From single-particle to many-body localisation in disordered systems.

Boris Altshuler
Physics Department, Columbia University
and NEC Laboratories America

The 4th Windsor Summer School on Condensed Matter Theory

Quantum Transport and Dynamics in Nanostructures

Great Park, Windsor, UK, August 6 - 18, 2007

Lectures 1,2

One-particle Localization

50 years of Anderson Localization

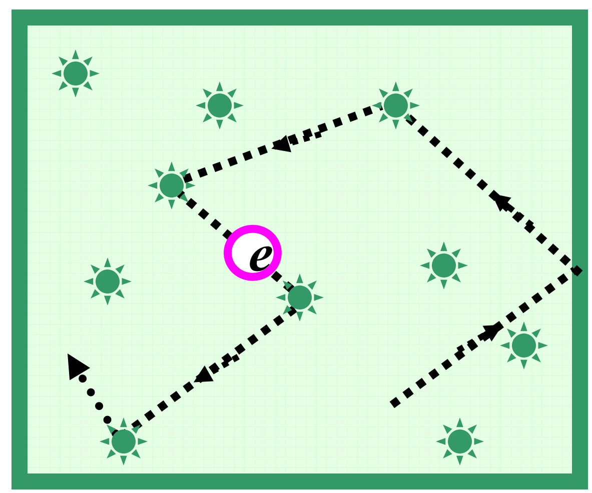
PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.



Philip W. Anderson

The Nobel Prize in Physics 1977

Nobel Lecture

Nobel Lecture, December 8, 1977

Local Moments and Localized States

I was cited for work both. in the field of magnetism and in that of disordered systems, and I would like to describe here one development in each held which was specifically mentioned in that citation. The two theories I will discuss differed sharply in some ways. The theory of local moments in metals was, in a sense, easy: it was the condensation into a simple mathematical model of ideas which. were very much in the air at the time, and it had rapid and permanent acceptance because of its timeliness and its relative simplicity. What mathematical difficulty it contained has been almost fully- cleared up within the past few years.

Localization was a different matter: very few believed it at the time, and even fewer saw its importance; among those who failed to fully understand it at first was certainly its author. It has yet to receive adequate mathematical treatment, and one has to resort to the indignity of numerical simulations to settle even the simplest questions about it .

Part 1. Introduction

Einstein Relation (1905)

$$\sigma = e^2 D \nu \qquad \nu \equiv \frac{dn}{d\mu}$$

Conductivity

Diffusion Constant

Density of states

No diffusion – no conductivity

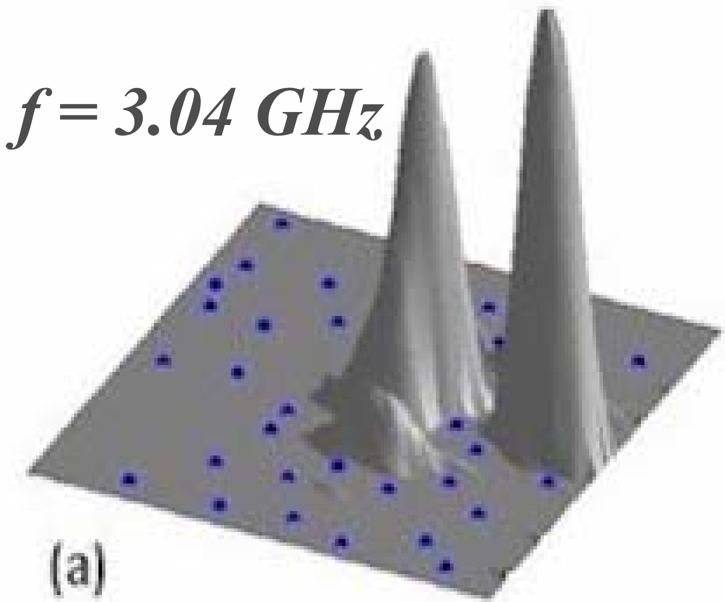
Localized states – insulator
Extended states – metal

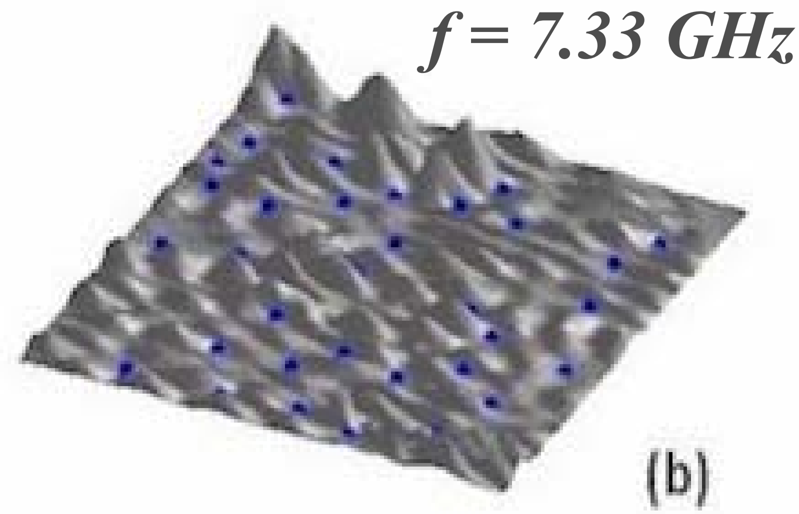
Metal – insulator transition

VOLUME 85, NUMBER 11 PHYSICAL REVIEW LETTERS 11 SEPTEMBER 2000

Correlations due to Localization in Quantum Eigenfunctions of Disordered Microwave Cavities

Prabhakar Pradhan and S. Sridhar
Department of Physics, Northeastern University, Boston, Massachusetts 02115
(Received 28 February 2000)

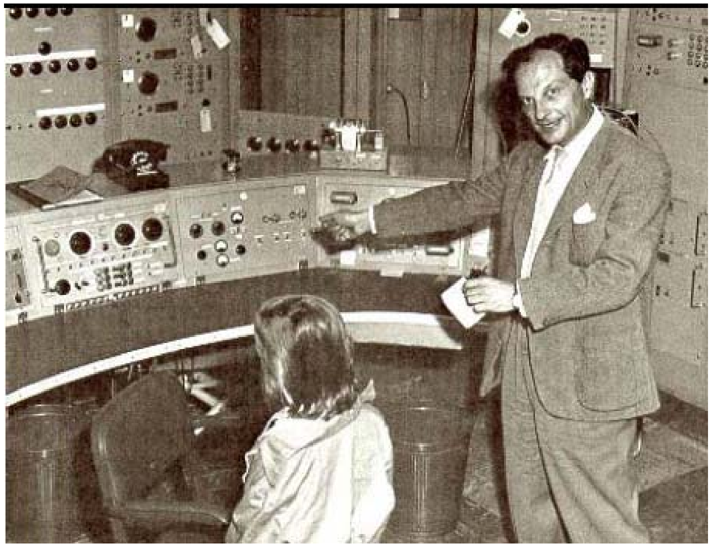




Anderson Insulator ***Anderson Metal***

Fermi Pasta Ulam 1955

Q: Will a nonlinear system (system of interacting particles) completely isolated from the outside world evolve to a microcanonical distribution (reach equipartition). ?



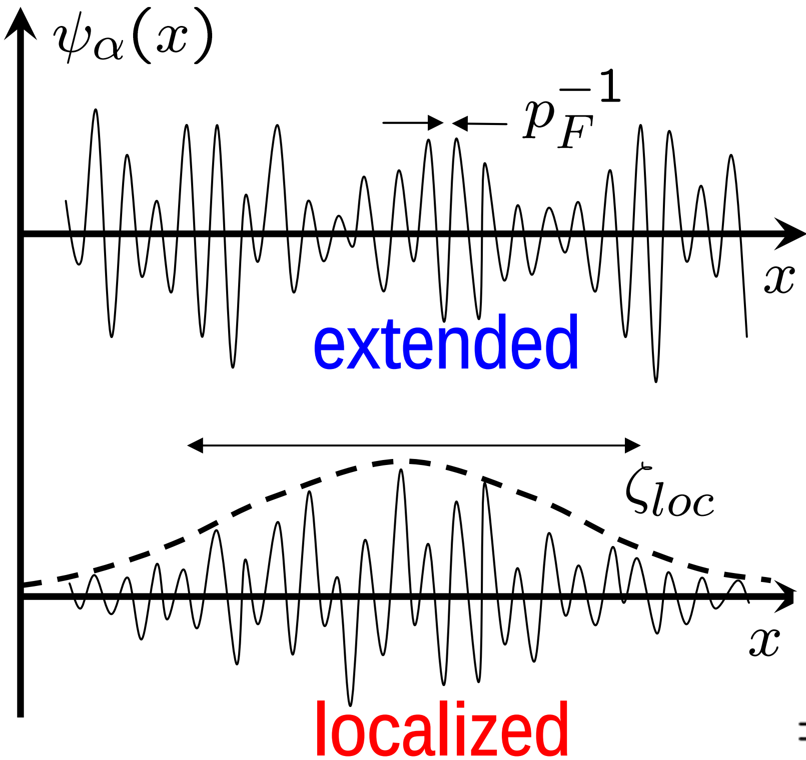
Anderson 1958

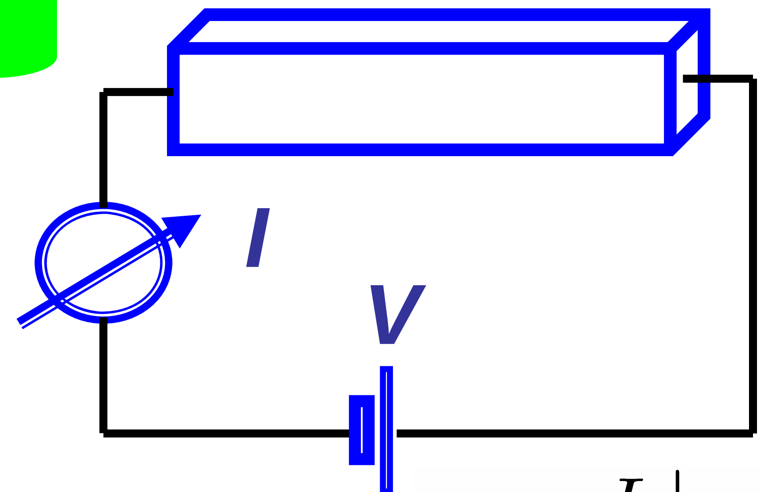
Q: Will a density fluctuation (a wave packet) in a system of quantum particles in the presence of disorder dissolve in the diffusive way. ?

Localization of single-electron wave-functions:

$$\left[-\frac{\boldsymbol{\nabla}^2}{2m} + U(\boldsymbol{r}) - \epsilon_F\right]\psi_\alpha(\boldsymbol{r}) = \xi_\alpha\psi_\alpha(\boldsymbol{r})$$

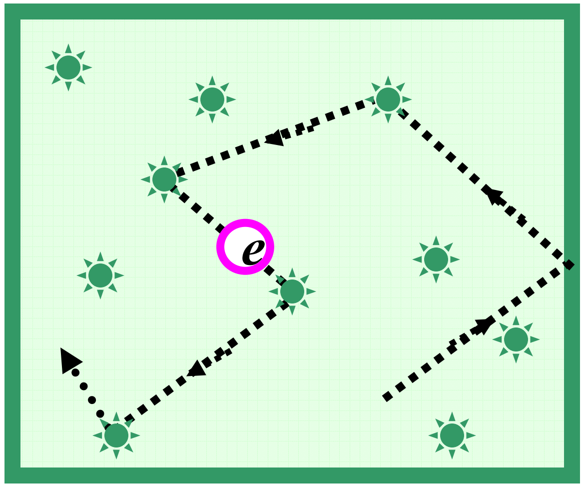
Disorder





Conductance $G = \left.\frac{I}{V}\right|_{V\to 0}$

$$= \begin{cases} \sigma\dfrac{L_x L_y}{L_z}; & \text{extended} \\ \propto \exp\left(-L_z/\zeta_{loc}\right); & \text{localized} \end{cases}$$



Scattering centers, e.g., impurities

Models of disorder:

Randomly located impurities

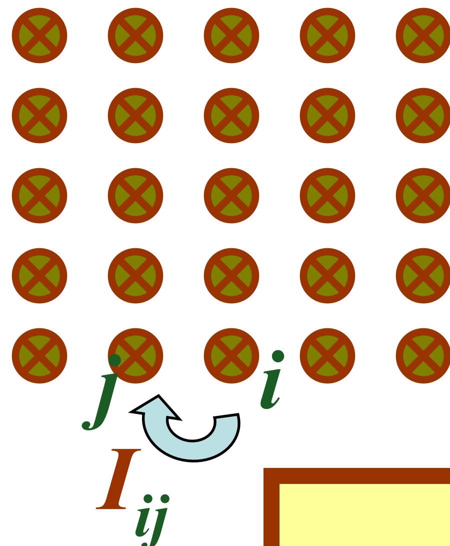
White noise potential

Lattice models

Anderson model

Lifshits model

Anderson Model



- *Lattice - tight binding model*
- *Onsite energies* ε_i *- random*
- *Hopping matrix elements* I_{ij}

$$-W < \varepsilon_i < W$$

uniformly distributed

$$I_{ij} = \begin{cases} I & i \text{ and } j \text{ are nearest neighbors} \\ 0 & \text{otherwise} \end{cases}$$

Anderson Transition

$$\frac{I_c}{W} \simeq \left(\frac{1}{2d}\right)\left(\frac{1}{\ln d}\right)$$

$$I < I_c$$

Insulator

All eigenstates are localized

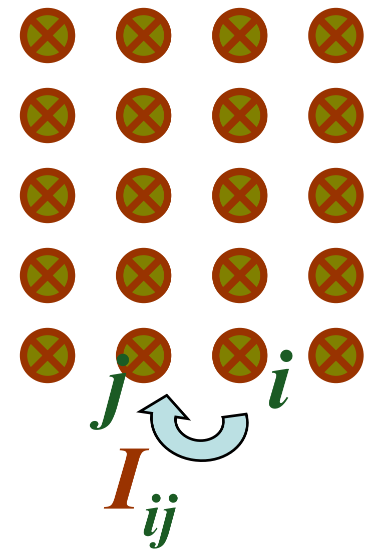
Localization length ξ

$$I > I_c$$

Metal

There appear states extended all over the whole system

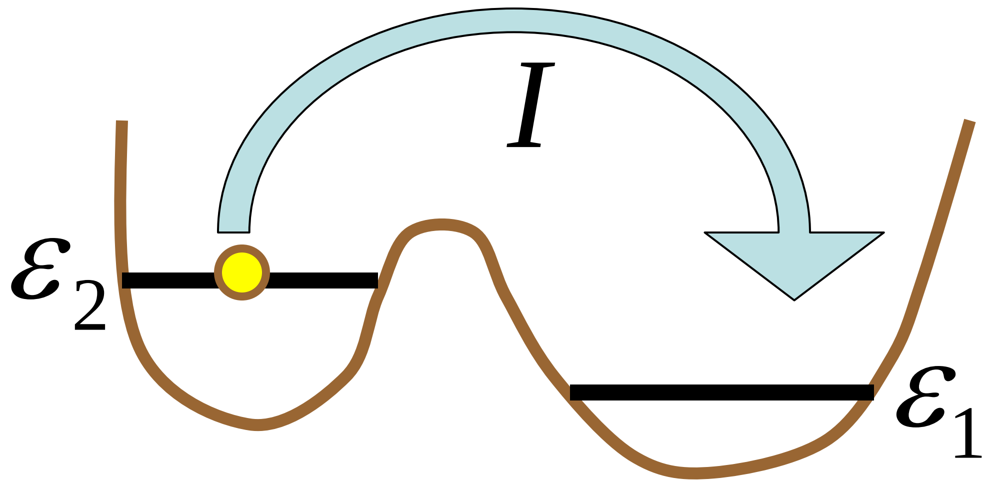
Q: Why arbitrary weak hopping I is not sufficient for the existence of the diffusion ?



Einstein (1905): Marcovian (no memory) process $\rightarrow$ diffusion

Quantum mechanics is not marcovian
There is memory in quantum propagation !

Why ?



Hamiltonian

$$\hat{H} = \begin{pmatrix} \varepsilon_1 & I \\ I & \varepsilon_2 \end{pmatrix} \xrightarrow{\text{diagonalize}} \hat{H} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

$$E_2 - E_1 = \sqrt{(\varepsilon_2 - \varepsilon_1)^2 + I^2}$$

$$\hat{H} = \begin{pmatrix} \varepsilon_1 & I \\ I & \varepsilon_2 \end{pmatrix} \xrightarrow{\text{diagonalize}} \hat{H} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

$$E_2 - E_1 = \sqrt{(\varepsilon_2 - \varepsilon_1)^2 + I^2} \approx \begin{matrix} \varepsilon_2 - \varepsilon_1 & \varepsilon_2 - \varepsilon_1 >> I \\ I & \varepsilon_2 - \varepsilon_1 << I \end{matrix}$$

von Neumann & Wigner "noncrossing rule"

Level repulsion

What about the eigenfunctions ?

$$\hat{H} = \begin{pmatrix} \varepsilon_1 & I \\ I & \varepsilon_2 \end{pmatrix}$$

$$E_2 - E_1 = \sqrt{(\varepsilon_2 - \varepsilon_1)^2 + I^2} \approx \begin{array}{ll} \varepsilon_2 - \varepsilon_1 & \varepsilon_2 - \varepsilon_1 >> I \\ I & \varepsilon_2 - \varepsilon_1 << I \end{array}$$

What about the eigenfunctions ?

$$\varphi_1 \varepsilon_1; \varphi_2 \varepsilon_2 \Leftarrow \psi_1, E_1; \psi_2, E_2$$

$$\varepsilon_2 - \varepsilon_1 >> I$$

$$\psi_{1,2} = \varphi_{1,2} + O\left(\frac{I}{\varepsilon_2 - \varepsilon_1}\right)\varphi_{2,1}$$

Off-resonance

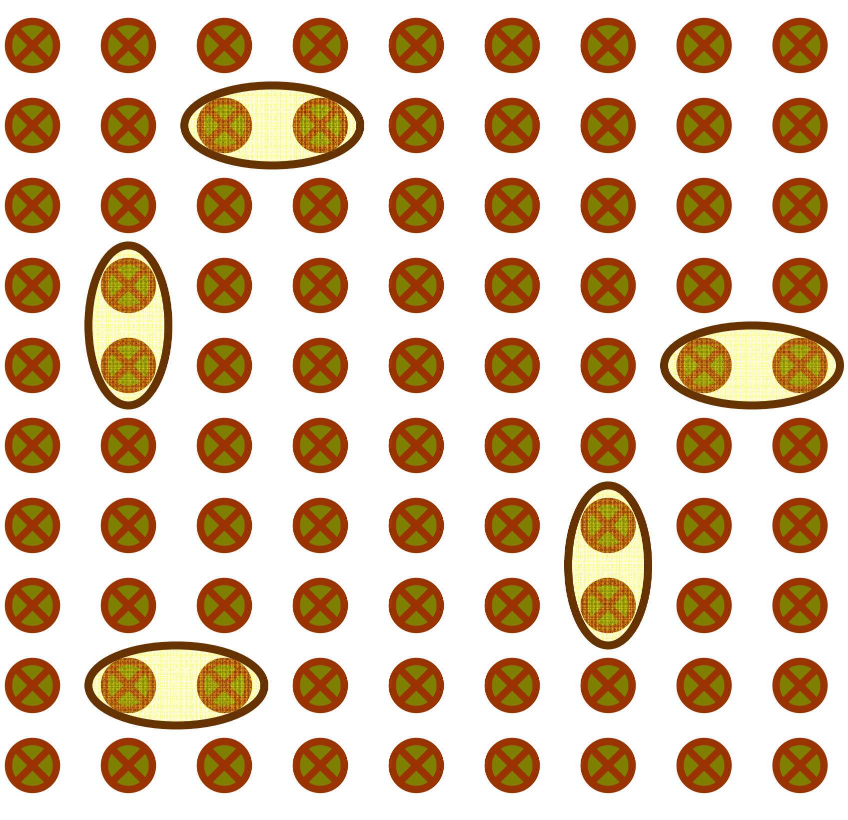
Eigenfunctions are close to the original on-site wave functions

$$\varepsilon_2 - \varepsilon_1 << I$$

$$\psi_{1,2} \approx \varphi_{1,2} \pm \varphi_{2,1}$$

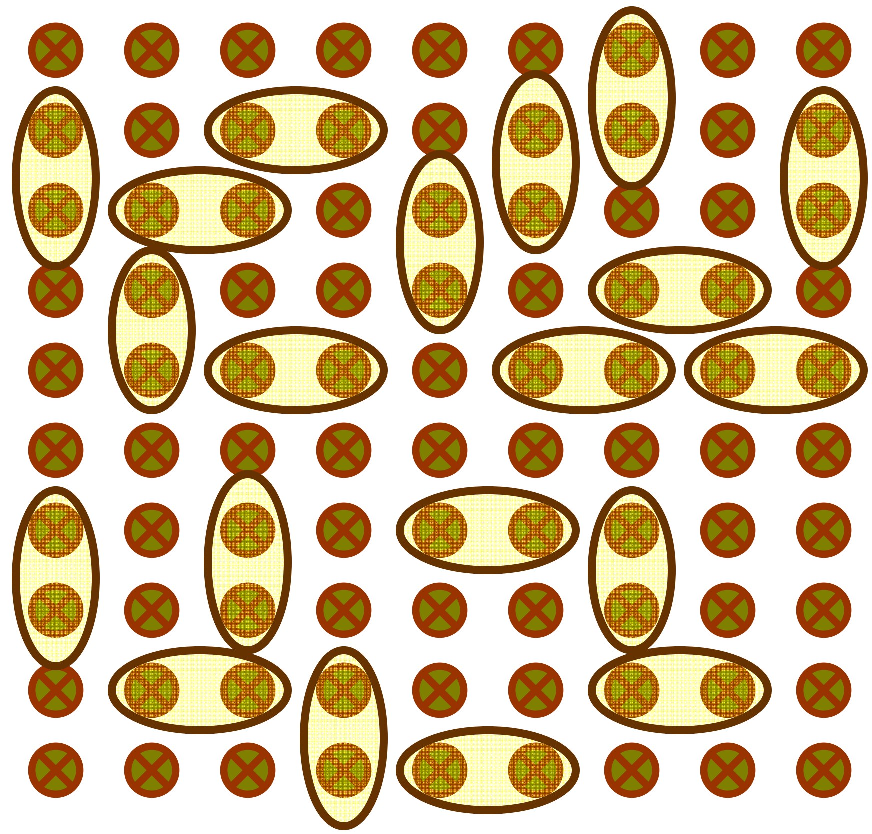
Resonance

In both eigenstates the probability is equally shared between the sites



Anderson insulator

Few isolated resonances



Anderson metal

There are many resonances and they overlap

Simplest example: Anderson Model Cayley tree:

J. Phys. C: Solid State Phys., Vol. 6, 1973. Printed in Great Britain. © 1973

A selfconsistent theory of localization

R Abou-Chacra†, P W Anderson‡§ and D J Thouless†

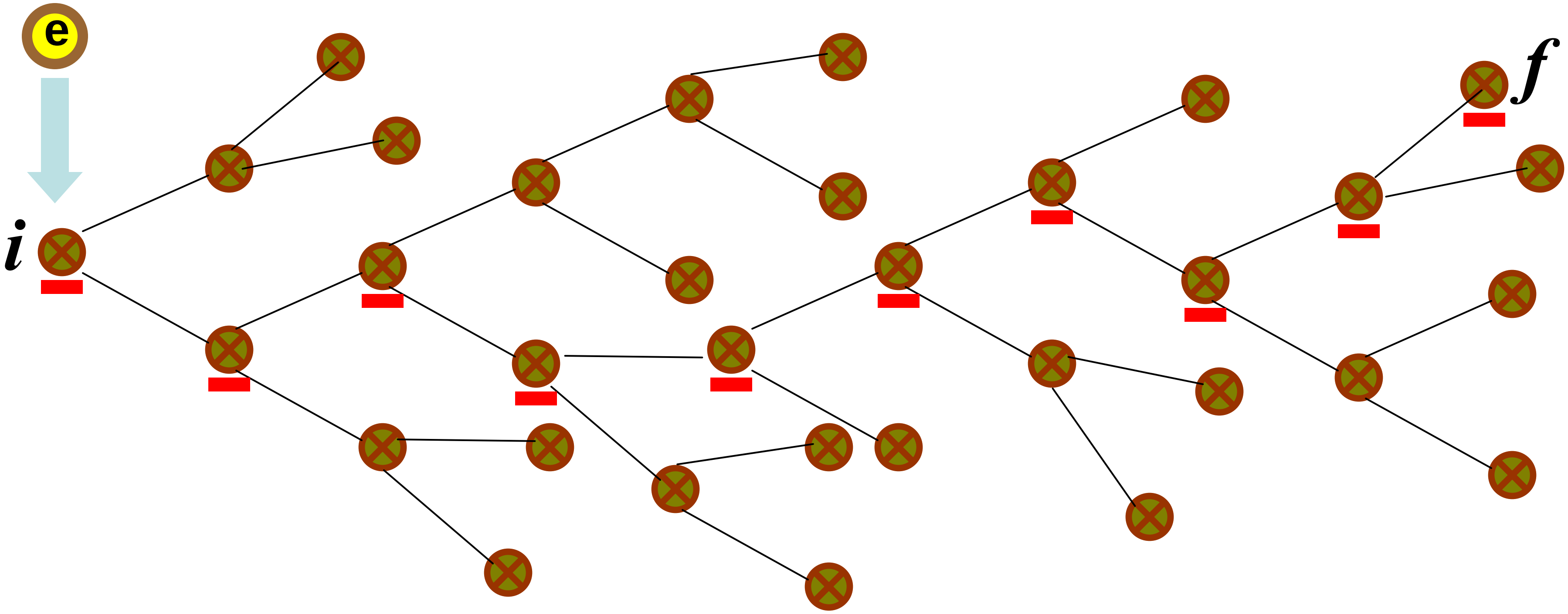
† Department of Mathematical Physics, University of Birmingham, Birmingham, B15 2TT

‡ Cavendish Laboratory, Cambridge, England and Bell Laboratories, Murray Hill, New Jersey, 07974, USA

Received 12 January 1973

Abstract. A new basis has been found for the theory of localization of electrons in disordered systems. The method is based on a selfconsistent solution of the equation for the self energy in second order perturbation theory, whose solution may be purely real almost everywhere (localized states) or complex everywhere (nonlocalized states). The equations used are exact for a Bethe lattice. The selfconsistency condition gives a nonlinear integral equation in two variables for the probability distribution of the real and imaginary parts of the self energy. A simple approximation for the stability limit of localized states gives Anderson's 'upper limit approximation'. Exact solution of the stability problem in a special case gives results very close to Anderson's best estimate. A general and simple formula for the stability limit is derived; this formula should be valid for smooth distribution of site energies away from the band edge. Results of Monte Carlo calculations of the selfconsistency problem are described which confirm and go beyond the analytical results. The relation of this theory to the old Anderson theory is examined, and it is concluded that the present theory is similar but better.

Simplest example: Anderson Model Cayley tree:



Parameters: ***I*, *W*** and branching number ***K*** (here ***K*=2**)

Crucial simplification: no loops

The probability amplitude to find the particle at a distance ***n*** is proportional to

$$A(n) \propto I^n \prod_{j=1}^{n} \frac{1}{\varepsilon - \varepsilon_j} \approx I^n \left(\frac{K}{W}\right)^n$$

The probability amplitude to find the particle at a distance $\boldsymbol{n}$ is proportional to

$$A(n) \propto I^n \prod_{j=1}^{n} \frac{1}{\varepsilon - \varepsilon_j} \approx I^n \left(\frac{K}{W}\right)^n$$

At each step among $\boldsymbol{K}$ site we can choose the one, which energy is the closest to $\boldsymbol{\varepsilon}$, i.e., $\left|\varepsilon - \varepsilon_j\right| \approx W/K$

K>1: Competition between exponentially small amplitude of each path and exponentially large number of paths.

Conclusion: for $I < I_c$, where $I_c \approx W/K$ the system is an insulator, because $A(n \to \infty) \to 0$ In the opposite case – metal

More precisely $I_c \approx W/(K \log K)$

$$A(n) \propto I^n \prod_{j=1}^{n} \frac{1}{\varepsilon - \varepsilon_j} \approx I^n \left(\frac{K}{W}\right)^n$$

Conclusion: for $I < I_c$, where $I_c \approx W/K$ the system is an insulator, because $A(n \to \infty) \to 0$ In the opposite case – metal

More precisely $I_c \approx W/(K \log K)$

$I > W/K$ Typically there is a resonance at every step

$W/(K \log K) < I < W/K$ The particle can travel infinitely far through the resonances of sites, which are not nearest neighbors

$I > W$ Typically each pair of nearest neighbors is at resonance

Part 2.

Localization and spectral statistics

Noncrossing rule (theorem)

Suggested by Hund (*Hund F. 1927 Phys. v.40, p.742*)

Justified by von Neumann & Wigner (*v. Neumann J. & Wigner E. 1929 Phys. Zeit. v.30, p.467*) . . . . .

Usually textbooks present a simplified version of the justification due to Teller (*Teller E., 1937 J. Phys. Chem 41 109).*

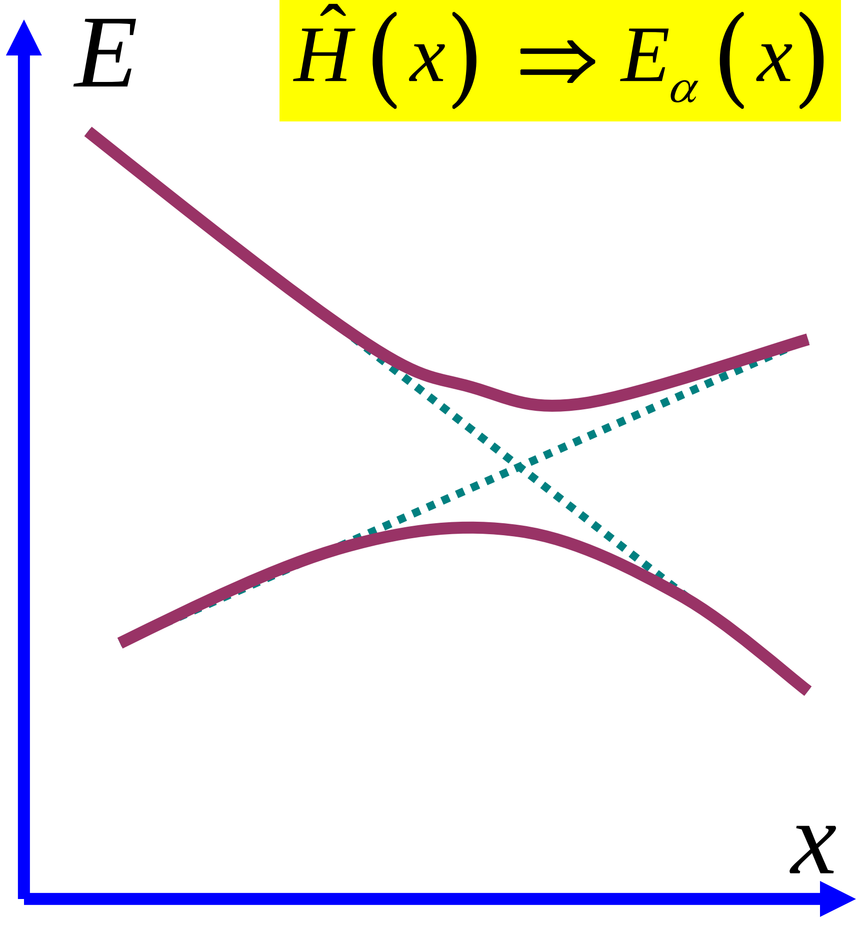
Arnold V. I., 1972 Funct. Anal. Appl.v. 6, p.94

Mathematical Methods of Classical Mechanics (Springer-Verlag: New York), Appendix 10, 1989

Arnold V.I., Mathematical Methods of Classical Mechanics (Springer-Verlag: New York), Appendix 10, 1989

In general, a multiple spectrum in typical families of quadratic forms is observed only for two or more parameters, while in one-parameter families of general form the spectrum is simple for all values of the parameter. Under a change of parameter in the typical one-parameter family the eigenvalues can approach closely, but when they are sufficiently close, it is as if they begin to repel one another. The eigenvalues again diverge, disappointing the person who hoped, by changing the parameter to achieve a multiple spectrum.

$$\hat{H}(x) \Rightarrow E_\alpha(x)$$



RANDOM MATRIX THEORY

Spectral statistics

$N \times N$ *ensemble of Hermitian matrices with random matrix element* $N \to \infty$

E_α - spectrum (set of eigenvalues)

$\delta_1 \equiv \langle E_{\alpha+1} - E_\alpha \rangle$ - mean level spacing

$\langle \ldots\ldots \rangle$ - ensemble averaging

$s \equiv \dfrac{E_{\alpha+1} - E_\alpha}{\delta_1}$ - spacing between nearest neighbors

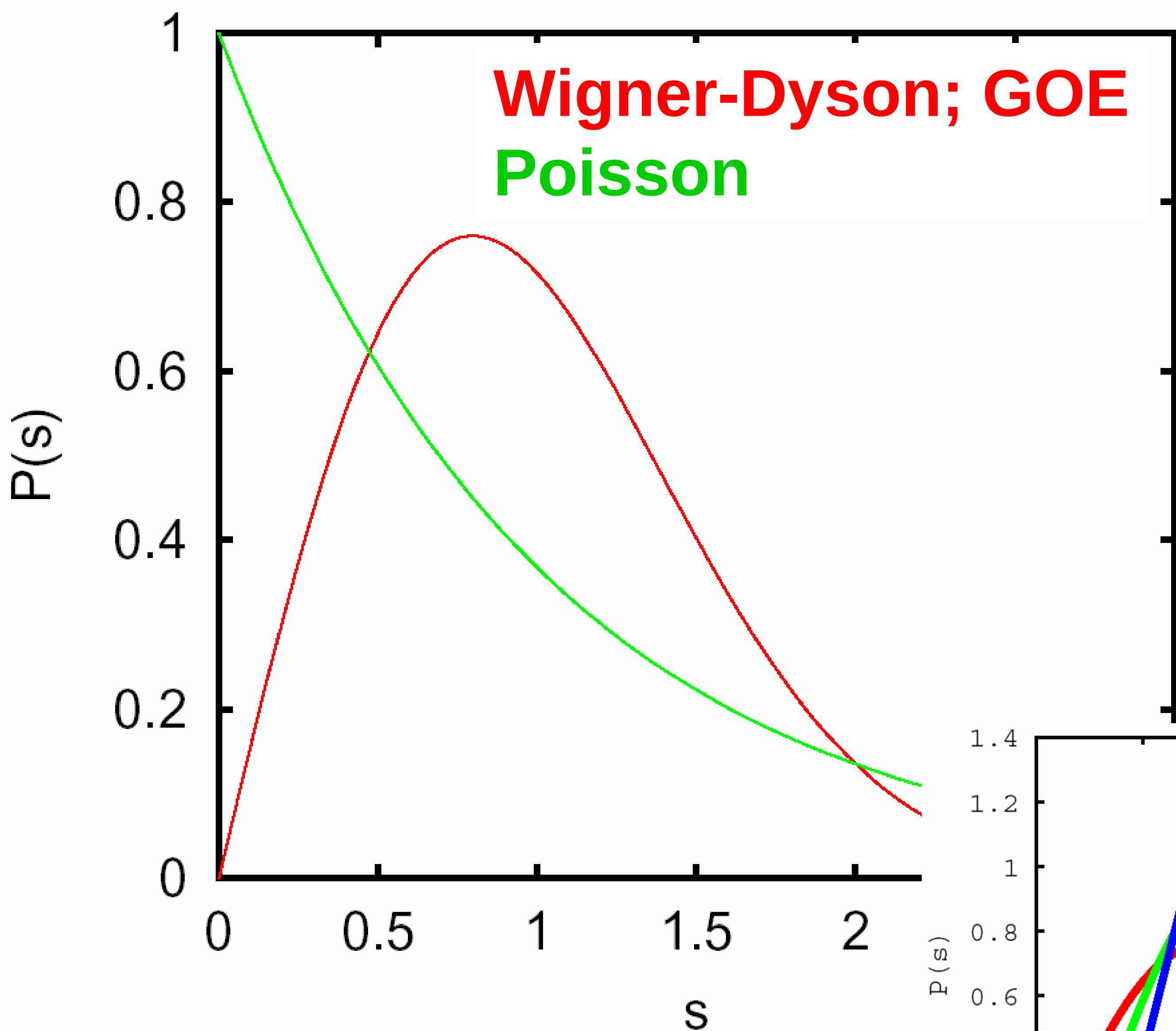
$P(s)$ - distribution function of nearest neighbors spacing between

Spectral Rigidity

Level repulsion

$$P(s=0)=0$$

$$P(s \ll 1) \propto s^\beta \qquad \beta = 1,2,4$$

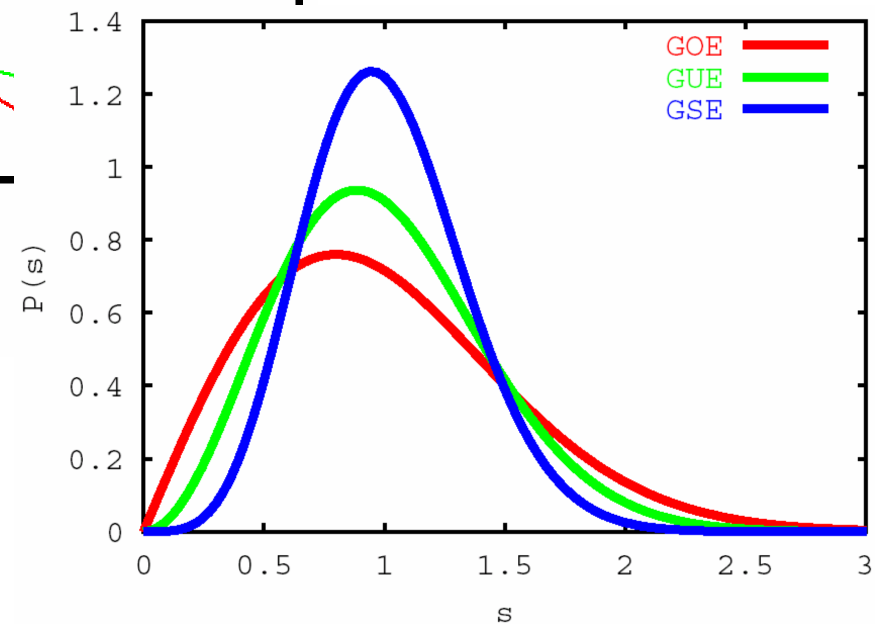


Gaussian
Orthogonal
Ensemble

Orthogonal
$\beta=1$

Unitary
$\beta=2$

Simplectic
$\beta=4$



Poisson – completely uncorrelated levels

RANDOM MATRICES

$N \times N$ matrices with random matrix elements. $N \to \infty$

Dyson Ensembles

Matrix elements	Ensemble	β	realization
real	orthogonal	1	T-inv potential
complex	unitary	2	broken T-invariance (e.g., by magnetic field)
2×2 matrices	simplectic	4	T-inv, but with spin-orbital coupling

Reason for $P(s) \to 0$ when $s \to 0$:

$$\hat{H} = \begin{pmatrix} H_{11} & H_{12} \\ H_{12}^{*} & H_{22} \end{pmatrix}$$

$$E_2 - E_1 = \sqrt{(H_{22} - H_{11})^2 + |H_{12}|^2}$$

small small small

1. The assumption is that the matrix elements are statistically independent. Therefore probability of two levels to be degenerate vanishes.

2. If H_{12} is real (orthogonal ensemble), then for s to be small two statistically independent variables ($(H_{22}- H_{11})$ and H_{12}) should be small and thus $P(s) \propto s \qquad \beta = 1$

$$\iint d\left(H_{11}-H_{22}\right)dH_{12}\,\delta\left(E_2-E_1-\sqrt{\quad}\right)P\left(H_{11}-H_{22}\right)P\left(H_{12}\right)$$

Reason for $P(s) \to 0$ when $s \to 0$:

$$\hat{H} = \begin{pmatrix} H_{11} & H_{12} \\ H_{12}^{*} & H_{22} \end{pmatrix}$$

$$E_2 - E_1 = \sqrt{(H_{22} - H_{11})^2 + |H_{12}|^2}$$

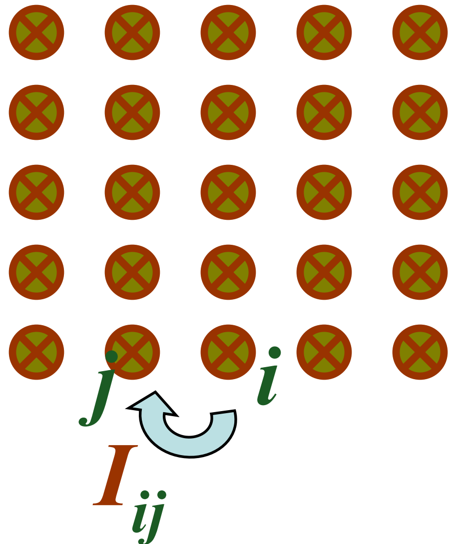
small small small

1. The assumption is that the matrix elements are statistically independent. Therefore probability of two levels to be degenerate vanishes.

2. If H_{12} is real (orthogonal ensemble), then for s to be small two statistically independent variables ($(H_{22}- H_{11})$ and H_{12}) should be small and thus $P(s) \propto s$ $\quad \beta = 1$

3. Complex H_{12} (unitary ensemble) $\Rightarrow$ both $Re(H_{12})$ and $Im(H_{12})$ are statistically independent $\Rightarrow$ three independent random variables should be small $\Rightarrow$ $P(s) \propto s^2$ $\quad \beta = 2$

Anderson Model



- *Lattice - tight binding model*
- *Onsite energies* ε_i - *random*
- *Hopping matrix elements* I_{ij}

$-W < \varepsilon_i < W$

uniformly distributed

Is there much in common between Random Matrices and Hamiltonians with random potential ?

Q: What are the spectral statistics of a finite size Anderson model **?**

Anderson Transition

Strong disorder

$$I < I_c$$

Insulator

All eigenstates are localized

Localization length ξ

The eigenstates, which are localized at different places will not repel each other

⇓

Poisson spectral statistics

Weak disorder

$$I > I_c$$

Metal

There appear states extended all over the whole system

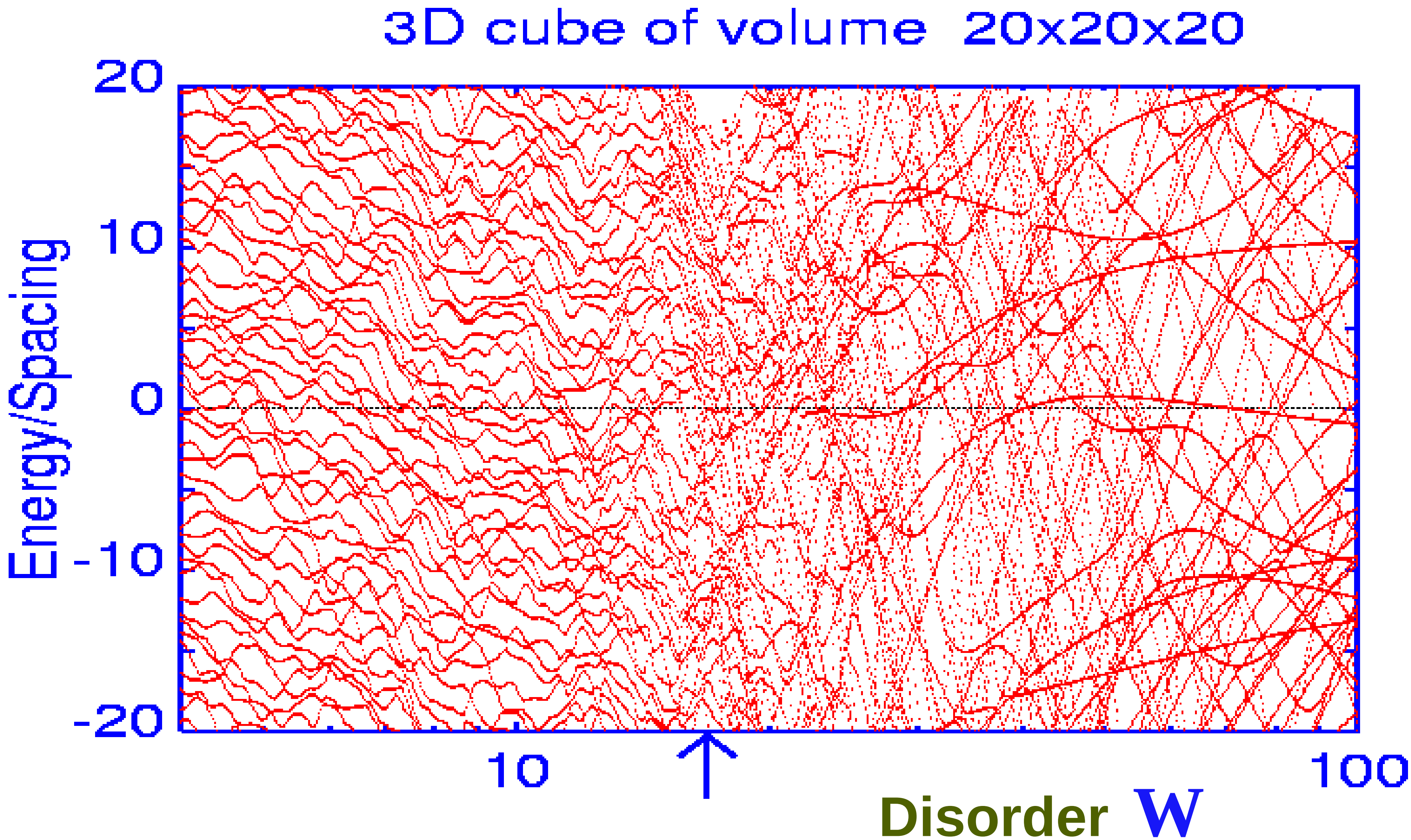
Any two extended eigenstates repel each other

⇓

Wigner – Dyson spectral statistics

Zharekeschev & Kramer.

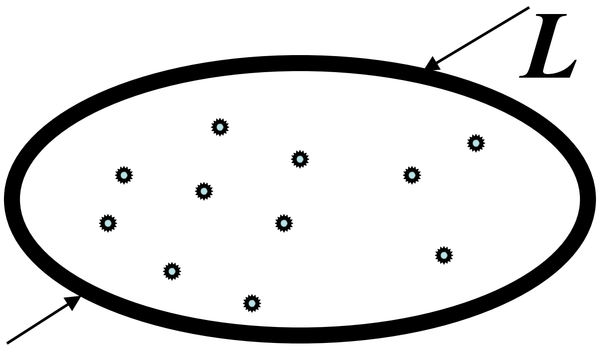
Exact diagonalization of the Anderson model

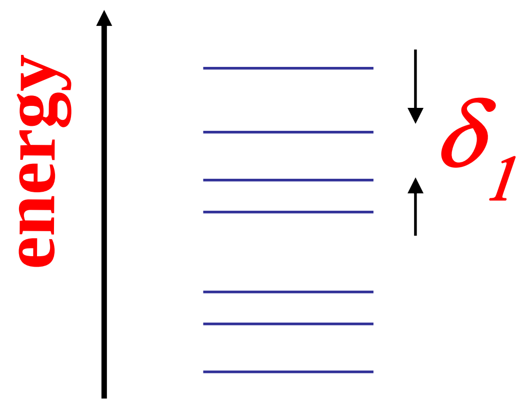


Energy scales in the localization problem. (*Thouless, 1972*)

1. Mean level spacing

$$\delta_1 = 1/\nu \times L^d$$





L *is the system size;*

d *is the number of dimensions*

This scale exists in the Random Matrix theory

Thouless Conductance and One-particle Spectral Statistics

Localized states
Insulator

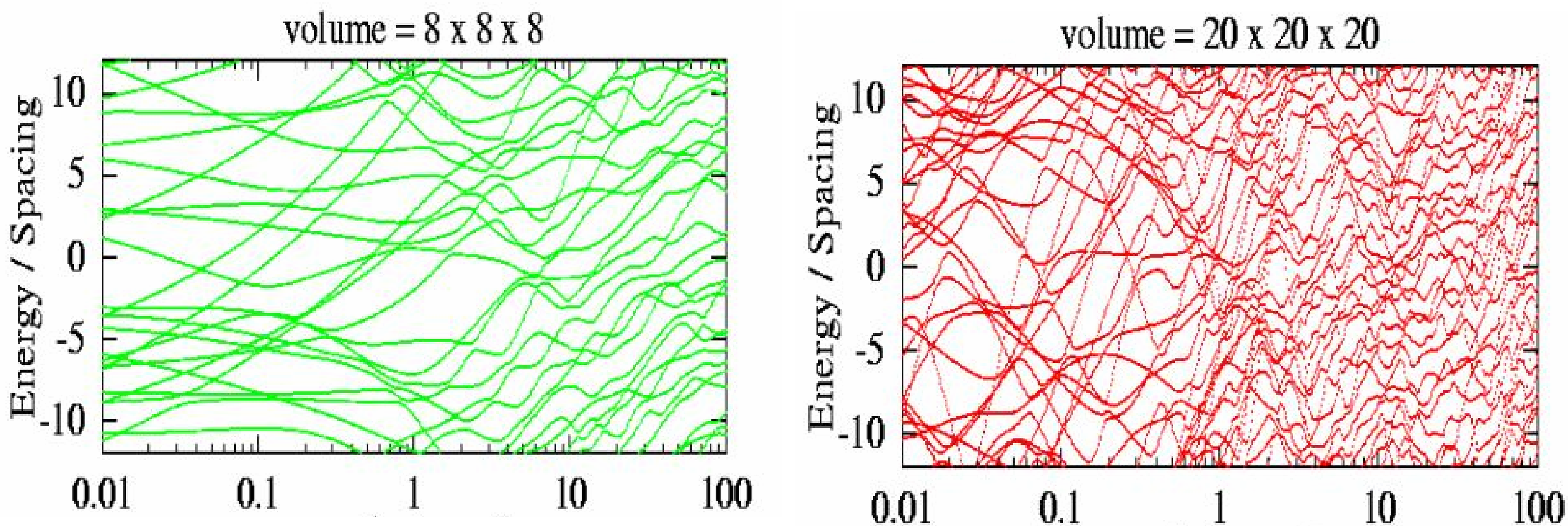
Poisson spectral statistics

Extended states
Metal

Wigner-Dyson spectral statistics

Transition at g~1.

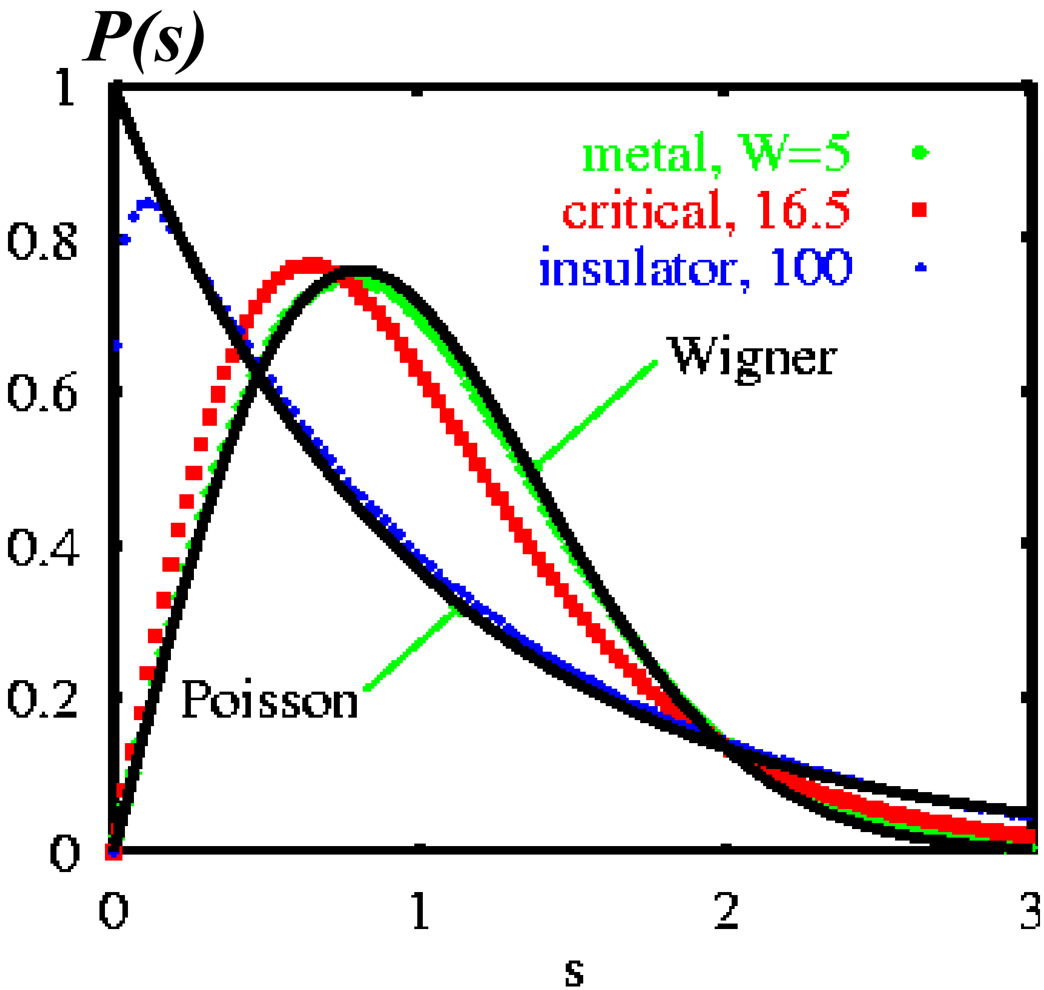
Is it sharp?

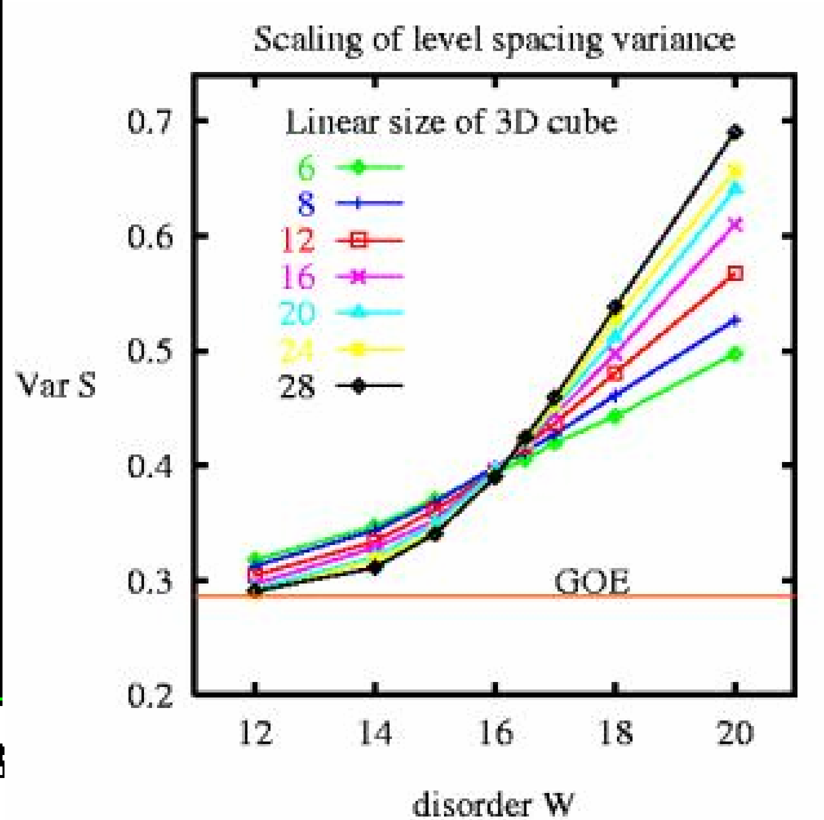


Conductance g

The bigger the system the sharper the transition

Anderson transition in terms of pure level statistics





Thouless Conductance and One-particle Spectral Statistics

0 — **1** — ***g***

Localized states	*Extended states*
Insulator	*Metal*
Poisson spectral statistics	**Wigner-Dyson spectral statistics**

***N × N* Random Matrices**

Quantum Dots with Thouless conductance g

$N \to \infty$

$g \to \infty$

The same statistics of the random spectra and one-particle wave functions (eigenvectors)

Part 3.

Quantum Chaos and Localization

Finite size quantum physical systems

Atoms
Nuclei
Molecules
.
.
.

Quantum Dots

ATOMS Main goal is to classify the eigenstates in terms of the quantum numbers

NUCLEI For the nuclear excitations this program does not work

E.P. Wigner: Study spectral **statistics** of a **particular** quantum system – a given nucleus

ATOMS Main goal is to classify the eigenstates in terms of the quantum numbers

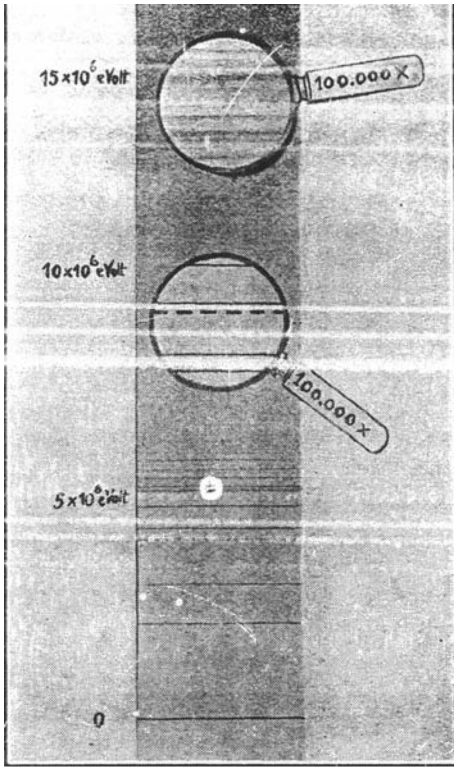
NUCLEI For the nuclear excitations this program does not work

E.P. Wigner: Study spectral **statistics** of a **particular** quantum system – a given nucleus

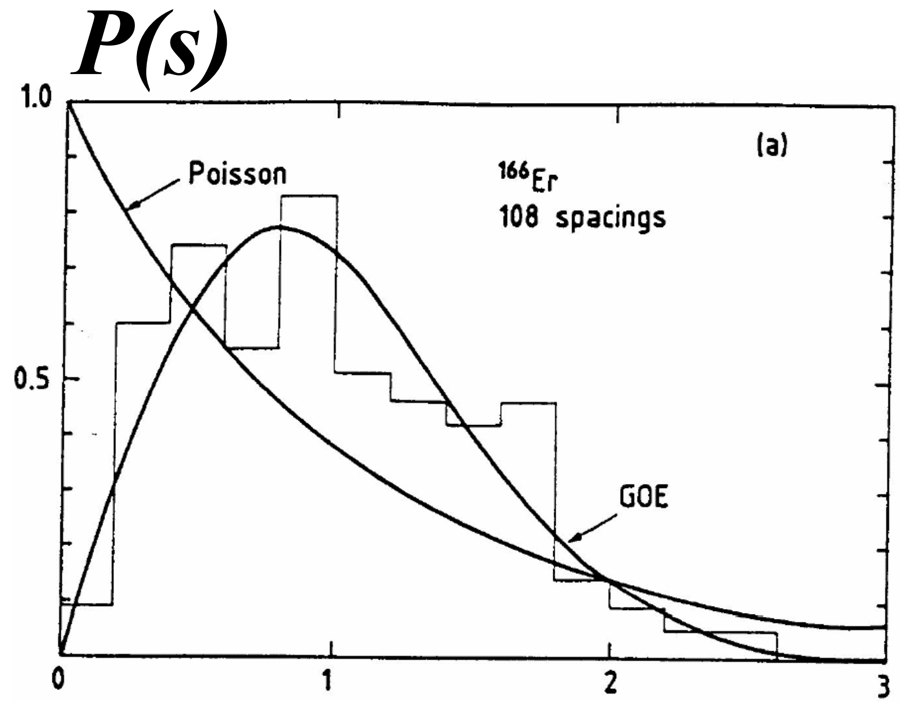
Spectra: $\{E_\alpha\}$

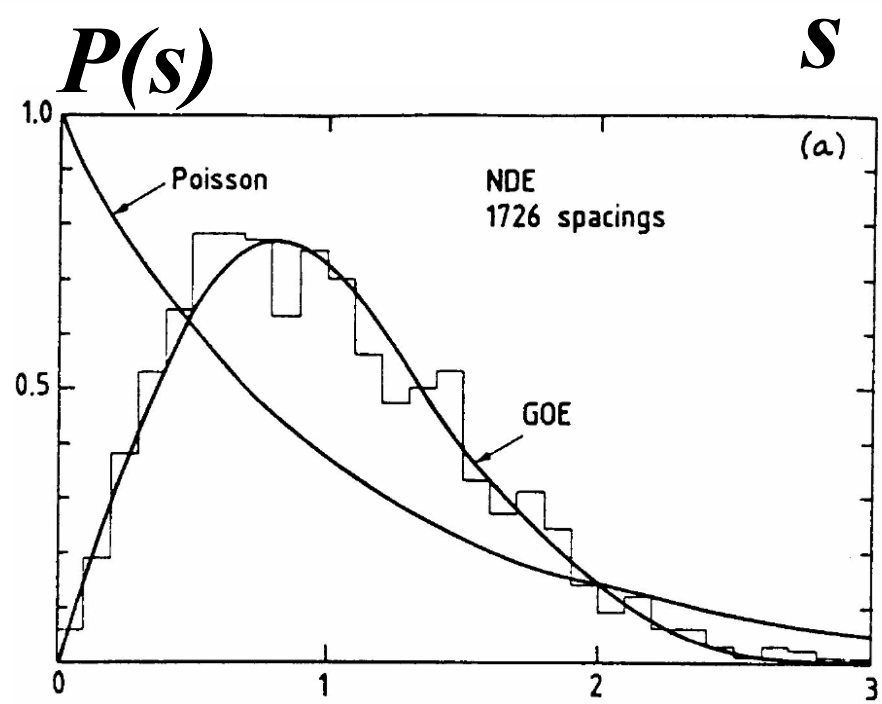
Random Matrices	Atomic Nuclei
• *Ensemble* • *Ensemble averaging*	•*Spectral averaging (over α)* •*Particular quantum system*

Nevertheless Statistics of the nuclear spectra are almost exactly the same as the Random Matrix Statistics



N. Bohr, Nature 137 (1936) 344.





Particular nucleus

^{166}Er

Spectra of several nuclei combined (after spacing) rescaling by the mean level

Q: *Why the random matrix theory (RMT) works so well for nuclear spectra* ?

Original answer:

These are systems with a large number of degrees of freedom, and therefore the "complexity" is high

Later it became clear that

there exist very "simple" systems with as many as 2 degrees of freedom (d=2), *which demonstrate RMT - like spectral statistics*

Classical ($\hbar$ =0) Dynamical Systems with d degrees of freedom

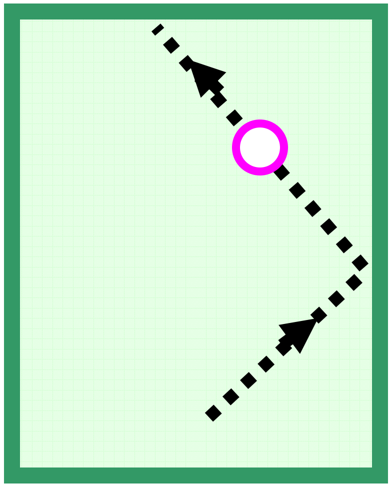
Integrable Systems

The variables can be separated and the problem reduces to d one-dimensional problems ⇒ d integrals of motion

Examples

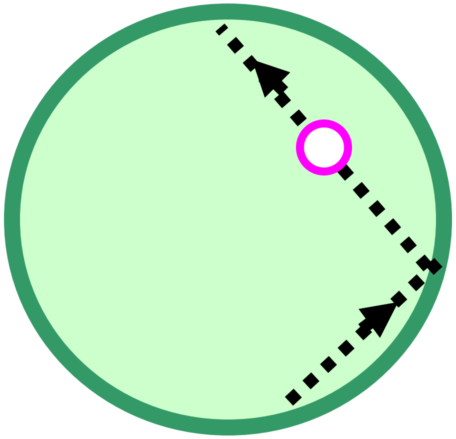
1. A ball inside rectangular billiard; $d=2$

- Vertical motion can be separated from the horizontal one
- Vertical and horizontal components of the momentum, are both integrals of motion



2. Circular billiard; $d=2$

- Radial motion can be separated from the angular one
- Angular momentum and energy are the integrals of motion



Classical Dynamical Systems with d degrees of freedom

Integrable Systems

The variables can be separated ⇨ d one-dimensional problems ⇨ d integrals of motion

Rectangular and circular billiard, Kepler problem, . . . ,
1d Hubbard model and other exactly solvable models, . .

Classical Dynamical Systems with *d* degrees of freedom

Integrable Systems

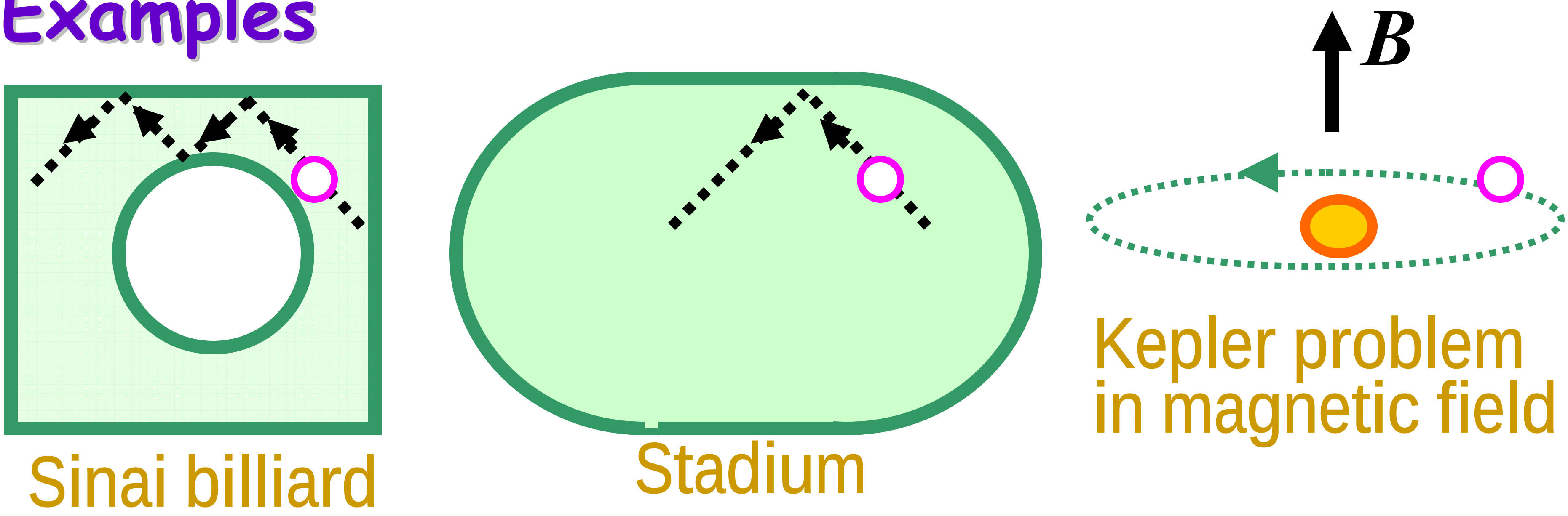
The variables can be separated ⇨ *d* one-dimensional problems ⇨ *d* integrals of motion

Rectangular and circular billiard, Kepler problem, . . . , 1d Hubbard model and other exactly solvable models, . .

Chaotic Systems

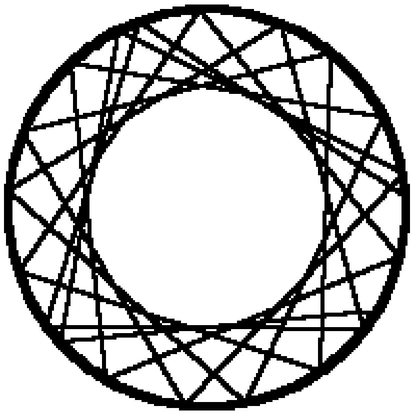
The variables can not be separated ⇨ there is only one integral of motion - energy

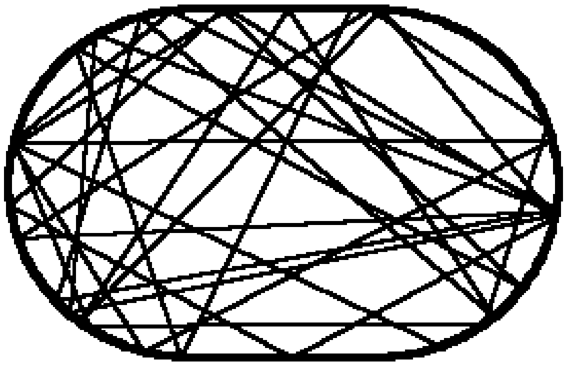
Examples



Classical Chaos
$\hbar = 0$

- *Nonlinearities*
- *Exponential dependence on the original conditions (Lyapunov exponents)*
- *Ergodicity*





Quantum description of any System with a finite number of the degrees of freedom is a linear problem – Shrodinger equation

Q: What does it mean Quantum Chaos ?

$\hbar \neq 0$ Bohigas – Giannoni – Schmit conjecture

VOLUME 52 2 JANUARY 1984 NUMBER 1

Characterization of Chaotic Quantum Spectra and Universality of Level Fluctuation Laws

O. Bohigas, M. J. Giannoni, and C. Schmit
Division de Physique Théorique, Institut de Physique Nucléaire, F-91406 Orsay Cedex, France

(Received 2 August 1983)

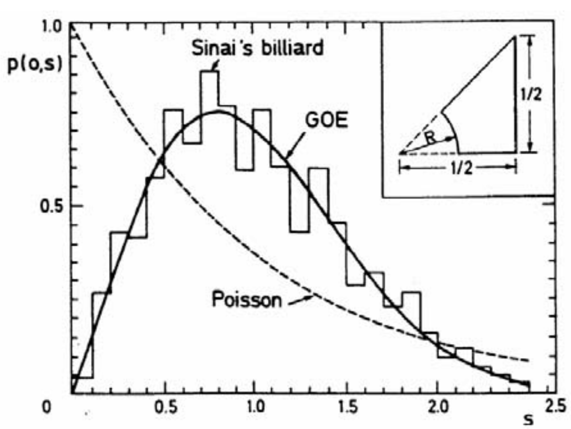
It is found that the level fluctuations of the quantum Sinai's billiard are consistent with the predictions of the Gaussian orthogonal ensemble of random matrices. This reinforces the belief that level fluctuation laws are universal.

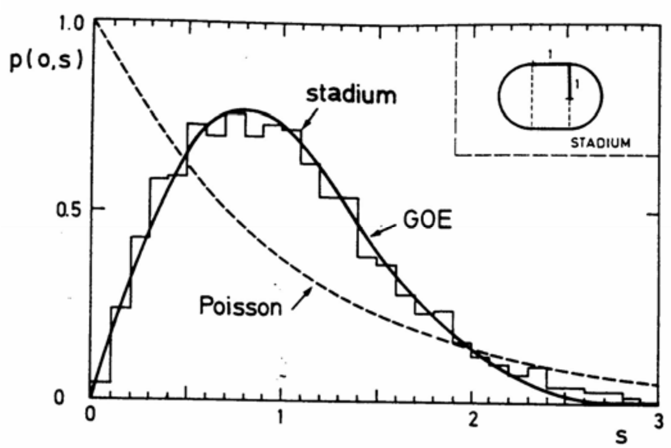
In summary, the question at issue is to prove or disprove the following conjecture: Spectra of time-reversal-invariant systems whose classical analogs are K systems show the same fluctuation properties as predicted by GOE

Chaotic classical analog

Wigner- Dyson spectral statistics

No quantum numbers except energy





Q: What does it mean Quantum Chaos ?

Two possible definitions

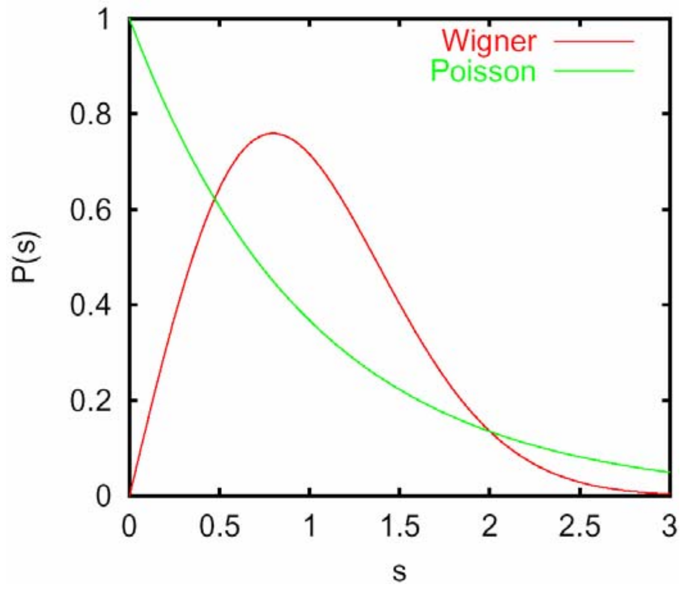
Chaotic classical analog

Wigner - Dyson-like spectrum

Classical Quantum

Integrable ? ⟺ Poisson

Chaotic ? ⟺ Wigner-Dyson

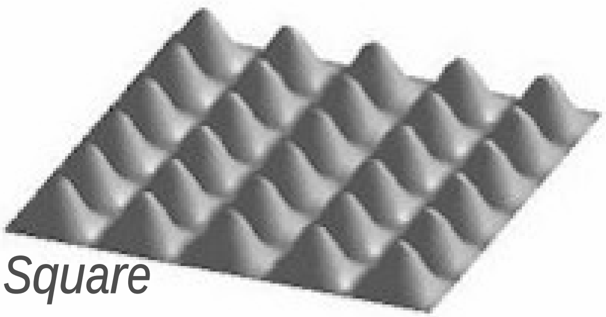


VOLUME 85, NUMBER 11 PHYSICAL REVIEW LETTERS 11 SEPTEMBER 2000

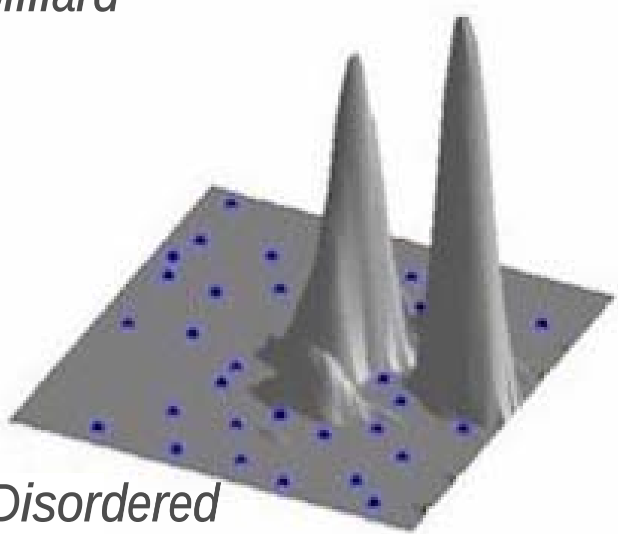
Correlations due to Localization in Quantum Eigenfunctions of Disordered Microwave Cavities

Prabhakar Pradhan and S. Sridhar
Department of Physics, Northeastern University, Boston, Massachusetts 02115
(Received 28 February 2000)

Integrable



Square billiard

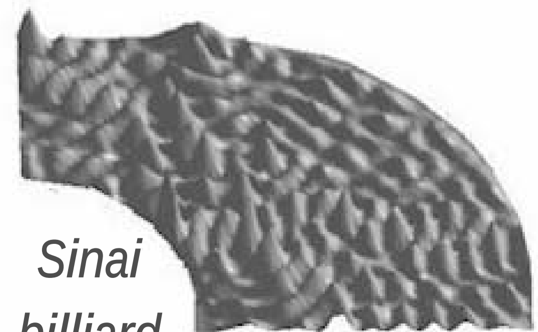


Disordered
localized

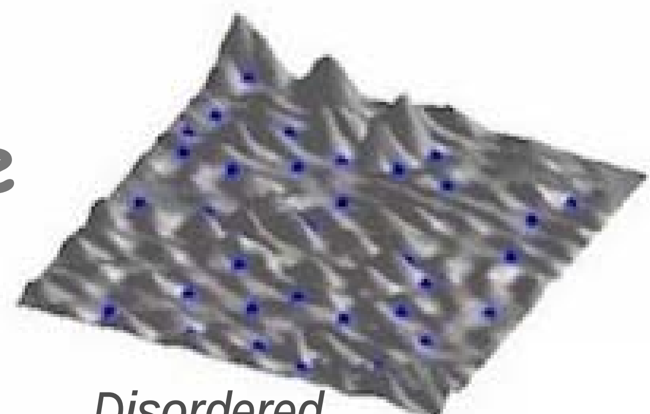
All chaotic systems resemble each other.

All integrable systems are integrable in their own way

Chaotic



Sinai billiard



Disordered
extended

Disordered Systems:

*Anderson **metal**;*
Wigner-Dyson spectral statistics

*Anderson **insulator**;*
Poisson spectral statistics

Q: *Is it a generic scenario for the Wigner-Dyson to Poisson crossover* **?**

Speculations

Consider an integrable system. Each state is characterized by a set of quantum numbers.

It can be viewed as a point in the space of quantum numbers. The whole set of the states forms a lattice in this space.

A perturbation that violates the integrability provides matrix elements of the hopping between different sites (Anderson model !?)

Q: ***Does Anderson localization provide a generic scenario for the Wigner-Dyson to Poisson crossover*** **?**

Consider an integrable system. Each state is characterized by a set of quantum numbers.

It can be viewed as a point in the space of quantum numbers. The whole set of the states forms a lattice in this space.

A perturbation that violates the integrability provides matrix elements of the hopping between different sites (Anderson model !?)

Weak enough hopping - Localization - Poisson
Strong hopping - transition to Wigner-Dyson

The very definition of the localization is **not invariant** – one should specify in which space the eigenstates are localized.

Level statistics **is invariant**:

Poissonian statistics $\exists$ basis where the eigenfunctions are localized

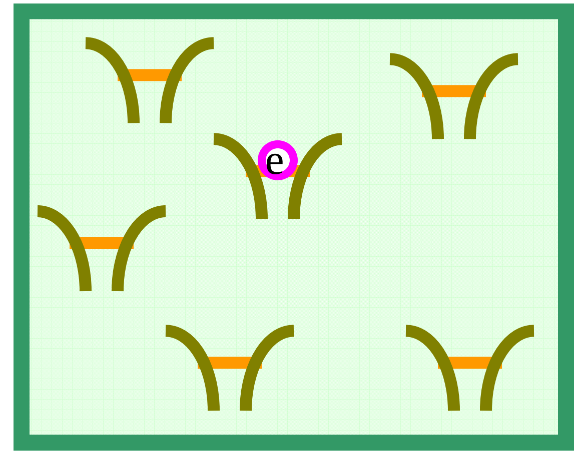
Wigner -Dyson statistics $\forall$ basis the eigenfunctions are extended

Example 1

Doped semiconductor

Low concentration of donors ⇒ Electrons are localized on donors ⇨ Poisson

Higher donor concentration ⇒ Electronic states are extended ⇨ Wigner-Dyson

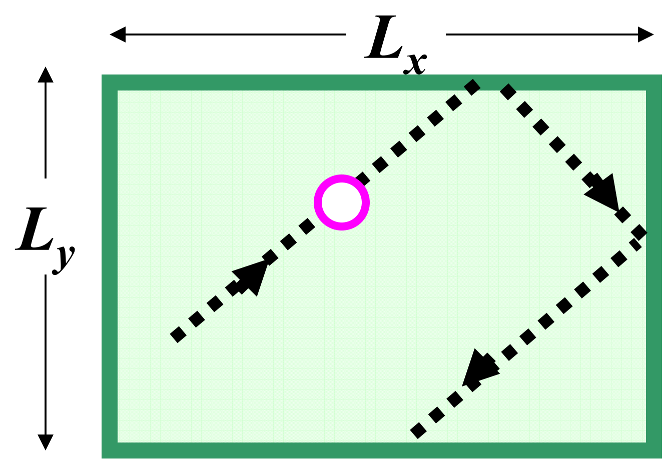


Example 2

Rectangular billiard

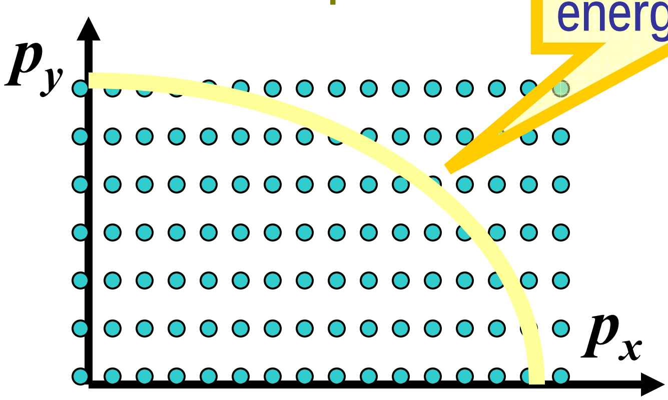
Two integrals of motion

$$p_x = \frac{\pi n}{L_x}; \quad p_y = \frac{\pi m}{L_y}$$



Lattice in the momentum space

Line (surface) of constant energy



Ideal billiard – localization in the momentum space ⇨ Poisson

Deformation or smooth random potential – delocalization in the momentum space ⇨ Wigner-Dyson

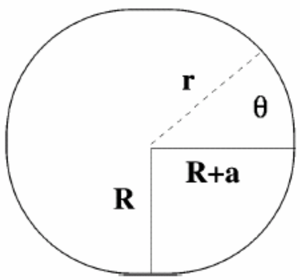
VOLUME 77, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECEMBER 1996

Diffusion and Localization in Chaotic Billiards

Fausto Borgonovi,[1,3,4] Giulio Casati,[2,3,5] and Baowen Li[6,7]
[1]Dipartimento di Matematica, Università Cattolica, via Trieste 17, 25121 Brescia, Italy
[2]Università di Milano, sede di Como, Via Lucini 3, Como, Italy
[3]Istituto Nazionale di Fisica della Materia, Unità di Milano, via Celoria 16, 22100, Milano, Italy
[4]Instituto Nazionale di Fisica Nucleare, Sezione di Pavia, Pavia, Italy
[5]Instituto Nazionale di Fisica Nucleare, Sezione di Milano, Milano, Italy
[6]Department of Physics and Centre for Nonlinear and Complex Systems, Hong Kong Baptist University, Hong Kong
[a7]Center for Applied Mathematics and Theoretical Physics, University of Maribor, Krekova 2, 2000 Maribor, Slovenia
(Received 29 July 1996)

Localization and diffusion in the angular momentum space

$$\varepsilon \equiv \frac{a}{R}$$



$\varepsilon > 0$ Chaotic stadium

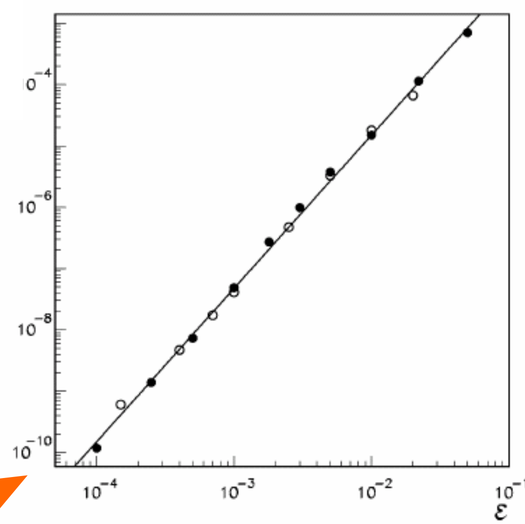
$\varepsilon \to 0$ Integrable circular billiard

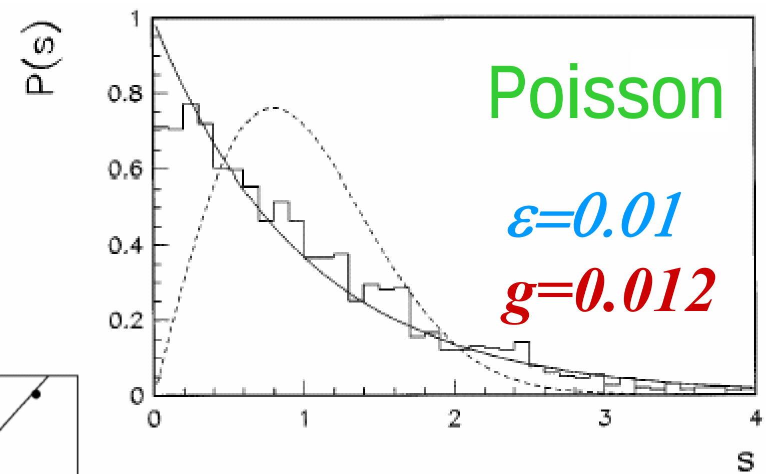
Angular momentum is the integral of motion

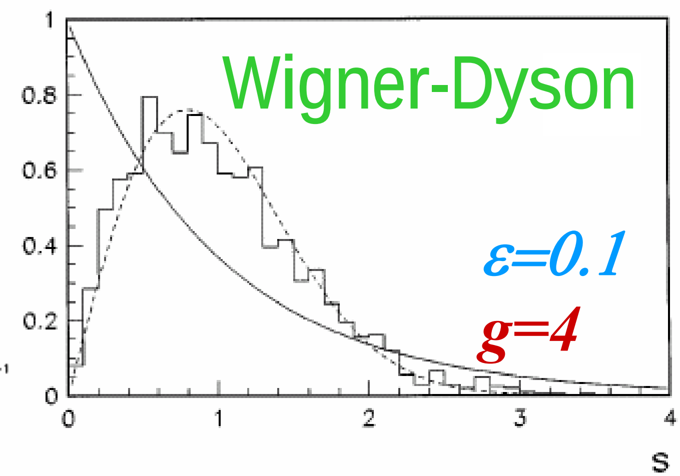
$\hbar = 0; \quad \varepsilon << 1$

Diffusion in the angular momentum space

$D \propto \varepsilon^{5/2}$







D.Poilblanc, T.Ziman, J.Bellisard, F.Mila & G.Montambaux

Europhysics Letters, v.22, p.537, 1993

1D Hubbard Model on a periodic chain

$$H = t\sum_{i,\sigma}\left(c^{+}_{i,\sigma}c_{i+1,\sigma} + c^{+}_{i+1,\sigma}c_{i,\sigma}\right) + U\sum_{i,\sigma} n_{i,\sigma}n_{i,-\sigma} + V\sum_{i,\sigma,\sigma'} n_{i,\sigma}n_{i+1,\sigma'}$$

Onsite interaction

n. neighbors interaction

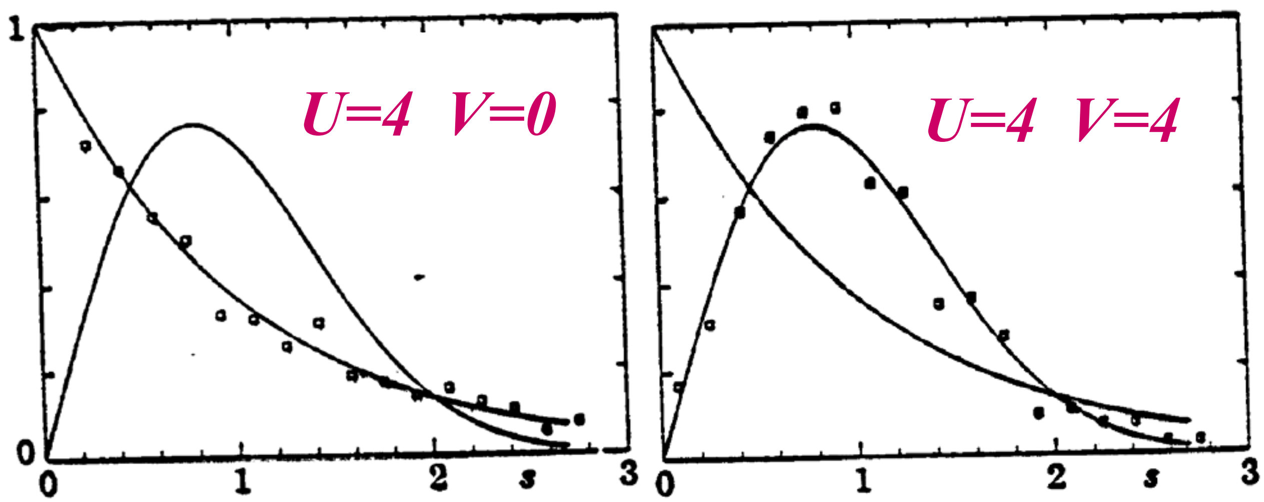
$V = 0$	Hubbard model	integrable
$V \neq 0$	extended Hubbard model	nonintegrable

12 sites

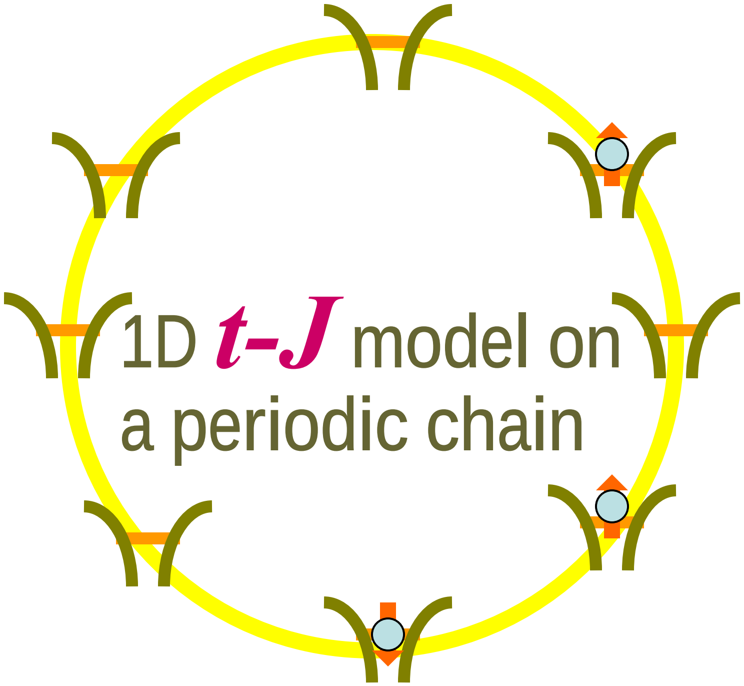
3 particles

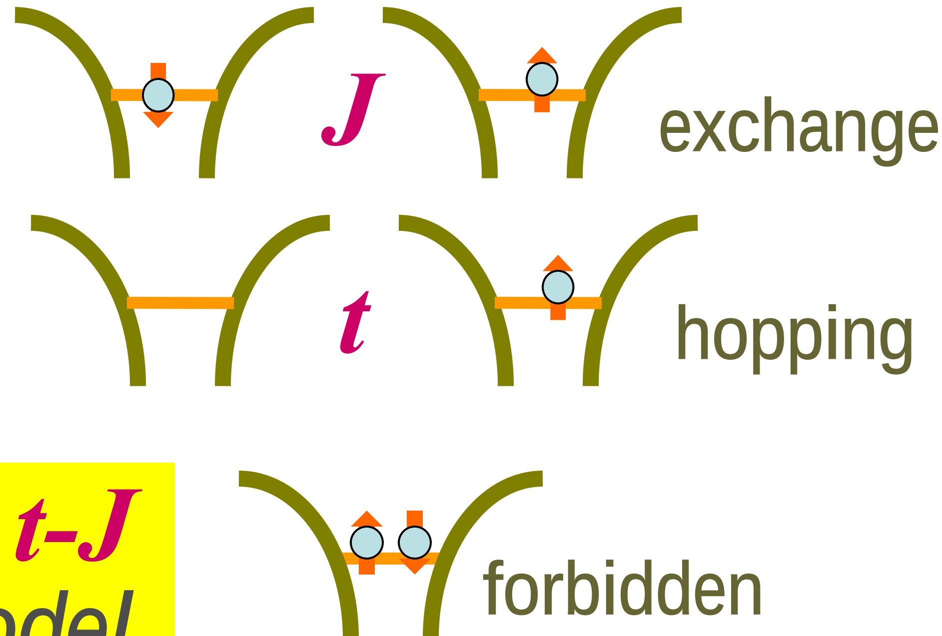
Zero total spin

Total momentum $\pi/6$

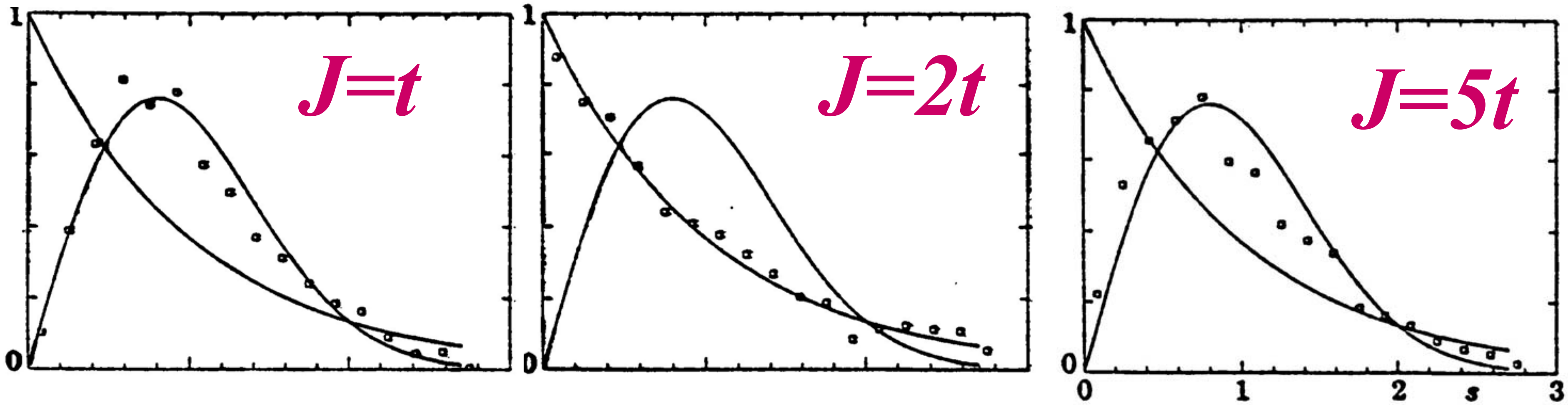


D.Poilblanc, T.Ziman, J.Bellisard, F.Mila & G.Montambaux
Europhysics Letters, v.22, p.537, 1993





1d *t-J* *model*



***N=16*; one hole**

Wigner-Dyson random matrix statistics follows from the delocalization.

Q: ***Why the random matrix theory (RMT) works so well for nuclear spectra*** **?**

Many-Body excitations are delocalized !

What does it mean ?