# Introduction to the Physics of Semiconductor Quantum Dots

M. A. Kastner, MIT

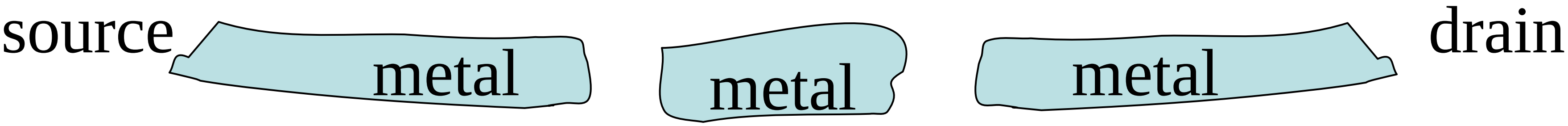
Windsor 2007

Massachusetts
Institute of
Technology

# Outline: Measuring Energy Scales

- Coulomb blockade energy U (Metal Single Electron Transistor)
- Energy level spacing $\Delta\varepsilon$ (Semiconductor SET)
- Coupling to the leads $\Gamma$ and $kT_K$
- Measuring charge instead of current
- Electron counting to determine $\Gamma$

# Two Barriers with Small Island



source

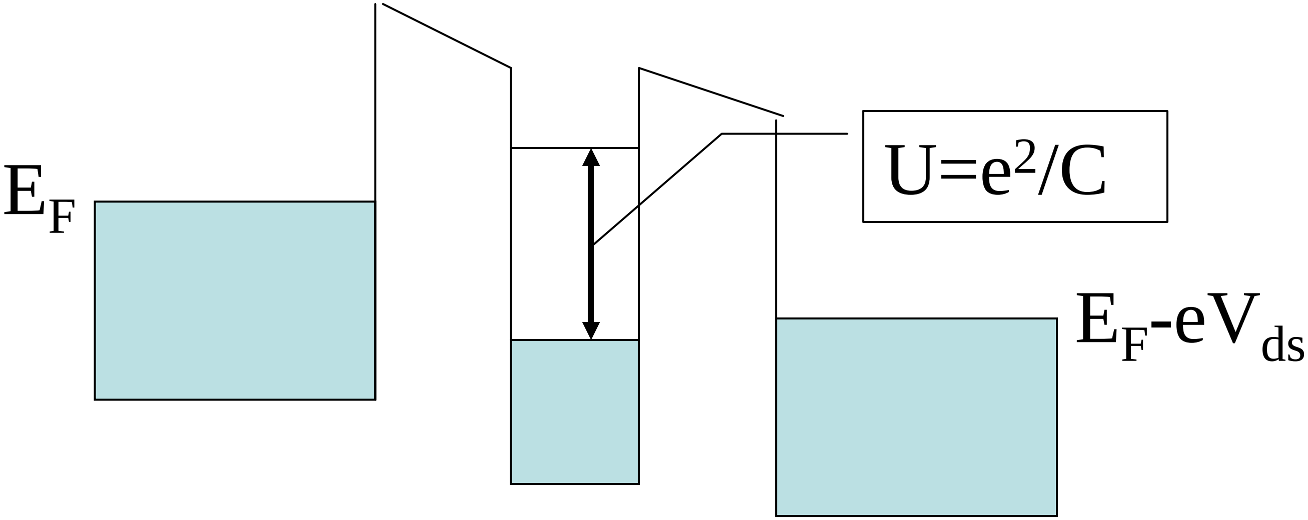
metal

metal

metal

drain

Small
Metal
Island



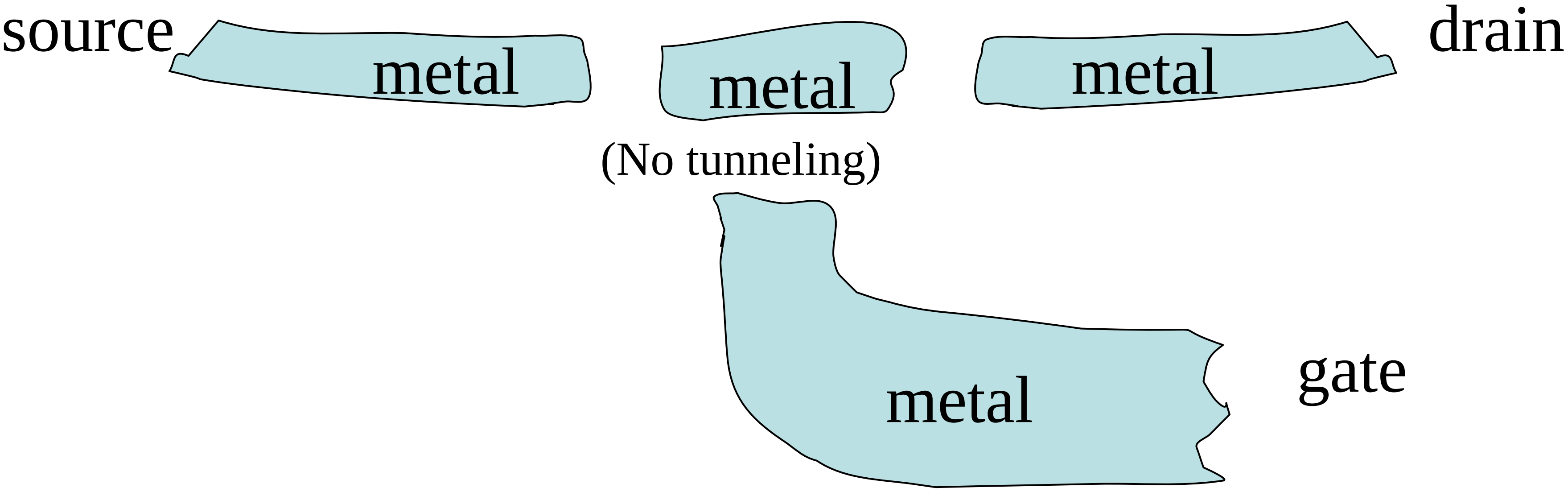
$E_F$

$U=e^2/C$

$E_F\text{-}eV_{ds}$

⇒ No current at zero temperature

# Schematic of Metal SET



source

metal

metal

metal

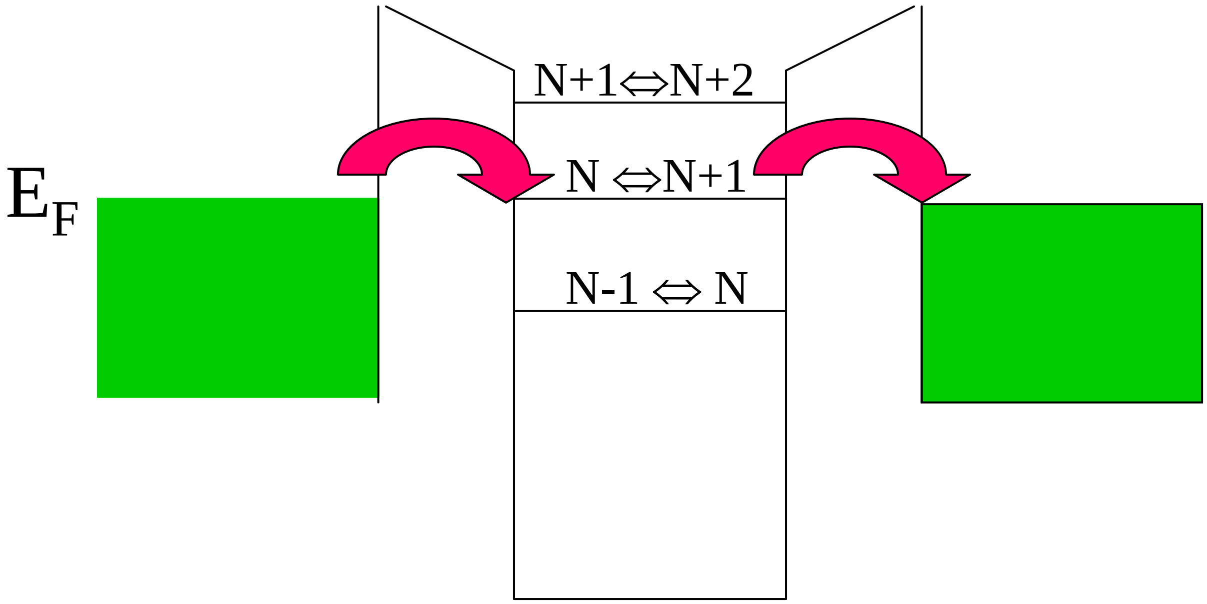
drain

(No tunneling)

metal

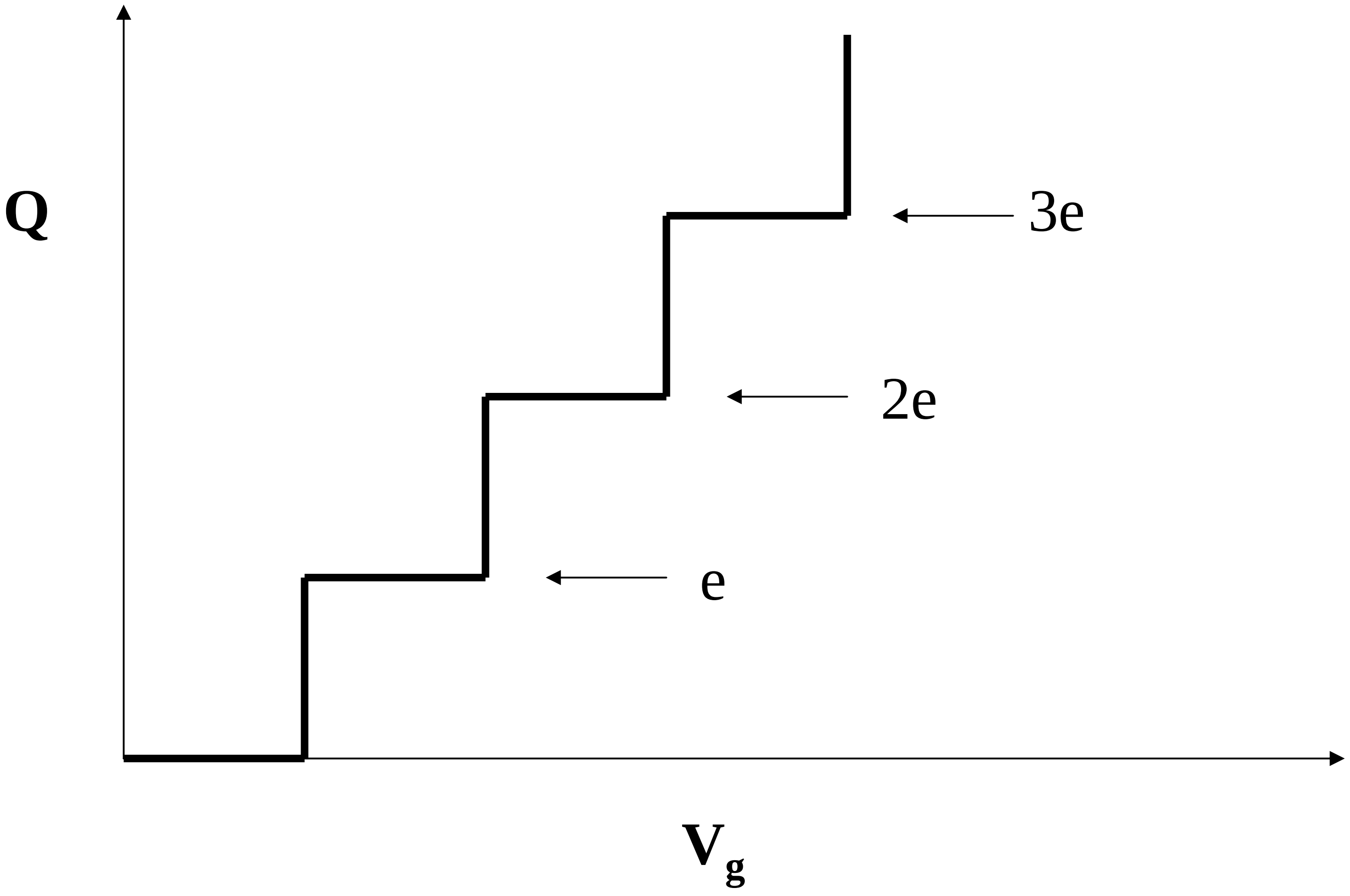
gate

# Sequential Charging



At low T and with very small $V_{ds}$ get one sharp peak for each electron added.

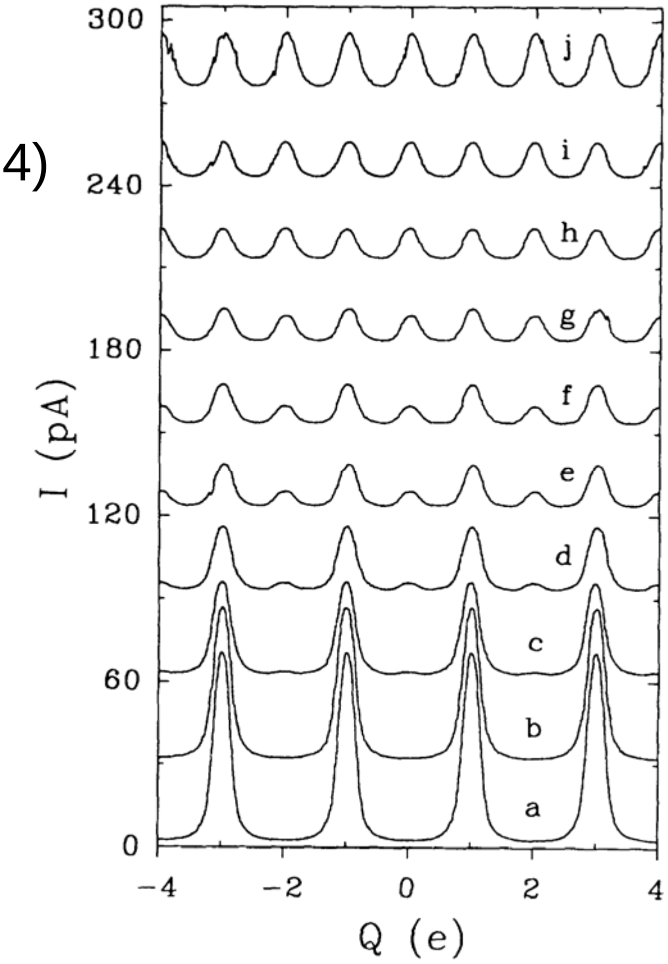
# Charge Quantization



# Current vs. Gate Voltage

Aluminum SET
A. Amar et al. PRL **72**, 3234 (1994)

Note: Because of superconductivity, below $T_C$ one gets a peak every time Cooper pair is added.

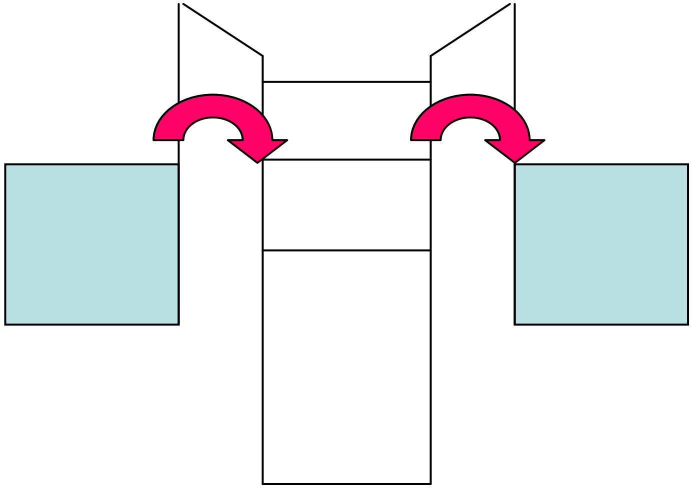


300 mK

50 mK

$Q = C_G V_G$

# Condition for Charge Quantization

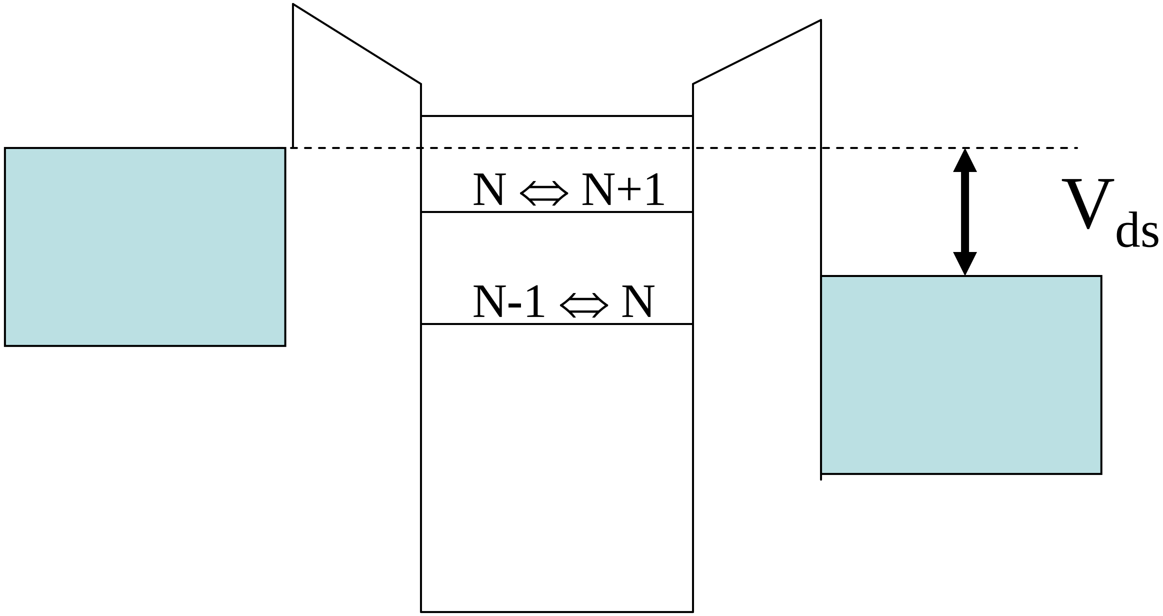


An extra electron stays on the island for time RC.
This time must be long enough that the uncertainty
in its energy is less than U.

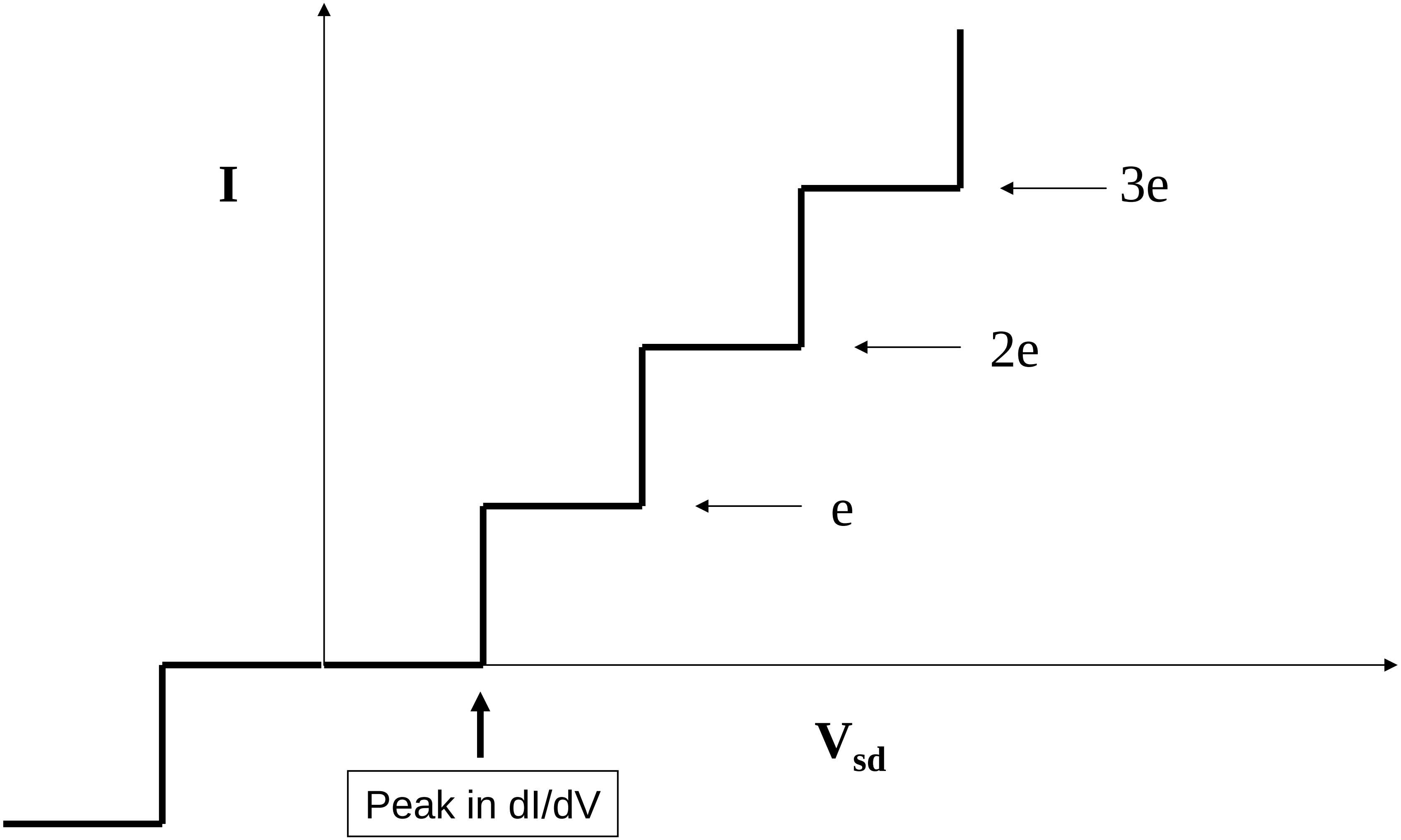
$U > h/RC$, but $U = e^2/C$

$R > h/e^2$ or $G < e^2/h$

# Adding Charge by Source-Drain

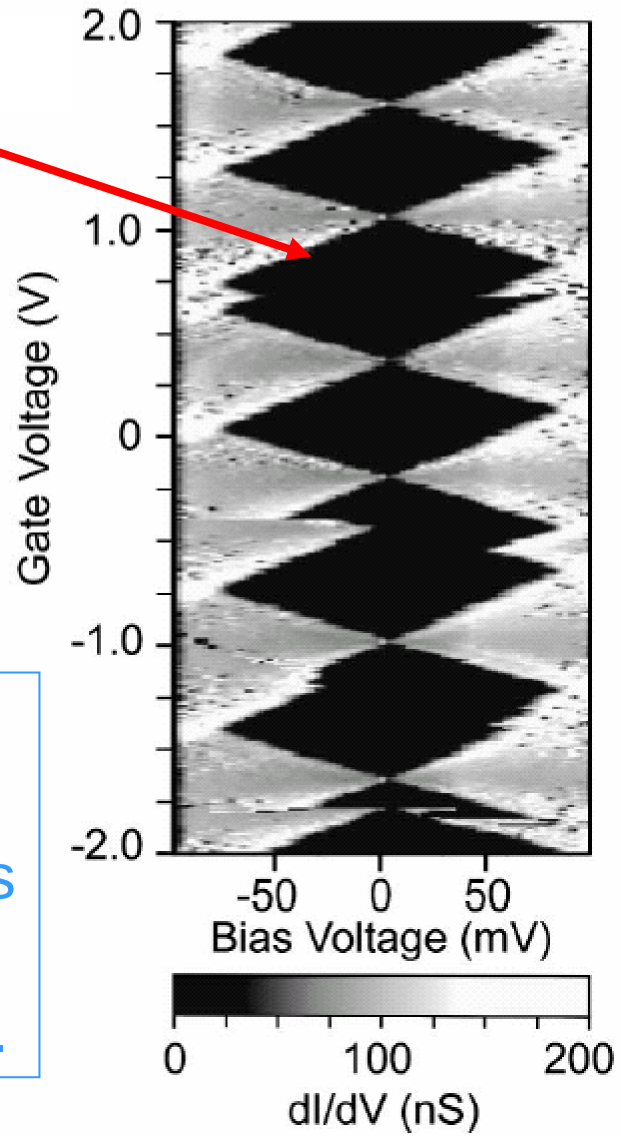


# Coulomb Staircase



# Coulomb Diamonds

First step in Coulomb staircase



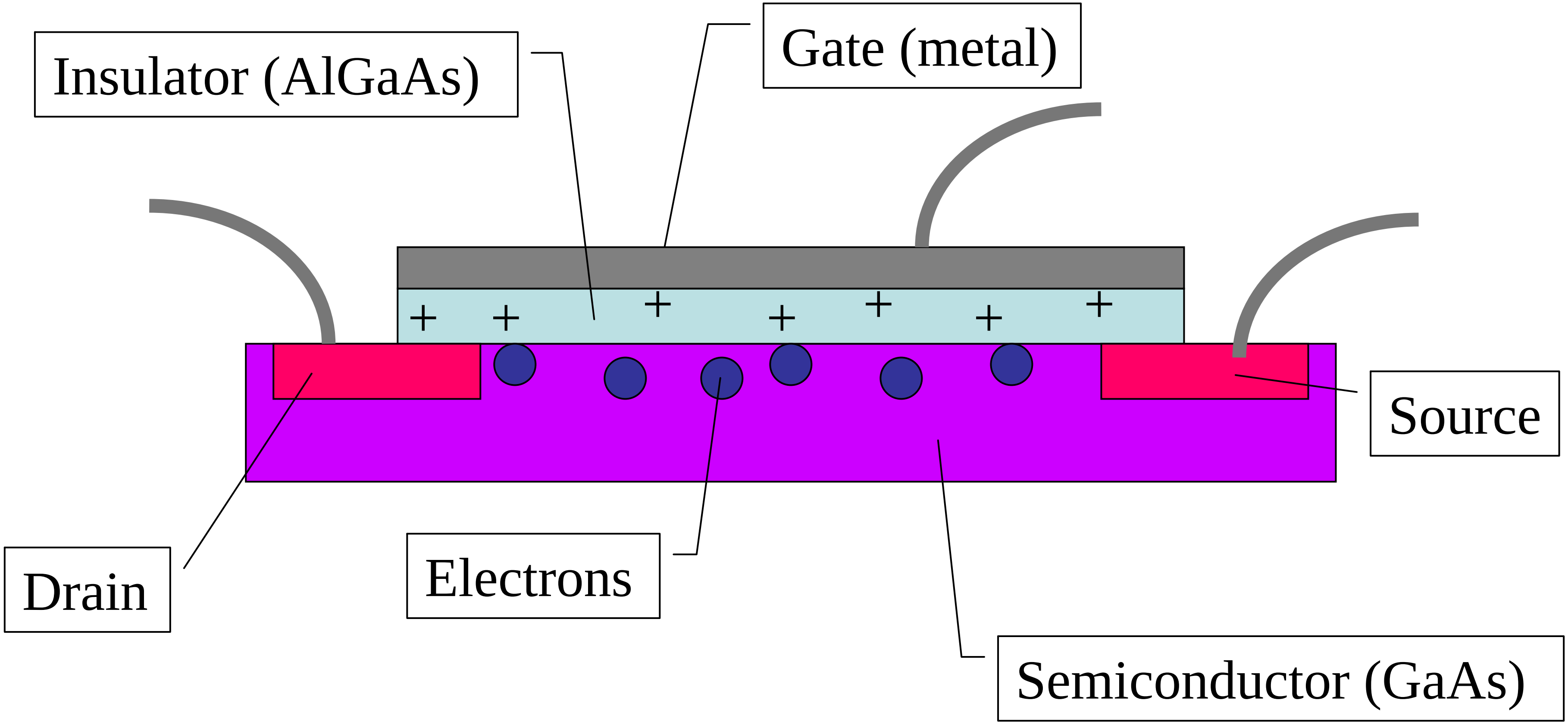
SET made with nano-particle, Bolotin et al. APL **84**, 3154 (2004)

Note: switching from nearby charges

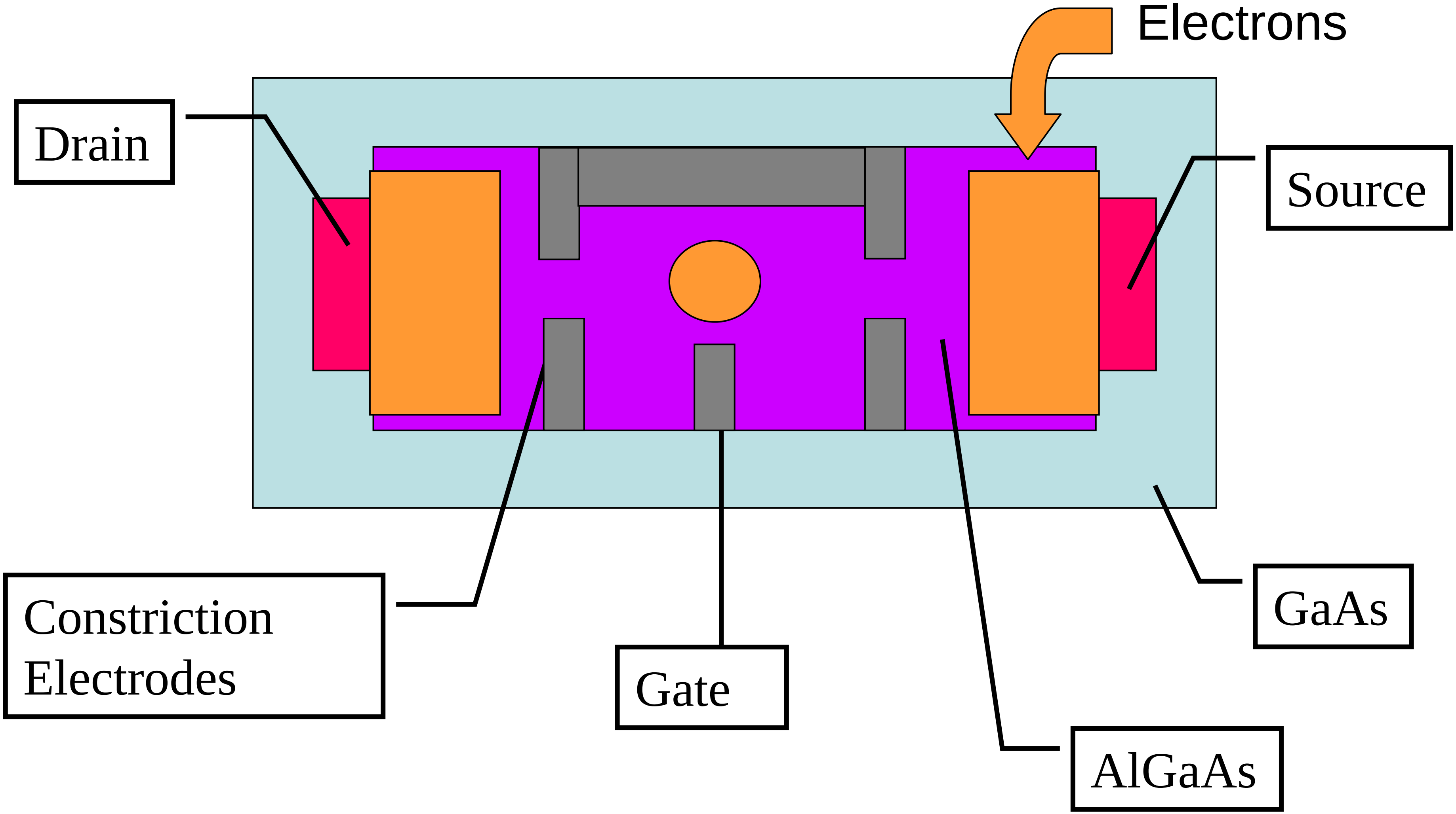
Slopes of diamonds give capacitance ratios which converts voltage to energy.

# Making Semiconductor SETs

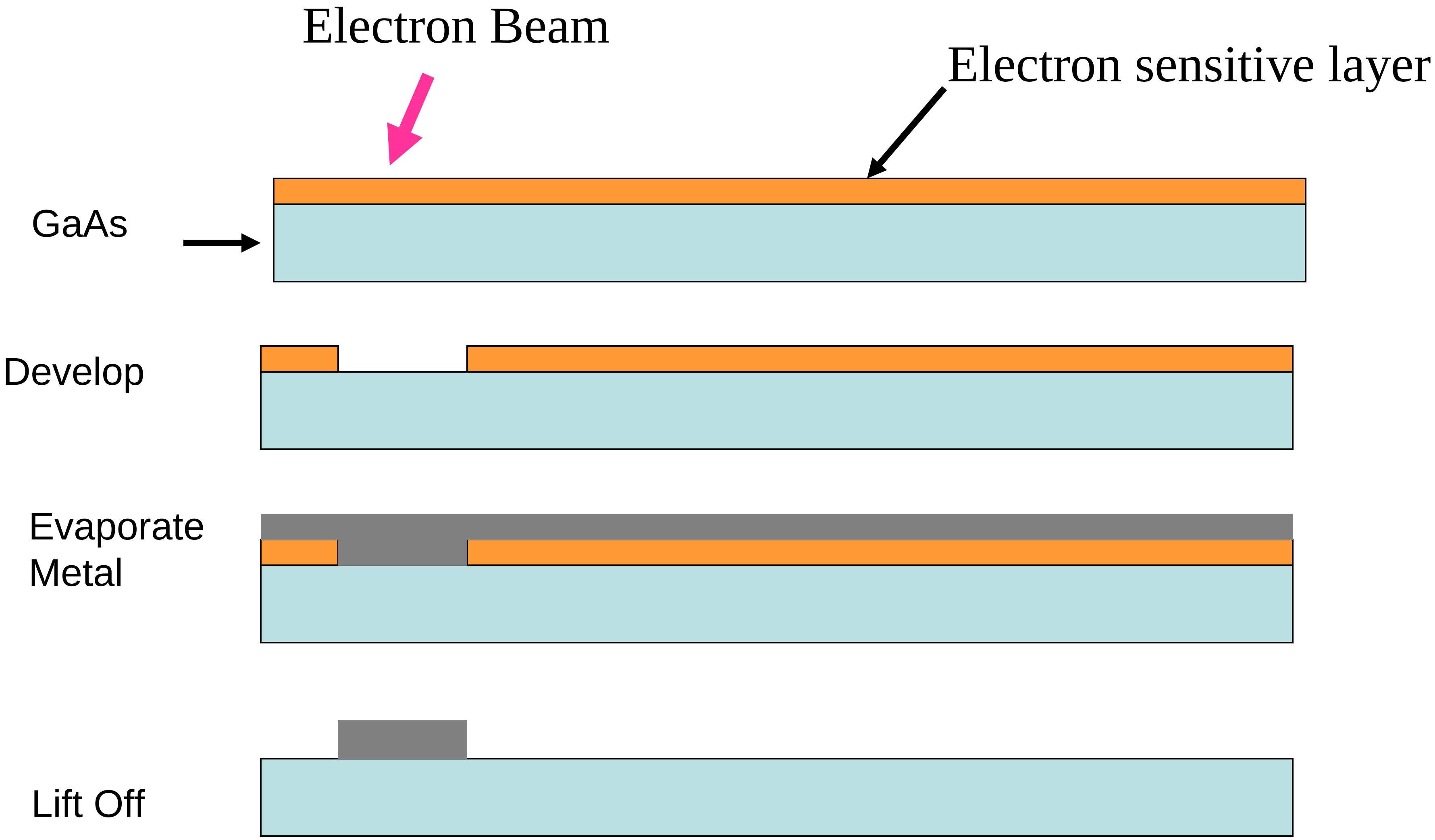
## GaAs Field Effect Transistor



# Schematic GaAs SET



# Electron Beam Lithography



Electron Beam

Electron sensitive layer

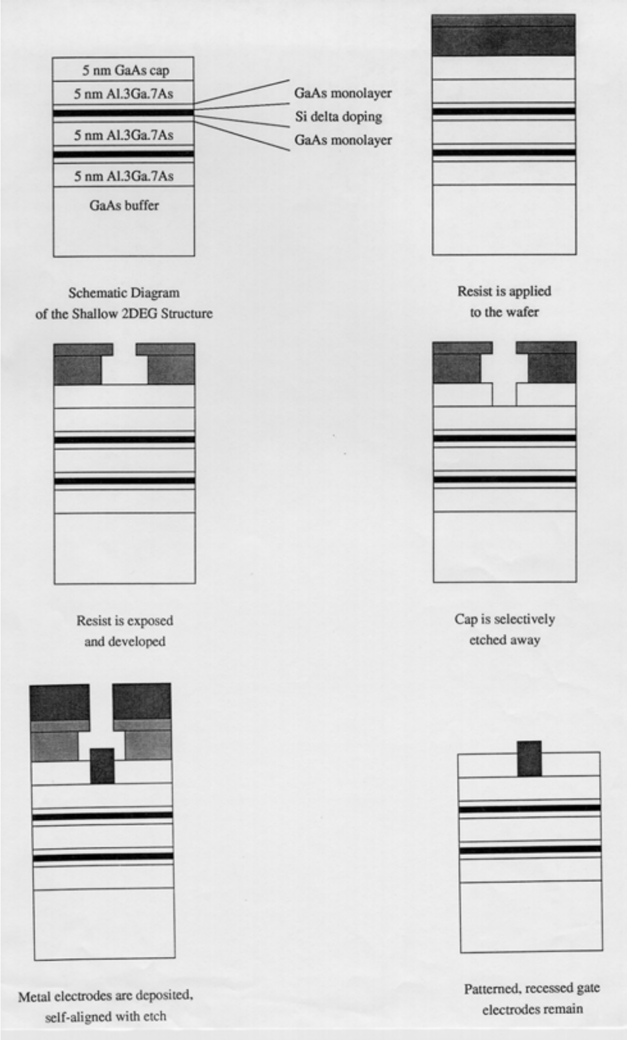
GaAs

Develop

Evaporate
Metal

Lift Off

# Actual Process



Schematic Diagram
of the Shallow 2DEG Structure

Resist is applied
to the wafer

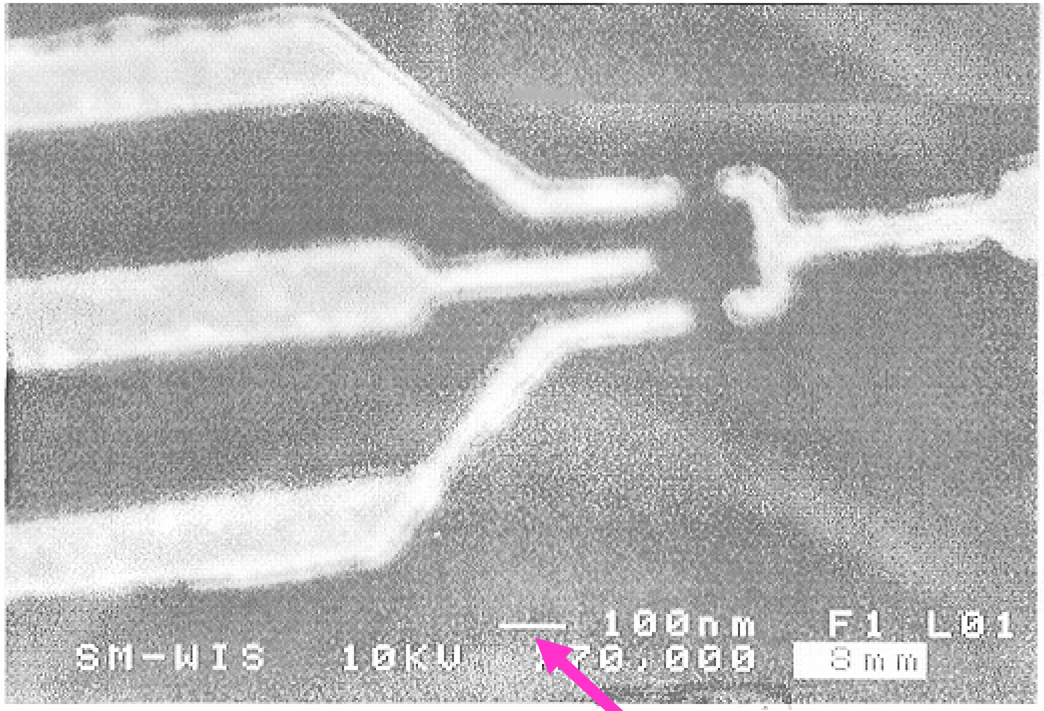
Resist is exposed
and developed

Cap is selectively
etched away

Metal electrodes are deposited,
self-aligned with etch

Patterned, recessed gate
electrodes remain

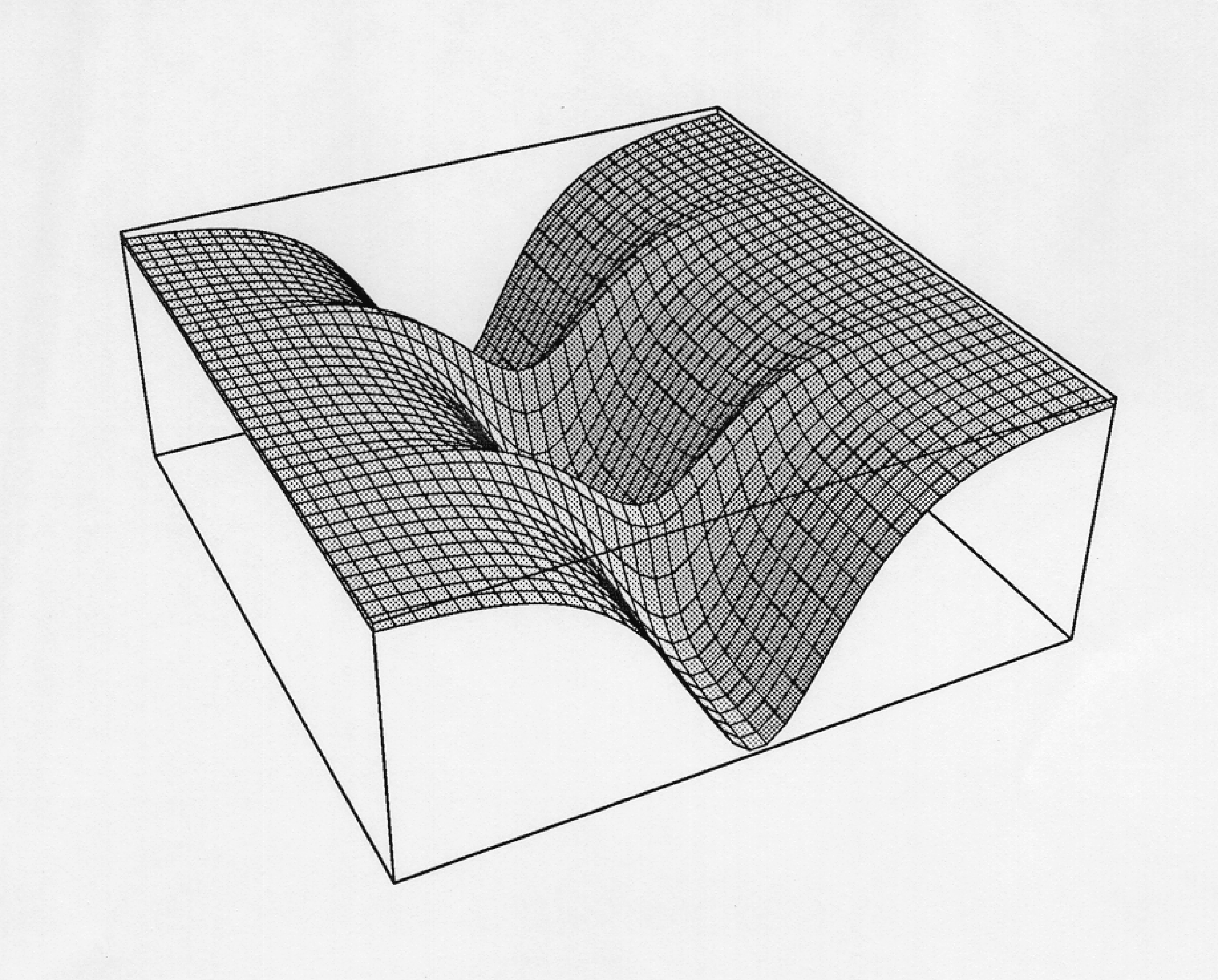
# GaAs SET



100nm = 1000Å

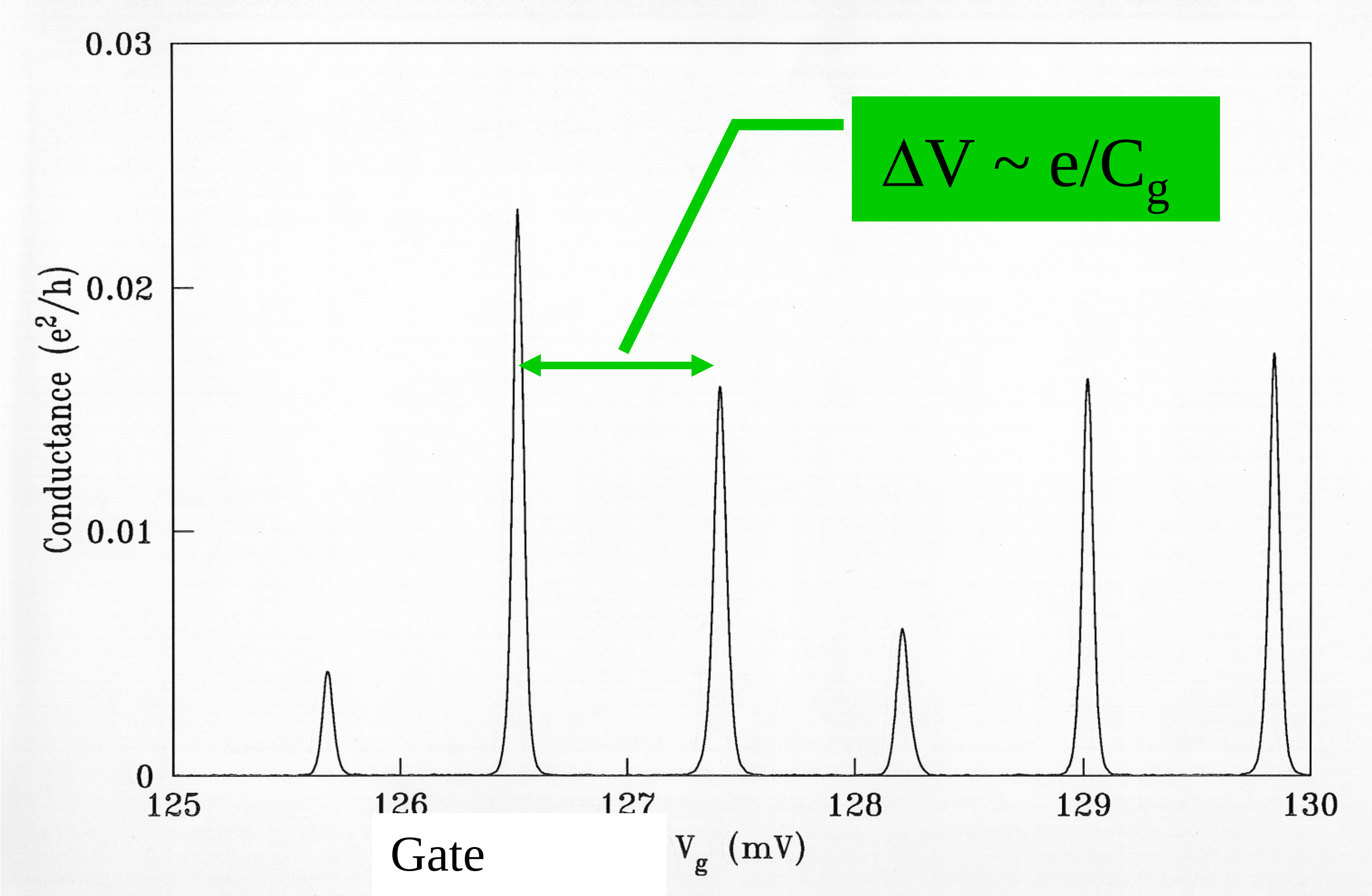
D. Goldhaber-Gordon et al, Nature, **391**, 156 (1998)

# Schematic Potential in SET



# Coulomb Charging Peaks

Data from Meirav et al. PRL **65**, 771 (1990).

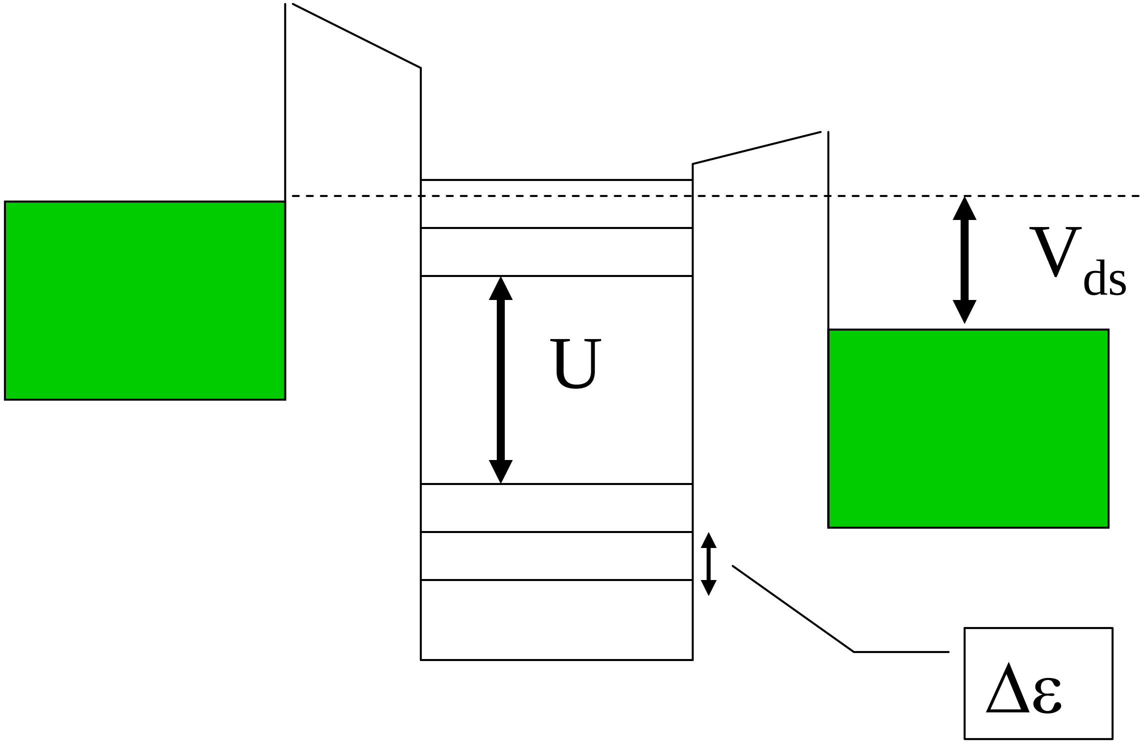


$\Delta V \sim e/C_g$

Gate Voltage

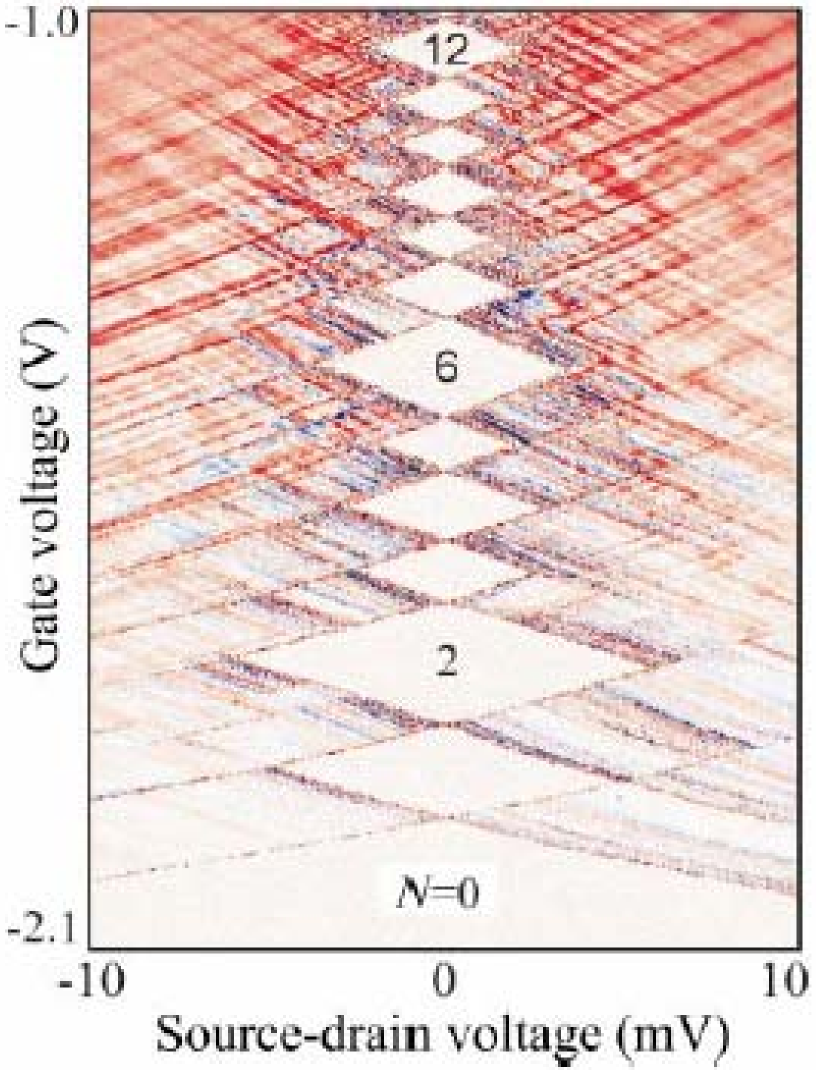
Note: Variation of peak height and spacing reflects individual levels.

# Quantized Energy Levels



There is a peak in $dI/dV_{sd}$ for every energy level. Although these have been detected in metal SET's it is hard because density of states is so large.

# Excited State Spectroscopy



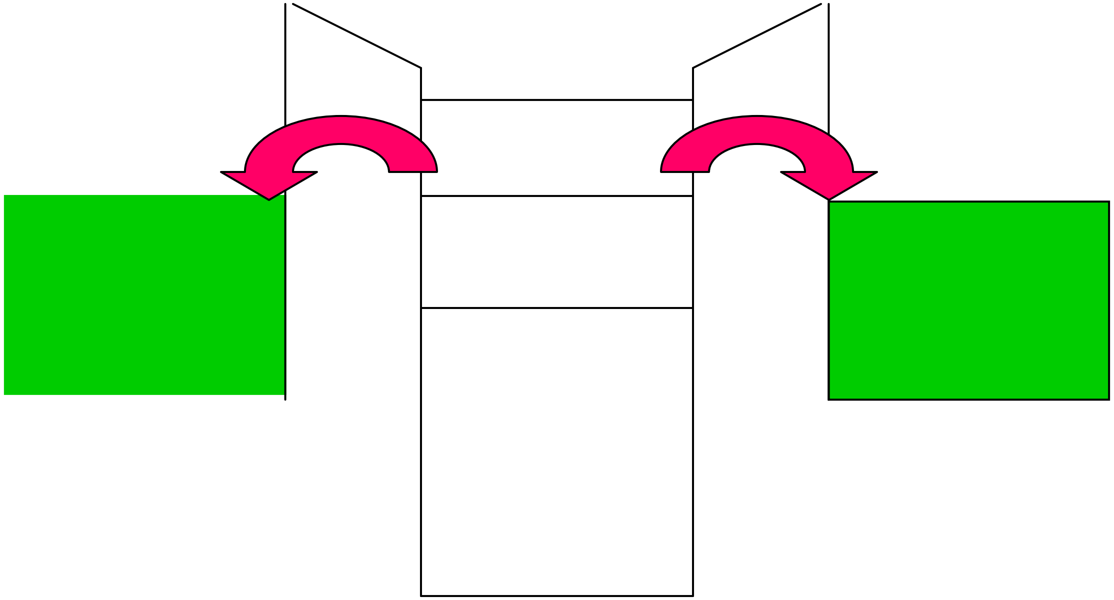
dI/dV$_{sd}$ has peak when level crosses E$_F$

Very small dot ⇒ peaks no longer periodic along V$_{sd}$ = 0

Electron interactions are more complicated than just U and involve exchange.

Kouwenhoven et al
Science **278**, 1788, 1997

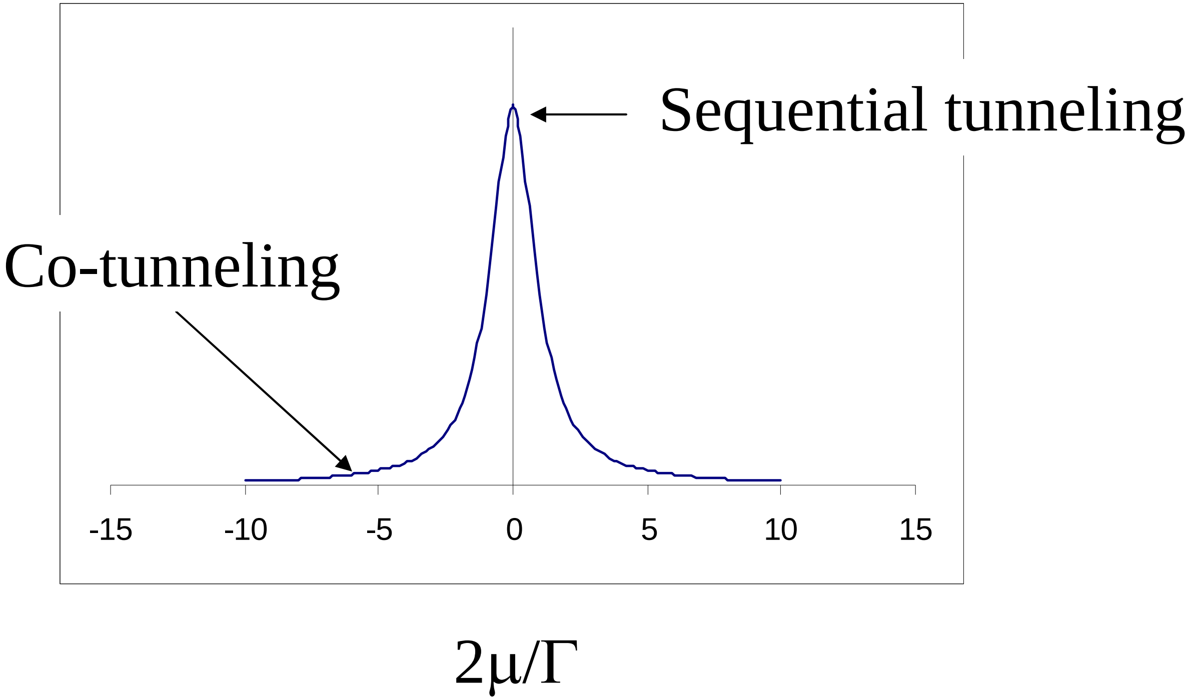
# Lifetime Broadening



Probability of electron remaining in a level on the dot decays as exp(-t/τ), so the level broadens into a Lorentzian with energy width

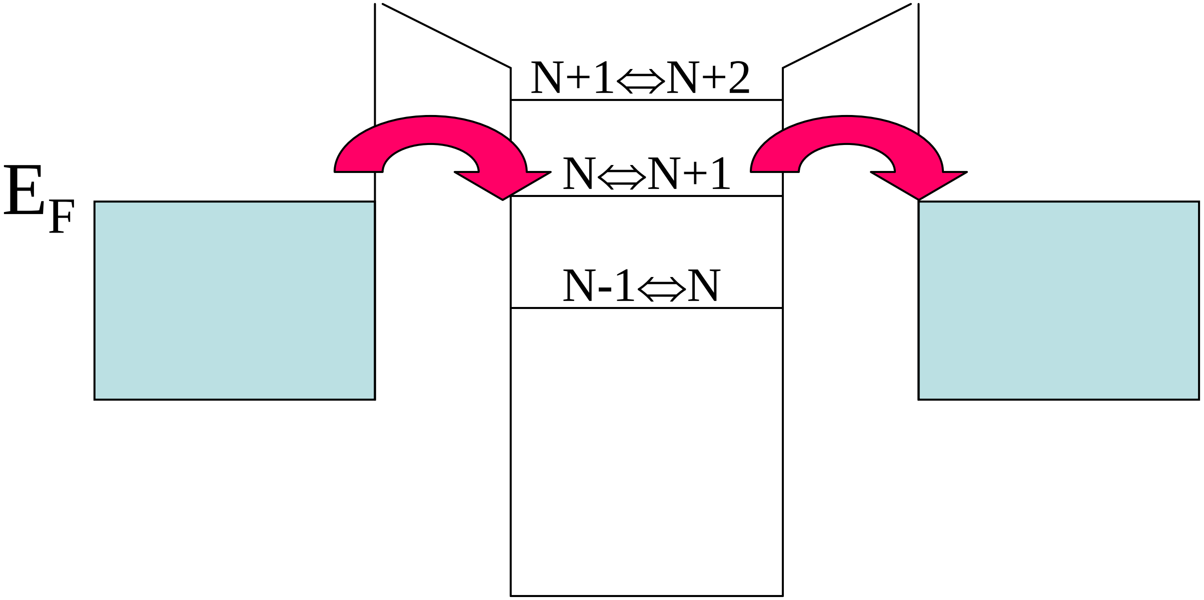
$\Gamma = h/\tau$

# Lorentzian Line Shape of Peaks vs Gate Voltage



The chemical potential $\mu$ is proportional to the gate voltage. The full width at half maximum is $\Gamma$. $\tau=h\Gamma^{-1}$ is the time for the electron to tunnel off.

# Temperature



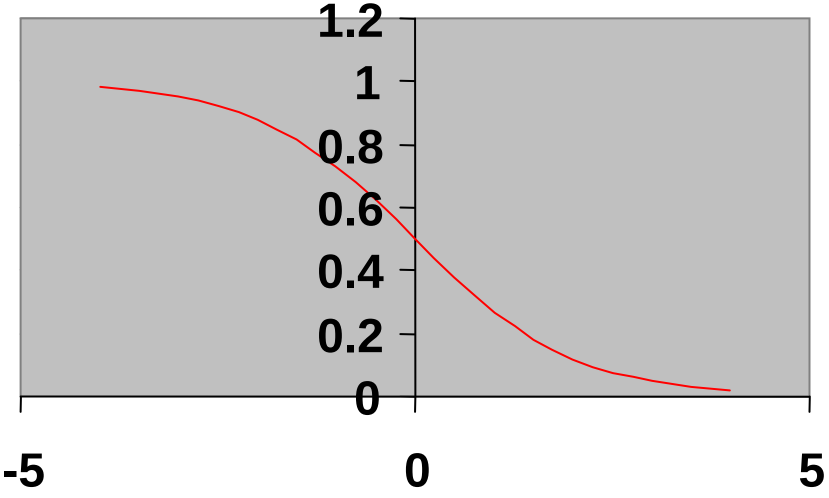
$$I \sim \int T(E)[f(E)-f(E-eV_{sd})]dE \qquad f(E) = \frac{1}{\exp[(E-E_F)/kT]+1}$$

For resonant tunneling near zero bias, i.e. $eV_{sd} < kT$, if $\Gamma$ is very small, $T(E) = \delta(E)$, $I = eV_{sd}\, df/dE$

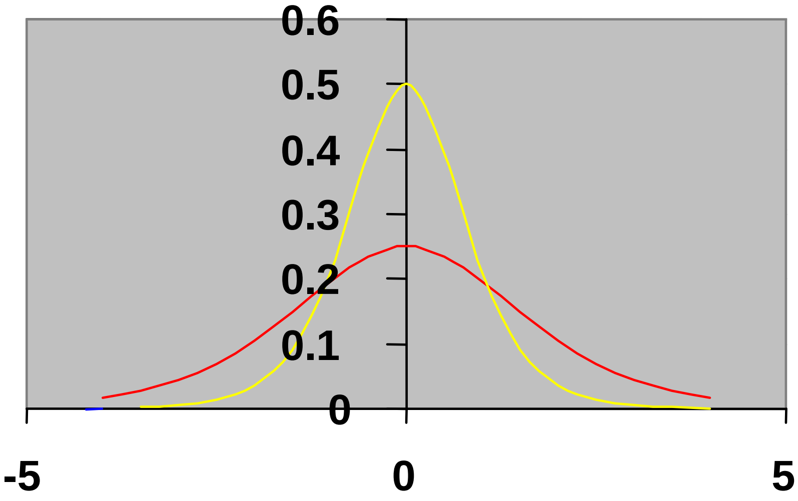
# Thermal Broadening

Fermi-Dirac
Distribution

f(E)



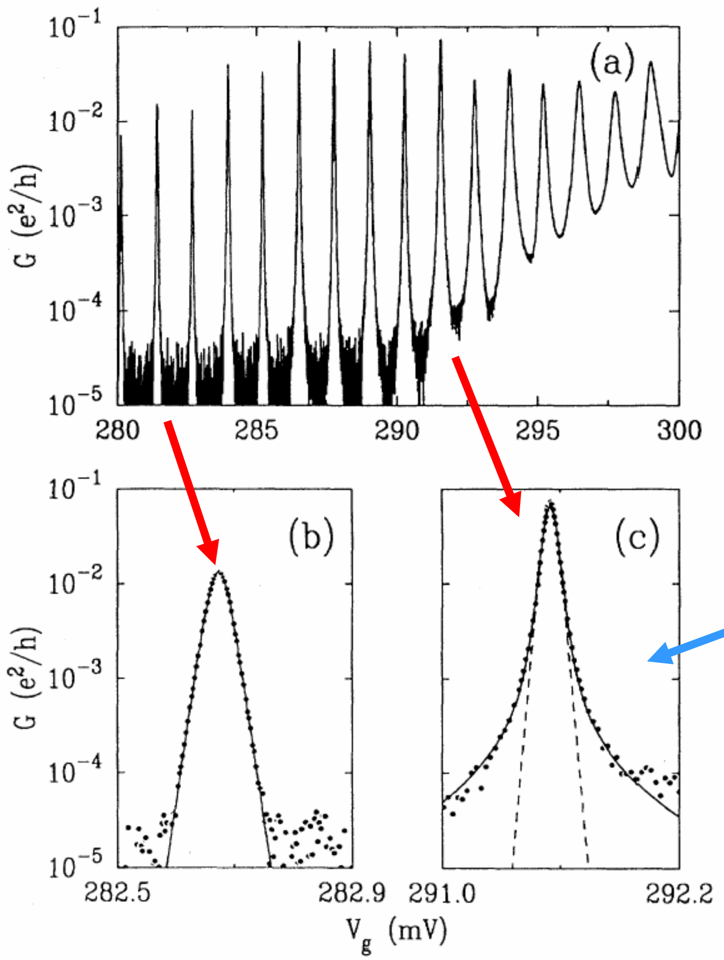
df/dE



Thermal broadening
gives width = 3.5kT
Height ~ 1/T

kT = 0.5
kT =1

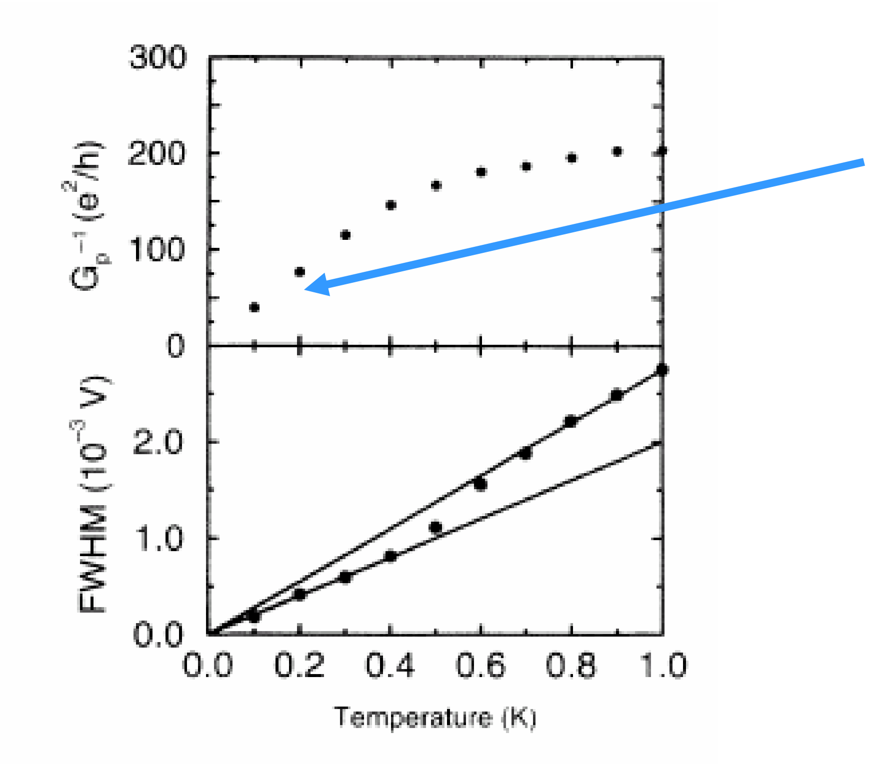
# Thermal and Intrinsic Broadening



Foxman et al. Phys. Rev B 47, 10020 (1992)

Dashed line from Fermi alone, solid includes Lorentzian

# Absolute Thermometer



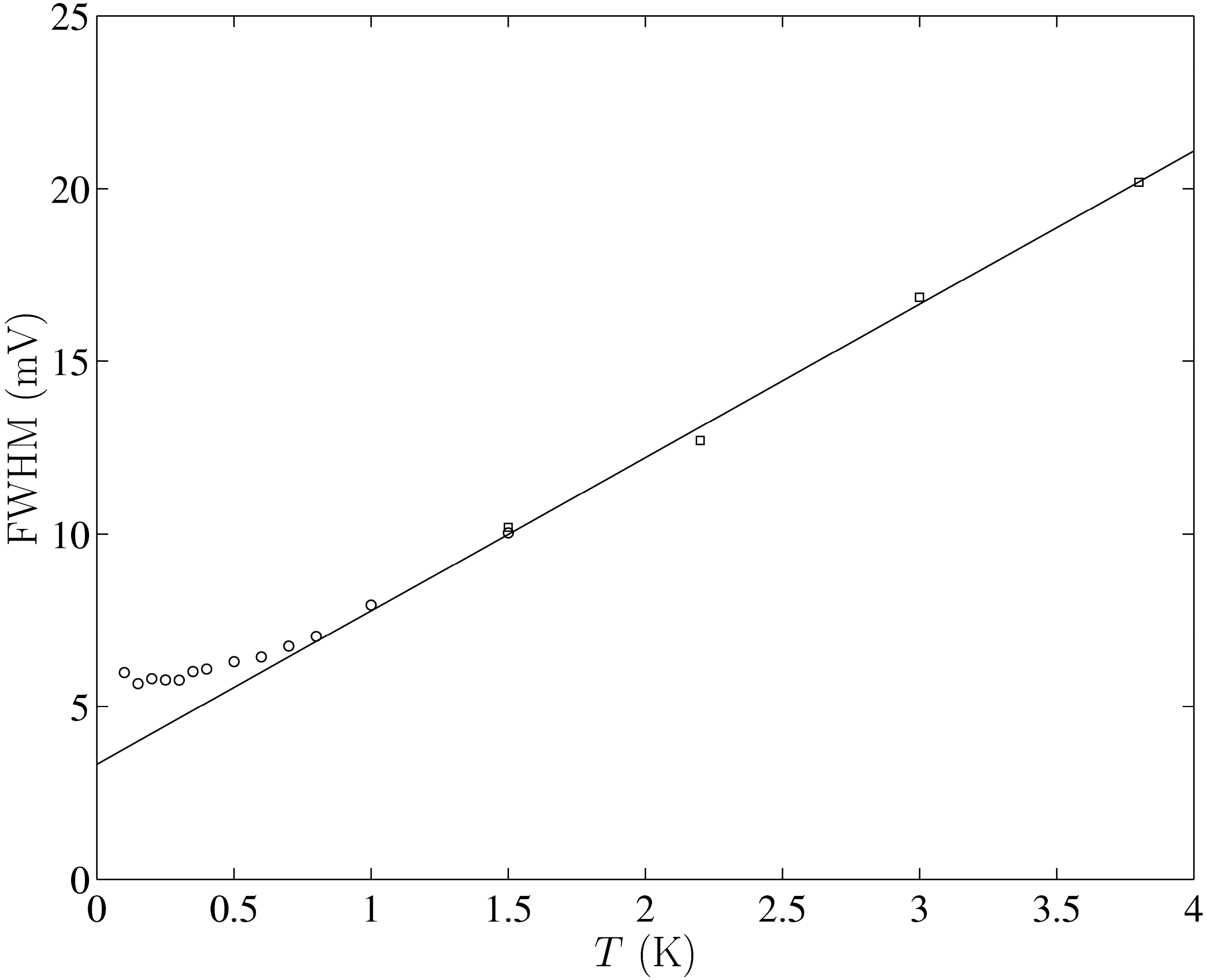
$1/G_p \sim T$

Full width at
half max. ~ T

When kT > $\Delta\varepsilon$ the peak conductance becomes constant
and the width changes slope slightly.

For thermometer application see Pekola et al. PRL **73**, 2903 (1994)

# Determining Γ from peak width



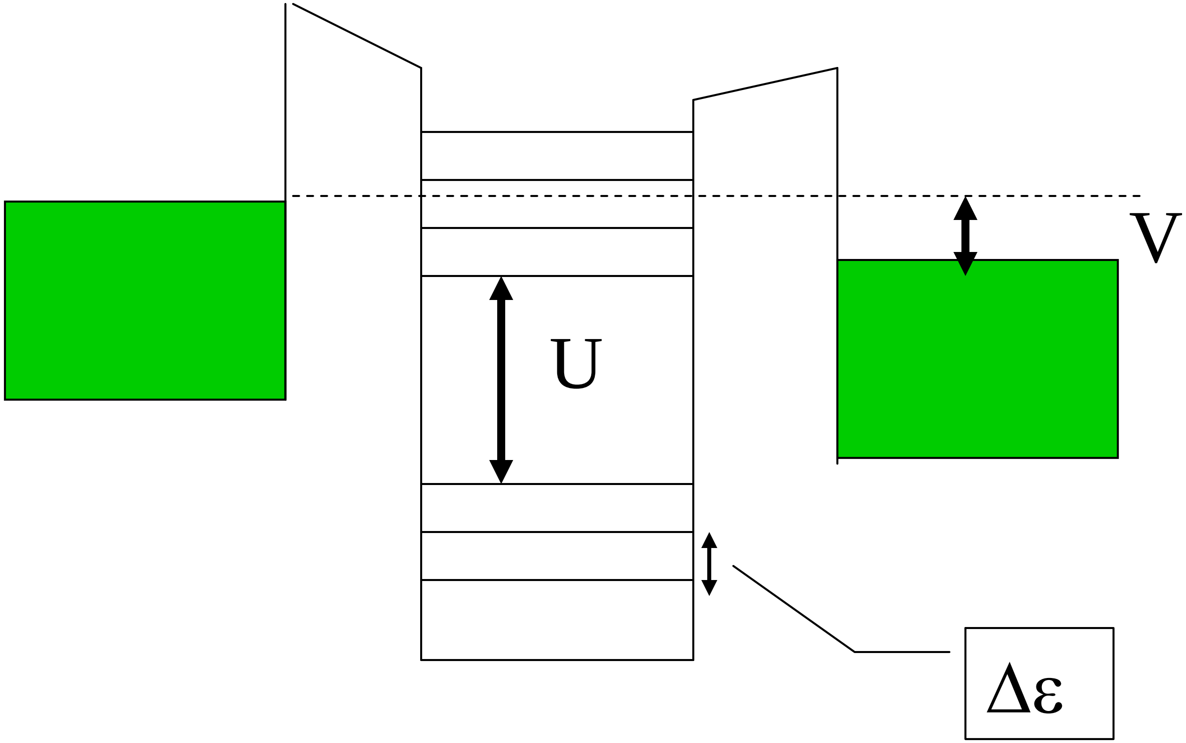
Slope gives conversion of voltage to energy.

T =0 intercept gives Γ

Note: in this case Γ/h~50GHz

Goldhaber-Gordon et al Phys. Rev. Lett. **81**, 5225 (1998)

# Condition for Charge Quantization is Condition for Level Separation

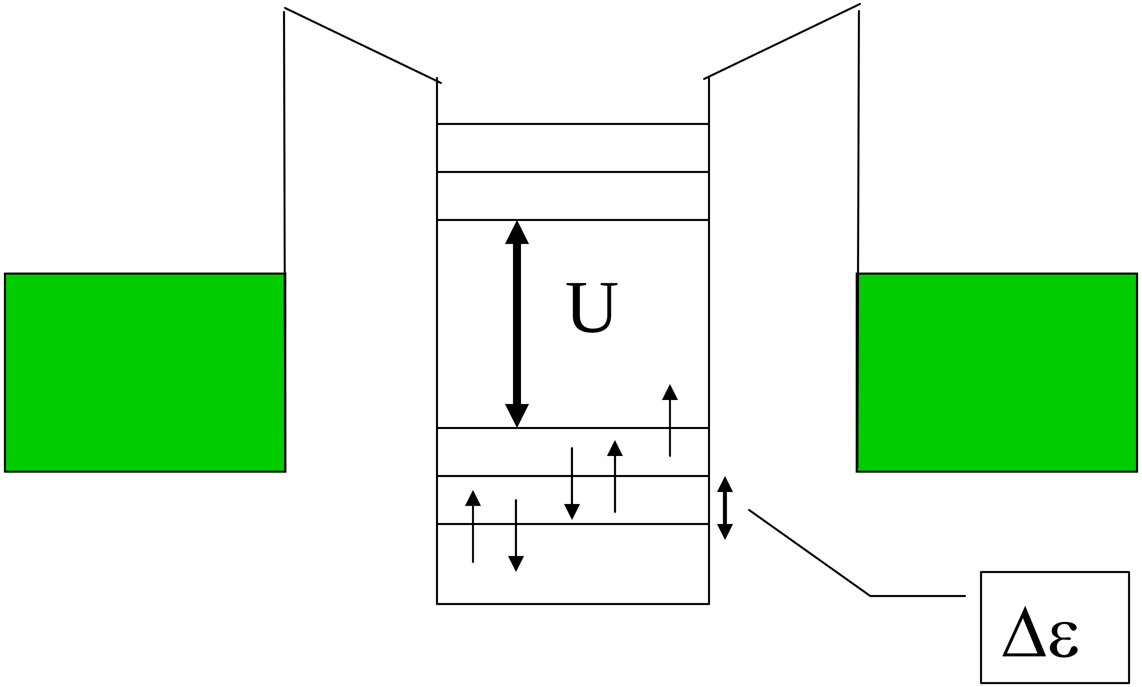


Above Coulomb gap, the current is $I = Ne/\tau$, $\tau = h\Gamma^{-1}$ and $N = eV/\Delta\varepsilon$

$$G = I/V = (e^2/h)(\Gamma/\Delta\varepsilon)$$

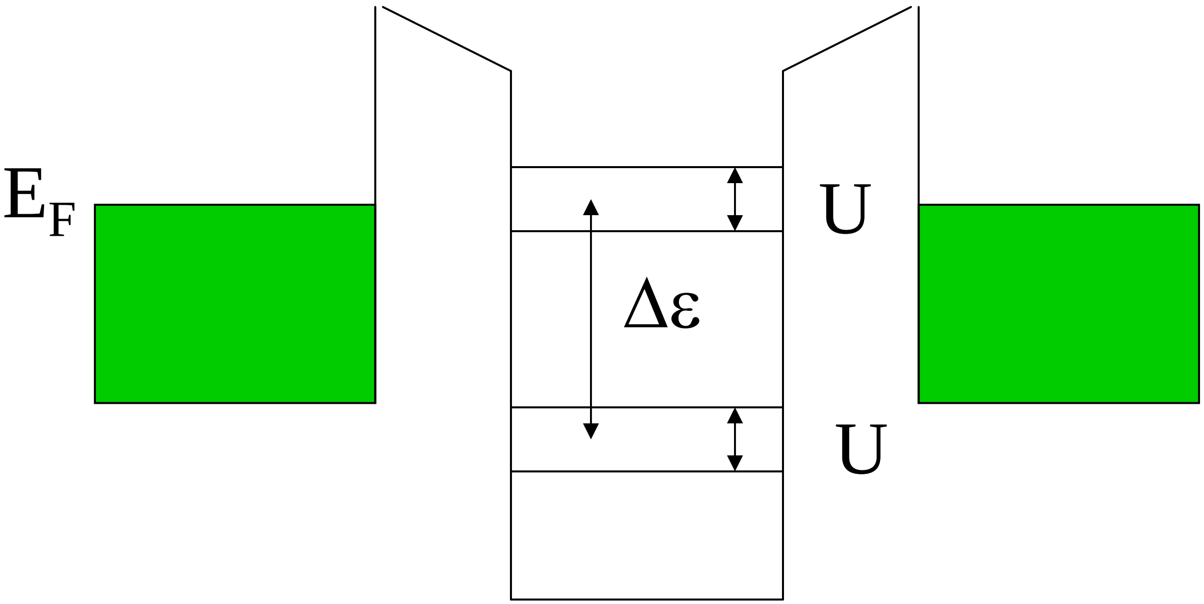
$$G < e^2/h \Rightarrow \Gamma < \Delta\varepsilon$$

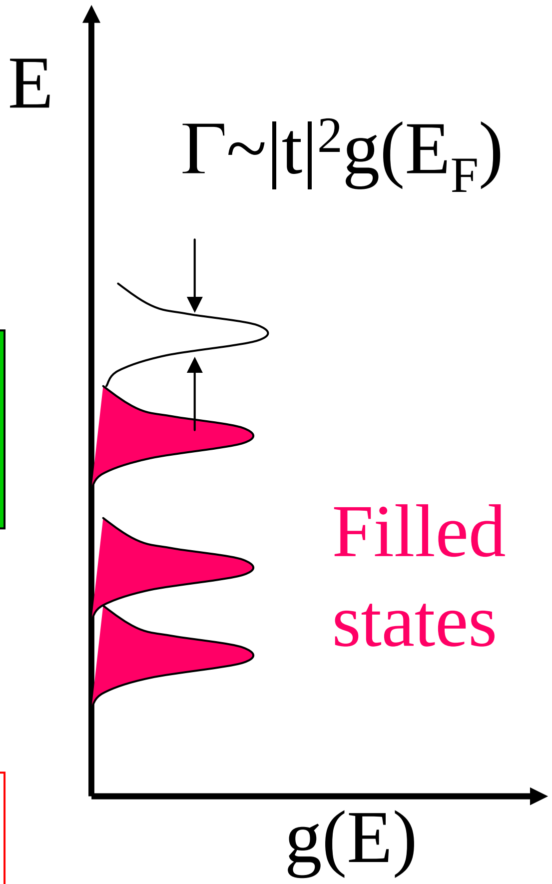
# Constant Interaction Model



Ignore interactions among electrons on artificial atom.
States fill two at a time.
Actually more complicated, but it is a useful starting point.

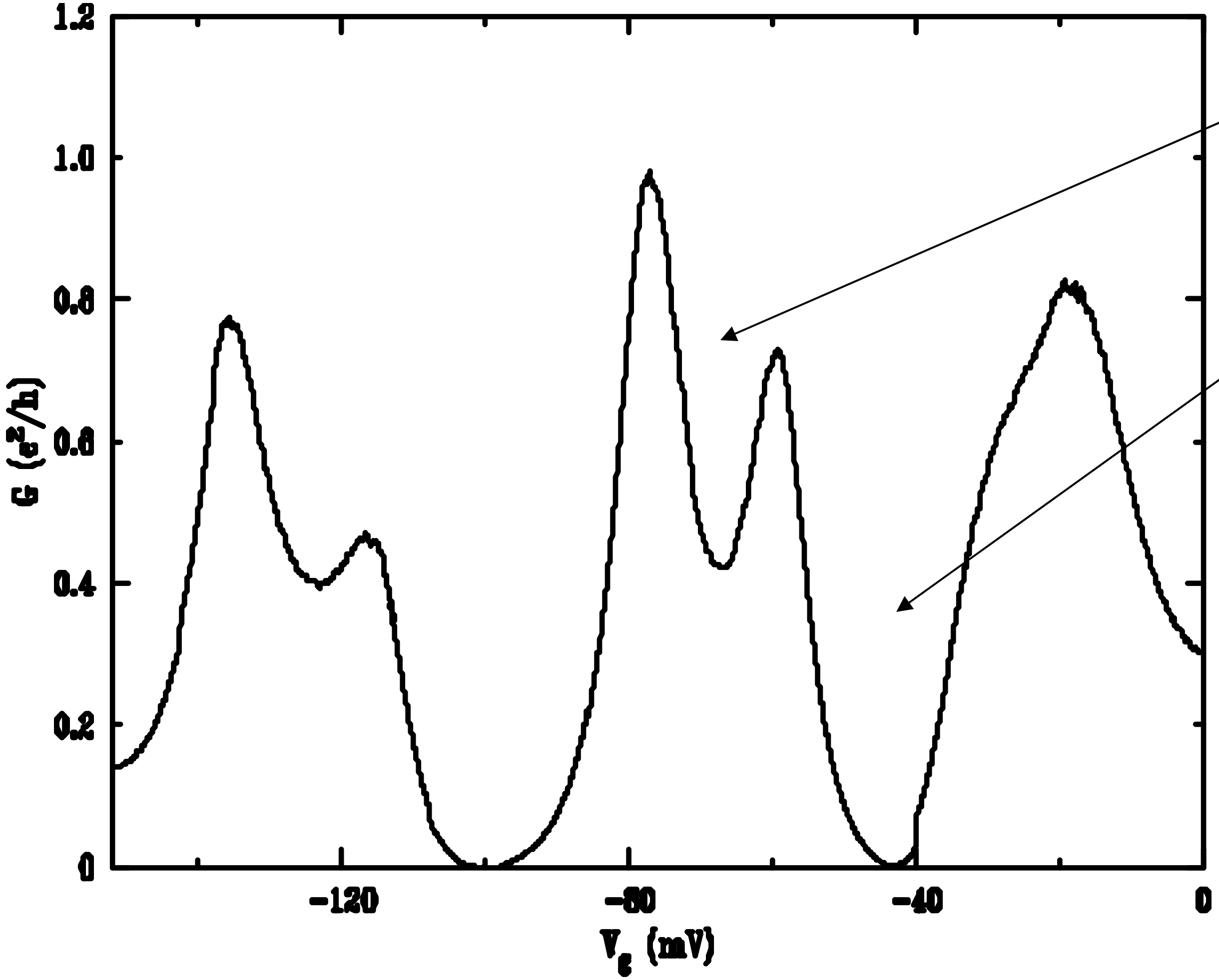
# Energy Scales in SET





Here $\Delta\varepsilon > U$ for simplicity
t is the hopping matrix element
between dot and leads

# Paired Peaks

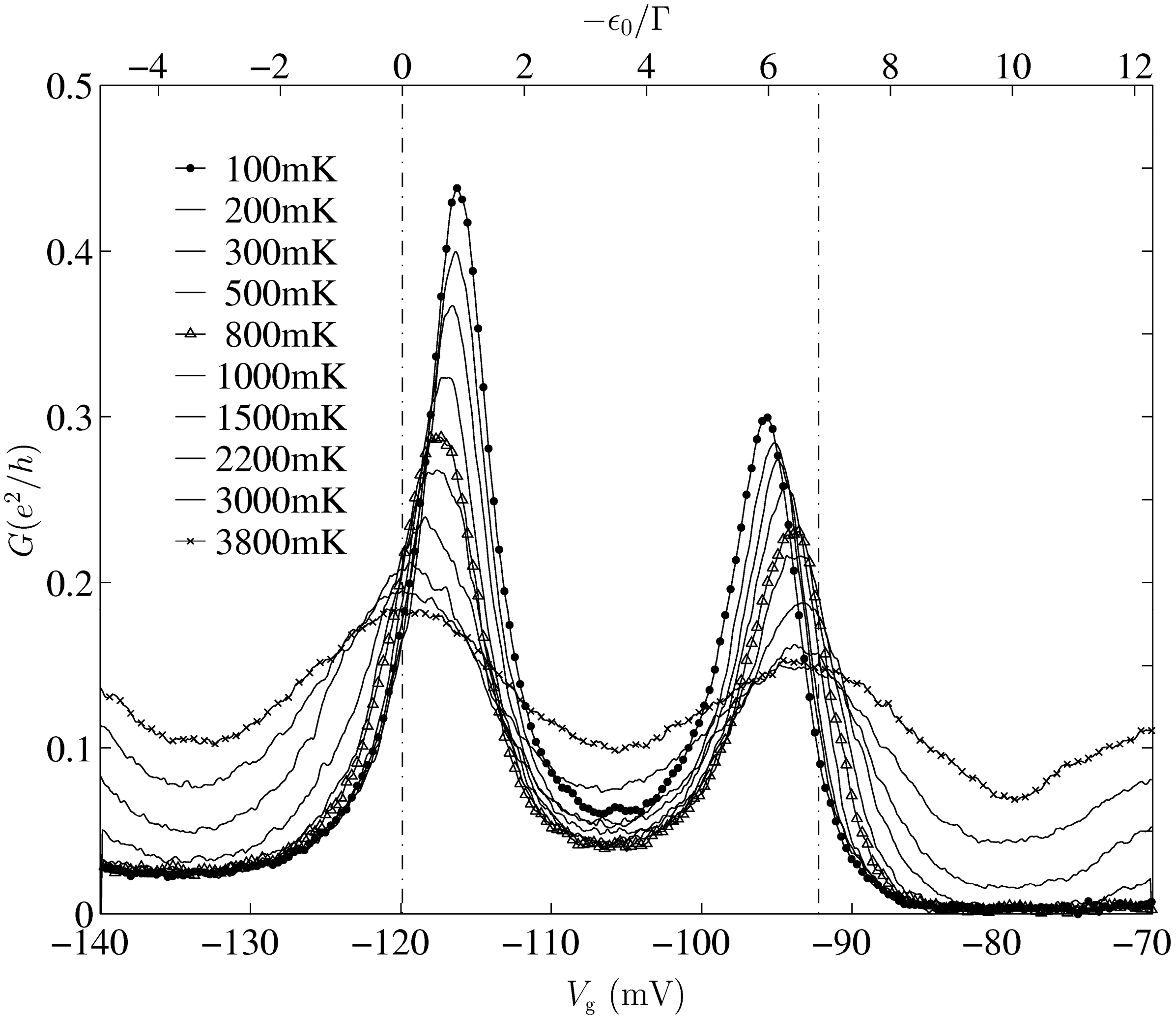


Odd N

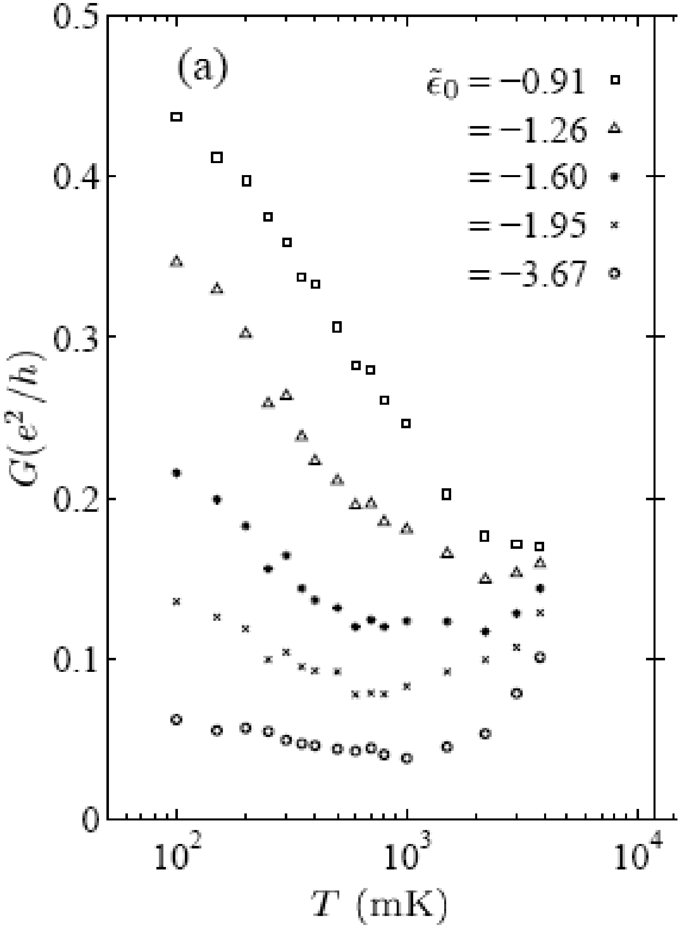
Even N

High conductance for odd N, low for even N.

# Temperature Dependence

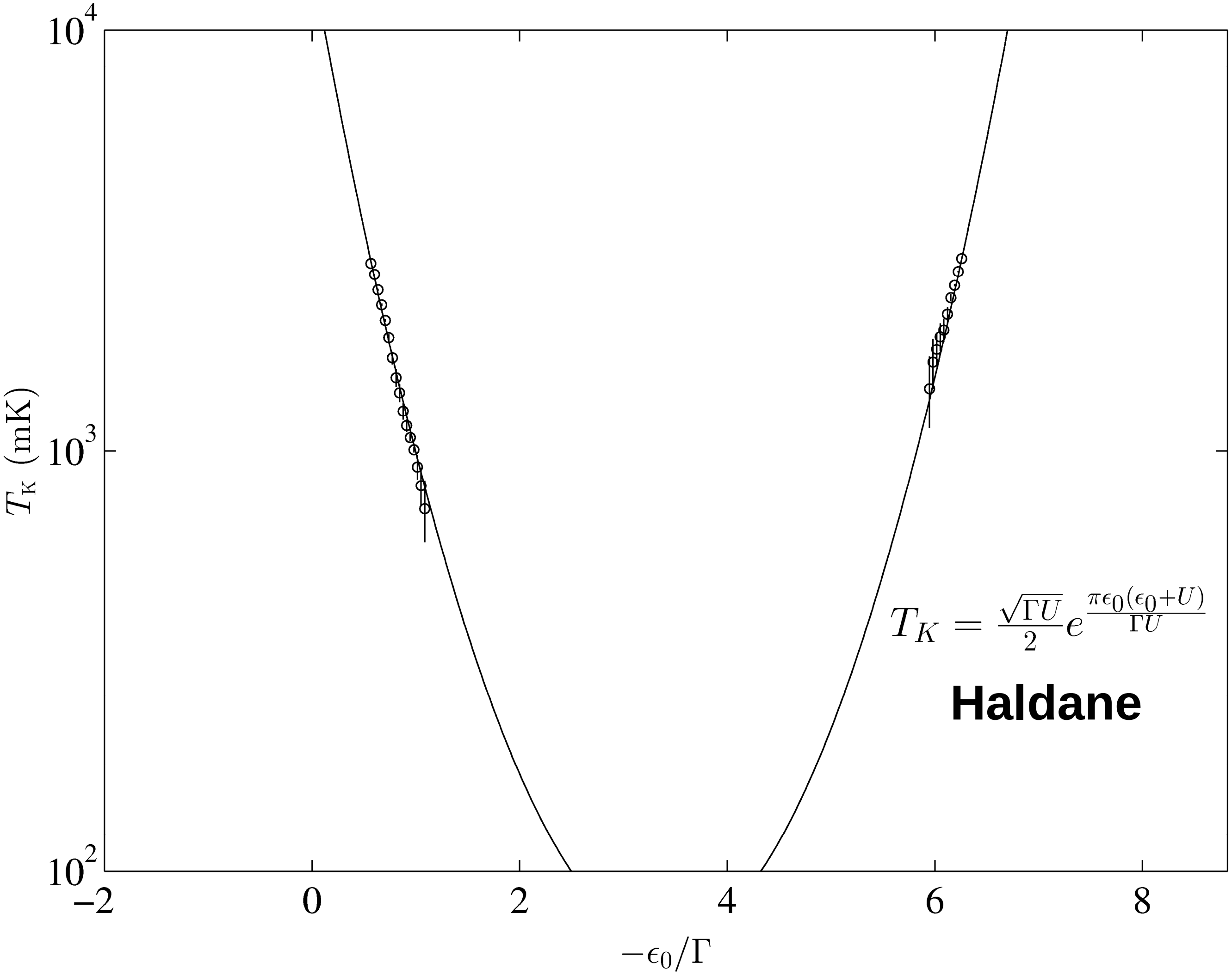


# T Dependence at Fixed $V_G$



Note logarithmic decrease of conductance with T

# Comparison with Scaling Theory



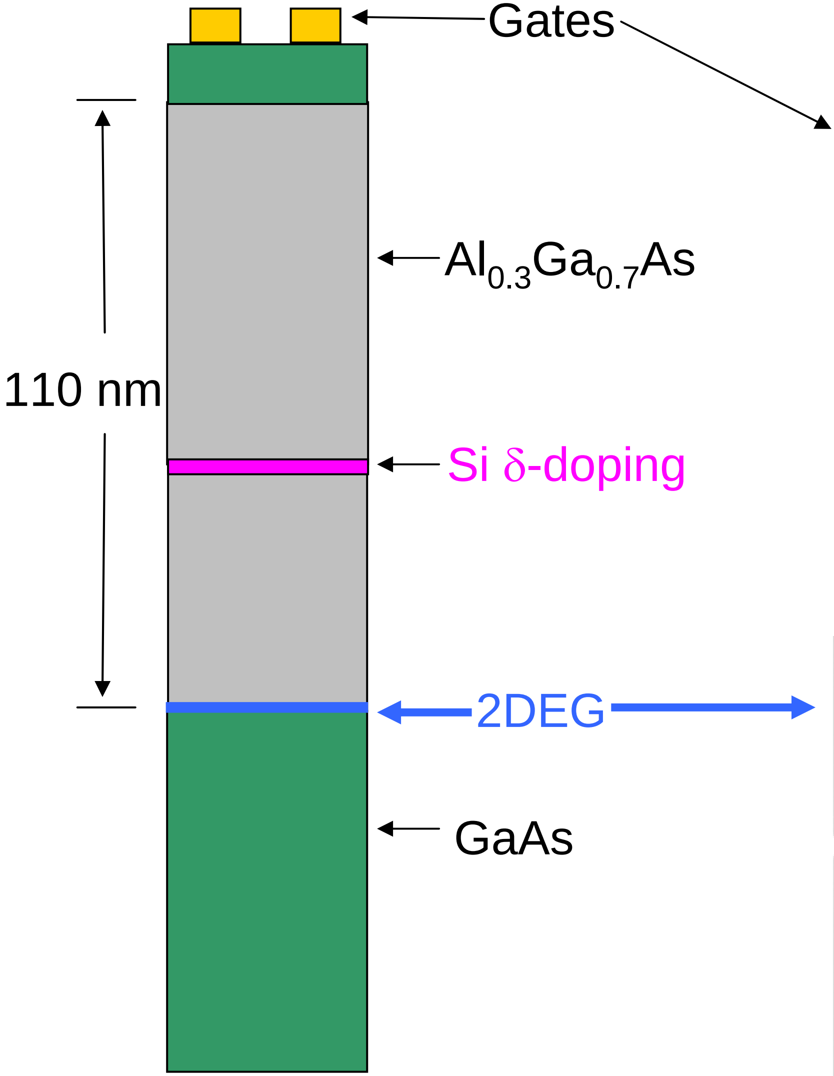
$$T_K = \frac{\sqrt{\Gamma U}}{2} e^{\frac{\pi\epsilon_0(\epsilon_0+U)}{\Gamma U}}$$

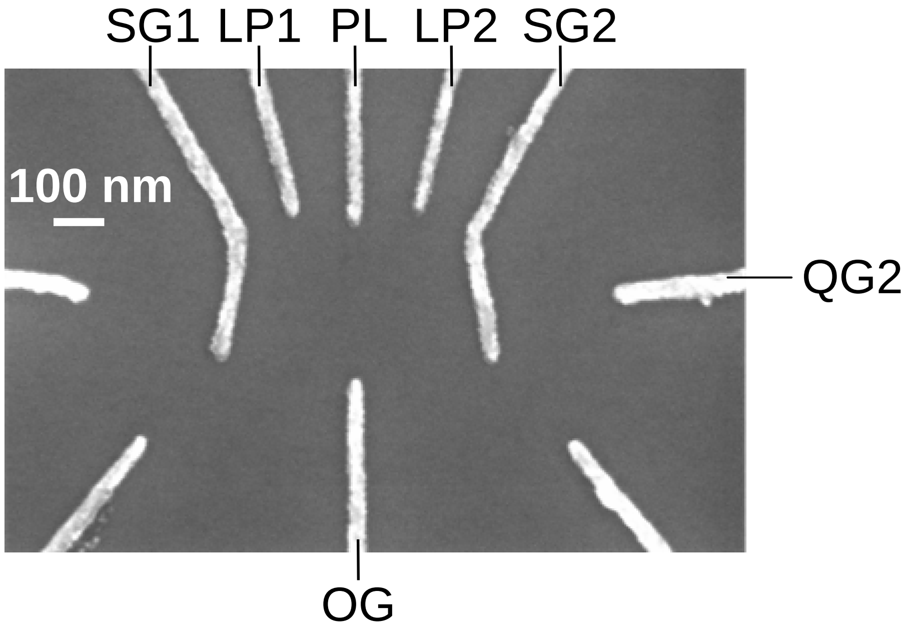
**Haldane**

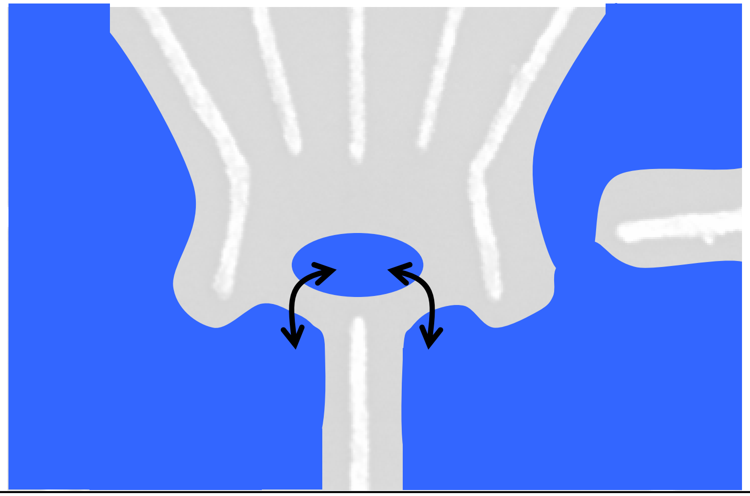
# Charge Measurement

Presentations of Sami Amasha
and Kenneth MaClean

# Laterally Gated Quantum Dots



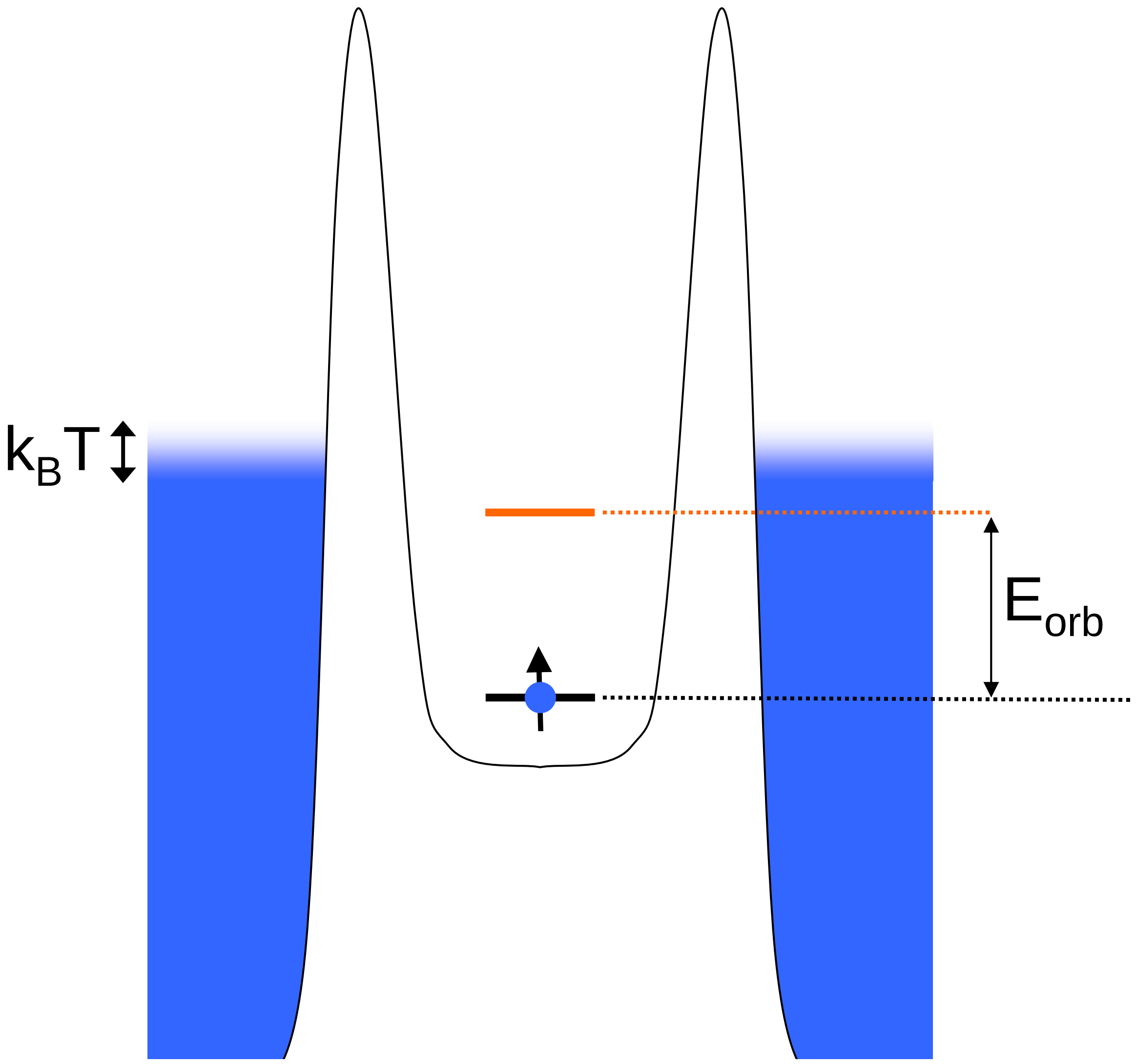


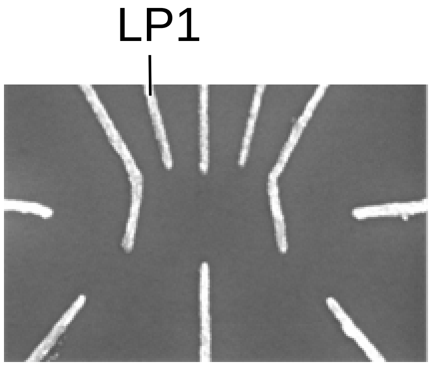


$n = 2.2\times10^{11}$ cm$^{-2}$
$\mu = 6.4\text{x}10^{5}$ cm$^{2}$/V•s

Can trap just one electron
Ciorga et al. PRB **61**, 16315 (2000)

# Energy Scales

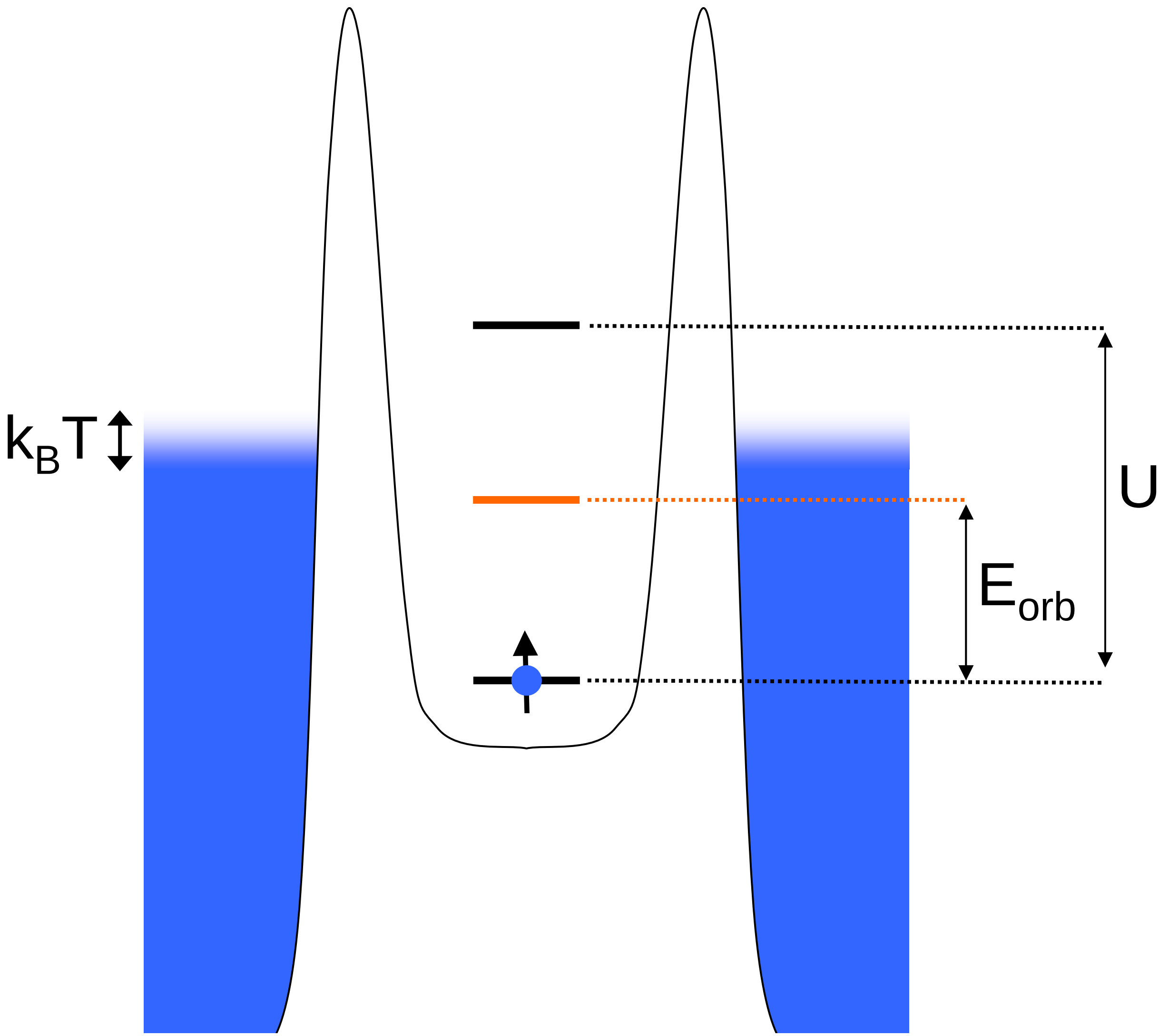


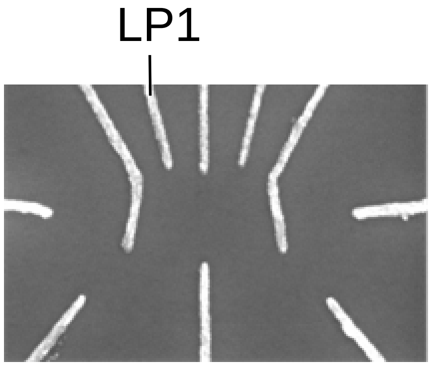


$k_BT = 10\ \mu eV$

$E_{orb} = 2\ meV$

# Energy Scales



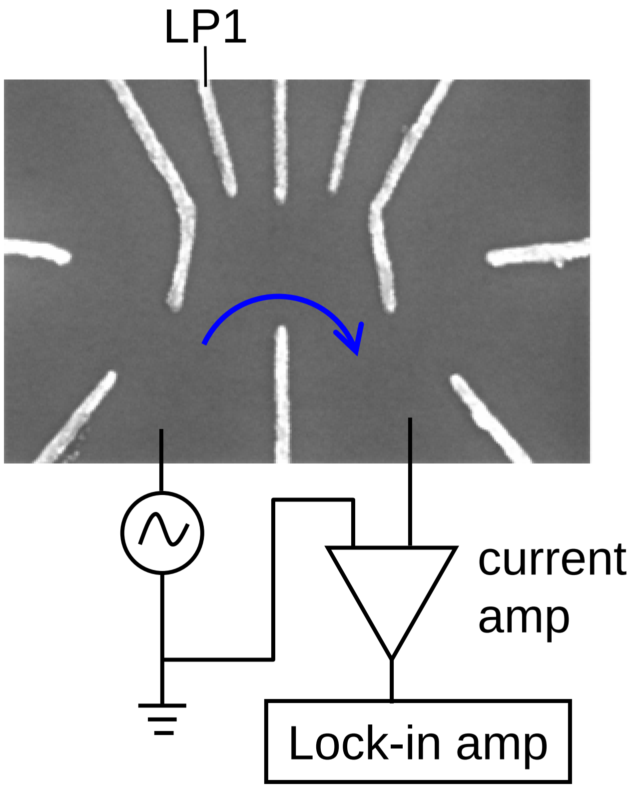


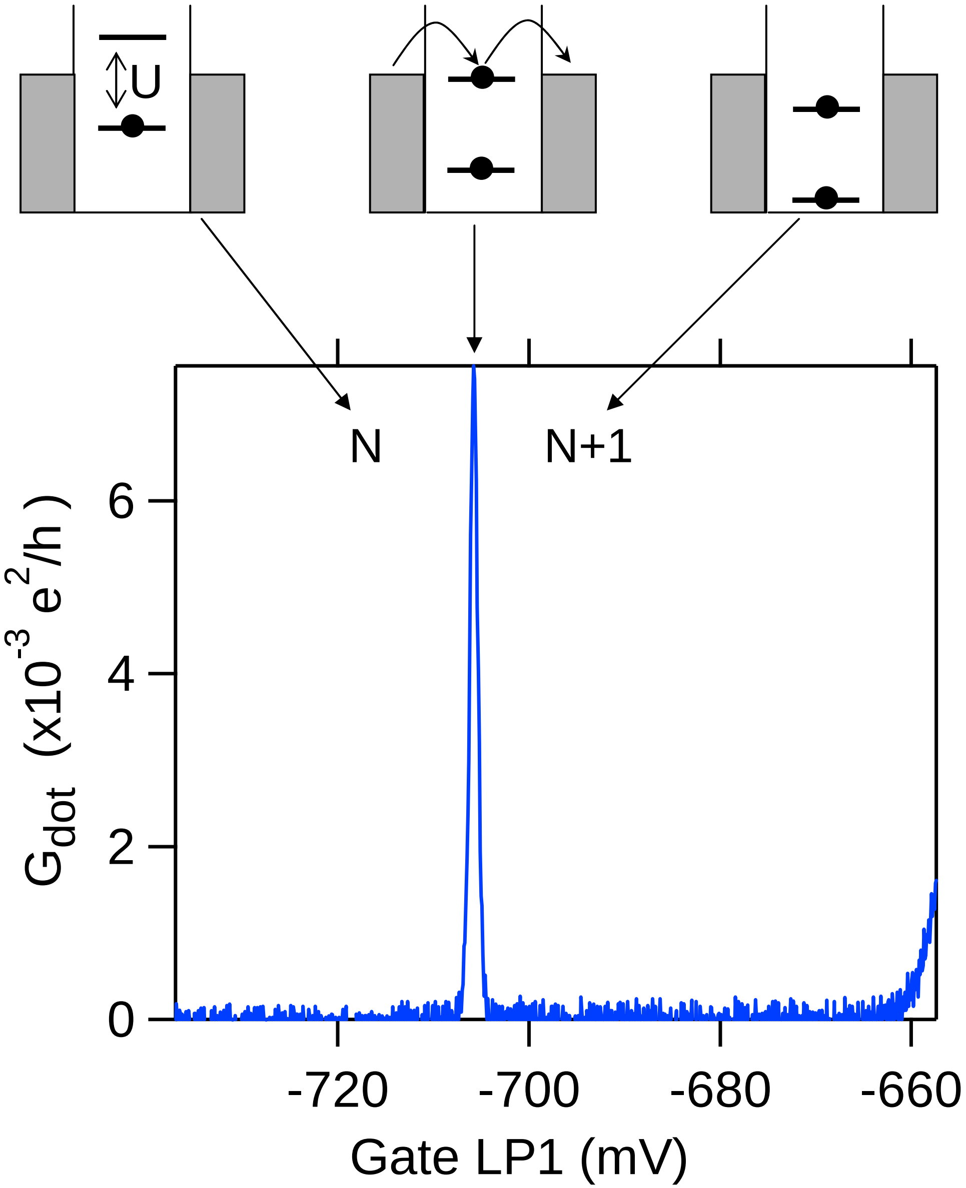
$k_BT = 10\ \mu eV$

$E_{orb} = 2\ meV$

$U = 4\ meV$

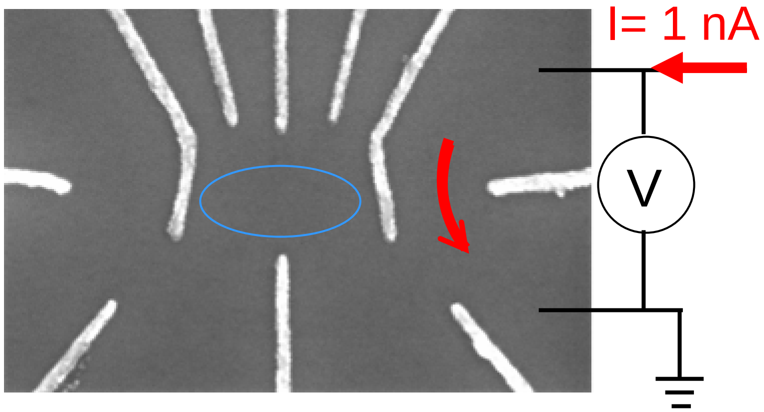
# Measurement of Current

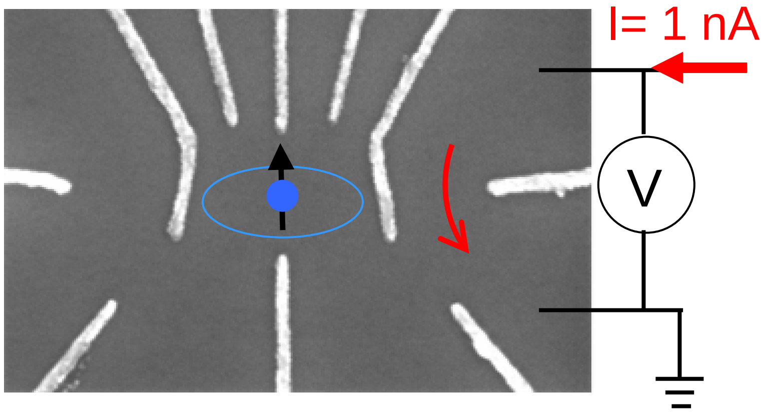


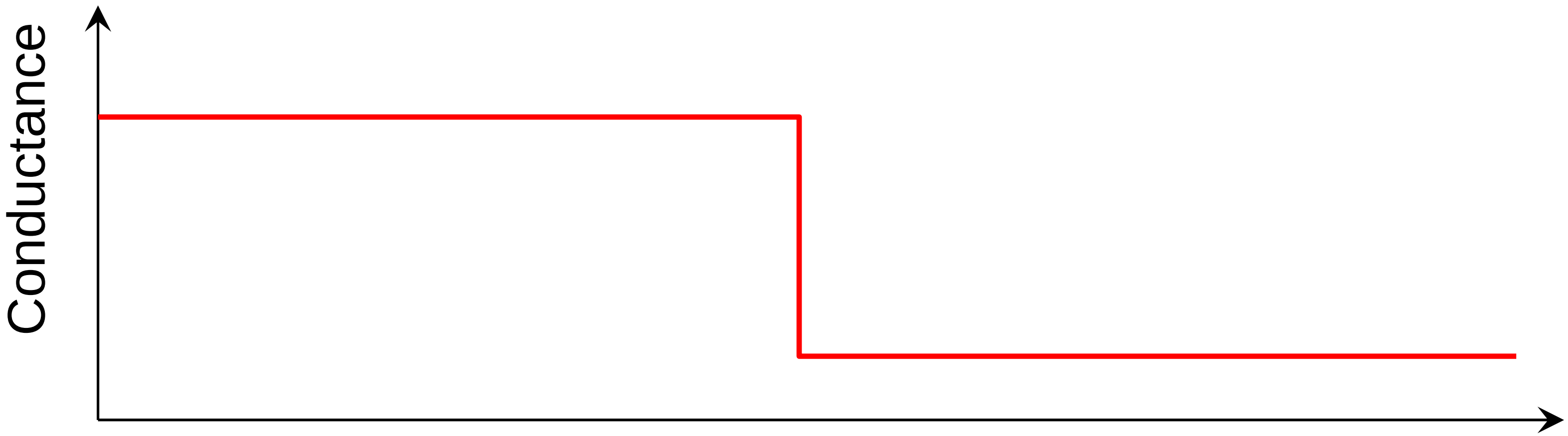


- $\Gamma$ tuned by gate
- $\Gamma \sim 0.01 - 100$ GHz
- $I \sim e\,\Gamma \sim$ 10fA – 10 nA

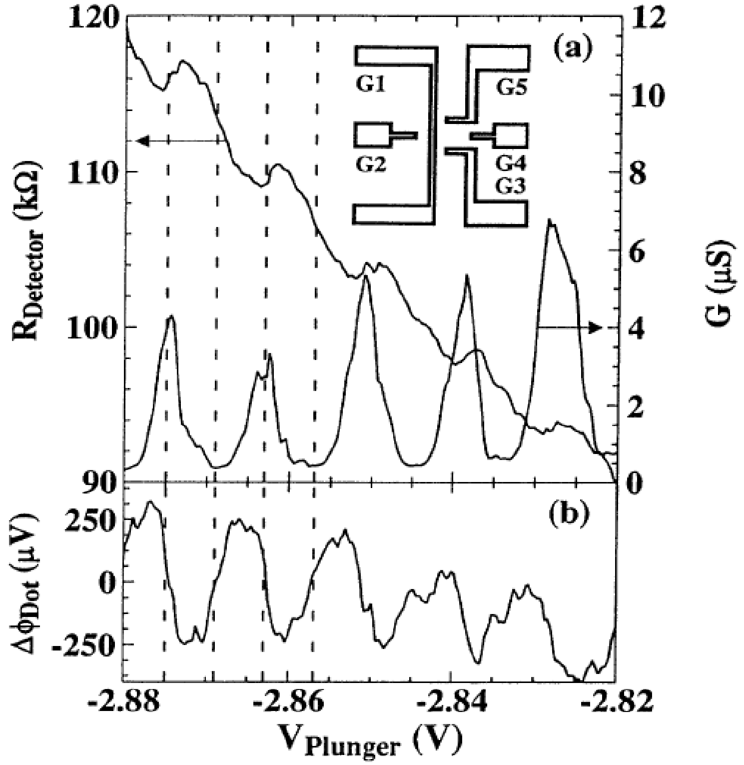
# Charge Sensing





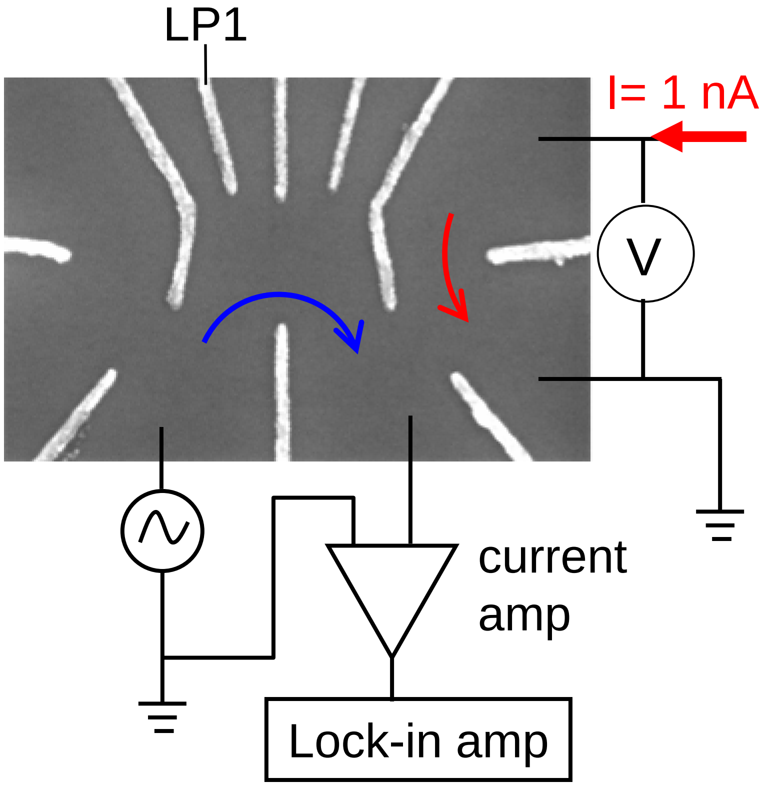


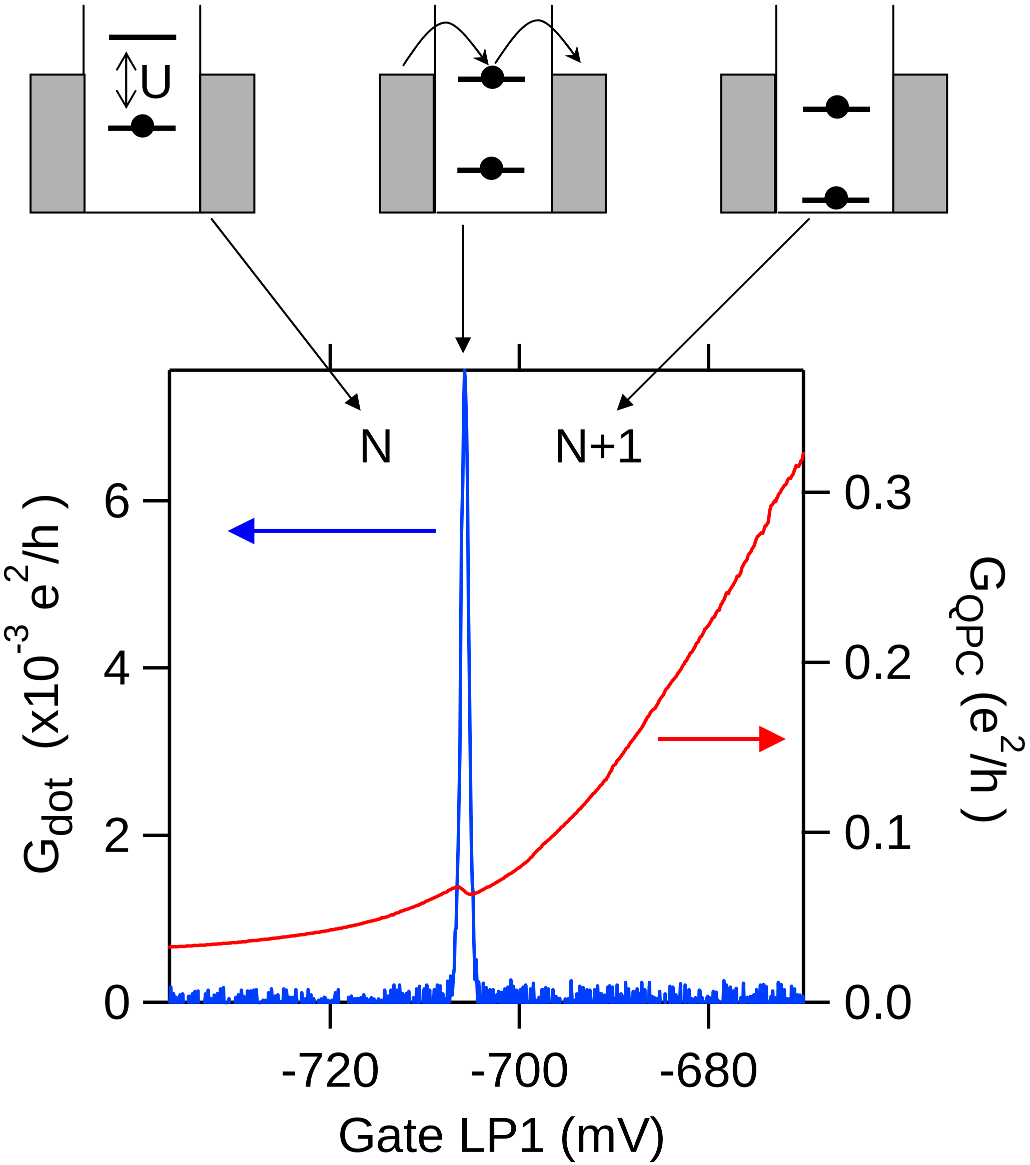
# Measuring Charge



Field et al PRL **70** 1311 (1993)

# Charge Sensing

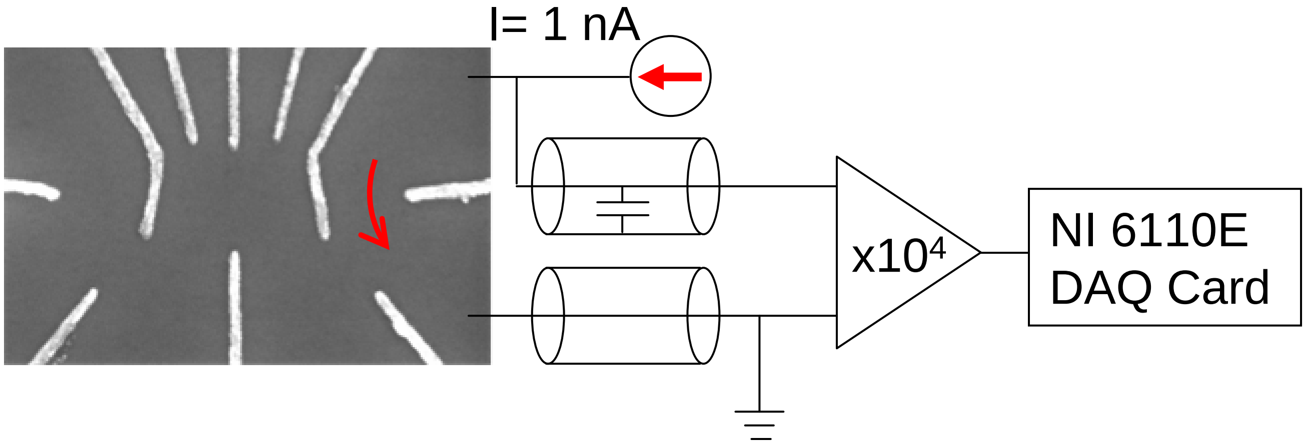




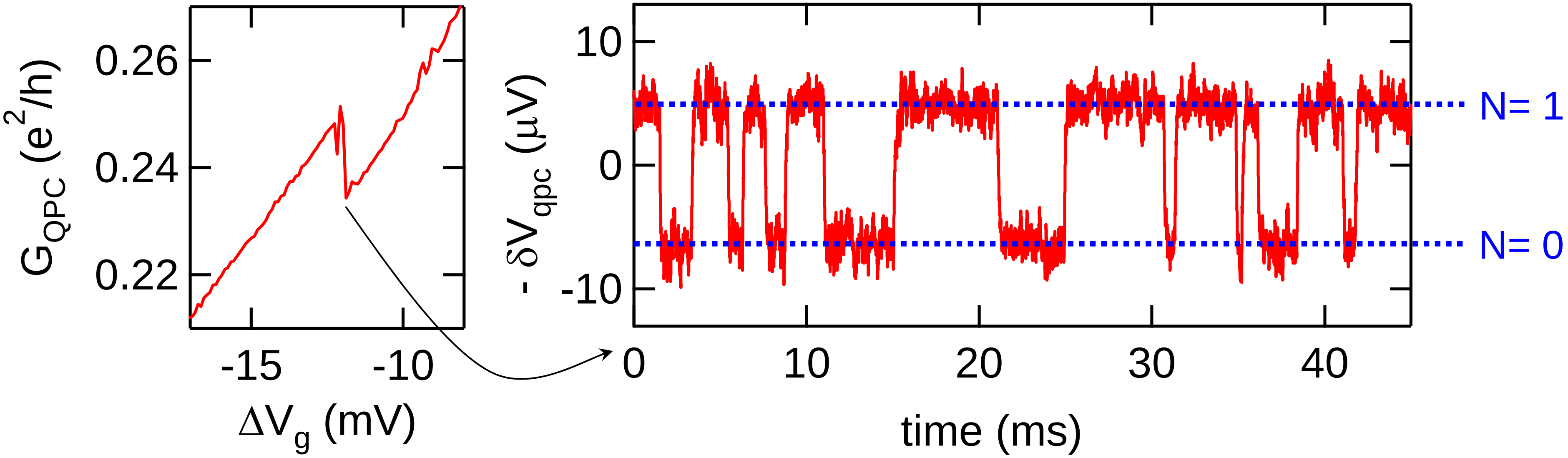
For large Γ current and charge can be measured simultaneously

# Real-time Charge Sensing

[ Lu *et al.*, Nature 2003 & Elzerman *et al.*, Nature 2004]

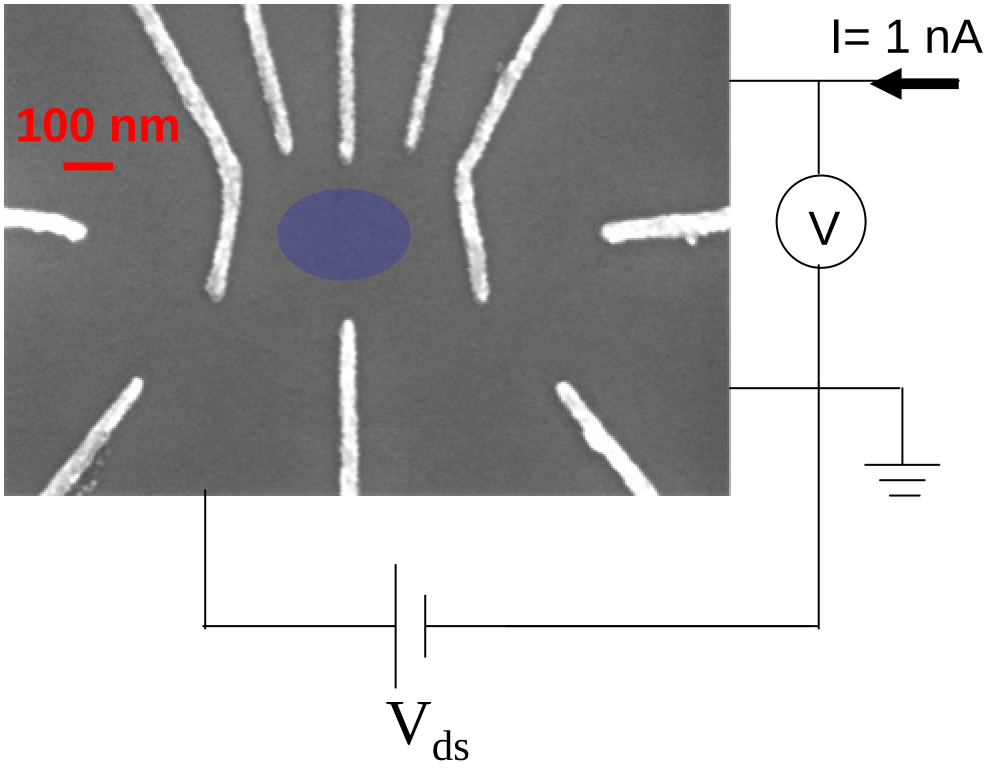


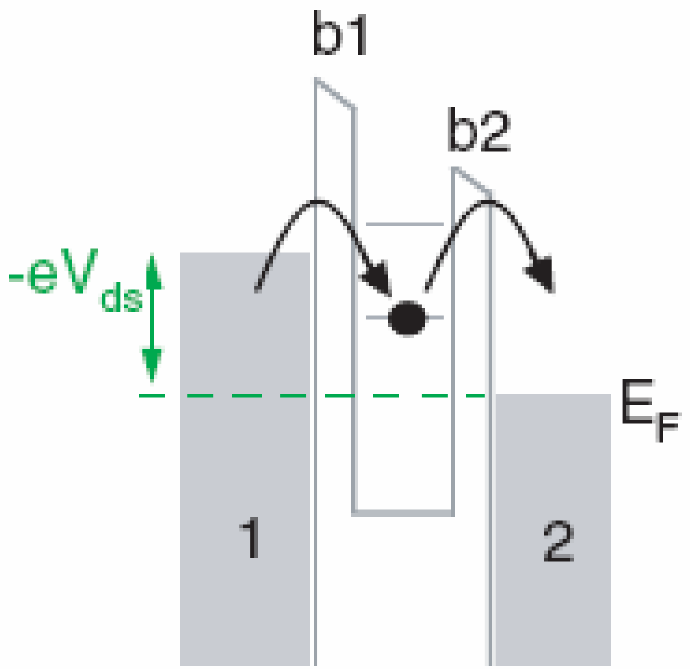
$R_{qpc} \approx 100\ \mathrm{k}\Omega$
$C_{coax} \approx 500\ \mathrm{pF}$
$\Downarrow$
$\tau_{rise} = 50\ \mu\mathrm{s}$



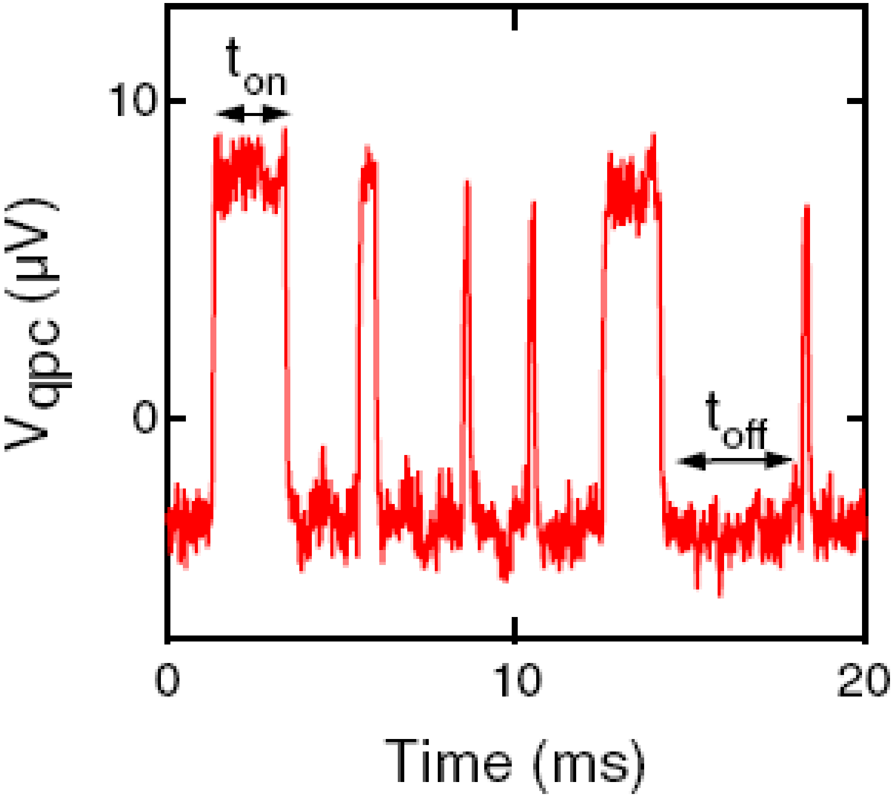
**Measure at small $\Gamma$**

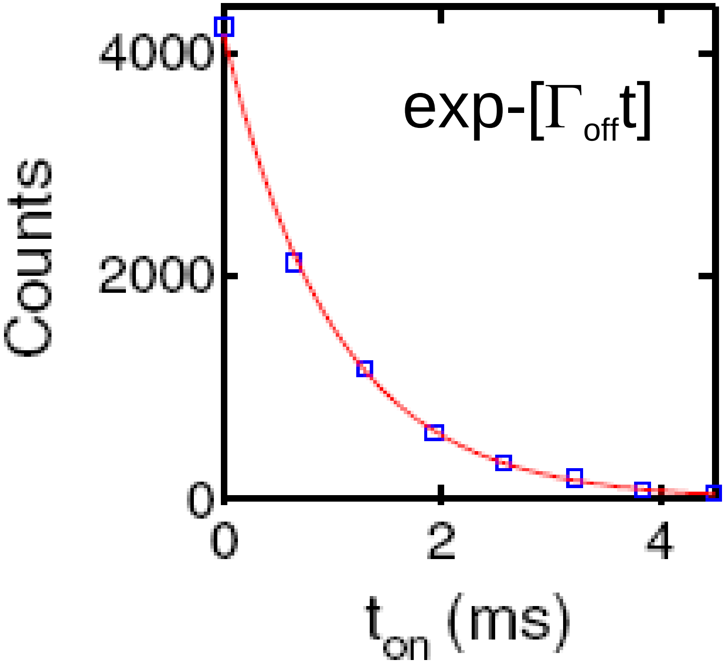
# Measuring Tunneling Rates





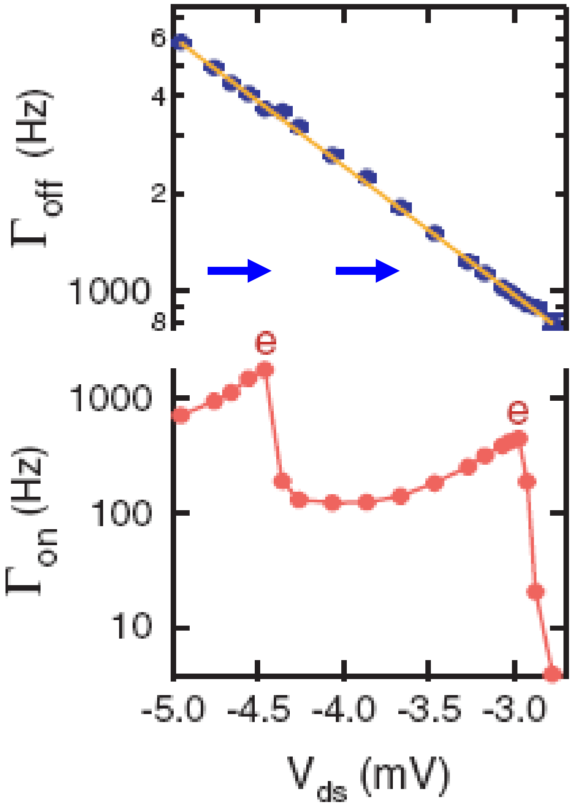
# Single-Electron Counting

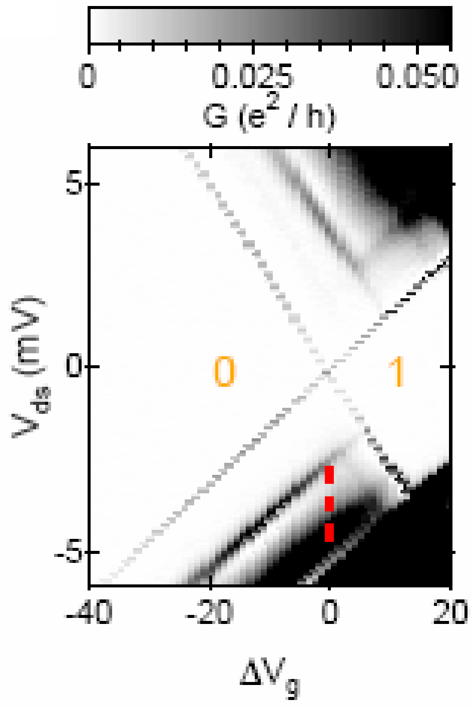


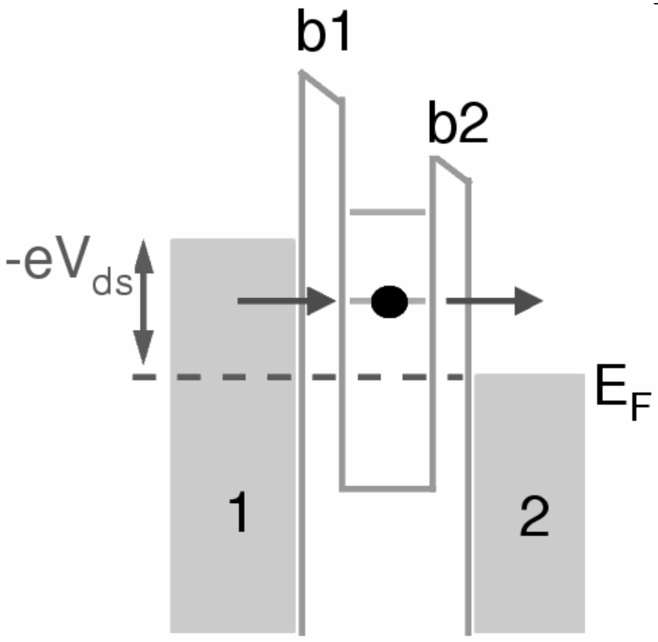


Γ/h can be measured
from 1-1000 Hz, compared to
10-50 GHz from peak shapes

# Dependence on Bias Voltage







Demonstrates: tunneling is elastic, exponential dependence on barrier height, ability to measure excited states.

MacLean et al. Phys. Rev. Lett. **98**, 036802 (2007)

# Summary

- Measure energy scales of quantum dots
  - U, $\Delta\varepsilon$ ( or $E_{orb}$), $\Gamma$, $kT_K$
- Measure charge instead of current
  - Access much smaller $\Gamma$
  - From charge with dc bias we see evidence for dominance of elastic tunneling