4 Waves with imperfectly conducting boundaries

4.1 WAVES INCIDENT NORMALLY ON A CONDUCTING SURFACE

In Chapter 2 the reflection of waves from perfectly conducting boundaries was discussed. In reality no boundaries (other than superconducting ones) are lossless so it is necessary to consider what the effects of imperfectly conducting boundaries are.

Consider the case of a plane wave in a lossless medium incident normally on a semi-infinite conducting slab as shown in Fig. 4.1. We will assume that the slab is a good conductor as defined in Section 1.5. Let the amplitudes of the incident, reflected and transmitted waves at the surface of the slab be E_i, E_r and E_t as shown. Then, from (1.84) and (1.85)

$$\frac{E_r}{E_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{4.1}$$

$$\frac{E_t}{E_i} = \frac{2Z_2}{Z_2 + Z_1}, \tag{4.2}$$

where the wave impedances of the two materials are

$$Z_1 = \sqrt{\left(\frac{\mu_1}{\varepsilon_1}\right)} \quad \text{and} \quad Z_2 = \frac{(1 + j)}{\sigma\delta} \tag{4.3}$$

from (1.29) and (1.45). Note that, because Z_2 is complex with a phase angle of 45°, the transmitted wave is out of phase with the incident wave.

The power absorbed by the surface can be calculated by a simple application of Poynting's theorem. The transmitted wave travels into the slab and is completely absorbed by it. The power absorbed per unit area is therefore equal to the power flowing into unit surface area of the slab which is just the magnitude of the Poynting vector. Thus, from (1.46)

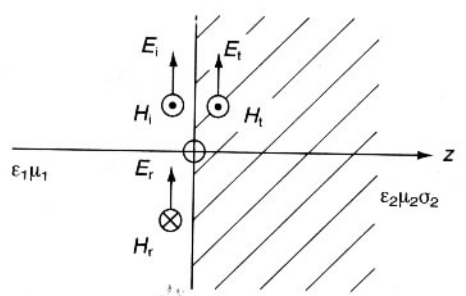

Fig. 4.1 Incident, reflected and transmitted waves for electromagnetic waves incident normally on the surface of a material having finite conductivity.

$$S_t = \tfrac{1}{4}\sigma\delta E_t^{\,2} \tag{4.4}$$

$$S_t = \tfrac{1}{4}\sigma\delta\left|\frac{2Z_2}{Z_1 + Z_2}\right|^2 E_i^{\,2}$$
$$= \tfrac{1}{2}\sigma\delta Z_1\left|\frac{2Z_2}{Z_1 + Z_2}\right|^2 P_i. \tag{4.5}$$

From this equation it is evident that the units of $\sigma\delta$ must be Ω^{-1} (siemens). It is convenient to define the *surface resistance* (or sheet resistivity) of the slab by

$$R_s = 1/\sigma\delta. \tag{4.6}$$

The wave impedance of the material can then be written

$$Z_2 = R_s(1 + \mathrm{j}). \tag{4.7}$$

From (4.5) the transmission loss at the interface is

$$L = -10\log_{10}\left(\frac{1}{2}\frac{Z_1}{R_s}\left|\frac{2Z_2}{Z_1 + Z_2}\right|^2\right). \tag{4.8}$$

Very often $|Z_1| \gg |Z_2|$ so that, to a good approximation,

$$L = -10\log_{10}\left[\frac{2|R_s(1+\mathrm{j})|^2}{R_s Z_1}\right]$$
$$= -10\log_{10}\left(\frac{4R_s}{Z_1}\right). \tag{4.9}$$

The concept of surface resistance can be given a simple physical interpretation. First we recall that, as shown in (1.44), a wave propagating through a good conductor decays exponentially with a decay constant equal to the skin depth δ. We recall also that, for a good conductor, δ is typically of the order of a millimetre or less. Consider therefore the approximation that

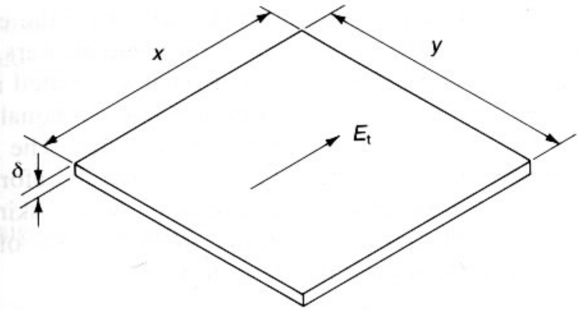

Fig. 4.2 Geometry of a thin sheet of resistive material.

the power dissipated in the material is dissipated uniformly in a surface layer of thickness δ as shown in Fig. 4.2. The resistance of the sheet measured in the x direction is

$$R = \frac{x}{\sigma\delta y} = \frac{x}{y}R_s. \tag{4.10}$$

For the special case of a square sheet it can be seen that $R = R_s$ regardless of the size of the square. For this reason surface resistance is usually written as so many 'ohms per square' sometimes abbreviated to $\Omega/\square$.

If all the current in the conductor is considered to flow in the skin depth then the boundary condition on the magnetic field is

$$H = 2H_i = A \tag{4.11}$$

where A is the current flowing in unit width of the surface. The almost complete reflection of the incident wave at the surface together with the accompanying reversal of the electric field vector means that the component of the magnetic field parallel to the surface is the sum of the tangential components of the incident and reflected magnetic field vectors (i.e. twice the incident field). The power dissipated per unit area is thus

$$P = \tfrac{1}{2}A^2R_s = \tfrac{1}{2}H^2R_s = 2H_i^2R_s. \tag{4.12}$$

Now from (4.2) making the approximation that $Z_2 \ll Z_1$

$$E_t = 2E_iR_s(1 + \mathrm{j})/Z_1$$

so

$$E_t^2 = 8R_s^2E_i^2/Z_1^2$$
$$= 8R_s^2H_i^2. \tag{4.13}$$

Substituting this in (4.4) gives (4.12) showing that the two approaches are consistent with each other.

It is frequently helpful to use this way of treating the absorption of power in a conducting surface. The use of the magnetic field and the accompanying surface current density is usually easier than the use of the electric field though both methods can be shown to give the same answers.

We may therefore think of all the power being dissipated in a surface layer, having a surface resistance R_s, whose thickness is equal to the skin depth. This approximation is not as crude as it seems. The power flow varies as the square of the electric field strength and therefore decays as $\exp -z/2\delta$ as the wave penetrates into the slab. At the skin depth the power is only 13.5% of that at the surface so that 86.5% of the power entering the slab is absorbed within the skin depth.

Example

Estimate the difference between the intensity of a 1 kHz radio wave received by a submarine on the surface of the sea and that received when the submarine is submerged to a depth of 10 m.

Solution

Because sea water is an electrical conductor it is not possible for radio waves to penetrate very far into it. This makes radio communication with submerged submarines very difficult. To make the skin depth as large as possible a very low carrier frequency must be used and that sets severe limitations on the possible rate of transmission of data.

The relative permitivity of sea water is about 81 and its conductivity about 4 S. Substituting these figures together with the given frequency into the right hand side of (1.37) yields

$$(\delta + j\omega\varepsilon) = (4 + 4.5 \times 10^{-6}\,j) \tag{4.14}$$

so that we are justified in treating sea water as a good conductor in this problem. The skin depth is, from (1.43),

$$\delta = \sqrt{(2/\omega\sigma\mu)} = 8.0\,\text{m}. \tag{4.15}$$

It is not sufficient merely to compute the decay of the signal as it passes through the water. We must also allow for the reflection of some of the incident power at the air–water interface. To simplify the calculation we will assume that the wave is incident normally on the sea surface. The wave impedances are

$$Z_1 = 377\,\Omega \quad \text{and} \quad Z_2 = 0.031(1 + j)\,\Omega. \tag{4.16}$$

The very large difference in the magnitudes of the two impedances allows us to use the approximate expression for the transmission loss given in (4.9). The transmission loss is therefore

$$L_t = -10\log_{10}\left[\frac{4 \times 0.031}{377}\right] = 35\,\text{dB}. \tag{4.17}$$

The signal power under the sea is proportional to the square of the electric field amplitude. At a depth of 10 m the power is

$$P = P_t \exp -\left[\frac{2 \times 10}{\delta}\right]$$
$$= 0.082 P_t \tag{4.18}$$

so that the signal transmission loss is

$$L = -10\log_{10}\left(\frac{P}{P_t}\right) = 11\,\text{dB}. \tag{4.19}$$

Combining the two transmission losses shows that the difference between the signal received on the surface and that received at a depth of 10 m is 46 dB.

4.2 TRANSMISSION THROUGH A THIN CONDUCTING SHEET

A very important practical case arises when the conducting material is in the form of a thin sheet rather than a thick slab as assumed in the previous section. 'Thin' in this context means 'having a thickness of the same order of magnitude as the skin depth'.

Figure 4.3 shows the arrangement of the problem. Because it is likely that some of the forward wave in the conductor will be reflected at B it is necessary to include this possibility in the calculations. The amplitudes of the forward and backward waves in each region are shown in the diagram.

If the thickness of the conductor is equal to the skin depth then the transmission loss is, from the previous section

$$L = -10\log(0.135) = 8.7\,\text{dB}. \tag{4.20}$$

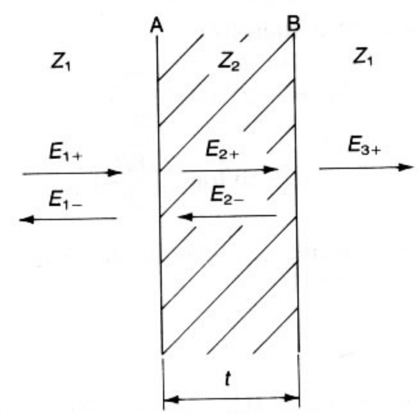

Fig. 4.3 Forward and backward waves for reflection by and transmission through a conducting sheet of finite thickness.

Thus, even if the forward wave is completely reflected at B and the backward wave completely reflected at A the doubly reflected signal has an amplitude which is 17.4 dB below that of the initial forward wave. If the thickness of the sheet is greater than the skin depth the effect of multiple reflections can be neglected. With this approximation the problem is reduced to the computation of the transmission losses at A and B and the transmission loss for the path A–B.

The transmission loss at A is given by (4.9) and the transmission loss through the sheet is

$$L_{AB} = -10 \log_{10} [\exp - (2t/\delta)]. \tag{4.21}$$

At B we have, by analogy with (4.2),

$$\frac{E_{3+}}{E_{2+}} = \frac{2Z_1}{Z_1 + Z_2} \simeq 2 \tag{4.22}$$

if $|Z_1| \gg |Z_2|$. This apparently surprising result occurs because the boundary condition at B is very nearly an open circuit. Very nearly all of the incident wave is reflected with a phase change of 180° making the electric field at the boundary twice that of the incident wave. The power in the transmitted wave is actually very small compared with the incident powder despite the magnitudes of the electric fields because of the differences in the impedances of the two media.

The incident power is

$$P_{2+} = |E_{2+}|^2/4R_s \tag{4.23}$$

from (1.46) and the transmitted power is

$$\begin{aligned} P_{3+} &= |E_{3+}|^2/2Z_1 \\ &= 2|E_{2+}|^2/Z_1. \end{aligned} \tag{4.24}$$

The transmission loss at B is therefore

$$\begin{aligned} L_B &= -10 \log_{10} \left(\frac{P_{3+}}{P_{2+}}\right) \\ &= -10 \log_{10} \left(\frac{8R_s}{Z_1}\right). \end{aligned} \tag{4.25}$$

The significance of these results can best be illustrated by an example.

Example

Calculate the attenuation of a 1 MHz radio wave by a sheet of aluminium 0.2 mm thick.

Solution

The conductivity of aluminium is $3.5 \times 10^7\,\mathrm{S\,m^{-1}}$ so the skin depth at 1 MHz is 0.085 mm (from (1.44)) and the surface resistance is $0.34 \times 10^{-3}\,\Omega/\square$. These figures show that we are justified in neglecting multiple reflections within the aluminium and in assuming that the wave impedance in the aluminium is much less than that in the air. The transmission losses are, from (4.9), (4.21) and (4.25),

$$L_A = 54\,\mathrm{dB}$$
$$L_{AB} = 20\,\mathrm{dB}$$
$$L_B = 51\,\mathrm{dB},$$

giving a total transmission loss of 125 dB.

4.3 ELECTROMAGNETIC SCREENING

In the example discussed in the previous section we saw how electromagnetic waves are attenuated by the presence of a conducting barrier. This leads naturally to a discussion of screening against electromagnetic interference. This topic is part of the subject known as electromagnetic compatibility (EMC). EMC is concerned with all the possible ways in which electronic equipment can be a source of interference or be susceptible to it. The steady increase in the number of possible sources of interference and the progress towards ever tighter packing densities in electronic equipment both make an understanding of the principles of EMC important to electronic engineers.

There is much more to the subject than finding ways of suppressing radio interference from cars and electric motors. In a modern aircraft, for example, a lot of electronic equipment is packed into an extremely small space. It is quite possible for signals radiated by a radar set to interfere with, say, the navigation system. If the interference is bad enough it could result in the loss of the aircraft. In addition to signals generated locally there is also the possibility of interference from high-power transmitters, from lightning and from solar storms. Perhaps the most dramatic example of all is the electromagnetic pulse (EMP) produced by a thermonuclear explosion. This pulse is powerful enough to destroy semiconductor devices in unprotected electronic equipment many miles from the explosion (Keiser, 1983).

In the previous section we saw that the screening effect of a conducting sheet can usually be divided into three parts, namely the reflection from the front of the sheet, the attenuation through it and the reflection from the back of the sheet. In addition, when the transmission loss through the sheet is small (typically less than 15 dB), multiple reflections can be important. The overall screening effectiveness can therefore be written

$$S = R + A + B, \tag{4.26}$$

where R is the transmission loss produced by both reflections, A is the attenuation loss for waves passing once through the sheet and B represents the effects of multiple reflections, all expressed in decibels. Note that B can be either positive or negative depending upon whether the multiply reflected waves interfere with each other constructively or destructively.

The reflection loss is, from (4.9) and (4.25),

$$R = -10 \log_{10} (4R_s/Z_1) - 10 \log_{10} (8R_s/Z_1). \tag{4.27}$$

Substituting for R_s and Z_1 we obtain, after a little manipulation,

$$R = 31.5 - 10 \log_{10} \left(\frac{f\mu}{\sigma}\right) \text{dB}. \tag{4.28}$$

The effects of reflections therefore decrease with increases in the frequency and of the permeability of the screen and increase with increases in conductivity.

The attenuation loss given by (4.21) may be written

$$A = -\frac{t}{\delta} 10 \log_{10} (e^{-2}) \tag{4.29}$$

which becomes

$$A = 15.4\sqrt{(f\sigma\mu)}\, t \,\text{dB}, \tag{4.30}$$

showing that the attenuation loss increases with the thickness of the sheet and also with the frequency, permeability and conductivity. This term changes faster than the logarithm in (4.28) so the overall screening effectiveness increases roughly as the attenuation loss.

If the sheet is very thin then the attenuation is negligible and all the screening is caused by the reflection loss. Such very thin screens can be made by evaporating metallic films on to dielectric surfaces. Multiple reflections cause the screening effectiveness to vary with frequency and it is a maximum when the film is a quarter wavelength thick. The reason for this can be seen by studying (4.1). The wave impedance of the film is much less than that of the surrounding space. This produces near short-circuit conditions at the front surface of the film and near open-circuit conditions at the back. Equation (4.1) shows that there is a phase reversal at the first reflection but not at the second. Thus if the film is a quarter wavelength thick the reflections from the two surfaces are in phase and the total reflection is a maximum.

The theory of screening by a conducting sheet described above involves a number of simplifying assumptions which need to be examined. The first of these is that the incident signal is a plane wave. This is only true if the screen is at least several wavelengths from the source. Sometimes, especially at low frequencies, this will not be the case. In the limit when the frequency is zero we know that an electric field can be completely excluded from a perfectly conducting closed box (a Faraday cage) (Carter, 1986). Such a box is, however, completely transparent to a static magnetic field. Similarly, a closed box of a high-permeability material such as mumetal can act as an effective screen against a static magnetic field but is less effective against electric fields because of the relatively poor conductivity of mumetal. This suggests that when the screen is close to the source its screening effectiveness will depend upon whether the source is an electric or a magnetic one.

As the frequency rises the eddy currents induced in a conducting screen tend to exclude the magnetic field from it so increasing its screening effectiveness. Conversely the currents produced by a changing electric field act as sources of electric field on the further side of the screen so reducing its effectiveness as the frequency increases. Thus it is useful to discuss separately the electric and magnetic screening effectiveness of an enclosure. Figure 4.4 shows their behaviour for a typical enclosure.

The next approximation which needs discussion is the application of the solution for the normal incidence of plane waves on an infinite sheet to practical cases involving finite enclosures of irregular shapes. The exact solution of practical problems is very difficult and can only be achieved in simple cases by numerical methods (Akhtarzad and Johns, 1975). Generally, though, we do not require very high accuracy in the estimation of the effectiveness of a screen. A useful approximation is to consider the screening effectiveness of a thin spherical shell of radius R and thickness t. The general formulae given by Field (1983) are

$$S_M = 20 \log_{10} \left[\cosh \gamma t + \frac{1}{3}\left[\frac{\gamma R}{\mu_r} + \frac{2(\mu_r - 1)}{\gamma R}\right] \sinh \gamma t\right], \tag{4.31}$$

where $\gamma = \sqrt{(j\omega\sigma\mu)}$, and

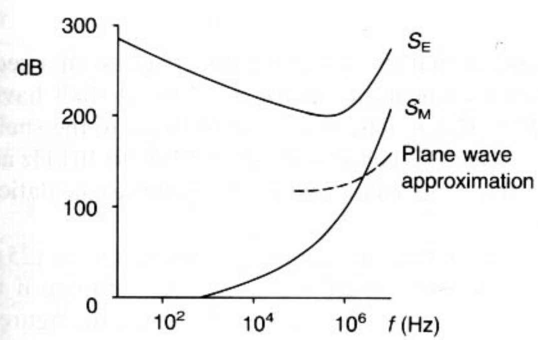

Fig. 4.4 Electric and magnetic screening effectiveness for an aluminium box as a function of frequency.

$$S_E = 20\log_{10}\left[\frac{2\gamma\sinh\gamma d}{3\omega^2\varepsilon_0\mu R}\right]. \tag{4.32}$$

These expressions are a little involved. Useful approximations are:

1. the magnetostatic case

$$S_M = 20\log_{10}\left[1 + \frac{2(\mu_r - 1)t}{3R}\right]; \tag{4.33}$$

2. the low-frequency approximations ($t \ll \delta$)

$$S_M = 10\log_{10}\left[1 + \omega^2\left(\frac{\mu_0\sigma R t}{3}\right)^2\right] \tag{4.34}$$

$$S_E = 20\log_{10}\left[\frac{2\sigma t}{3\omega\varepsilon_0 R}\right]; \text{ and} \tag{4.35}$$

3. the high-frequency approximations ($t \gg \delta$)

$$S_M = 20\log_{10}\left[\frac{R}{3\sqrt{2}\mu_r\delta}\,e^{t/\delta}\right] \tag{4.36}$$

$$S_E = 20\log_{10}\left[\frac{\sigma\delta}{3\sqrt{2}\omega\varepsilon_0 R}\,e^{t/\delta}\right]. \tag{4.37}$$

To see what these formulae imply let us consider an example.

Example

Estimate the screening effectiveness of a rectangular aluminium box 0.2 mm thick whose dimensions are 50 mm × 100 mm × 200 mm over the frequency range 10 Hz to 10 MHz.

Solution

Field (1983) suggests that the screening effectiveness of a rectangular box can be estimated by calculating it for a spherical shell having the same volume. Thus R = 62 mm. The skin depth is equal to the shell thickness at 36 kHz so we can use (4.34) and (4.35) from 10 Hz to 10 kHz and (4.36) and (4.37) from 100 kHz to 10 MHz. The results of these calculations are shown in Table 4.1.

These figures suggest that the screening effectiveness of 125 dB calculated using the plane-wave approximation is an over-estimate if the source of interference is a magnetic source. It is evident that the figures in the table follow the pattern shown in Fig. 4.4.

Table 4.1

f	δ (mm)	S_M (dB)	S_E (dB)
10 Hz	12	0	283
100 Hz	3.8	0.06	263
1 kHz	1.2	3.6	243
10 kHz	0.38	21.2	223
100 kHz	0.12	56	204
1 MHz	0.038	97	205
10 MHz	0.012	206	274

Example

What is the effect of replacing the aluminium enclosure of the previous example with one made of mumetal having the same dimensions?

Solution

Mumetal (a special magnetic screening alloy) has a relative permeability of 80 × 10^3 and a conductivity of 1.74 × 10^6 S m^{-1}. The shell thickness is equal to the skin depth at 45 Hz. This time it is possible to calculate the magnetic screening effectiveness under d.c. conditions using (4.33). Table 4.2 shows the results of the calculations.

Table 4.2

f	δ (mm)	S_M (dB)	S_E (dB)
0	–	65	–
10 Hz	0.42	56	277
100 Hz	0.13	34	317
1 kHz	0.042	72	315
10 kHz	0.013	175	648

These figures again show the same pattern as Fig. 4.4 but this time the magnetic screening is much better at low frequencies. The reduction in the electric screening effectiveness at low frequencies is because the impedance mismatch at the surface of the mumetal is less than that at the surface of the aluminium.

The values of screening effectiveness calculated in the preceding examples are so high that it appears that effective screening of electronic equipment should present no problems. The fact that very real problems do exist is a consequence of the other assumptions which have been made in the calculations. These are:

1. the enclosure is not resonant at any frequency within the range of interest; and
2. the enclosure is perfect with no hole in it of any kind.

Assumption 1 is equivalent to saying that the enclosure is small compared with the wavelength of the waves within it at all frequencies which matter. This is not necessarily the case when the frequencies are in the gigahertz region (including any signal harmonics), or when the circuits are mounted on high-permittivity substrates (e.g. alumina) or encapsulated in epoxy resin. We shall return to this point when resonant cavities are discussed in Chapter 6.

Assumption 2 is almost impossible to satisfy in any real system. Since a circuit must do something it must exchange energy or information with the outside world. This can only be achieved by making holes in the screen to allow wires or optical fibres to pass through. We shall see in the next chapter that the effect of such holes is a dramatic reduction in screening effectiveness. It follows that very great care is necessary in the design and construction of enclosures if they are not to be degraded in this way.

The final assumption which needs to be explored is that the enclosure is made wholly out of metal sheet. In many cases it is inconvenient to use sheet and other possibilities exist such as conducting plastics and metals in woven, braided or expanded mesh forms. It is to be expected that these materials will be less effective as screens than sheet metal but they can still provide useful screening. For a fuller discussion of the subject consult Field (1983) and Keiser (1979).

4.4 WAVES INCIDENT OBLIQUELY ON A CONDUCTING SURFACE

So far we have only considered the case where a wave is incident normally on a conducting surface. We now turn to the general case.

The wave impedance for waves incident obliquely on a surface with E parallel to the surface is

$$Z = Z_1/\cos\theta \tag{4.38}$$

from (1.97). Thus

$$\frac{E_r}{E_i} = \frac{Z_2\cos\theta_1 - Z_1\cos\theta_2}{Z_2\cos\theta_1 + Z_1\cos\theta_2} \tag{4.39}$$

and

$$\frac{E_t}{E_i} = \frac{2Z_2\cos\theta_1}{Z_2\cos\theta_1 + Z_1\cos\theta_2} \tag{4.40}$$

from (4.1) and (4.2) with Z_1 and Z_2 given by (4.3). The refraction of the waves at the boundary must obey Snell's Law (1.91) so

$$\frac{\sin\theta_2}{\sin\theta_1} = k_1\delta. \tag{4.41}$$

In any good conductor the skin depth is much less than the free-space wavelength so the right-hand side of (4.41) is normally very small. It follows that θ_2 is very close to zero so that the transmitted wave can be regarded as travelling normal to the boundary. Making this approximation and substituting for Z_2 gives

$$\frac{E_r}{E_i} = \frac{(1+j)R_s\cos\theta_1 - Z_1}{(1+j)R_s\cos\theta_1 + Z_1} \simeq -1, \tag{4.42}$$

since Z_1 is normally much greater than R_s. The wave is, therefore, almost completely reflected with a phase reversal just as in the case of a perfect conductor. The transmitted wave amplitude is

$$\frac{E_t}{E_i} = \frac{2(1+j)R_s\cos\theta_1}{(1+j)R_s\cos\theta_1 + Z_1} \simeq \frac{2(1+j)R_s}{Z_1}\cos\theta_1, \tag{4.43}$$

showing that the power absorbed decreases as the angle of incidence increases.

When the incident wave has its magnetic field parallel to the boundary the wave impedances are given by (1.94) and we obtain the following expressions for the reflected and transmitted wave amplitudes with the same assumptions as before:

$$\frac{E_r}{E_i} = \frac{(1+j)R_s - Z_1\cos\theta_1}{(1+j)R_s + Z_1\cos\theta_1} \tag{4.44}$$

and

$$\frac{E_t}{E_i} = \frac{2(1+j)R_s}{(1+j)R_s + Z_1\cos\theta_1}. \tag{4.45}$$

For most angles of incidence the wave impedance of medium 1 is much greater than that of the conductor and the wave is almost completely reflected with a phase reversal as before. However, when there is grazing incidence the two terms on the top line of (4.44) are of comparable magnitudes and an appreciable part of the incident power may be absorbed. The condition for mimimum reflection is

$$Z_1 \cos \theta_1 = R_s \tag{4.46}$$

and then

$$\frac{E_t}{E_i} = \frac{j}{2 + j} \tag{4.47}$$

and

$$\frac{E_r}{E_i} = \frac{2(1 + j)}{2 + j}. \tag{4.48}$$

Note carefully that it is never possible for all the power to be absorbed because of the phase difference between the impedances of the two media. When condition (4.46) is satisfied the incident and reflected waves are no longer in antiphase. To get a feel for the numbers involved consider the case of a 1 MHz wave in air incident on an aluminium sheet. The surface resistance of aluminium is $0.34 \times 10^{-3}\,\Omega$ so the angle of incidence for minimum reflection would differ from 90° by only a few millionths of a degree.

4.5 LOSSES IN TRANSMISSION LINES AND WAVEGUIDES

The theory of the preceding section finds its application in the calculation of losses in transmission lines and waveguides. In a TEM transmission line the electric field is normal to the surface of the conductors and the magnetic field is tangential. The loss per unit area of the conductors is given by (4.12).

Example

Find the attenuation per unit length at 10 GHz of a 50 Ω semi-rigid coaxial cable whose centre conductor is 1 mm in diameter. The conductors are copper and the dielectric is polythene.

Solution

The characteristic impedance of a coaxial cable is given by

$$Z_0 = \frac{1}{2\pi} \sqrt{\left(\frac{\mu}{\varepsilon}\right)} \ln \left(\frac{b}{a}\right), \tag{4.49}$$

where b and a are the radii of the outer and inner conductors, respectively (see Carter, 1986, p. 120). The relative permittivity of polythene is 2.25 so for a 50 Ω cable the inside diameter of the outer conductor must be 3.5 mm. The amplitude of the magnetic field is given by

$$H = \frac{I}{2\pi r}. \tag{4.50}$$

The power absorbed per unit length of the outer conductor is

$$P_b = \frac{1}{2} \left(\frac{I}{2\pi b}\right)^2 R_s\, 2\pi b, \tag{4.51}$$

from (4.12) and (4.50). Similarly, the power absorbed per unit length by the inner conductor is

$$P_a = \frac{1}{2} \left(\frac{I}{2\pi a}\right)^2 R_s\, 2\pi a. \tag{4.52}$$

Thus the total loss per metre in the conductors is

$$P_L = \frac{I^2 R_s}{4\pi} \left(\frac{1}{a} + \frac{1}{b}\right), \tag{4.53}$$

so that the effective series resistance of the line is

$$R = \frac{R_s}{2\pi} \left(\frac{1}{a} + \frac{1}{b}\right). \tag{4.54}$$

For a low-loss line the attenuation constant $\alpha = R/2Z_0$ (see Appendix A). So the attenuation per metre is

$$L = 20 \log_{10} [\exp - (R/2Z_0)]. \tag{4.55}$$

Now the conductivity of copper is $5.8 \times 10^7\,\mathrm{S\,m^{-1}}$ so the skin depth at 10 GHz is 0.66 μm and the surface resistance is 0.026 Ω/□. The loss in the line is found, by substituting these figures into (4.55), to be $0.46\,\mathrm{dB\,m^{-1}}$. The loss in real lines is about three times this figure because the skin depth is much less than the surface roughness.

Another possible source of loss in the line is dielectric loss. The loss tangent of polythene at microwave frequencies is 0.0003. The dielectric loss per unit length is therefore from (1.52)

$$P_D = 20 \log_{10} \{\exp - [\tfrac{1}{2}\omega\sqrt{(\varepsilon'\mu)}\delta]\} \tag{4.56}$$

(where δ is the loss tangent) and, substituting the numbers, we find that the dielectric loss is $0.2\,\mathrm{dB\,m^{-1}}$.

Provided that the losses are small it is justifiable to treat them separately and add the results. They can be modelled by a series resistance in the equivalent circuit of the line.

The attenuation of signals by other waveguiding systems can, in principle, be calculated in the same way. The non-uniform distribution of the currents in the conductors may make this difficult in practice. For further information on transmission line losses see Ramo *et al.* (1965) for waveguides and Edwards (1981) for microstrip.

The theory of wave propagation on lossy transmission lines is beyond the scope of this book. A useful treatment is given by Collin (1966). Fortunately it is usually possible either to neglect transmission-line losses or to treat them as lumped at a single point on the line. It should be noted that the characteristic impedance of a lossy line is complex and frequency dependent.

4.6 MICROWAVE ATTENUATORS

In the previous section we considered the effects of conductor and dielectric losses on the propagation of waves on transmission lines. These losses are generally small and can often be neglected. Sometimes, however, it is necessary to introduce much larger losses. Two common examples are attenuators (used to adjust signal levels) and loads (used to provide matched terminations). Microwave loads are discussed in the next section.

Consider first the realization of a fixed attenuator in a two-wire line. Superficially it would seem that all that is needed is to insert a lossy section into one of the conductors. Figure 4.5(a) shows this arrangement in coaxial line. The equivalent circuit of Fig. 4.5(b) shows the problem with this approach. The characteristic impedance of the lossy section is different from the rest of the line so there are mismatches at both ends of it. The reflections from these mismatches would beat with each other as the frequency changed, giving a variable transmission of power to the load. The alternative possibility of putting a thin resistive disc across the line shown in Fig. 4.5(c) has the equivalent circuit Fig. 4.5(d). This also introduces a mismatch into the line.

The solution is to use a combination of series and shunt elements as shown in Fig. 4.6. Figure 4.6(a) shows a combination of a lossy central conductor and a lossy disc in a coaxial line. Figure 4.6(c) shows a stripline attenuator having one series element and two shunt elements. The corre-

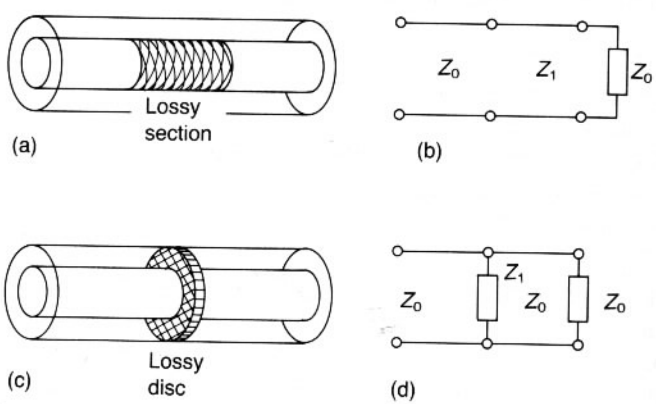

Fig. 4.5 Simple coaxial line attenuators and their equivalent circuits: series resistance (a) and (b), and shunt resistance (c) and (d).

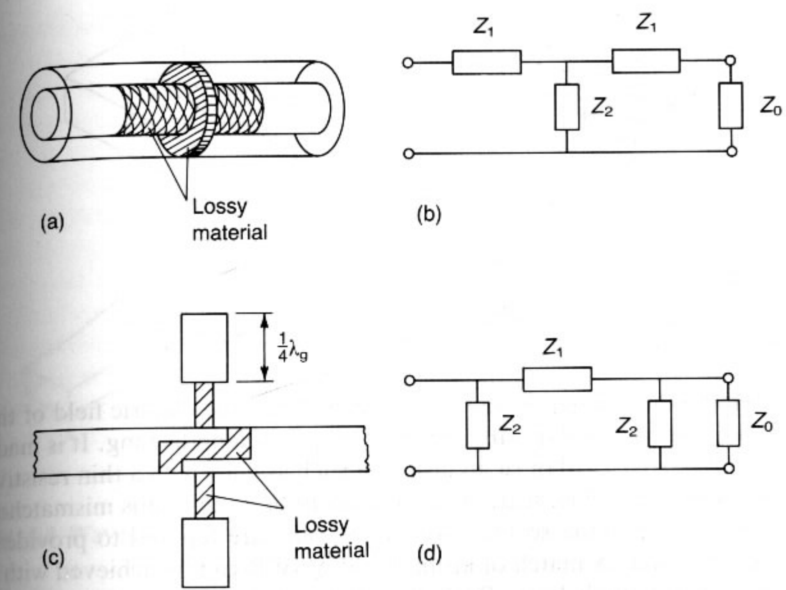

Fig. 4.6 Attenuators having both series and shunt elements with their equivalent circuits: coaxial line (a) and (b), and microstrip (c) and (d).

sponding equivalent circuits are shown in Fig. 4.6(b) and (d). By a suitable choice of the component values both these circuits can be matched to the transmission line. Provided that the physical sizes of the lossy elements are small (less than one eighth of a wavelength at the highest frequency) they can be regarded as lumped components. In that case the match depends only on the component values and not on their dimensions and it is possible to make an attenuator which has a good match and constant attenuation over a very wide frequency band. Coaxial attenuators are commercially available with attenuation flat to within ±1 dB from d.c. to 18 GHz.

An interesting feature of the stripline design shown in Fig. 4.6(c) is the use of open-circuited quarter-wave sections of line to provide short-circuit terminations for the shunt resistors. This technique obviously limits the frequency band over which the attenuator will work correctly. The alternative would be to connect shorting wires through holes drilled in the substrate; this technique requires skilled manual operations.

Where variable coaxial attenuators are required two approaches are in common use. One method is to use a set of fixed attenuators with mechanical or PIN switches to select them. The other, the piston attenuator, employs the attenuation of a cut-off waveguide.

In waveguides a rather different approach is used to construct attenuators. Figure 4.7 shows two common arrangements. In both cases a resistive vane

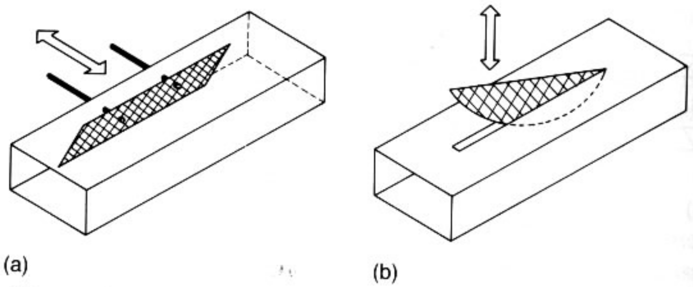

Fig. 4.7 Waveguide variable attenuators.

is arranged along the guide so that it is parallel to the electric field of the TE mode. The vane is typically around three wavelengths long. It is made either of a resistive card or of an insulator such as glass with a thin resistive film deposited on it. The section of guide with the vane in it is mismatched to the rest of the guide so the ends of the vane are tapered to provide a gradual transition. A match of around 1.05 VSWR can be achieved with a taper one wavelength long. Both the attenuators shown in Fig. 4.7 are variable. In Fig. 4.7(a) the vane is moved across the guide from the region of weak electric field at the side of the guide to the region of strong field at its mid-plane. In Fig. 4.7(b) the vane is lowered through a slot in the centre of the broad wall. Because the electrical length of the vanes varies with frequency the attenuation varies with frequency by a few decibels over an octave frequency band. The attenuation can be varied from zero to around 40 dB. One interesting variant of Fig. 4.7(b) has a dielectric vane with a neoprene tube fastened along its curved edge. Water is passed through the tube as a microwave absorber so making a high-power attenuator. Another type of high-power attenuator is described in Chapter 5.

The attenuators described in the previous paragraph are useful for level setting especially over narrow frequency bands. They have the advantage of being simple and relatively cheap. They can be calibrated if necessary but are not really suitable for use as attenuation standards for measuring purposes. Figure 4.8 shows a rotary vane attenuator. This type has very little variation of attenuation with frequency and its attenuation can be calculated so making it suitable for use as a reference. The device is symmetrical about its centre. The incoming rectangular waveguide A is connected via a transition B to a section of cylindrical waveguide C. B contains a fixed resistive vane arranged parallel to the broad wall of A. The purpose of this vane is to absorb any modes having horizontal electric field components so that the wave travelling from B into C is a pure TE_{11} mode polarized with its electric field vertical.

Section C contains a centrally placed attenuating vane and is capable of being rotated about its axis. The field of the incoming wave can be resolved

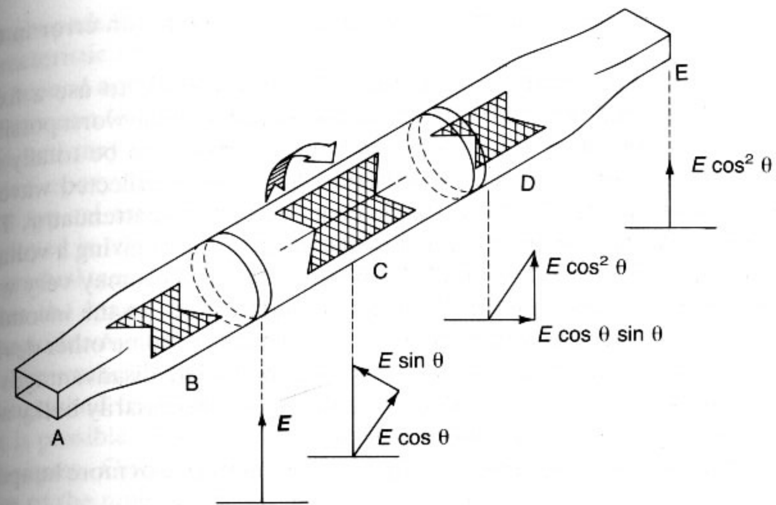

Fig. 4.8 Waveguide rotary vane attenuator.

into components $E\cos\theta$ normal to the plane of the vane and $E\sin\theta$ parallel to it. The vane is designed to provide very high attenuation for the component parallel to it. Thus the wave at the exit from C is polarized in the plane normal to the vane and has an amplitude $E\cos\theta$.

Section D is a transition from cylindrical to rectangular waveguide containing a fixed horizontal attenuating vane. The incoming wave can be resolved into a vertical component with amplitude $E\cos^2\theta$ and a horizontal component with amplitude $E\cos\theta\sin\theta$. The horizontal component is completely absorbed by the vane. Thus the amplitude of the wave transmitted to the output rectangular waveguide E is $E\cos^2\theta$. The attenuation is therefore

$$A = -40\log_{10}(\cos\theta). \tag{4.57}$$

This depends only upon the angle of the vane in C so it is independent of frequency. These attenuators are usually calibrated to be read directly.

Compared with the moving-vane attenuators described earlier rotary-vane attenuators are bulky and expensive. They generally have excellent matches to the input and output guides because of the use of tapered ends to the vanes and tapered rectangular to circular waveguide transitions.

4.7 MICROWAVE LOADS

It is often important to have some means of providing a matched termination for a transmission line or waveguide. This is especially true in microwave measuring systems where any reflected signal will produce an error in the measurement.

One simple way of providing a matched termination is to use a fixed attenuator. For example if a 20 dB attenuator is used then the worst possible case is for the signal transmitted through the attenuator to be totally reflected by either a short circuit or an open circuit. This reflected wave is attenuated by a further 20 dB on passing back through the attenuator. The reflected signal is therefore 40 dB below the incident signal giving a voltage reflection coefficient of 0.01 and a VSWR of 1.02 : 1. This may very well be a better match than the match of the attenuator itself to the incoming wave. An attenuator can be used to improve the match of some other device in just the same way. It is then described as a 'pad'. The disadvantages of using an attenuator as a matched load are that it is unnecessarily bulky and may not be able to dissipate much power.

Matched loads for stripline circuits generally employ one or more lumped-

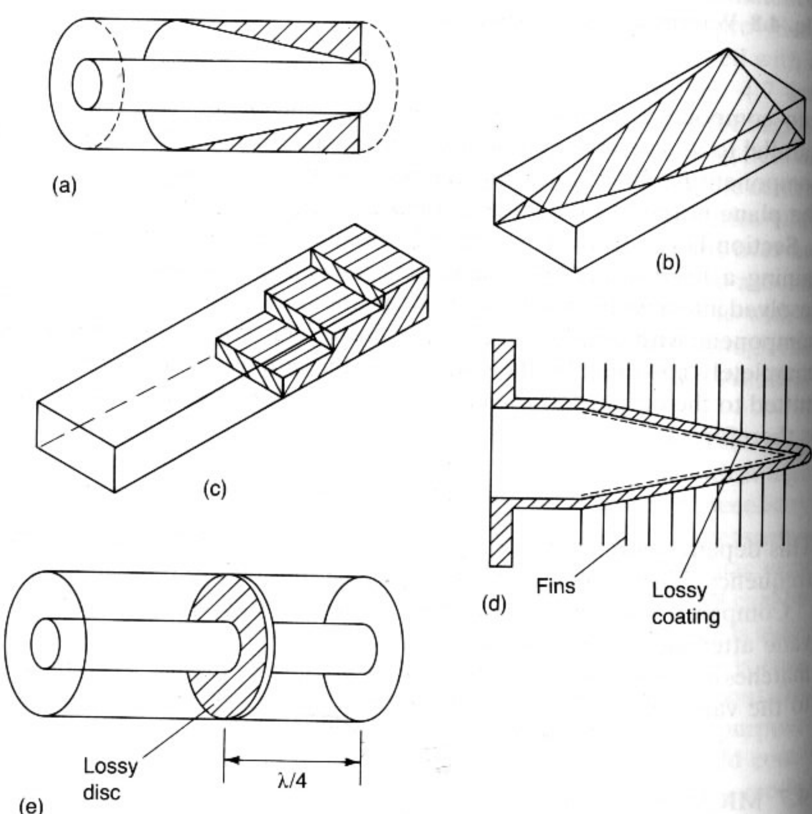

Fig. 4.9 Microwave loads: (a) coaxial line, (b) waveguide, (c) compact wavguide, (d) high-power waveguide and (e) compact coaxial line loads.

element pads followed by a lumped resistor whose resistance is equal to the characteristic impedance of the line.

A second approach is based on the one described above. The end of the transmission line or waveguide is filled with a tapered lossy section whose wide end may completely fill the space. Possible materials include iron-loaded epoxy resin and carbon-loaded ceramic. Figures 4.9(a) and (b) show typical forms for use in coaxial line and waveguide. The principle employed is to make the load material as lossy as possible and to rely on the gradual taper to ensure a satisfactory match. For high-precision measurements the load can be left free to slide inside the transmission line or waveguide. As it is moved backwards and forwards the phase of the residual mismatch changes. In this way the mismatch of the load can be distinguished from that of the device under test.

It is possible to make more compact loads by more sophisticated design techniques. One of these is the use of a series of quarter-wave steps in place of the taper as shown in the waveguide load of Fig. 4.9(c). It can be shown that such an arrangement has a better match than a taper of the same length. It also has the advantage that the load itself no longer has a thin, fragile, tip.

High-power loads are sometimes made by tapering the guide itself, coating the interior with a temperature-resistant material such as flame-sprayed Kanthal, and providing cooling fins on the outside. This arrangements is shown in Fig. 4.9(d).

Very compact loads can be made using a sheet of resistive card matched to the wave impedance of the incoming wave. This impedance appears in parallel with the impedance of the transmission line beyond it. By putting a short circuit a quarter wavelength behind the card that impedance is close to an open circuit over a range of frequencies. Figure 4.9(e) shows this arrangement in coaxial line. A similar technique can be used to make compact waveguide loads.

Absorbent materials are manufactured which are designed to absorb TEM waves in free space. They generally take the form of arrays of pyramids which present a gradual transition from the impedance of free space to that of the absorber. These materials are used for lining the anechoic ('echo-free') chambers employed for testing antennas and for electromagnetic compatibility measurements (Keiser, 1979).

4.8 CONCLUSION

In this chapter we have examined the effects on electromagnetic waves of real conducting boundaries which cause some loss. We have seen that metallic boundaries are usually very badly mismatched to the wave impedance of the incoming wave so that there is almost total reflection. The wave impedance of the conductor is complex implying loss in a wave travelling through it and a phase change in the wave reflected from its surface. The combination of reflection loss and transmission loss makes it possible to use thin metal sheets to screen electronic equipment against electromagnetic interference. The reflection of waves which are obliquely incident on a conducting surface was examined leading to a discussion of the attenuation of signals in transmission lines and waveguides. Finally techniques for deliberately introducing loss in the form of attenuators and matched loads were described.

EXERCISES

4.1 Calculate the skin depth and surface resistance for brass ($\sigma = 1.1 \times 10^7\,\mathrm{S\,m^{-1}}$) and gold ($\sigma = 4.1 \times 10^7\,\mathrm{S\,m^{-1}}$) at frequencies of 200 MHz, 2 GHz and 20 GHz.

4.2 It is proposed to make a room secure against electronic 'bugging' at frequency of 100 MHz by coating the windows with a film of copper ($\sigma = 5.7 \times 10^7\,\mathrm{S\,m^{-1}}$) 10 μm thick. If the window glass is 4 mm thick and has a relative permittivity of 4.0 estimate the change in the transmission loss produced by the copper film.

4.3 Estimate the electric and magnetic screening effectiveness of box whose dimensions are 20 mm × 50 mm × 200 mm which is made of brass ($\sigma = 1.1 \times 10^7\,\mathrm{S\,m^{-1}}$) 0.5 mm thick over the frequency range 10 Hz to 100 MHz.

4.4 Estimate the loss per unit length in a section of WG16 waveguide made of copper ($\sigma = 5.7 \times 10^7\,\mathrm{S\,m^{-1}}$) at a frequency of 10 GHz.