Circuit QED:

Quantum Optics and Quantum Computation
with
Superconducting Electrical Circuits
and
Microwave Photons

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EXPERIMENT

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Outline

Lecture 1: ATOMIC PHYSICS:
   Superconducting Circuits as artificial atoms
   -charge qubits

Lecture 2: QUANTUM OPTICS
   Circuit QED -- microwaves are particles!
   --many-body physics of microwave polaritons

Lecture 3: QUANTUM COMPUTATION
   Multi-qubit entanglement
   and a quantum processor
   -Bell inequalities
   -GHZ states
   -Grover search algorithm
Merger of AMO and CM physics

• Atoms and lasers ► Many-body physics
  - many microscopic d.o.f.
  - tunable interactions
  - switch lattice on/off
  - long coherence times
  - readout by optical imaging

• Nanofab and electronics ► Quantum optics
  - macroscopic d.o.f.
  - tunable Hamiltonian
  - modest coherence times
  - electrical readout

Recently: Cavity QED with a BEC!
Brennecke et al., arXiv:0706.3411v1 [quant-ph]
Atoms for 2-level systems

Requirements:
- anharmonicity (natural!)
- long-lived states
- good coupling to EM field
- preparation, trapping etc.

Rydberg atoms & microwave cavities
(Haroche et al.)

Excited atoms with one (or several) e⁻ in very high principal quantum number (n)
- long radiative decay time ($\sim 3 \times 10^{-2}$ s),
- very large dipole moments
- well-defined preparation procedure

Alkali atoms trapped in optical cavities
(Kimble et al.)

can trap single atom inside optical cavity, manipulate and read out its state with lasers!
Quantum Bits and Information

2-level quantum system (two distinct states $|0\rangle$, $|1\rangle$) can exist in an infinite number of physical states intermediate between $|0\rangle$ and $|1\rangle$.

System can be in ‘both states at once’ just as it can take two paths at once.
**Bloch sphere, qubit superpositions**

**Bloch sphere**: geometric representation of qubit states as points on the surface of a unit sphere.

Any superposition state: represented by arrow (called ‘spin’) pointing to a location on the sphere.

State $|1\rangle$

State $|0\rangle$

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad |\alpha|^2 + |\beta|^2 = 1$$

ignoring global phase factor

$$|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

equiv. $$|\Psi\rangle = e^{-i\varphi/2}\cos\frac{\theta}{2}|0\rangle + e^{i\varphi/2}\sin\frac{\theta}{2}|1\rangle$$

Latitude and longitude on the ‘Bloch sphere’

$$0 \leq \theta \leq \pi, \quad 0 \leq \varphi < 2\pi$$

nice discussion: [http://www.vcpc.univie.ac.at/~ian/hotlist/qc/talks/bloch-sphere.pdf](http://www.vcpc.univie.ac.at/~ian/hotlist/qc/talks/bloch-sphere.pdf)
Superconductivity, Josephson junctions and Artificial Atoms
Recent Reviews

‘Wiring up quantum systems’
R. J. Schoelkopf, S. M. Girvin

‘Superconducting quantum bits’
John Clarke, Frank K. Wilhelm

*Quantum Information Processing* **8** (2009)
ed. by A. Korotkov

‘Circuit QED and engineering charge based superconducting qubits,’
S M Girvin, M H Devoret, R J Schoelkopf
Proceedings of Nobel Symposium 141
Superconducting Qubits
Nonlinearity from Josephson junctions (Al/AIO$_x$/Al)

- 1st superconducting qubit operated in 1998 (NEC Labs, Japan)
- “long” coherence shown 2002 (Saclay)
- two examples of C-NOT gates (2003, NEC; 2007, Delft and UCSB)
- Bell inequality violations (2009, UCSB, Yale, Saclay)
- Grover search algorithm (2009, Yale)
- GHZ 3 qubit entanglement (2010, UCSB, Yale)

Classical E-M fields and atomic physics with circuits

Goal: interaction w/ quantized fields
Building Quantum Electrical Circuits

The Josephson Junction is the only known non-linear non-dissipative circuit element.

ingredients:
- nonlinearity
- low temperatures
- small dissipation
- isolation from environment

Different types of SC qubits

- Charge qubit (CPB)
  - Nakamura et al., NEC Labs
  - Vion et al., Saclay
  - Devoret et al., Schoelkopf et al., Yale, Delsing et al., Chalmers

- Flux qubit
  - Lukens et al., SUNY
  - Mooij et al., Delft
  - Orlando et al., MIT
  - Clarke, UC Berkeley

- Phase qubit
  - Martinis et al., UCSB
  - Simmonds et al., NIST
  - Wellstood et al., U Maryland
  - Koch et al., IBM

Reviews:

Yu. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. 73, 357 (2001)

Nonlinearity from Josephson junctions

...and more...
New member of the menagerie: ‘Fluxonium’
Topologically same as phase and flux qubits but acts like a charge qubit
arXiv:0902.2980
Charging effects in the inductively shunted Josephson junction
Jens Koch, V. Manucharyan, M. H. Devoret, L. I. Glazman
What is superconductivity?

vanishing dc resistivity
Meissner effect
signature in heat capacity (phase transition!)
isotope effect e-ph coupling!

Fermi sea unstable for attractive e^-e^- interaction
e^- pairing in k space
Cooper pairs
Cooper pairs form coherent state (BCS)

Complex order parameter (like BEC)

$$\psi \sim \langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \sim \Delta$$

$$\psi = |\psi| e^{i\varphi}$$

SC gap
SC phase
What is superconductivity?

- clean crystal
- momentum space

► can use dirty materials for superconductors!

general case: coupling of time-reversed states

impurity
Why superconductivity?

Wanted:
► electrical circuit as artificial atom
► atom should not spontaneously lose energy
► anharmonic spectrum

Superconductor
► dissipationless!
► provides nonlinearity via Josephson effect

Superconductor

(Al, Nb)

E

“forest” of states

2\Delta \sim 1 \text{ meV}

superconducting gap
Collective Quantization easiest (?) to understand for charge qubits

An isolated superconductor has definite charge.

For an even number of electrons there are no low energy degrees of freedom!

Unique non-degenerate quantum ground state.

No degrees of freedom left! (oops…)

Normal State

Superconducting State

$2\Delta$

$N(2e)$
Low energy dynamics requires **two** SCs

- couple two superconductors via oxide layer
- oxide layer acts as tunneling barrier
- superconducting gap inhibits e\(^{-}\) tunneling
  Cooper pairs CAN tunnel!

▶ **Josephson tunneling**
(2\(^{nd}\) order with virtual intermediate state)
- FGR does NOT apply. Discrete states!

![Diagram](image)

\[ |N_1\rangle \otimes |N_2\rangle \]

\[ |N_1 - 1, N_2 + 1\rangle \]

\[ |N_1 - 2, N_2 + 2\rangle \]
Characterize basis states by number of Cooper pairs that have tunneled:

\[ |n\rangle := |N_1 - 2n, N_2 + 2n\rangle, \quad n \in \mathbb{Z} \]

Tunneling operator for Cooper pairs:

\[
\hat{H}_T = -\frac{E_J}{2} \sum_{n=-\infty}^{\infty} \left[ |n + 1\rangle \langle n| + |n\rangle \langle n + 1| \right]
\]

Note: \( E_J \sim \Delta \) \quad \text{NOT} \quad E_J \sim 1/\Delta
Tight binding model:

\[ \hat{H}_T = -\frac{E_J}{2} \sum_{n=-\infty}^{\infty} |n + 1\rangle \langle n| + |n\rangle \langle n + 1| \]

Diagonalization:

\[ |\varphi\rangle = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} e^{i\varphi n} |n\rangle \quad \leftrightarrow \quad \frac{1}{\sqrt{V}} \sum_j e^{ikx_j} |x_j\rangle \]

\('position' \quad x_j \leftrightarrow n\)

\('wave vector' \quad k \leftrightarrow \varphi\)

\('plane wave eigenstate'\)
\[\hat{H}_T |\varphi\rangle = -\frac{E_J}{2} \sum_{n'=-\infty}^{\infty} \left[ |n' + 1\rangle \langle n'| + |n'\rangle \langle n' + 1| \right] \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} e^{in\varphi} |n\rangle\]

\[= -\frac{E_J}{2} \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} e^{i\varphi n} \left[ |n + 1\rangle + |n - 1\rangle \right]\]

\[= -\frac{E_J}{2} \frac{1}{\sqrt{2\pi}} \left[ e^{-i\varphi} \sum_{n=-\infty}^{\infty} e^{i\varphi(n+1)} |n + 1\rangle + e^{i\varphi} \sum_{n=-\infty}^{\infty} e^{i\varphi(n-1)} |n - 1\rangle \right]\]

\[= -E_J \cos \varphi |\varphi\rangle\]
Supercurrent through a JJ

Wave packet group velocity

\[ \frac{dx}{dt} = v_g = \frac{d\omega}{dk} \leftrightarrow \frac{dn}{dt} = \frac{1}{\hbar} \frac{dH_T}{d\varphi} = \frac{E_J}{\hbar} \sin \varphi \]

current:

\[ I = (2e) \frac{dn}{dt} = \frac{2e}{\hbar} E_J \sin \varphi \]

\[ = I_c \sin \varphi \]

critical current \( I_c = \frac{2e}{\hbar} E_J \)

Josephson equation: current-phase relation

\[ \hat{H}_T|\varphi\rangle = -E_J \cos \varphi |\varphi\rangle \]

‘position’ \( x_j \leftrightarrow n \)

‘wave vector’ \( k \leftrightarrow \varphi \)

the only non-linear non-dissipative circuit element!
Charging Energy

Transfer of Cooper pairs across junction:
\[ |N_1 - 2n, N_2 + 2n \rangle = |n\rangle, \quad n \in \mathbb{Z} \]

charging of SCs
- junction also acts as capacitor!

\[ U = \frac{Q^2}{2C} = \frac{(2e)^2}{2C}n^2 \quad \Rightarrow \quad \hat{H}_U = 4E_c\hat{n}^2 \]

with \( E_c = \frac{e^2}{2C} \)

charging energy
Josephson tunneling + charging: the Cooper pair box

Combine Josephson tunneling and charging:

$$\hat{H}_{\text{CPB}} = \hat{H}_U + \hat{H}_T$$

$$= 4E_C\hat{n}^2 - \frac{E_J}{2} \sum_{n=-\infty}^{\infty} \left[ |n+1\rangle\langle n| + |n\rangle\langle n+1| \right]$$

the **Cooper pair box** (CPB) Hamiltonian

**crucial parameter:**

$$E_J/E_C$$
CPB Hamiltonian
in charge and phase basis

\[ \hat{H}_{\text{CPB}} |\Psi\rangle = E |\Psi\rangle \]

projections:

\[ \Phi(n) = \langle n | \Psi \rangle, \quad \Psi(\varphi) = \langle \varphi | \Psi \rangle \]

probability amplitude for number
probability amplitude for phase

charge basis:

\[ 4E_cn^2 \Phi(n) - \frac{E_J}{2} \left[ \Phi(n + 1) + \Phi(n - 1) \right] = E\Phi(n) \]

numerical diagonalization

phase basis:

\[ \hat{n} \rightarrow i \frac{d}{d\varphi} \]

\[ \left[ 4E_c \left( i \frac{d}{d\varphi} \right)^2 - E_J \cos \varphi \right] \Psi(\varphi) = E\Psi(\varphi) \]

exact solution with Mathieu functions

\[ \Psi_m(\varphi) = \frac{1}{\sqrt{2}} \me^{-2m} \left( \frac{E_J}{2E_c}, \frac{\varphi}{2} \right) \]
**CPB: the simplest solid-state atom**

Josephson junction with capacitive voltage bias:

\[
\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_J \cos \varphi + \cdots
\]

3 parameters:

- \( n_g = \frac{Q_r}{2e} + \frac{C_g V_g}{2e} \) \quad offset charge (tunable by gate)
- \( E_J \) \quad Josephson energy (tunable by flux in split CPB)
- \( E_C = \frac{e^2}{2C_\Sigma} \) \quad charging energy (fixed by geometry)
Fabrication of CPB charge qubits

split CPB (like SQUID) allows tuning of Josephson energy by external magnetic flux:

\[ E_J \rightarrow E_J | \cos(\pi \Phi / \Phi_0) | \]

fabrication:
- e-beam lithography (mask)
- 2 angle evaporation of Al (total thickness ~180nm)

typical junction capacitances:

\[ C \sim 500 \text{ aF} \ldots 2000 \text{ aF} \]
\[ E_C/k_B \sim 0.5K \ldots 2K \]
Temperature requirements

$$T \ll T_c \sim 1\, \text{K (Al)}$$

$$\omega_a \sim 10\, \text{GHz} \sim 0.5\, \text{K} \gg T$$

- work with dilution refrigerators
  - base temp. $\sim 30\, \text{mK}$

$1\, \text{K} = 21\, \text{GHz}$

photo: Matthew Gibbons
Cooper Pair Box: charge limit

Charge limit: \( \frac{E_J}{E_C} \ll 1 \)

\[
\hat{H} = 4E_C(\hat{n} - n_g)^2 - \frac{E_J}{2} \sum_{n=-\infty}^{\infty} \left[ |n+1\rangle \langle n| + |n\rangle \langle n+1| \right]
\]

For small perturbation, the charge limit is approximately:

\[ n_g = \frac{C_g V_g}{2e} \]
The crux of designing qubits

- Need good coupling!
- Need to be uncoupled!
Relaxation and dephasing

- Relaxation – time scale $T_1$
- Dephasing – time scale $T_2$

- Qubit transition
- Fast parameter changes: sudden approx, transitions
- Slow parameter changes: adiabatic approx, energy modulation

$$\hat{H} = \hat{H}(x_1, x_2, \cdots)$$

- Transition $|1\rangle \rightarrow |0\rangle$
- Random switching

$$\omega_a \rightarrow \omega_a + \Delta\omega_a(t)$$
- Phase randomization $e^{-i\omega_a t}$
Imperfections of junction parameters

charged impurities

tunnel channels

electric dipoles

Al

AlO$_x$

Al

1nm
Junction parameter fluctuations

Random part of offset charge most dangerous

\[
\begin{align*}
Q_r &= Q_r^{\text{mean}} + \Delta Q_r(t) \\
E_J &= E_J^{\text{mean}} + \Delta E_J(t) \\
E_C &= E_C^{\text{mean}} + \Delta E_C(t)
\end{align*}
\]

\[
S(f) = \frac{A^2}{f} \quad \text{1/f noise}
\]


\[\begin{array}{|c|c|c|}
\hline
\text{param.} & \text{dispers.} & \text{noise} & \text{fluct. @ 1 Hz} \\
\hline
Q_r^{\text{mean}} & \text{random!} & \Delta Q_r/2e & \sim 10^{-3}\text{Hz}^{-1/2} \\
E_J^{\text{mean}} & 10\% & \Delta E_J/E_J & 10^{-5} - 10^{-6}\text{Hz}^{-1/2} \\
E_C^{\text{mean}} & 10\% & \Delta E_C/E_C & < 10^{-6}\text{Hz}^{-1/2} \? \\hline
\end{array}\]

\[\triangleright \text{ reduce sensitivity to charge noise!} \]

**2 solutions:**
- control offset charge with gate
  - CPB (charge qubit), sweet spot
- make \( E_J/E_C \) large
  - Transmon (optimized CPB)
  - RF-SQUID (flux qubit)
  - Cur. biased J. (phase qubit)
Phase coherence sweet spot

\[ n_g(t), \quad \hat{H} = \hat{H}(n_g), \quad n_g = n_g(t) \quad \text{charge fluctuations} \]

\[ \omega_a = \omega_a(t) \]

Best CPB performance @ sweet spot:

\[ T_1 \sim 7\mu s, \quad T_2 > 500\, \text{ns} \quad \text{(Schoelkopf Lab)} \]


Vion et al., Science 296, 886 (2002)
In a nutshell: the transmon

- Effects of increasing $E_J/E_C$:
  - Anharmonicity decreases...
  - Flatter energy levels, become **insensitive to** charge noise!
  - ...only algebraically
  - ...exponentially!

island volume ~1000 times bigger than conventional CPB island

$E_J/E_C = 1$: 

$E_J/E_C = 5$: 

$E_J/E_C = 10$: 

$E_J/E_C = 50$: 

sweet spot everywhere!

cond-mat/0703002, PRA in print
Relaxation and dephasing

relaxation – time scale $T_1$

dephasing – time scale $T_2$

$qubit$

$T_1 = \text{excited state lifetime}$

$T_2 = \text{superposition phase coherence lifetime}$

$\omega_a \rightarrow \omega_a + \Delta \omega_a(t)$

$\uparrow \text{phase randomization}$

\[
\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\varphi}
\]
Pi pulse to measure $T_1$

$2$

$\omega_{01}$

$0$

6.1 GHz

6.5 GHz

$3$ nano-secs

$V = \Omega_{Rabi}^x(t) \cos(\omega_{01} t) \sigma^x$

$+ \Omega_{Rabi}^y(t) \sin(\omega_{01} t) \sigma^y$

$Fidelity = 99%$

Chow et al., PRL(2009)

$T_1 = 1.5 \mu s$
Test of Quantum Phase Coherence: Ramsey Fringe Experiment for $T_2$

Interference between Two possible paths:

flip + flip
no-flip + no-flip
Ramsey Fringe and Qubit Coherence

\[ V = \Omega_{\text{Rabi}}(t) \cos(\omega_{01} t) \sigma^x \]

Fidelity = 99%

J. Chow et al., PRL (2009):

\[ T^*_2 = 3.0 \mu s \]
Coherence in Transmon Qubit

Random benchmarking of 1-qubit ops

Chow et al. *PRL* 2009:
Technique from Knill et al. for ions

Similar error rates in phase qubits (UCSB):
Lucero et al. *PRL* 100, 247001 (2007)
‘Moore’s Law’ for Charge Qubit Coherence Times

Next goal

MIT/NEC

T2 now limited largely by T1

$T_\varphi \geq 30\,\mu s$
Yale circuit QED team members ‘10
Lecture 2: Introduction to Circuit QED

SC Qubits interacting with microwave photons