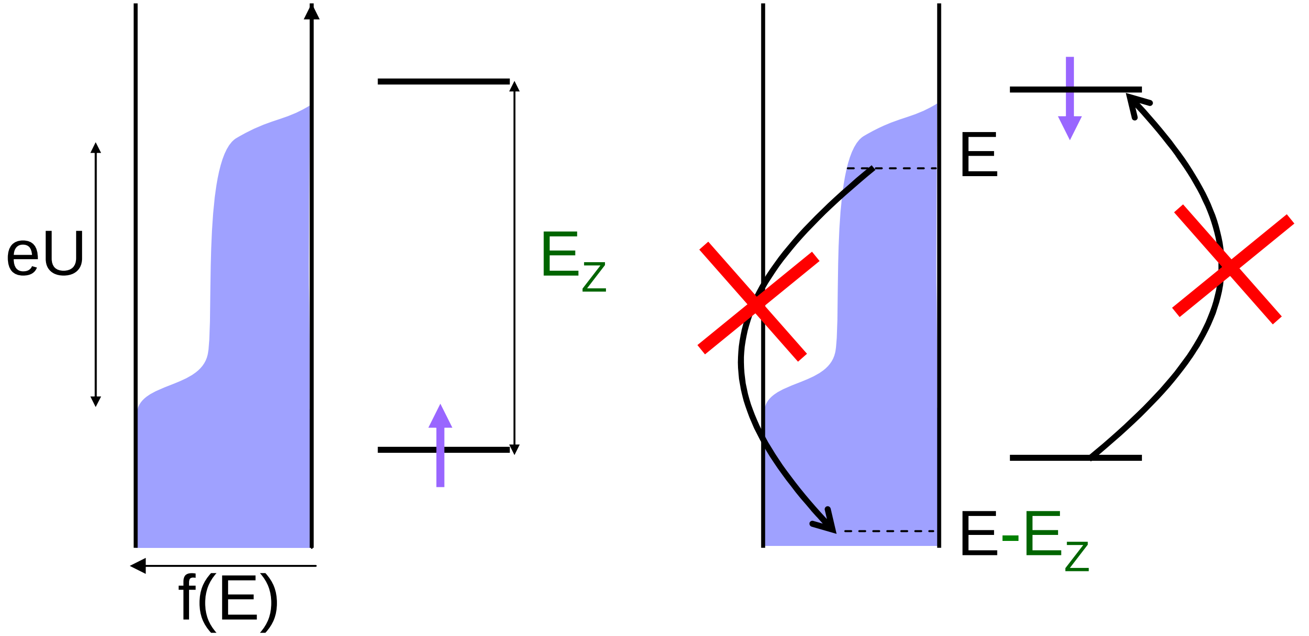
Reduced rate

Spin-flip scattering on a magnetic impurity : effect of a *small* magnetic field



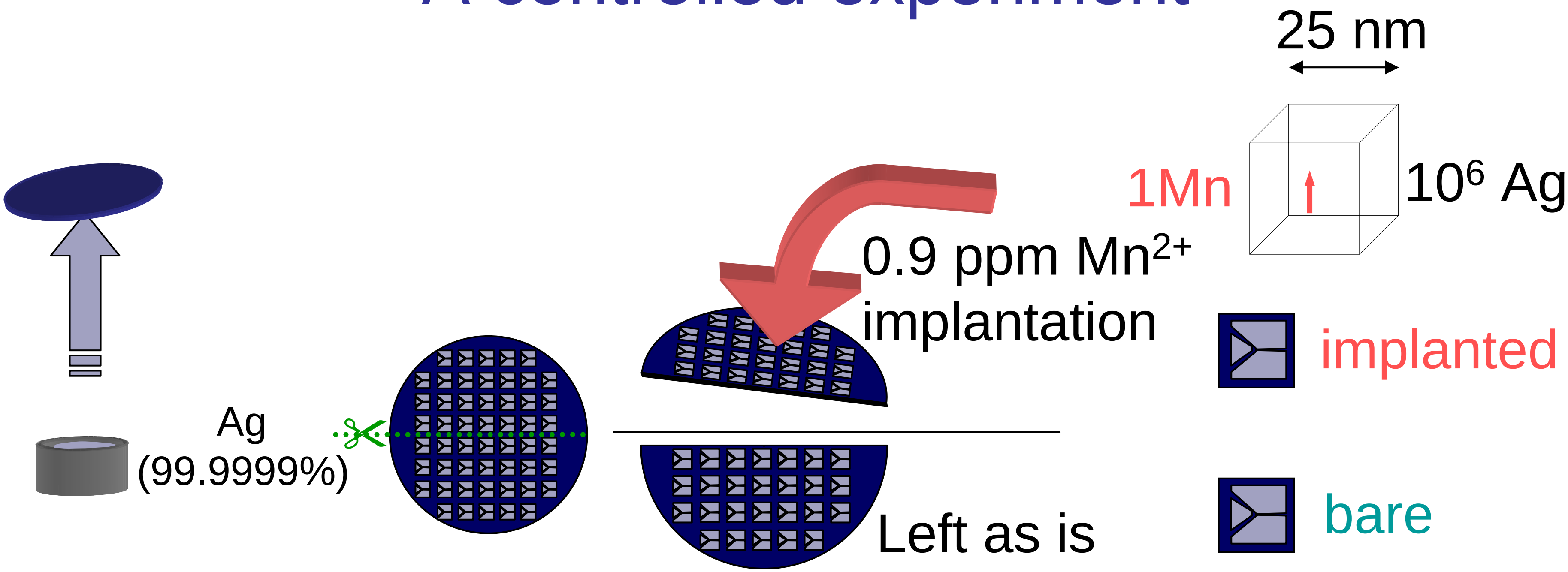
Leads to energy exchange

Spin-flip scattering on a magnetic impurity : effect of a *strong* magnetic field



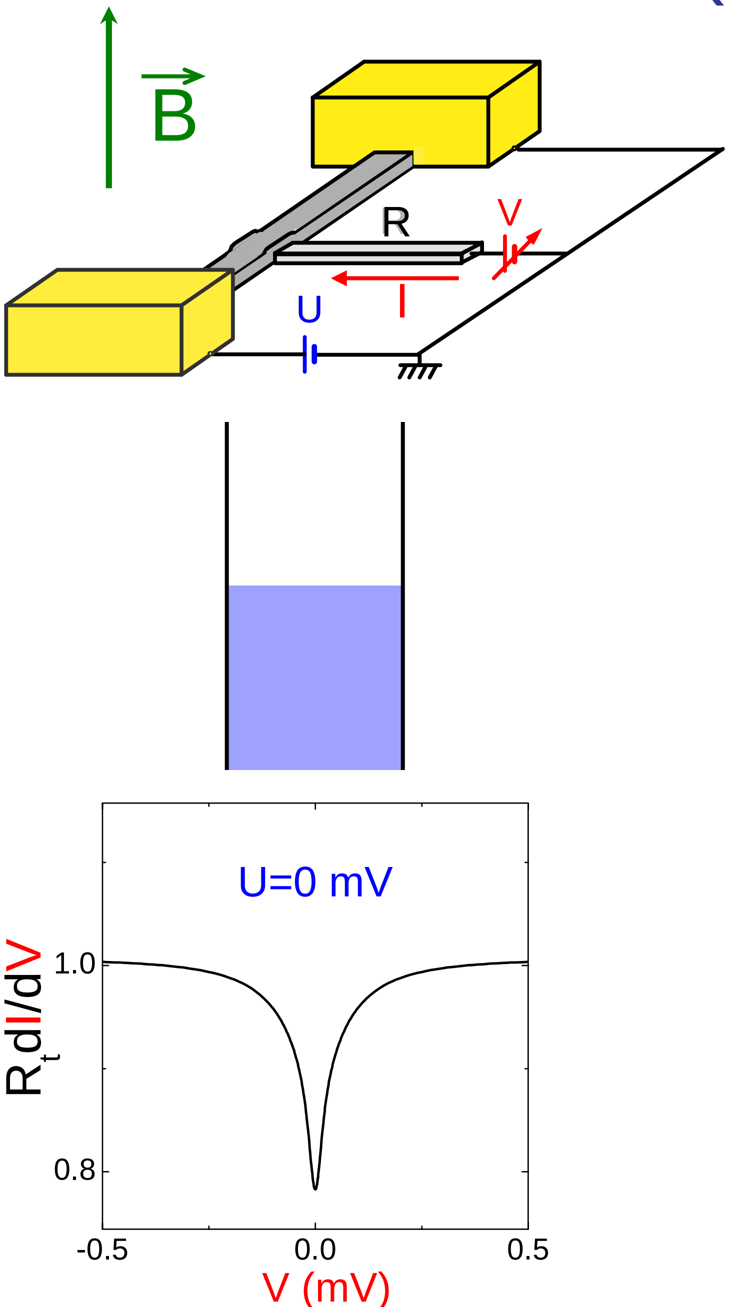
Freezing of impurities

A controlled experiment

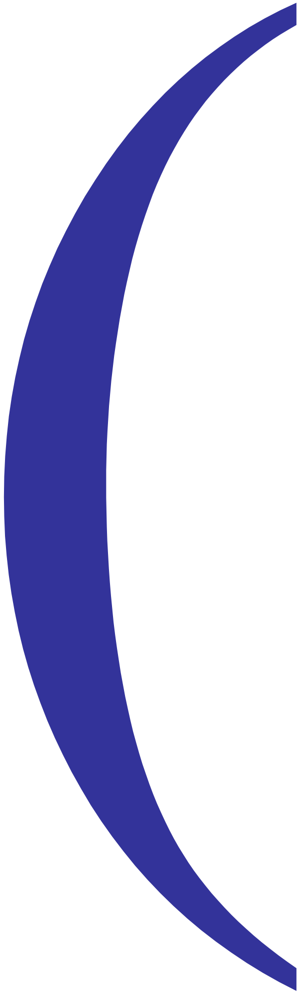


Effect of 1 ppm Mn on interactions ?

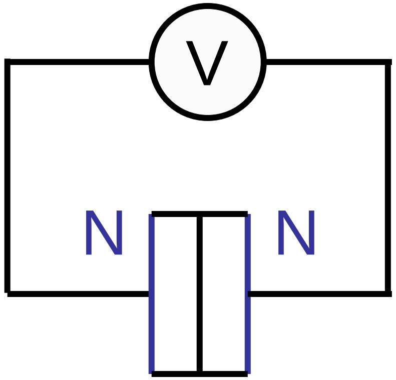
Measurement of f(E) in presence of a mag. field



Dynamical Coulomb blockade (ZBA)

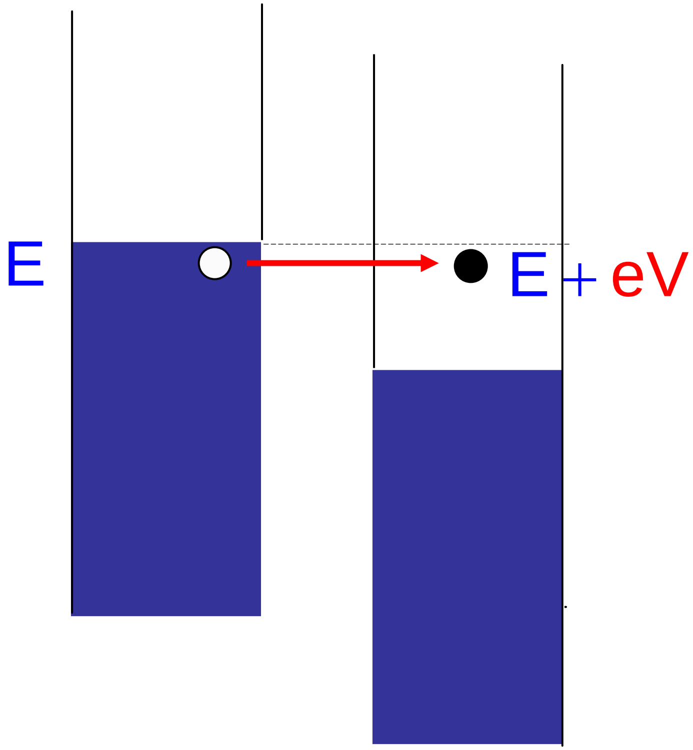


Conductance of an N-N junction

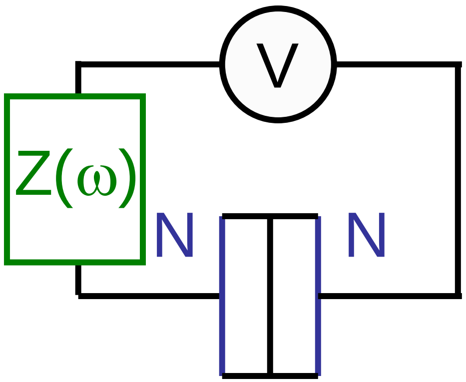


$$I = \frac{1}{eR_T} \int dE \left(f_L(E) - f_R(E + eV) \right)$$

$$\frac{dI}{dV} = \frac{1}{R_T}$$

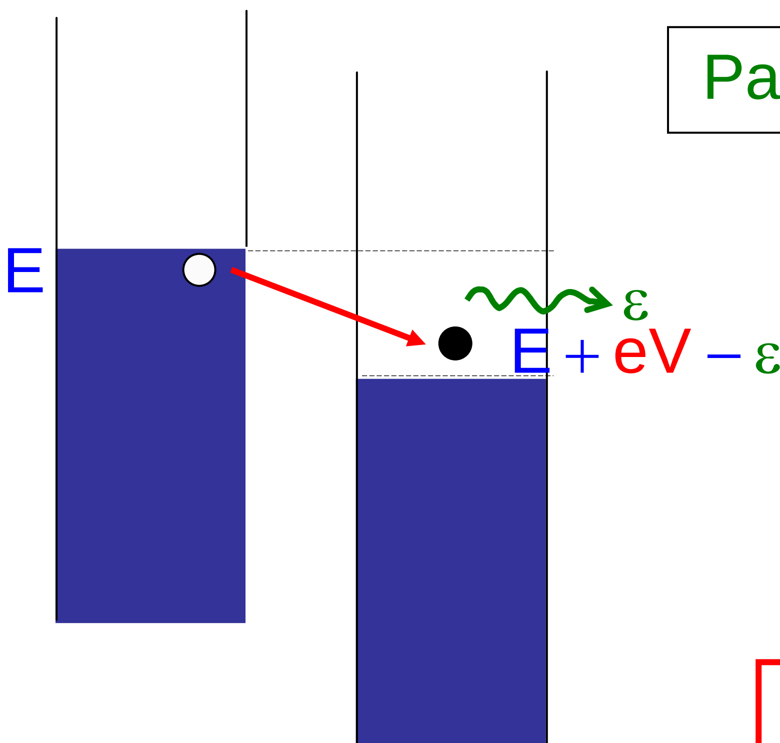


Conductance of an N-N junction
effect of an external impedance



$$I = \frac{1}{eR_T}\iint dE\, d\varepsilon \left(f_L(E) - f_R(E + eV - \varepsilon) \right) P(\varepsilon)$$

$$\frac{dI}{dV} = \frac{1}{R_T}\int_0^{eV} d\varepsilon\, P(\varepsilon) = \frac{1}{R_T}\left(1 - \int_{eV}^{\infty} d\varepsilon\, P(\varepsilon)\right)$$

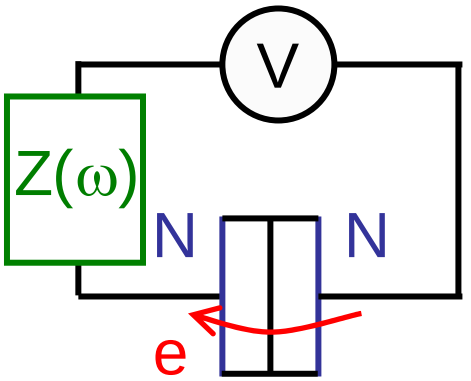


Part of the energy is dissipated in Z(ω)

Reduction of the phase space for QPs

Reduction of the conductance

Conductance of an N-N junction
perturbative calculation of P(ε)



For one electron tunneling :

$$I(t) = e\,\delta(t)$$

Energy dissipated in the impedance :

$$E = \int dt\, V(t)\, I(t)$$

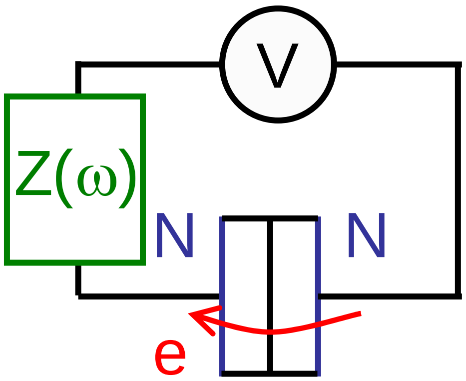
$$= eV(t=0)$$

$$= e\int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, v(\omega)$$

$$= e\int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, Z(\omega)\, i(\omega)$$

$$= \frac{e^2}{\pi}\int_{0}^{\infty} d\omega\, \mathrm{Re}\big(Z(\omega)\big)$$

Conductance of an N-N junction
perturbative calculation of $P(\varepsilon)$



For one electron tunneling :

$$I(t) = e\,\delta(t)$$

Energy dissipated in the impedance :

$$E = \frac{e^2}{\pi}\int_0^\infty d\omega\,\mathrm{Re}\left(Z(\omega)\right)$$

$$= \int_0^\infty d\varepsilon\,\varepsilon\,P\left(\varepsilon\right)$$

$$\Rightarrow \quad P\left(\varepsilon\right) = \frac{2\,\mathrm{Re}\left(Z(\varepsilon/\hbar)\right)}{\varepsilon R_K}$$

Conductance of an N-N junction
Perturbative result

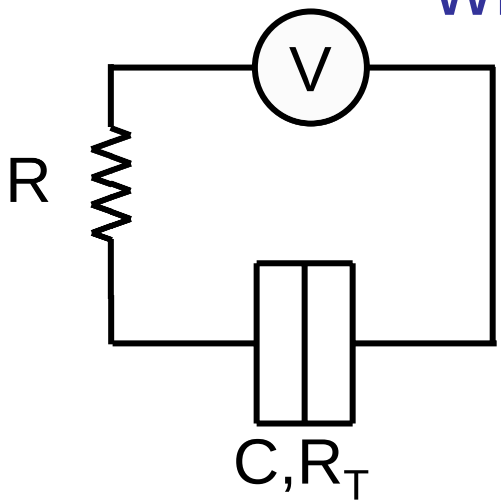
$$\frac{dI}{dV} = \frac{1}{R_T}\left(1 - \int_{eV}^{\infty} d\varepsilon\, P(\varepsilon)\right)$$

$$P(\varepsilon) = \frac{2\,\mathrm{Re}\left(Z(\varepsilon/\hbar)\right)}{\varepsilon R_K}$$

$$\frac{dI}{dV} = \frac{1}{R_T}\left(1 - 2\int_{eV/\hbar}^{\infty} \frac{\mathrm{Re}\,Z(\omega)}{R_K}\frac{d\omega}{\omega}\right)$$

Non-perturbative, finite T: see Devoret *et al.*, PRL **64**, 1824 (1990)
Joyez & Esteve, PRB **68**, 1828 (1997)

Dynamical Coulomb blockade with a resistive environment



$$Z(\omega) = R // C = \frac{R}{1 + jRC\omega}$$

$$\mathrm{Re}\left(Z(\omega)\right) \approx R \text{ for } RC\omega \ll 1$$

$$\approx 0 \text{ for } RC\omega \gg 1$$

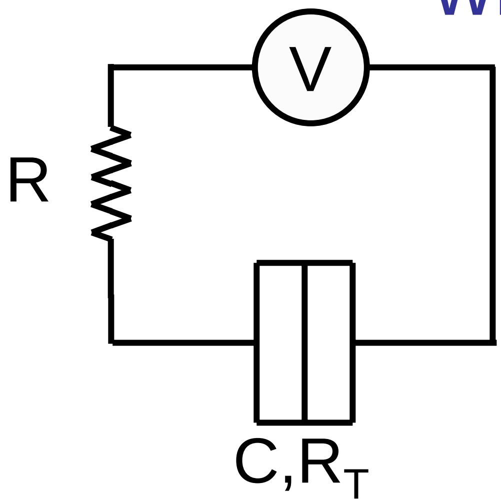
$$\frac{dI}{dV} \approx \frac{1}{R_T}\left(1 - 2\int_{eV/\hbar}^{(RC)^{-1}} \frac{R}{R_K}\frac{d\omega}{\omega}\right)$$

$$\approx \frac{1}{R_T}\left(1 + \frac{2R}{R_K}\log\frac{eV}{\hbar/RC}\right)$$

Perturbative result

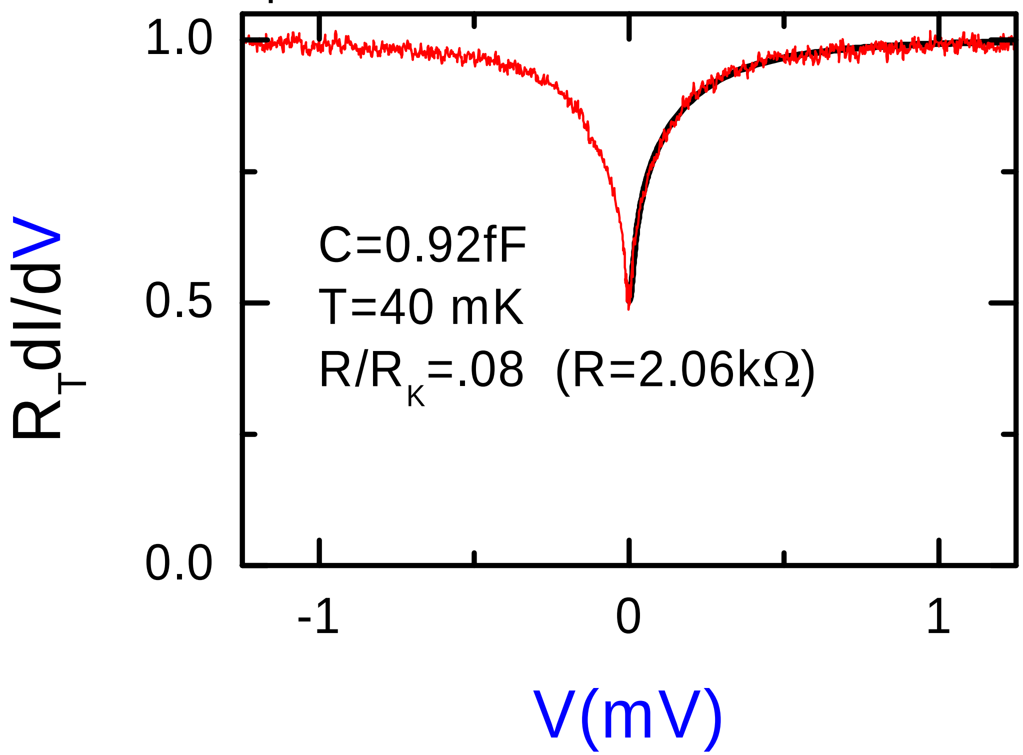
Non-perturbative result : for $k_B T \ll eV \ll \frac{\hbar}{RC}$ $\quad \frac{dI}{dV} \propto \left(V^{\frac{2R}{R_K}} + \text{cst.}\right)$

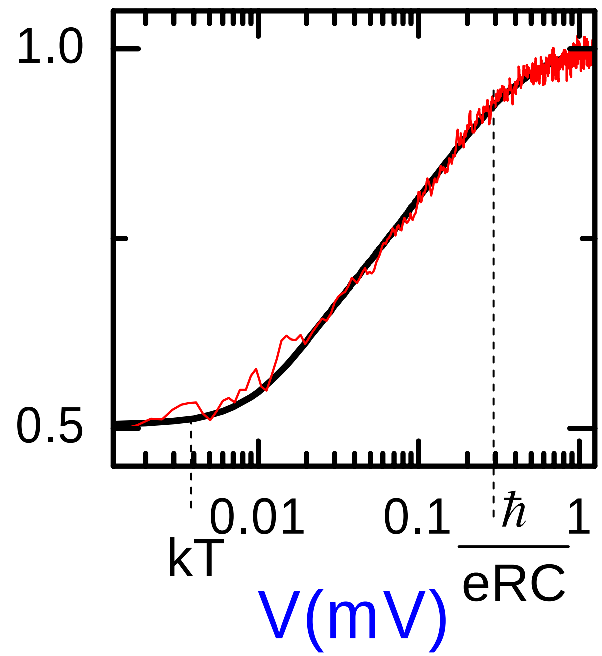
Dynamical Coulomb blockade with a resistive environment



for $k_B T \ll eV \ll \frac{\hbar}{RC}$

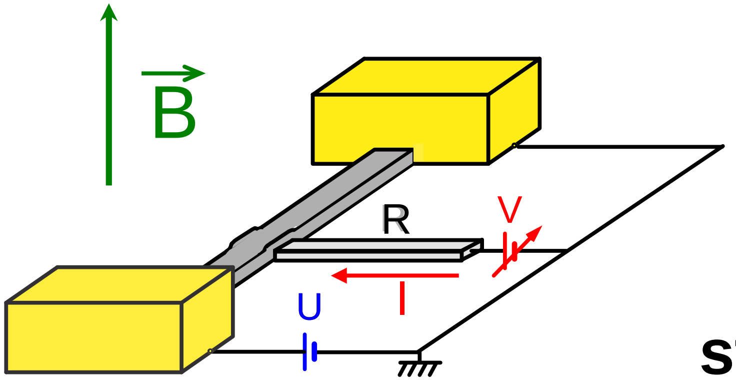
$$\frac{dI}{dV} \propto \left(V^{\frac{2R}{R_K}} + cst. \right)$$





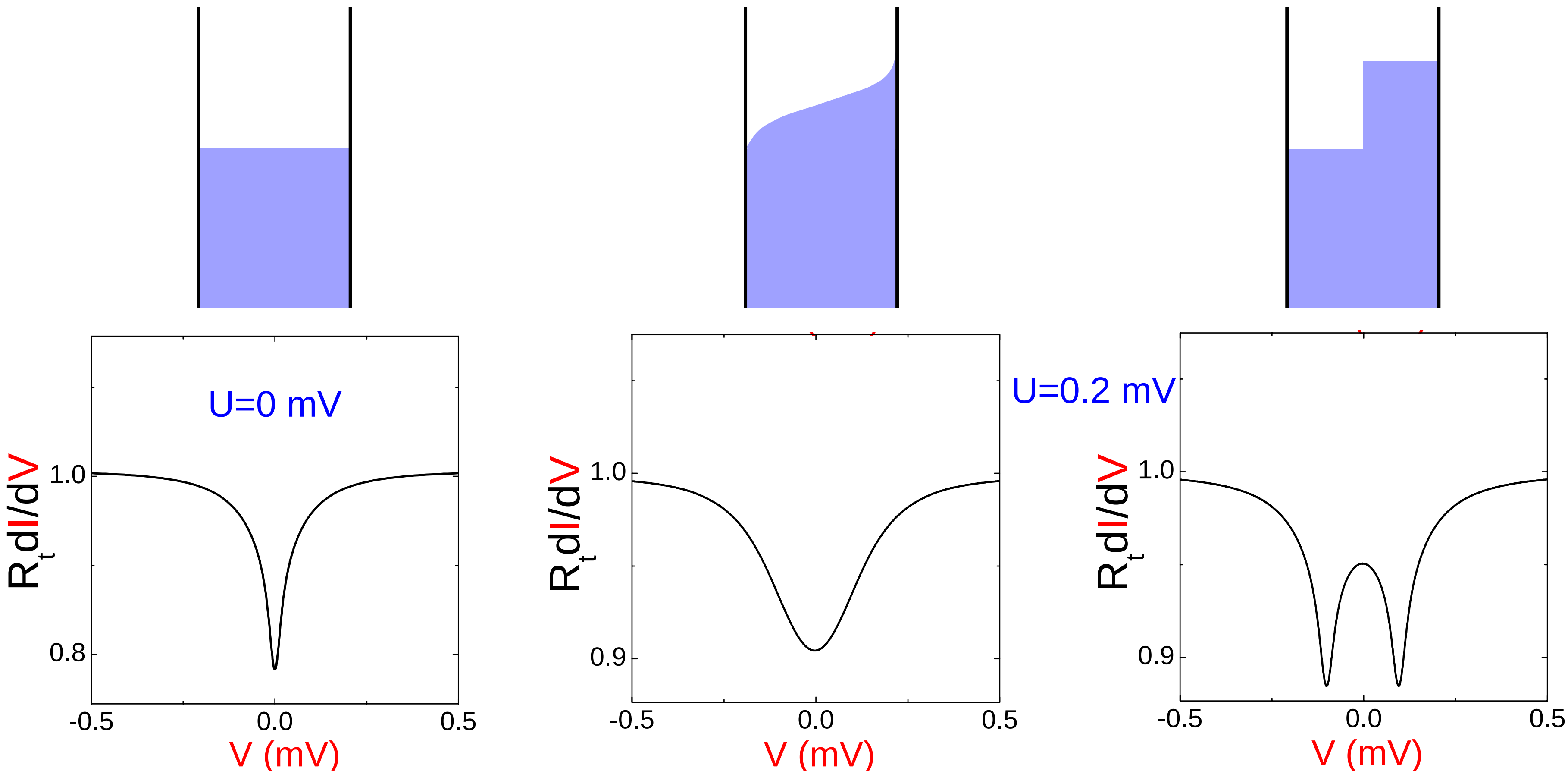
)

Measurement of f(E)



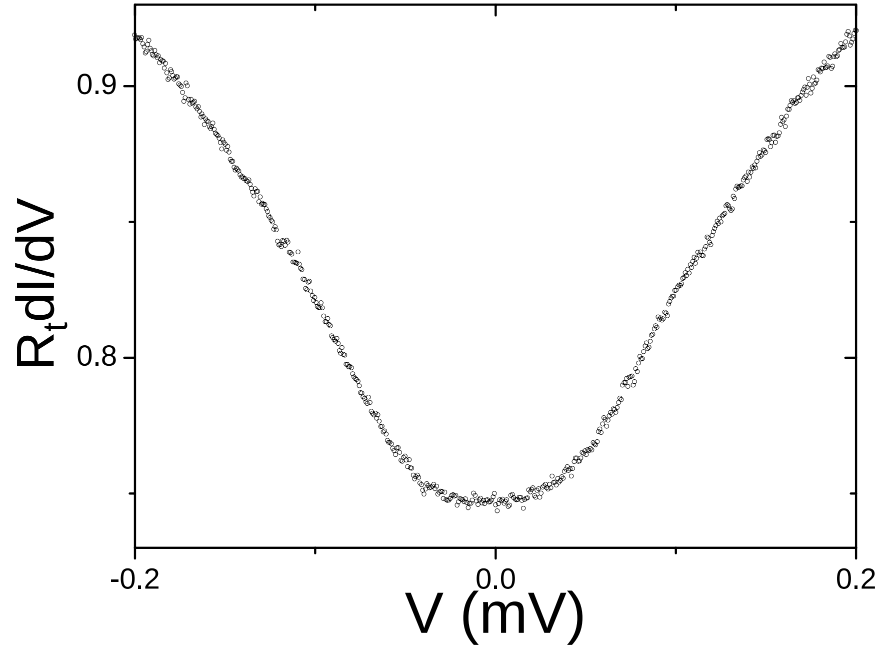
Dynamical Coulomb blockade (ZBA)

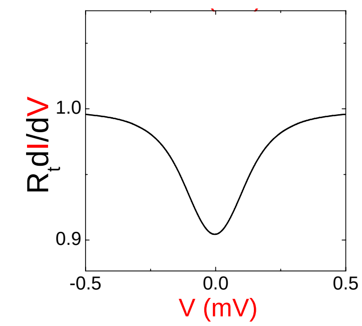
strong interaction **weak interaction**



Experimental data at weak B

implanted

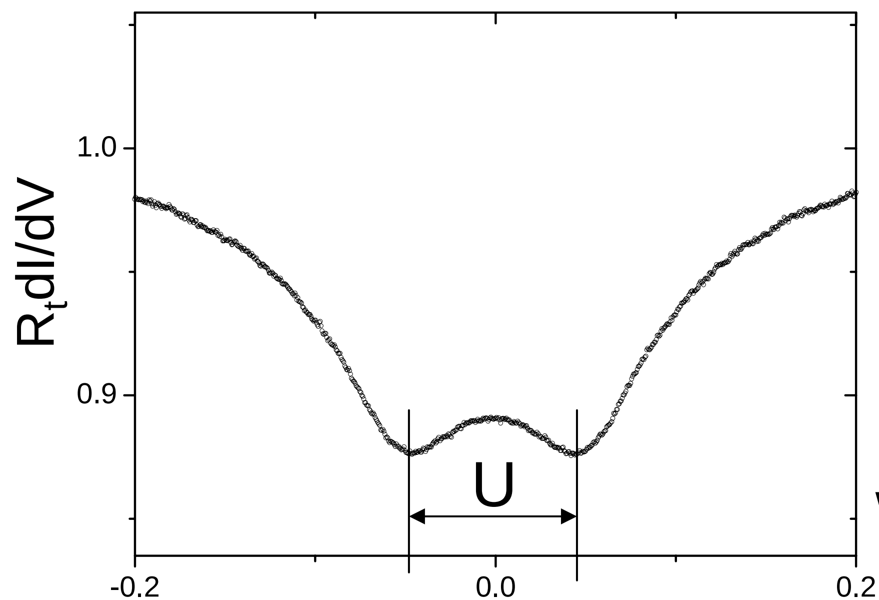


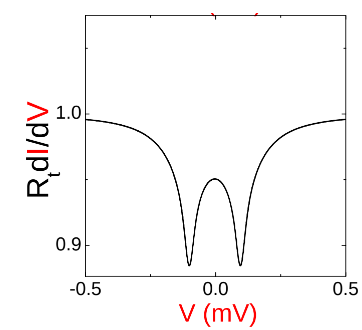


strong interaction

U = 0.1 mV
B = 0.3 T

bare





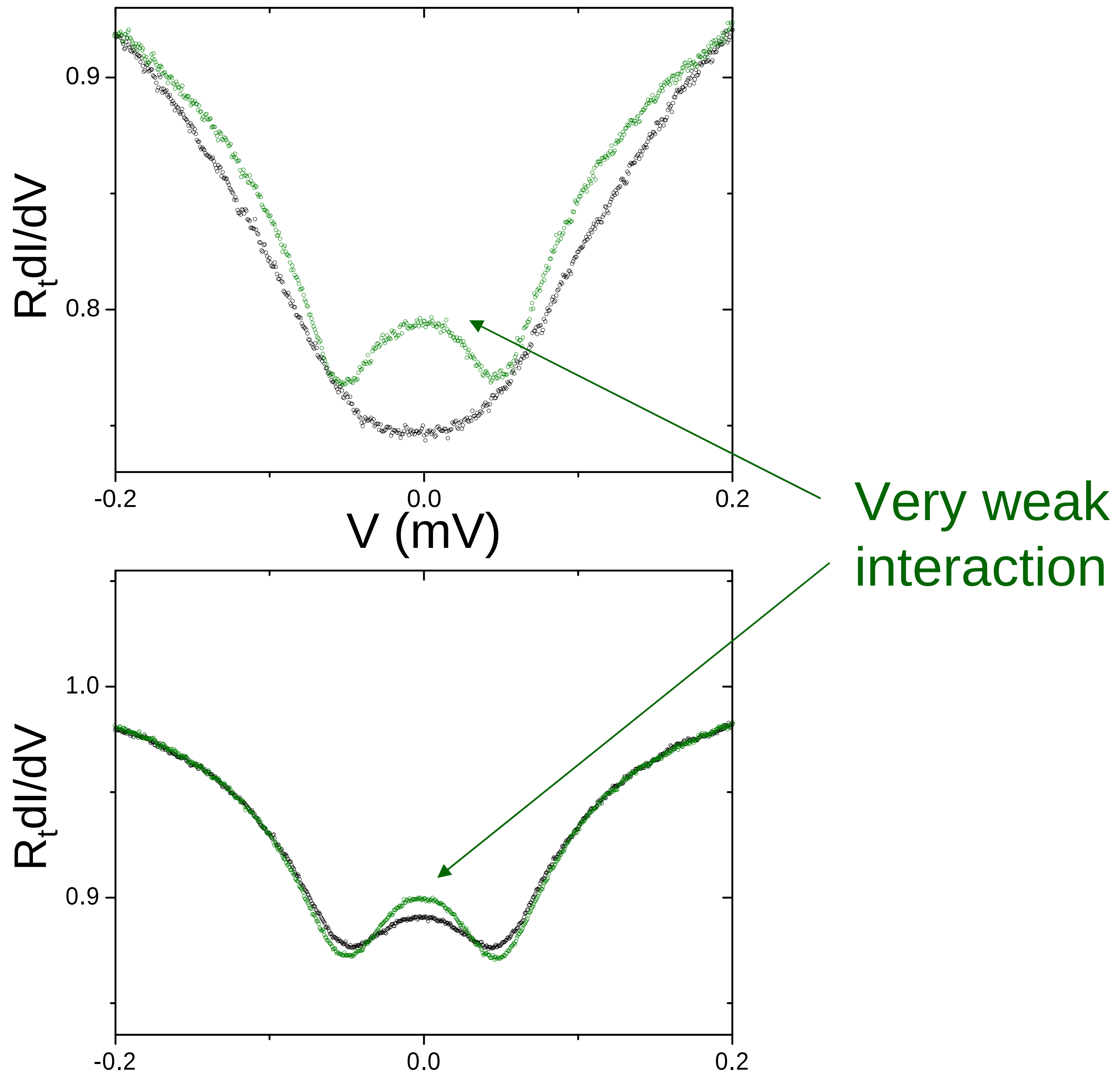
weak interaction

Experimental data at weak and at strong B

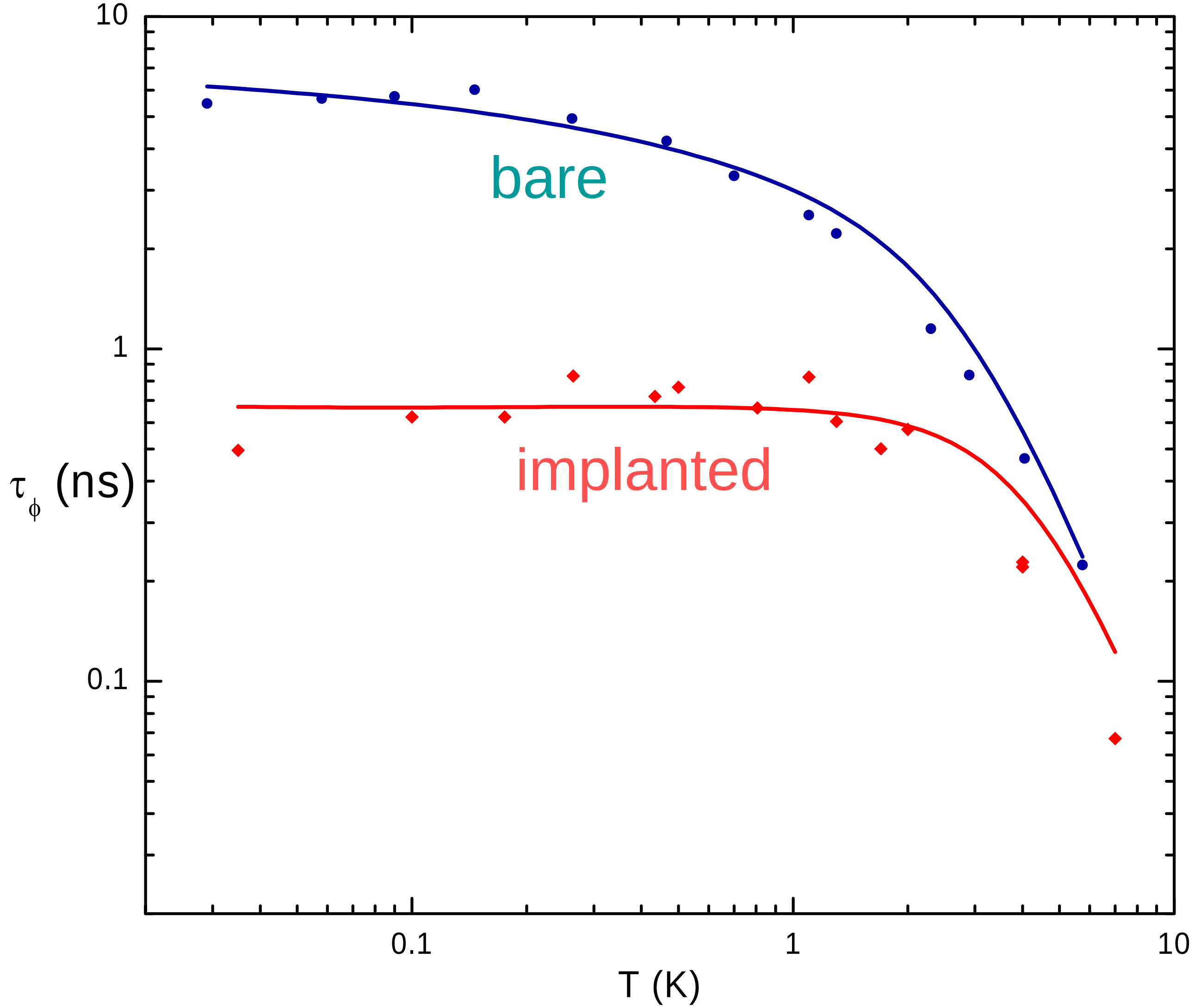
implanted

U = 0.1 mV
B = 0.3 T
B = 2.1 T

bare



Coherence time measurements on the same 2 samples

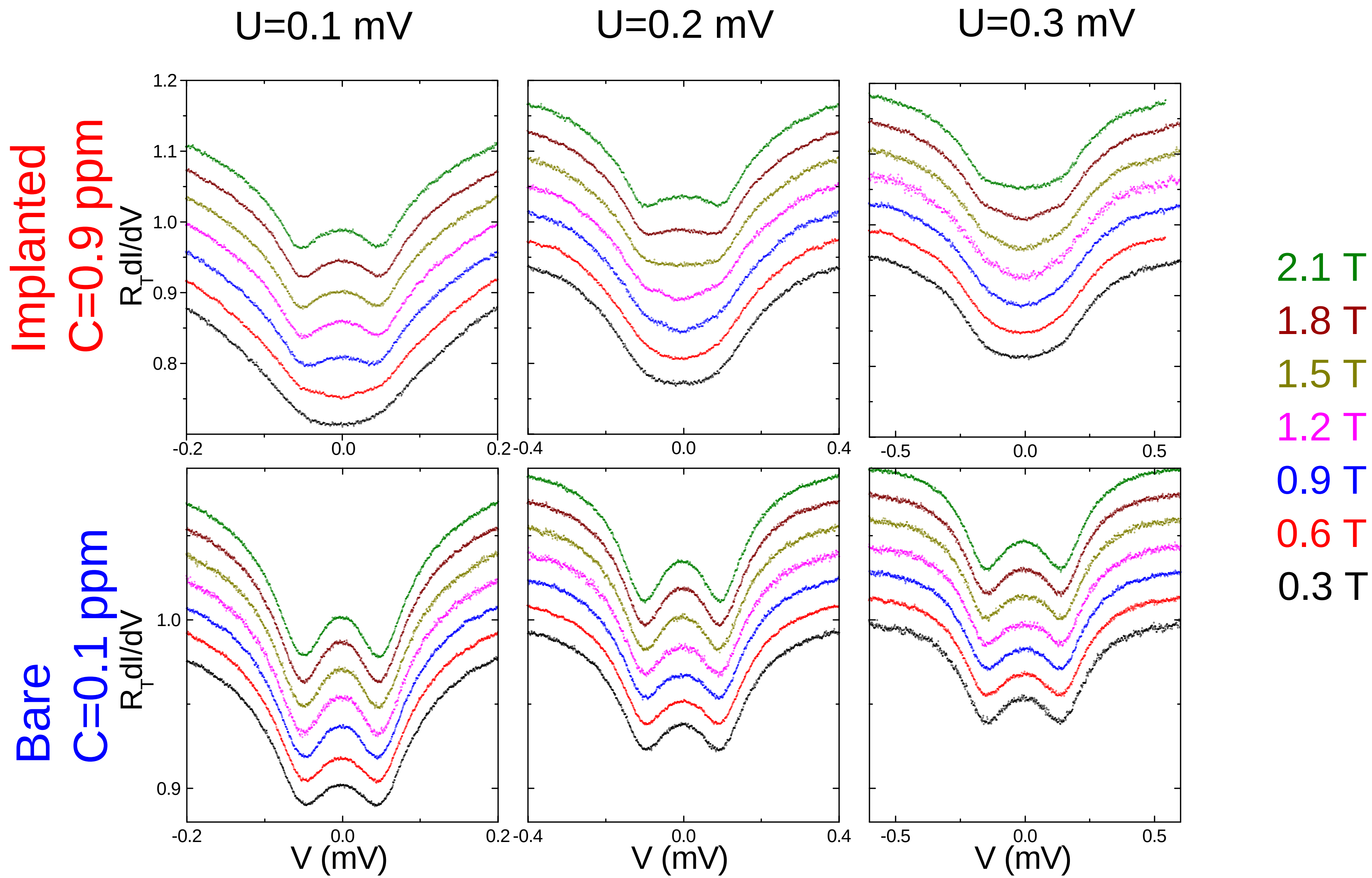


Fits:

C_{bare} =0.1 ppm

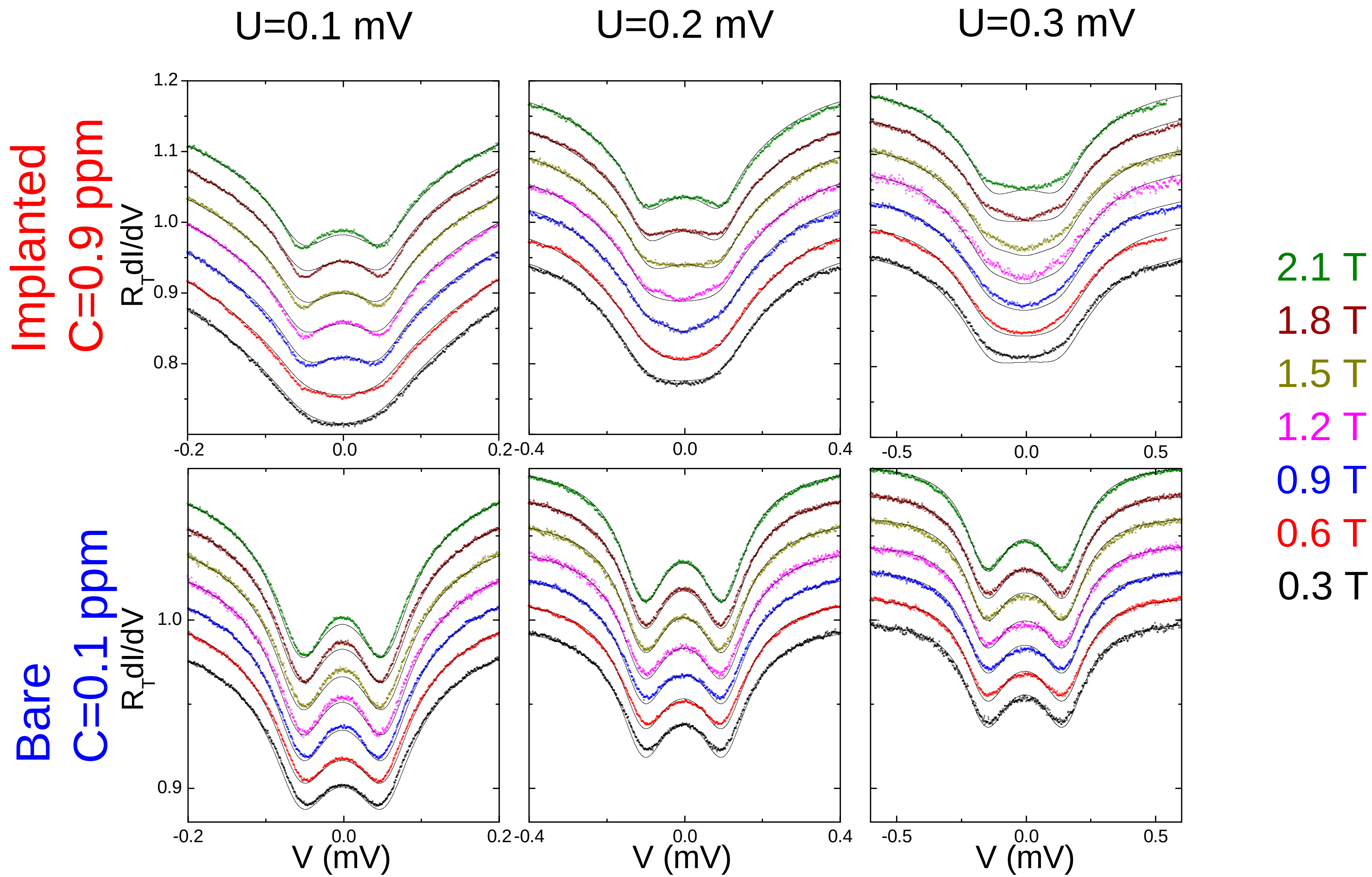
$C_{implanted}$=0.9 ppm

Full U,B dependence



Comparaison with theory $\left(S = \frac{1}{2}\right)$

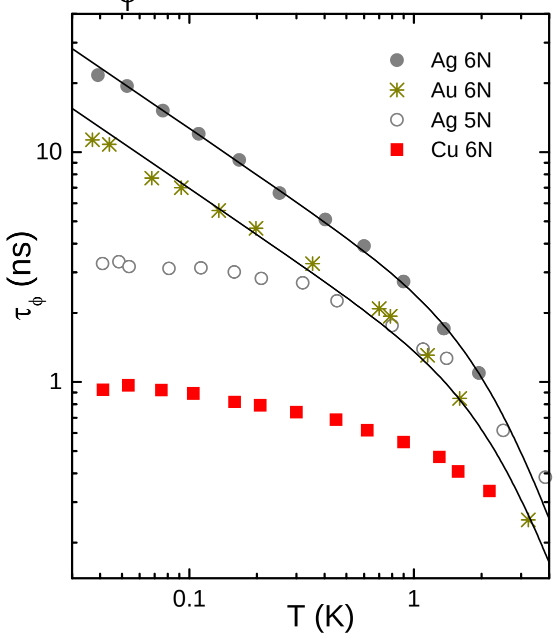
Goeppert, Galperin, Altshuler and Grabert, PRB 64, 033301 (2001)



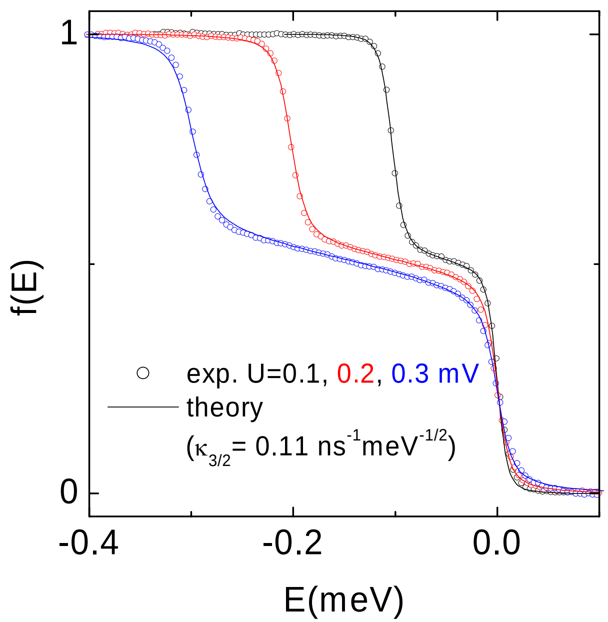
Conclusions

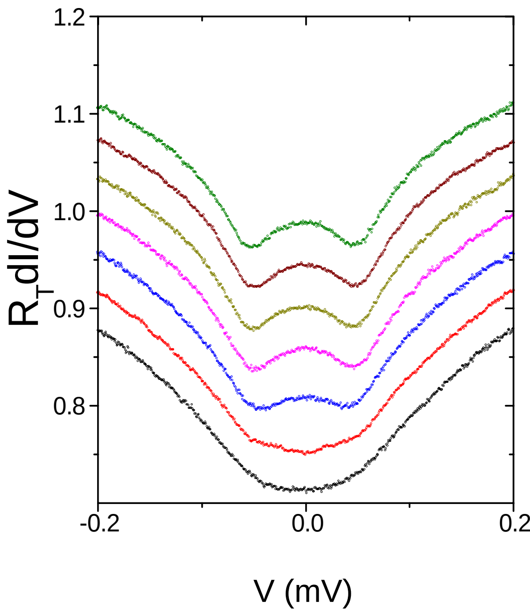
Two methods to investigate interactions in wires

$\tau_\phi(T)$



f(E)





Metal purity matters :

impurities with low T_K at ppm concentrations rule the game