

# Electoral Rule Disproportionality and Platform Polarization

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## Abstract

We analyze the effect of electoral rule disproportionality on the degree of platform polarization by the means of a unidimensional spatial model with policy motivated parties. We identify two distinct channels through which disproportionality affects polarization: First, polarization is decreasing in the level of disproportionality (direct channel) and increasing in the number of competing parties. Second, the number of competing parties itself is decreasing in the level of disproportionality when parties strategically decide whether to enter the electoral race or not. Therefore, an increase in the level of disproportionality may further decrease polarization by decreasing the number of competing parties (indirect channel). By constructing a large and homogeneous database we provide empirical evidence in support of our theoretical findings: Electoral rule disproportionality is the major determinant of polarization while the number of competing parties has limited explanatory power possibly due to its dependency on the level of disproportionality.

**Keywords:** proportional representation; disproportional electoral systems; polarization; policy-motivated parties; number of parties; Duvergerian predictions

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# 1 Introduction

Cox (1990) by the means of an electoral competition model with purely office-motivated candidates argues that the degree of disproportionality of the electoral system affects platform polarization in a negative manner; the more disproportional the electoral system is the smaller the degree of platform polarization.<sup>1</sup> Moreover, he argues that the number of competing parties is also positively related to the degree of platform polarization. Cox's (1990) formal results, though, are not sufficient to perfectly back up these intuitive ideas. He specifically states that "these results only tell what will happen *if there is an equilibrium*; they do not guarantee that an equilibrium will exist".

This paper surmounts the complexities of establishing equilibrium existence in such a framework by considering that parties are mainly - but not necessarily purely - policy motivated in the spirit of Wittman (1977).<sup>2</sup> Introduction of policy motives in the analysis does not only help us prove that an equilibrium exists but, more importantly, it allows us to further analyze the effect of electoral rule disproportionality on parties' equilibrium political platforms choices.

Regarding the comparative analysis of different electoral systems, this paper departs from pairwise comparisons of electoral systems and introduces a continuum of "disproportionalities" that essentially includes any rule which lies between a purely proportional and a "winner-take-all" electoral rule.<sup>3</sup> While in winner-take-all elections the implemented

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<sup>1</sup>The provisions of an electoral rule which determine its degree of disproportionality are usually related to the magnitude (see for example Jackman 1987; Cox 1990) and to the composition of electoral districts (see for example Coate and Knight 2007; Besley and Preston 2007; Bracco 2013).

<sup>2</sup>This route to overcome problems of possible non-existence of equilibria in electoral competition models is not novel in the literature. Groseclose (2001), for example, used this framework to deal with the non-existence of pure strategy equilibria in competition of two office-motivated candidates of unequal valence.

Recently, Calvo and Hellwig (2011) also point to the direction of Cox's (1990) conclusions. The authors neatly address the issue of identifying equilibria using the "probabilistic" voting model (see Adams et al. 2005) at the cost of assuming that voters vote for every candidate with positive probability (even for their least preferred one). As data suggest a great majority of voters vote for the candidate they rank first (sincere voting), a small minority votes second ranked candidates in order to affect the result in the direction they regard most profitable for them (strategic voting) while no voters (at least no detectable measure of them) votes for candidates they dislike the most. In the Comparative Study of Electoral Systems (CSES) Module 2 the average of sincere voting is around 85% while the remaining 15% votes for candidates they do not rank neither first nor last so as to maximize the impact of their vote.

<sup>3</sup>The continuum of disproportionalities also permits the comparison of different electoral rules that

policy is determined by the winning party (Wittman, 1977, 1983; Calvert, 1985; Roemer, 1994; Hansson and Stuart, 1984; Grofman, 2004), elections under a more-or-less proportional representation system produce a policy that is collectively determined by the platforms of the competing parties and their “political power” (e.g. Ortuño-Ortín 1997; Austen-Smith and Banks 1988; Llavador 2006; De Sinopoli and Iannantuoni 2007, 2008; Merrill and Adams 2007). If a party’s political power is understood as the proportion of the parliamentary seats it holds, and given that parties’ parliamentary seats are jointly determined by the electoral outcome and the rule that maps the latter to a parliamentary seat-distribution, the relevance of the electoral rule disproportionality on parties’ choice of political platforms and on platform polarization becomes more than evident.

In order to fully demonstrate why electoral disproportionality acts as a centripetal force, hence resulting in low levels of polarization, we first model a two-party election in a unidimensional policy space considering that parties are mainly policy-motivated. The *leftist* party has its preferred policy at the extreme left of the policy space while the *rightist* party has its preferred policy at the extreme opposite.<sup>4</sup> First, parties announce their platforms. Second, voters observe the announced platforms and vote for the party that proposed the platform closer to their ideal policy. Finally, a policy is implemented according to the parliamentary-mean model.<sup>5</sup> In such model the implemented policy is a weighted average of parties’ announced platforms where each party’s weight is determined by its seat-share in parliament. Hence, the policy outcome is a function of the announced

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belong in the same family but whose disproportionality may vary significantly (e.g. in the family of proportional representation (PR) systems the electoral rule in Italy is much more disproportional than in Netherlands).

For important pairwise comparisons between first-past-the-post and proportional systems see for example Lizzeri and Persico (2001) on public good provision, Morelli (2004) on party formation, Austen-Smith (2000) on redistribution, Persson et al. (2003) on corruption, Iaryczower and Mattozzi (2013) on campaign spending and the number of competing candidates. For a pairwise comparison between plurality and runoff elections see Osborne and Slivinski (1996). Finally, Myerson (1993a,b) offers pairwise comparisons between PR, approval voting, FPTP and Borda rule mainly on the issues of corruption and campaign promises.

<sup>4</sup>As we show in section 3.5 our results do not hinge on this since our main predictions are robust to non-extreme parties when we impose some mild assumptions on the distribution of voters.

<sup>5</sup>We borrow the name of this model from Merrill and Adams (2007). For earlier approaches on proportional systems using this model see among others Ortuño-Ortín (1997); Llavador (2006); De Sinopoli and Iannantuoni (2007).

platforms, the electoral outcome, and the disproportionality of the electoral system.

According to our results, parties' platforms in equilibrium converge towards the ideal policy of the median voter as the electoral system becomes more disproportional (in favor of the winner). The intuition is clear. On one hand, a move towards the median harms a party since it proposes a platform further away from its ideal policy. On the other hand, as parties move towards the median they increase their vote-share and hence their weight in the implemented policy. As the disproportionality of the electoral system increases proposing a moderate platform may be worthwhile since the incentives to obtain some extra votes are amplified.

We conduct the same analysis with three parties (a leftist, a centrist and a rightist one) and we show that in equilibrium platform polarization not only moves as in the two-parties case (higher electoral rule disproportionality leads to less platform polarization) but is also slightly higher compared to the two-parties case given the presence of the centrist party.<sup>6</sup> Hence, *ceteris paribus*, higher electoral disproportionality leads to a lower degree of platform polarization and, *ceteris paribus*, a larger number of competing parties increases platform polarization.

Finally, we consider the case of an endogenous number of competing parties by introducing an entry stage in the three-party version of our model. In stage one, the three parties strategically decide whether to enter the costly electoral race or not. In stage two, parties that have entered the race strategically select their policy platforms. Considering that parties have, on top of their main policy concerns, some secondary office-holding concerns, we show that the equilibrium number of competing parties is decreasing in the level of electoral disproportionality (a result in line with the Duvergerian predictions where proportional rules tend to favor multiparty systems while disproportional rules lead to two-party systems). This result indicates that electoral rule disproportionality affects polarization through a second indirect channel. Disproportionality does not only

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<sup>6</sup>In the three party case polarization is measured by the distance between the two most distant platforms. Our conclusions are robust to considering alternative measures of platform polarization that are used in the relevant (mainly empirical) literature.

provide the centripetal forces previously described but it may further amplify them by reducing the number of competing parties.

In light of contradicting conclusions of recent empirical research regarding the significance of the disproportionality of the electoral rule on the level of platform polarization (Erzow, 2008; Dow, 2011; Calvo and Hellwig, 2011; Curini and Hino, 2012) we consider that empirical verification of the model’s main theoretical predictions becomes not only relevant but also necessary. Our empirical analysis departs from recent literature in terms of estimation methodology and, mainly, in terms of survey design (Curini and Hino, 2012; Dow, 2011, 2001; Andrews and Money, 2009; Ezrow, 2008; Dalton, 2008; Budge and McDonald, 2006) and provides strong support in favor of our main theoretical result; the level of platform polarization is decreasing in the level of electoral disproportionality. The number of competing parties seems to have a limited significant direct impact on platform polarization and it is found to be negatively correlated with the level of electoral rule disproportionality (as the endogenous number of parties version of the theoretical model suggests).

The rest of the paper is organized as follows: In section two we present our theoretical model. In section three we provide our main set of results while in section four we consider an endogenous number of parties. In section five we perform our empirical analysis while in section six we present our concluding remarks. Formal proofs and descriptive statistics are incorporated in the appendix.

## 2 The Model

We construct a formal model in line with the *parliamentary-mean* model (e.g. Ortuño-Ortín 1997; Llavador 2006; De Sinopoli and Iannantuoni 2007; Merrill and Adams 2007).<sup>7</sup>

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<sup>7</sup>In existing approaches, the implemented policy is assumed to be a convex combination of the proposed platforms weighted by a function that depends solely on parties’ vote-shares. As Ortuño-Ortín (1997) states “Clearly, the specific function used should [also] depend on many institutional and cultural factors”. In this paper we consider the effect of one of the most important institutional factors, namely the disproportionality of the electoral system.

The innovation of our benchmark model of electoral competition is that the implemented policy does not only depend on parties' vote-shares and proposed platforms but also on the (dis)proportionality of the electoral system.<sup>8</sup> We first consider two parties ( $j = L, R$ ) that compete in an election and announce their platforms. Voters observe these platforms and vote for one of the two parties. Given parties' vote-shares ( $V_L$  and  $V_R$ ), the announced platforms ( $t_L$  and  $t_R$ ), and the (dis)proportionality of the electoral system ( $n$ ) a policy  $\hat{t}$  is implemented.

The policy space is assumed to be continuous, one-dimensional, and represented by the interval  $T = [0, 1]$ . We assume that there is a continuum of voters. Voters are distributed on the policy space according to a continuous and twice-differentiable probability measure  $F$  on  $T$ . Let  $\tau_i \in T$  denote the ideal policy of individual  $i$ . We assume that each voter cares about the remoteness (but not the direction) of the proposed platform  $t_j$  from his ideal policy  $\tau_i$ . Specifically, individual  $i$ 's preferences are represented by a utility function  $u : T \rightarrow \mathbb{R}$ , where  $u$  is a continuous, strictly decreasing function with respect to the distance between  $t_j$  and  $\tau_i$ . This utility function is "symmetric", that is for all  $\tau_i$  and for all  $x$  such that  $\tau_i + x \in T, \tau_i - x \in T$  then it holds that  $u(\tau_i + x; \tau_i) = u(\tau_i - x; \tau_i)$ . Voters support the party that proposes the platform closer to their ideal point. Formally, a voter  $i$  with ideal policy  $\tau_i$  votes for party  $j$  if  $u(t_j; \tau_i) > u(t_{-j}; \tau_i)$  and splits his vote if  $u(t_L; \tau_i) = u(t_R; \tau_i)$ . We denote as  $\bar{\tau}$  the ideal policy of the indifferent voter, that is the ideal policy of the voter for whom it holds that  $u(t_L; \bar{\tau}) = u(t_R; \bar{\tau})$ . Given that preferences are assumed to be symmetric, the location of the indifferent voter is always half distance between the platforms proposed by the two parties. Formally,  $\bar{\tau} = (t_L + t_R)/2$  and therefore parties' vote-shares are given by

$$V_L(t_L, t_R) = \begin{cases} F(\bar{\tau}) = F\left(\frac{t_L + t_R}{2}\right), & \text{if } t_L < t_R \\ \frac{1}{2}, & \text{if } t_L = t_R \\ 1 - F(\bar{\tau}) = 1 - F\left(\frac{t_L + t_R}{2}\right), & \text{if } t_L > t_R \end{cases}$$

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<sup>8</sup>Merrill and Adams (2007) also consider that the implemented policy depends on parties' parliament seat-shares but they analyze strategic behavior of parties only under a purely proportional rule. That is, in their case the parliament seat-share of a party coincides with its vote-share.

and

$$V_R(t_L, t_R) = 1 - V_L(t_L, t_R).$$

Parties are policy motivated. Their payoffs depend on the implemented policy rather than on an exogenous given office value for winning the election. Each party  $j$  has an ideal policy  $\tau_j \in T$ . We assume that parties have an ideal policy at the extremes of the policy line, that is,  $\tau_L = 0$  and  $\tau_R = 1$  and that party's  $j$  preferences over policies are the same as the preferences of a voter with the same ideal policy. Formally, party's  $j$  payoffs when policy  $\hat{t}$  is implemented are defined as  $\pi_j(\hat{t}) = u(\hat{t}; \tau_j)$ .

The implemented policy is a function of parties' power in the parliament (parties' seat-shares are denoted by  $(S_L, S_R)$ ) and parties' proposed platforms  $(t_L, t_R)$ . Notice that parties' seat-shares are of course a function of parties' vote-shares  $(V_L, V_R)$  which ultimately are a function of the proposed platforms and the disproportionality of the electoral system denoted by  $n$ , where  $n \geq 1$ .<sup>9</sup> We formally define the implemented policy function as:

$$\hat{t}(t_L, t_R, n) = S_L(t_L, t_R, n) * t_L + S_R(t_L, t_R, n) * t_R$$

This function captures the post electoral compromise between parties' platforms depending on parties' parliamentary power.<sup>10</sup> The way parties' vote-shares ( $V_L$  and  $V_R$ ) translate into seat-shares ( $S_L$  and  $S_R$ ) in the parliament depending on the disproportion-

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<sup>9</sup>The analysis could incorporate the case of degressive proportionality where  $n < 1$  and the electoral system is disproportional in favor of the loser (Koriyama et al., 2013; Bracco, 2013). In order to apply the term disproportionality as it is conventionally understood, that is in favor of the winner, we assume throughout the paper that  $n \geq 1$ . This helps in the clarity and intuitive understanding of our results.

<sup>10</sup>For two-party compromise models under pure PR elections where  $S_L = V_L$  and  $S_R = V_R$  see Llavador (2006); Ortuño-Ortín (1997). For multiparty models see De Sinopoli and Iannantuoni (2007, 2008); Gerber and Ortuño-Ortín (1998); Austen-Smith and Banks (1988); Merrill and Adams (2007).

An alternative way of modeling PR elections is by assuming that each party implements its proposed platform with probability equal to its vote (or seat) share (Faravelli and Sanchez-Pages, 2012; Iaryczower and Mattozzi, 2013; Merrill and Adams, 2007). Merrill and Adams (2007) provide a further comparison among the two ways of modeling PR systems and their effect on equilibrium proposed platforms. In our model both ways of modeling are equivalent and provide the same results and intuition if parties are risk neutral.

ality of the electoral system ( $n$ ) follows Theil (1969):<sup>11</sup>.

$$\frac{S_L}{S_R} = \left( \frac{V_L}{V_R} \right)^n$$

Through the above formula and  $n = 1$  one captures a purely proportional representation system where no distortions are present. Letting  $n = 3$  the seat allocation is based on the famous “*cube law*” which is used in the literature as a good approximation of the distortions created in favor of the winner in first-past-the-post (FPTP) elections. In general as  $n$  increases the electoral system is more disproportional in favor of the winner of the election. Since we know that  $S_L + S_R = 1$  and that  $S_L/S_R = (V_L/V_R)^n$  we can rewrite the seat-shares as follows:  $S_L = V_L^n / (V_L^n + V_R^n)$  and  $S_R = V_R^n / (V_L^n + V_R^n)$ .<sup>12</sup> Therefore the implemented policy function can be rewritten as:

$$\hat{t}(t_L, t_R, n) = \frac{V_L(t_L, t_R)^n}{V_L(t_L, t_R)^n + V_R(t_L, t_R)^n} * t_L + \frac{V_R(t_L, t_R)^n}{V_L(t_L, t_R)^n + V_R(t_L, t_R)^n} * t_R$$

In existing models of pure proportional representation (e.g. Ortuño-Ortín 1997; Llavador 2006; Merrill and Adams 2007) the associated weights to parties’ platforms are assumed to be proportional to parties’ vote-shares (captured by our function by assuming  $n = 1$ ). By allowing  $n$  to take values larger than one, and for a given electoral outcome, we increase the weight put on the policy proposed by the winner of the election. This may well be the consequence of the disproportional power the winner enjoys in case

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<sup>11</sup>Taagepera (1986) offers a further analysis of the above formula and empirical estimations of parameter  $n$ . For an overview of measures of bias and the proportionality in the relationship between vote-shares and seat-shares see Grofman (1983). Gallagher (1991) provides an analysis of indices and empirical measures of disproportionality. For recent applications of this formula see Calvo and Hellwig (2011); Ergun (2010); King (1990)

<sup>12</sup>As the familiar reader with the literature on contests may have already observed, the weights determined through this specific functional form are identical to the seminal contest success function introduced by Tullock (1980, pg. 97-112). Remember though that since we assume that  $n \geq 1$ , and as depicted in figure 1, the weight function is convex for values smaller than one-half, and concave for values larger than one-half. Hence, in the electoral contest winning the elections makes a difference. The electoral system gives disproportional weight to the party that obtains the majority of the votes. Up to what extent the winner is favored is determined by the disproportionality parameter  $n$ .



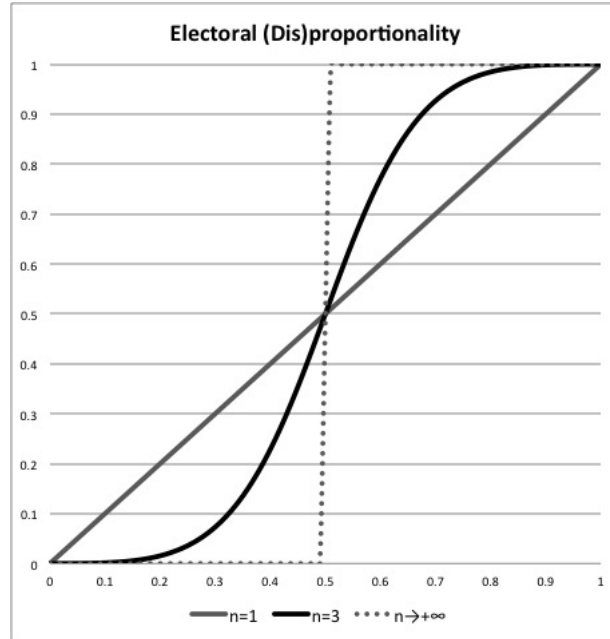


Figure 1: The weight of a party's proposal (i.e. its seat-share) as a function of its vote-share for the cases where  $n = 1$ ,  $n = 3$ , and  $n \rightarrow \infty$ .

of disproportional electoral systems. As depicted in figure 1, under pure PR ( $n = 1$ ) if one party wins the election with sixty percent of the votes, then the associated weight to its platform is 0.6, with the loser affecting the policy with a weight of 0.4. In a more disproportional electoral system, where for example  $n = 3$ , the winner of sixty percent of the votes influences the implemented policy with a larger weight of 0.77 (hence the loser affects the policy only with a weight of 0.23). If  $n \rightarrow \infty$  then parties actually compete in a winner-take-all election where the implemented policy converges to the winner's proposed platform.

As in most spatial models of this type individuals vote for one of the parties once they observe the announced platforms. Hence, parties are the actual players of the game. Parties strategically announce their platforms, which are enough to determine the voters' behavior and hence the outcome of the game and the corresponding payoffs. The equilibrium concept we apply is Nash equilibrium in pure strategies.

PROPOSED PLATFORMS AND ELECTION ( $\tau_L=0, \tau_R=1$ )	DISPROP.	PARLIAMENTARY POWER	POLICY
	$n=1$		$\hat{t}=0.5$
	$n=1$		$\hat{t}=0.52$
	$n=3$		$\hat{t}=0.38$

Figure 2: The incentives for policy convergence. Figure 2a and 2b if  $n = 1$ , 2c if  $n = 3$ .

### 3 Results

For clarity of presentation we first analyze the case of uniformly distributed voters and then we provide our general result.

#### 3.1 Uniformly distributed voters

Consider first a pure PR system (where  $n = 1$ ) depicted in Figure 2a. Let parties' platforms coincide with their ideal policies, hence they propose platforms at the extremes of the policy space ( $t_L = \tau_L = 0$  and  $t_R = \tau_R = 1$ ). The voter who is located at one-half is indifferent between the two proposed platforms ( $\bar{\tau} = 0.5$ ). Voters on the left of the indifferent voter support the platform of the left party, while voters on the right of the indifferent voter support the right party. Given that voters are uniformly distributed each party obtains 50% of the votes. Combining this vote-share with the proportional system ( $n = 1$ ) each party's platform has an equal (50%) weight on the implemented policy. Hence, the implemented policy is one-half (which coincides with both the median and the ideal policy of the indifferent voter).

Let us first exploit the possible incentives for any of the two parties to deviate from

this strategy and converge towards the median (Figure 2b). Let for example party  $L$  announce platform  $t_L = 0.2$ . Now the indifferent voter moves further to the right (now  $\bar{\tau} = 0.6$ ). Hence, party  $L$  obtains 60% of the votes while party  $R$  obtains 40% of the votes. When the implemented policy is determined party  $L$  now has a larger weight than before (now 60% compared to 50%), but at the same time proposes a policy further from its ideal point ( $t_L = 0.2$  compared to  $t_L = 0$ ). Therefore the implemented policy is  $\hat{t} = 0.52$  and hence party  $L$  has no incentives to deviate from its initial strategy since by doing so the implemented policy moves further from its ideal point and is worse off.

Consider now that the electoral system is more disproportional, hence it favors the winner of the election (let for example  $n = 3$ ) as depicted in Figure 2c. As before, if party  $L$  announces platform 0.2 rather than zero, it obtains 60% of the vote-share. Because of the disproportionality such vote-share translates to a disproportionately high weight of 77% on its proposed platform. In contrast to the case when  $n = 1$  such weight now compensates the loss because of proposing a moderate platform. It turns out that the implemented policy is 0.38. Hence, party  $L$  has incentives to converge towards the median since it brings the implemented policy closer to its ideal point.

The general mechanism which provides incentives to parties to propose less differentiated platforms when the degree of disproportionality of the electoral rule increases must have been made clear by now. Analyzing a uniform distribution of voters, though, does not only formally prove the negative direction of this relationship but further guarantees a closed form solution regarding parties' equilibrium strategies. That is, it enhances our understanding on quantitative aspects of the relationship between platform polarization and electoral rule disproportionality other than its direction.

**Proposition 1.** *Let  $\tau_i \sim U[0, 1]$ . Then: (i) There exists a unique equilibrium  $(t_L^{**}, t_R^{**}) = (\frac{n-1}{2n}, \frac{n+1}{2n})$  (ii) The distance between  $t_R^{**}$  and  $t_L^{**}$  is decreasing in  $n$  (iii)  $\hat{t} = \bar{\tau} = 0.5$ .*

The unique equilibrium  $(t_L^{**}, t_R^{**})$  of two party competition for different values of electoral disproportionality  $n \in [1, 20]$  and a graphical interpretation of proposition 1 is

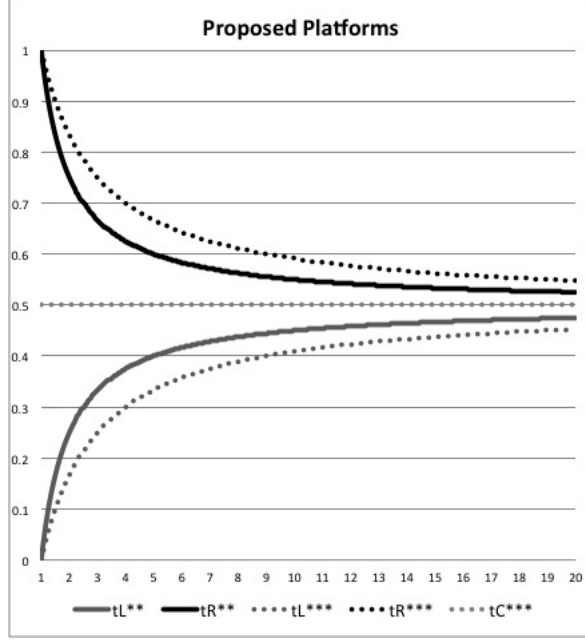


Figure 3: The effect of electoral disproportionality  $n \in [1, 20]$  on proposed platforms for two  $(t_L^{**}, t_R^{**})$  and three-party competition  $(t_L^{***}, t_R^{***}, t_C^{***})$  as characterized in propositions 1 and 4.

depicted in figure 3. If  $n = 1$  (i.e. no distortions are present) both parties' platforms coincide with their ideal policies, hence they stick at the the extremes of the policy space ( $t_L^{**} = \tau_L = 0$  and  $t_R^{**} = \tau_R = 1$ ). It is easy to calculate that the unique equilibrium when  $n = 3$  is  $(t_L^{**}, t_R^{**}) = (1/3, 2/3)$ . In the extreme case of a winner-take-all election ( $n \rightarrow \infty$ ) where the winner can perfectly implement his platform we obtain the standard result of full convergence to the median.

### 3.2 The General Result

The following result generalizes the case of a uniform distribution to any cumulative distribution function  $F$ .

**Proposition 2.** *Let  $m$  denote the median of any c.d.f.  $F$ . Then: (i) There exists a unique equilibrium  $(t_L^*(n), t_R^*(n))$  where  $t_L^*(n) < t_R^*(n)$  (ii) If for some  $n_0$  we have that  $0 < t_L^*(n_0) < t_R^*(n_0) < 1$  then  $\hat{t}(n_0) = \bar{\tau}(n_0) = m$  and for every  $n_1$  and  $n_2$  such that  $n_0 < n_1 < n_2$  we have  $|t_L^*(n_1) - t_R^*(n_1)| > |t_L^*(n_2) - t_R^*(n_2)|$  (iii)  $t_L^*(n) \rightarrow t_R^*(n)$  when*

$n \rightarrow \infty$  (iv) If  $m = 0.5$  then  $t_L^* = 1 - t_R^*$ .

For any degree of disproportionality there exists a unique equilibrium where the leftist party proposes a policy at the left of the median and the rightist party proposes a policy at the right of the median voter. In equilibrium the implemented policy coincides with the median voter's ideal point while both parties propose platforms that diverge from the median by the same distance. Despite our analysis allowing any possible distribution of voters' ideal points, as we show, parties propose platforms that are symmetric with respect to the median no matter how skewed the distribution is. The asymmetry in terms of the distribution is of course reflected in parties' payoffs.

From a comparative perspective, as the disproportionality of the electoral system increases parties have incentives to moderate their policies and converge towards the median. Full convergence to the median is predicted in the case of winner-take-all elections ( $n \rightarrow \infty$ ). The intuition behind parties' incentives to moderate or not their platforms is similar to the case of uniformly distributed voters. If the disproportionality increases parties have incentives to deviate from the initial equilibrium and moderate their proposals aiming at a higher vote-share since the latter further translates to a disproportionately higher weight in the implemented policy. Such higher weight may well compensate the losses entailed because of proposing more moderate platforms. Consequently, in the unique equilibrium, parties' platforms tend to differentiate less as the disproportionality of the electoral system increases.

### 3.3 Further Results

We now discuss three possible directions of enrichening our model that provide an intuition in line with our main result and the centrifugal incentives present in proportional systems.

### 3.4 Office motivated parties

So far we have considered purely policy motivated parties. Remember that party's  $j$  payoffs when policy  $\hat{t}$  was implemented were defined as  $\pi_j(\hat{t}) = u(\hat{t}; \tau_j)$ . Let us modify that payoff specification by considering that parties have mixed motives and also care for holding office. Formally we define party's  $j$  payoffs as  $\pi_j = \alpha S_j - (1 - \alpha)|\tau_j - \hat{t}|$  where  $\alpha \in [0, 1)$  denotes the importance parties attach to the appropriation of office rents.<sup>13</sup> We do not allow parties to be purely office motivated (that is  $\alpha = 1$ ) since this would always yield the standard Downsian prediction of full convergence to the median no matter the electoral disproportionality (Downs, 1957). When parties are both office and policy motivated the following result holds:

**Proposition 3.** *Let  $\tau_i \sim U[0, 1]$ . Then: (i) There exists a unique equilibrium  $(t_L^\alpha, t_R^\alpha) = (\min\{\frac{n-1+\alpha}{2n(1-\alpha)}, \frac{1}{2}\}, \max\{\frac{1}{2}, \frac{n+1-\alpha(2n+1)}{2n(1-\alpha)}\})$  (ii) The distance between  $t_R^\alpha$  and  $t_L^\alpha$  is decreasing in  $n$  (iii) The distance between  $t_R^\alpha$  and  $t_L^\alpha$  is decreasing in  $\alpha$  (iv)  $\hat{t} = \bar{\tau} = 0.5$ .*

As before in the unique equilibrium parties' proposed platforms are symmetric around the median. If parties are purely policy motivated ( $\alpha = 0$ ) the result replicates the one of proposition 1. If parties have mixed motives and are partially interested in extracting office rents then office rents act as a centripetal force. Proportional electoral systems on the contrary still provide centrifugal incentives. If office motives are not strong enough then parties in equilibrium propose diverged platforms that depend on the proportionality of the electoral system. If office motives are strong enough such that they compensate the centrifugal incentives provided by the electoral system then both parties propose the same platform ( $t_L^\alpha = t_R^\alpha = 0.5$  if  $\alpha > 1/(n + 1)$ ).

Notice that despite the presence of office motives proportional electoral systems still provides the aforementioned centrifugal incentives. As  $n$  decreases parties differentiate

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<sup>13</sup>Notice that we implicitly assume that the value for holding office is one. Moreover, while in our general results we used a general utility function  $u(\hat{t}; \tau_j)$  here we assume that parties have a linear utility function when they value the implemented policy. Different utility specifications (for example a quadratic utility function) or different positive values for office would pronounce one of the motives versus the other. Intuition would remain the same no matter our assumptions. Office motives act as a centripetal force while proportionality remains an active centrifugal force.

more. The centrifugal incentives provided by the electoral system are however less intense than in proposition 1 since the presence of office motives always acts as a balancing centripetal force (that is  $t_L^\alpha \geq t_L^{**}$  and  $t_R^\alpha \leq t_R^{**}$ ).

### 3.5 Non-extreme parties

The main result of our paper as presented in proposition 2 incorporates a large degree of generality since it does not impose any restrictions on the distribution of voters' ideal points. As we have shown, no matter how asymmetric the society is and whether one party is favored compared to its opponent, both parties have incentives to converge towards the median as the disproportionality of the electoral system increases. This general result can be generalized regarding parties' ideal policies.

It can be shown that our general result (Proposition 2) holds in the case of non-extreme parties ( $0 < \tau_L < m < \tau_R < 1$ ) and voters that are uniformly distributed. For more general distributions some further structure is required. In particular, if we define party's  $L$  weight on the implemented policy as a function  $S_J$  where  $S_J(x, n) = \frac{F(x)^n}{F(x)^n + (1-F(x))^n}$  and assume that  $S_J$  is log-concave in  $x$  then if for some  $n_0$  we have that  $0 < t_L^*(n_0) < t_R^*(n_0) < 1$  then for every  $n_1$  and  $n_2$  such that  $n_0 < n_1 < n_2$  we can show that there exists a unique equilibrium where  $|t_L^*(n_1) - t_R^*(n_1)| > |t_L^*(n_2) - t_R^*(n_2)|$ . Hence we can show that the more disproportional the electoral system is the less polarized the equilibrium.<sup>14</sup>

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<sup>14</sup>Notice that the assumption of log-concavity is usually imposed on the distribution of voters' ideal points  $F$  (for example in Llavador 2006; Ortuño-Ortín 1997). However, here we require that the weight function  $S_J$  is log-concave. Intuitively this means that as a party proposes a more moderate platform then its seat-share and hence its weight in the implemented policy increases at a decreasing rate. In order to relate the log-concavity of  $S_J$  with the primitives of the model it can be shown that  $S_J$  is log-concave for a large family of  $F$ 's. A general example guaranteeing the log-concavity of  $S_J$  is when voters are distributed according to any unimodal beta distribution (that is  $\tau_i \sim Beta(\alpha, \beta)$  with  $\alpha \geq 1, \beta \geq 1$ ).

### 3.6 The Three Parties Case

We now consider the case of a three-party electoral race (a leftist, a centrist and a rightist party).<sup>15</sup> It is straightforward that the complexity of the analysis increases several orders in magnitude when we increase the cardinality of the set of players from two to three. For example, the game is no longer strictly competitive as when only two-parties compete and, hence, the equilibrium characterization cannot follow from a standard combination of the popular properties that strictly competitive games have (see proof of Proposition 2). Hence, to guarantee tractability some further assumptions are in order. We consider that a) the ideal policy of the *centrist* party is at one-half (that is  $\tau_C = 0.5$ ) and the leftist and the rightist party are modeled as in the two-party scenario analyzed above ( $\tau_L = 0$  and  $\tau_R = 1$ ) and b) an equilibrium in this case is a pure-strategy Nash equilibrium such that the distribution of policy proposals is symmetric about the center of the policy space. Since now three parties compete in the election each party's seat-share is given by the following expression (Theil, 1969; Taagepera, 1986):

$$S_J = \frac{V_J^n}{V_L^n + V_C^n + V_R^n}$$

and therefore the implemented policy function for the three-party model is accordingly defined as:

$$\begin{aligned} \hat{t}(t_L, t_C, t_R, n) &= \frac{V_L(t_L, t_R)^n}{V_L(t_L, t_R)^n + V_C(t_L, t_R)^n + V_R(t_L, t_R)^n} * t_L \\ &+ \frac{V_C(t_L, t_R)^n}{V_L(t_L, t_R)^n + V_C(t_L, t_R)^n + V_R(t_L, t_R)^n} * t_C \\ &+ \frac{V_R(t_L, t_R)^n}{V_L(t_L, t_R)^n + V_C(t_L, t_R)^n + V_R(t_L, t_R)^n} * t_R \end{aligned}$$

The following result holds:<sup>16</sup>

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<sup>15</sup>The result trivially extends to  $k$  parties where two parties have ideal policies at the two extremes (that is at 0 and 1) and  $k - 2$  parties have an ideal policy at 0.5.

<sup>16</sup>Without providing a formal definition of voters' strategies it is necessary to mention that each voter  $i$  supports party  $j$  that proposes the closest platform to his ideal policy. If a voter is indifferent between two or even three platforms then he randomizes his vote.



**Proposition 4.** *Let  $\tau_i \sim U[0, 1]$ . Then: (i) There exists a unique equilibrium  $(t_L^{***}, t_C^{***}, t_R^{***}) = (\frac{n-1}{2(n+1)}, 0.5, \frac{n+3}{2(n+1)})$  (ii) The distance between  $t_R^{***}$  and  $t_L^{***}$  is decreasing in  $n$  (iii)  $\hat{t} = \bar{\tau} = 0.5$ .*

As we observe the centrist party proposes a platform equal to its ideal policy ( $t_C^{***} = \tau_C = 0.5$ ). The two other parties differentiate and propose more extreme platforms. The extent to what parties differentiate depends on the level of disproportionality (see figure 3 depicting the proposed platforms  $(t_L^{***}, t_R^{***})$  for different values of electoral disproportionality  $n \in [1, 20]$ ). The more proportional the electoral system is, the higher are the centrifugal forces, and hence polarization increases. As it was the case for our general result and two-party competition electoral proportionality acts as a centrifugal force. Clearly, if parties compete in a winner-take-all election ( $n \rightarrow \infty$ ) then the standard result of convergence to the median applies (that is  $(t_L^{***}, t_C^{***}, t_R^{***}) = (0.5, 0.5, 0.5)$ ). Note that the qualitative implications of the above proposition directly extend to a general class of distributions (at least for the class of log-concave distributions) and are not restricted to the uniform case.

**Corollary 1.** *Polarization in a three-party election is larger than in a two-party election ( $t_R^{***} - t_L^{***} \geq t_R^{**} - t_L^{**}$  with the equality holding for  $n = 1$ )*

Notice that the centrifugal force identified in the proportionality of the electoral system is now amplified as the number of competing parties increases. Comparing the proposed platforms by the leftist and rightist party we observe that in the unique equilibrium of the three-party election they polarize more than in the two-party election (for any  $n > 1$  it holds that  $t_L^{***} < t_L^{**}$  and that  $t_R^{***} > t_R^{**}$ ). The presence of a third party makes competition for centrist voters tougher and hence parties have less incentives to moderate their policies in the hunt of more votes. This comparative result can be visualized in Figure 3 where for every value  $n$  polarization is higher three-party than two-party competition ( $t_R^{***} - t_L^{***} \geq t_R^{**} - t_L^{**}$ ).

Finally, note that inclusion of minor-to-moderate office-holding concerns in this three-

parties version of the model has the same effect on the equilibrium level of platform polarization (understood as the distance between the two most distant platforms) as in the two-parties case presented above; the more office-motivated parties are the smaller the degree of platform polarization.<sup>17</sup>

## 4 Endogenous Number of Parties

Given the importance of the effect of electoral systems on the number of competing parties dating back to Duverger's Law (FPTP lead to a two-party system) and Duverger's Hypothesis (PR lead to multi-party systems) we now discuss an endogenous entry decision by political parties.

Consider again the three parties ( $L, C$  and  $R$ ) with ideal policies  $(\tau_L, \tau_C, \tau_R) = (0, 0.5, 1)$ . In order to incorporate parties' entry decision we consider the following two stage game: In the first stage parties decide whether to enter the electoral race or not. In the second stage each competing party observes which other parties entered the race and strategically selects its political platform  $t_j \in [0, 1]$ . Payoffs are subsequently realized.

We consider that parties have lexicographic preferences.<sup>18</sup> Remaining in the spirit of policy motivated parties their first priority is to minimize the distance between the implemented policy and their ideal point. Formally they aim at maximizing  $\pi_j(\hat{t}) = -|\hat{t} - \tau_j|$ . If multiple strategies offer them the same maximal value of  $\pi_j(\hat{t})$  then they refine their choice by their second priority. Their second priority is related to their office motives and they aim at maximizing  $\psi_j = S_j - c$  where  $c = \hat{c} > 0$  if they enter the race and  $c = 0$  if they do not. In order to guarantee that the entry cost is not the determinant

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<sup>17</sup>Unfortunately, one cannot derive a closed form solution of such equilibrium. Computational results though which validate all these claims may be provided by the corresponding author upon request.

<sup>18</sup>Let us stress that lexicographic preferences do not determine the qualitative characteristics of our results. We could have instead assumed that parties have mixed motives, as described in a previous section, with sufficiently low weight on the office concerns dimension. Given the analytical complexities of such framework (see previous footnote) we can only derive computational results for this (more general) case that are available by the corresponding author upon request. Nevertheless, these computational results are in line with the formal results that we obtain in the lexicographic preferences setup presented here.

of the qualitative features of the equilibrium we assume that  $\hat{c} < 0.25$ .

Regarding the implemented policy we assume that a) if only party  $j$  enters then its platform is implemented ( $\hat{t}(t_j) = t_j$ ), b) if two parties enter then the implemented policy is determined as in the benchmark two-party case and c) if all three parties enter then the implemented policy is determined as in the three-party case presented in the previous section. For simplicity we assume that if no party enters a status quo policy  $q \in [0, 1]$  is implemented and that this status quo policy is known to all parties (this assumption can be relaxed). We can now state the main proposition regarding parties' equilibrium entry and platform decisions.

**Proposition 5.** *If  $\tau_i \sim U[0, 1]$  then there exists a unique subgame perfect equilibrium and a unique  $\hat{n} > 0$  such that: (i) for  $n < \hat{n}$  all three parties enter and their platform choices are  $(t_L^{***}, t_C^{***}, t_R^{***}) = (\frac{n-1}{2(n+1)}, 0.5, \frac{n+3}{2(n+1)})$  (ii) for  $n > \hat{n}$  only parties  $L$  and  $R$  enter and their platform choices are  $(t_L^{**}, t_R^{**}) = (\frac{n-1}{2n}, \frac{n+1}{2n})$ .*

The above result is depicted in figure 4. For a given entry cost if the electoral system is sufficiently proportional (low values of  $n$ ) all three parties have incentives to pay the entry cost and compete in the election. In the case of three-party competition the centrist party sticks to its ideal policy ( $t_C^{***} = 0.5$ ) while the two other parties announce platforms that diverge symmetrically around that point. Since three parties have entered the race their policy choices are identical to the ones described in proposition 4 and the policy implemented coincides with the one of the median ( $\hat{t} = 0.5$ ).

As the electoral system becomes more disproportional the two extreme parties converge towards the center and suppress the seat-share of the centrist party. From a point on (when  $n > \hat{n}$ ) the seat-share of the centrist party is suppressed sufficiently such that the centrist party is better off by not paying the entry cost. Notice that no matter whether two or three parties compete the implemented policy coincides with the ideal point of the centrist party and hence the latter competes only to maximize its office benefits (its second priority). Hence for sufficiently disproportional electoral systems only two parties remain active in the political arena.

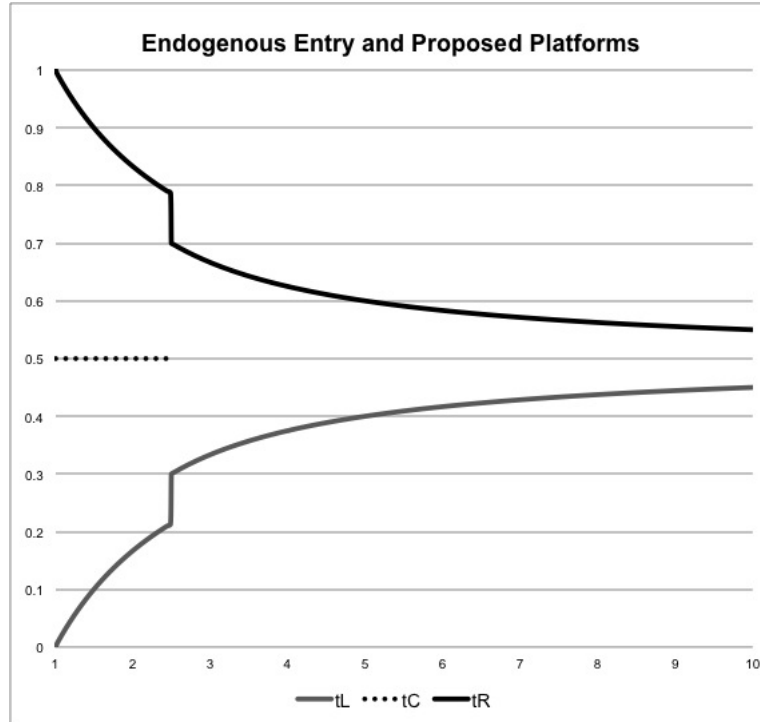


Figure 4: The effect of electoral disproportionality  $n \in [1, 10]$  on the number of competing parties and proposed platforms as characterized in proposition 5. In the graph we depict the case of  $c = 0.22$  which implies that  $\hat{n} = 2.5$ .

As clearly depicted in the graph notice that as the disproportionality increases  $t_L$  and  $t_R$  converge smoothly towards the center up to the point when  $n = \hat{n}$ . This is the direct effect of electoral disproportionality on polarization described in Proposition 4. Once disproportionality is sufficiently high ( $n = \hat{n}$ ) we observe an indirect effect of disproportionality on polarization that is attributed to the the depicted jump provoked by the decision of the centrist party not to enter the political race. Hence we observe that the direct effect is further amplified by the indirect effect and the fact that disproportionality affects the number of competing parties.

## 5 Empirical Analysis

We formulate the two following institutional hypotheses in order to test the main theoretical predictions of our model: Disproportionality has a direct effect on platform polarization and an indirect one through its effect on the number of competing parties.

**(H1)** *Electoral System Hypothesis (Propositions 1 and 2): Platform polarization (distance between parties's platforms) is decreasing in the disproportionality of the electoral rule ( $n$ ).*

**(H2)** *The Number of Parties Hypothesis (Corollary 1 to Proposition 4): Platform polarization is increasing in the number of parties participating in the election.*

Both hypotheses have been explored by a number of related studies in the past yielding mixed empirical findings. While most approaches fail to garner enough support for **H1** (e.g., Budge and McDonald 2006; Ezrow 2008; Dalton 2008; Curini and Hino 2012) few of them provide some (conditional) evidence in favor of either **H1** (e.g., Calvo and Hellwig 2011; Dow 2011) or **H2** (Andrews and Money, 2009).<sup>19</sup> Let us mention that most existing studies utilized small and unbalanced data sets (see Table A.1) and were therefore restricted to only exploit cross-country variation thus making level comparisons across countries. On top of it, by including very few observations for many of these countries (in some instances only one) they are, in fact, estimating those level effects using just a single observation for each country. That is, they just provide a snapshot of polarization at a given point in time (which incidentally is not the same across different countries) and compare it. This additional limitation does not allow to disentangle between variation in polarization that is related to electoral rules and variation that is due to other sources (e.g. time or country specific shocks). Therefore, by considering an enlarged and balanced sample, that includes at least ten observations for each country, our work is an improvement on both fronts: not only we introduce some within country variation in the electoral rule disproportionality (though admittedly limited) but we also improve significantly the cross-country comparison, thus obtaining a more accurate picture of the

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<sup>19</sup>For a complete comparative presentation of the empirical literature, hypotheses tested and their findings see Table A1 in the Appendix. Curini and Hino (2012) find support for the voters' hypotheses and two additional institutional hypotheses: the *cabinet-parties conditional hypothesis* and the *electoral spill-over hypothesis*. Including the relevant variables in the institutional controls we do not find a significant effect of *spill-overs* while some specifications provide partial support regarding the *cabinet-parties conditional hypothesis*. In addition to their full model, Curini and Hino (2012) also test a simpler version that includes only H1 and H2 and which is directly comparable to our model in columns 1 and 2 in Table A4. The fact that we find a statistically significant effect of disproportionality on polarization while they do not cannot therefore be attributed to the exclusion of the hypotheses mentioned above.

effects of interest instead of a snapshot.

## 5.1 Data Description and Measurement

We construct a balanced panel that combines electoral, political, institutional, socio-economic, and demographic data for more than 300 elections from 23 OECD countries during the period from 1960 to 2006 (on average 13 elections in each country). We describe our data and the main variables thereafter and provide summary statistics in Table A2 of the Appendix.

### The Dependent Variable: Platform Polarization

Our dependent variable, platform polarization, is constructed using data from the Volkens et al. (2012) Comparative Manifesto Project (CMP) dataset compiled by the Wissenschaftszentrum Berlin für Sozialforschung (WZB). The latter records the ideological position of the platforms proposed by hundreds of political parties since 1946 (in line with our theoretical model we consider a unidimensional policy space in a 0 – 10 scale where zero stands for extreme left and ten for extreme right).<sup>20</sup> In order to maintain consistency with recent literature we follow Dalton (2008) and formally define the index of platform polarization ( $IP_i$ ) in election  $i$  as:<sup>21</sup>

$$IP_i = \sqrt{\sum_j V_j \left( \frac{P_{ji} - \bar{P}_i}{5} \right)^2}$$

where  $\bar{P}_i$  denotes the weighted mean of parties' position on the left-right dimension (each party  $j$  is weighted by its vote-share  $V_j$ ) and  $P_{ji}$  is the ideological position of party  $j$  in

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<sup>20</sup>Technically, the CMP provides parties' positions on a –100 to 100 scale. We perform an affine, monotonic, order preserving transformation of the index by adding 100 to each party's position then, dividing the sum by 20. Other studies that use the CMP to measure platform polarization are: Budge and McDonald (2006); Ezrow (2008); Andrews and Money (2009).

<sup>21</sup>Curini and Hino (2012) also use the Dalton index while Ezrow (2008); Dow (2011) use a very close analogue that incorporates all parties' positions weighted by their vote-shares. In Table A3 of the Appendix we replicate our estimates using another measure of polarization (as in Budge and McDonald 2006; Andrews and Money 2009): the distance between the two most extreme parties. The use of such index not only serves as a robustness check but further allows for an one-to-one correspondence between our theoretical predictions and the empirical estimation. Our results are robust to different measures of polarization.

election  $i$  (0-10 scale), normalized by the mid-point ideology position (5).

The index takes value zero when all parties converge to a single position and ten when parties are equally split between the two extreme positions. Two important properties of this index is that it is relatively independent of the number of competing parties and that weighting for the electoral size of each party implies that a large party at one of the extremes implies greater polarization than a marginal party in the same position (Curini and Hino, 2012).

### **The Main Explanatory Variables: Electoral Rule Disproportionality and the Number of Competing Parties**

Our key explanatory variable is the measure of electoral rule (dis)proportionality. By combining data from two different sources (Armingeon et al. 2012 (CPDS I) and the Carey 2012 archive) we use two alternative measures, one continuous and one binary, that provide qualitatively identical results. In models sub-indexed with an  $a$  we consider a binary variable that takes value zero whenever the *FPTP* rule with single-member districts applies and one otherwise. In models sub-indexed with  $b$  we control for the mean magnitude of electoral districts: larger magnitude reduces the effective threshold required for a party to occupy a parliamentary seat, hence, making the electoral system more proportional (Taagepera, 1986).

The use of the two alternative measures increases the robustness of our findings and allows us to address any concerns related to limited within country variation of our independent variable.<sup>22</sup> On one hand, the binary variable records only a radical change from a *FPTP* rule to a more proportional one and vice versa. Since such radical changes are not frequent this is a slow-moving institutional variable.<sup>23</sup> On the other hand, the continuous version of our independent variable is more sensitive to changes in the electoral rule and allows for some within country variation. Both specifications provide results that

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<sup>22</sup>Incidentally, limited within country variation can also ease to some extent, albeit not completely eradicating, any potential concerns over endogeneity and reverse causality.

<sup>23</sup>In most countries in the sample such radical change occurred at most once (save for Greece, Italy and New Zealand) while in some countries no such change ever occurred (e.g. USA, UK, and the Netherlands).

are equally robust, statistically significant and identical in the direction of the effect.

We test the *number of competing parties hypothesis* (**H2**) by using two different measures of the explanatory variable. The first one is the Effective Number of Parties (*ENP*) index.<sup>24</sup> The second is the actual number of parties participating in the election. Since our findings in all specifications do not vary significantly when we substitute the *ENP* with the actual number of parties we present all our results controlling for the *ENP* in order to guarantee consistency with the literature. The results when we control for the actual number of parties are presented only for Model 1 (See Table 1).<sup>25</sup>

## 5.2 Empirical Estimation

Model 1 jointly tests our two primary institutional hypotheses: the impact of the electoral rule (dis)proportionality (**H1**) and the number of competing parties (**H2**) in determining polarization. Since empirical evidence (e.g. Gallagher 1991) and theoretical literature (e.g. Duverger 1954) suggest that electoral rules may also affect polarization through the structure of the party-system (e.g. number of parties) we test our first two hypotheses jointly (e.g. Cox 1990) in order to prevent a biased estimation.<sup>26</sup> Model 1 serves as our benchmark since most of the literature tests these two hypotheses (e.g. Dalton 2008; Ezrow 2008; Andrews and Money 2009; Dow 2011; Calvo and Hellwig 2011). Formally, we estimate the following model:

$$IP_{it} = \alpha_0 + \beta_1 (PROPORTIONALITY)_{it} + \beta_2 (NUMBEROFPARTIES)_{it} + \lambda_t + \gamma_i + \epsilon_{it} \quad (1)$$

According to **H1**, we expect  $\beta_1$  to be positive as more proportional rules should lead to more polarization. From **H2**, we also expect  $\beta_2$  to be positive. To fully exploit the advantages of a large balanced panel we introduce country ( $\gamma_i$ ) and time ( $\lambda_t$ ) dummies

<sup>24</sup>Laakso and Taagepera (1979) define the *effective number of political parties* as  $1/\sum_j (V_j)^2$ .

<sup>25</sup>Similar to Andrews and Money (2009) we also control for the Log ENP (see Table A3 in the Appendix).

<sup>26</sup>Also note that our result in Proposition 5 suggests a close link between the proportionality of the electoral rule and the number of competing parties.



(fixed effects).

In Model 2 we control for a set of other relevant institutional variables: coalition habits dummy, number of parties participating in government/cabinet and their interaction, type of political regime (presidentialism vs. parliamentarianism), degree of institutional constraints, years of consolidated democracy, a dummy variable indicating government change, the ideological distance between incumbent and past government. Model 3 further controls for some socioeconomic variables: unemployment rate, GDP growth rate, government spending (as % of GDP), Gini coefficient of inequality. Finally, in tables A.3 and A.4 we estimate Models 4, 5 and 6 that incorporate some additional robustness checks using alternative measures of our dependent and independent variables as well as a model with random effects (Model 6).

## Results

Table 1 contains the combined results of the econometric estimations that provide strong support for our main hypothesis (**H1**) at any conventional level of significance and irrespective of how the (dis)proportionality of the electoral system is measured. In all specifications, we find that more proportional electoral rules are associated with increased levels of polarization. This finding is robust to various alterations of the model, including different ways of measuring both the dependent and independent variables (as presented in the Appendix) and the inclusion of more control variables (Models 2 and 3). Hence, our empirical analysis verifies the main theoretical prediction of the model on the effect of electoral rule (dis)proportionality on polarization.

This result contrasts with existing literature where electoral institutions have no impact on polarization (e.g. Dow 2001; Budge and McDonald 2006; Dalton 2008; Ezrow 2008; Andrews and Money 2009; Curini and Hino 2012) and reinforces recent work by Dow (2011) and Calvo and Hellwig (2011) who find some conditional support for the Electoral System (**H1**) hypothesis. According to our results not only is the coefficient  $\beta_1$  positive (as predicted) and statistically significant, but it is also large in magnitude. Our estimates can associate a switch from a FPTP to a PR rule (electoral system becomes

**Table 1: The Impact of Electoral Rule Disproportionality on Platform Polarization in OECD Democracies (from 1960-2007)**

Dependent Variable		Dalton Index of Party-System (Platform) Polarization											
		Model 1.a		Model 1.b		Model 2.a		Model 2.b		Model 3.a		Model 3.b	
Explanatory Variables		PR rule		District Magnitude		PR rule		District Magnitude		PR rule		District Magnitude	
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
<b>H1</b>	Electoral Rule Dummy (PR = 1)	1.659 (0.181)***	1.743 (0.193)***	--	--	1.326 (0.268)***	--	1.263 (0.283)***	--				
<b>H1</b>	Log Avg. Electoral District Magnitude	--	--	0.264 (0.071)***	0.280 (0.074)***	--	0.195 (0.094)**	--	0.222 (0.068)***				
<b>H2</b>	Effective Number of Parties (ENP)	0.009 (0.064)	--	0.033 (0.079)	--	-0.012 (0.151)	0.002 (0.154)	-0.136 (0.181)	-0.113 (0.181)				
<b>H2</b>	Actual Number of Parties	--	-0.040 (0.041)	--	-0.033 (0.046)	--	--	--	--				
	Country Dummies?	✓	✓	✓	✓	✓	✓	✓	✓				
	Year Dummies?	✓	✓	✓	✓	✓	✓	✓	✓				
	Other Institutional Controls?	No	No	No	No	Yes	Yes	Yes	Yes				
	Economic Controls?	No	No	No	No	No	No	Yes	Yes				
	R <sup>2</sup>	0.39	0.39	0.41	0.41	0.43	0.41	0.63	0.62				
	N	307	307	255	255	237	237	123	123				

\*  $p < 0.10$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$

Robust standard errors clustered at the country level reported in parentheses. Country and year dummies (fixed effects) are included in all specifications. Other institutional controls include: a (dummy) variable indicating strong coalition habits and its interaction with ENP, the number of parties participating in government/cabinet, the type of political regime (presidentialism/parliamentarism), the degree of institutional constraints (a categorical variable taking values from 0 - 6), years of consolidated democracy, a (dummy) variable indicating government change and the ideological distance between incumbent and past government. Economic controls include: unemployment rate (in %), GDP growth rate (in %), government spending (as % of GDP), Gini coefficient of inequality. Model 3 has less observations due to missing economic data from 1960-1980. Models 1.b and 2.b also have some missing values for the average electoral district size. In all models the dependent variable (polarization) is constructed as in Dalton (2008).

more proportional) with a two standard deviations increase in polarization. In other words, a one-standard-deviation increase in the average district magnitude is associated with a half standard deviation increase in polarization.

On the other hand, the number of parties hypothesis (**H2**) is confirmed under few specifications since the number of political parties does not appear to have a consistent significant effect on polarization regardless of the measurement technique (neither the ENP nor the actual number of parties).<sup>27</sup> One plausible explanation, in line with our theoretical predictions (Proposition 5), is that any potential effect that the number of parties may have on polarization operates via the electoral system itself (e.g., Duverger 1954). To better see this point, we have estimated the correlation between the number of parties (actual and effective) and the electoral rule (dis)proportionality (using both the binary and the continuous variable). In all cases the correlation coefficient is of the

<sup>27</sup>The models where **H2** is validated are presented in Table A3 of the Appendix is Model 4 where for robustness we control for the log of ENP following Andrews and Money (2009) and in Table A4 in two of the specifications (columns 2 and 8) where where we use the random effects estimator.

magnitude of 0.50 and is also statistically significant at any conventional level.

Overall, our empirical results yield robust support for the main prediction of the theoretical model (**H1**) on the effect of electoral rule (dis)proportionality on platform convergence. The combination of our theoretical and empirical findings suggest that far from being “the dog that did not bark” (Ezrow 2008) the (dis)proportionality of the electoral rule is the most important institutional determinant of platform polarization.

## Discussion

Our empirical analysis complements previous literature (summarized in Table A1 in the Appendix) in two main aspects. First, by combining different sources of data (CPDS I, CMP and the Carey archive) we construct a balanced panel of 23 advanced democracies (OECD states) with a large number of electoral observations (more than 300) over a 50-year-long time frame. Second, our study simultaneously considers: a) both continuous and categorical measures of the electoral rule (dis)proportionality; b) two different measures of platform polarization (one that uses information on vote-share distribution and one that does not); c) alternative measures for the number of competing parties; and d) both country and year fixed effects.

Our analysis addresses a common concern associated with studies that, given data limitations, use unbalanced panels that often contain few observations for a large number of countries and are forced to group together a set of diverse countries that share different and divergent institutional history and traditions (e.g., Curini and Hino 2012; Dow 2011).<sup>28</sup> Such unbalanced panels may result not only to observable differences in electoral institutions but also to important unobservable, or difficult to measure at best, differences among various other institutional features.

Regarding our model specification itself, while country and year dummies (fixed effects) help us account both for idiosyncratic country-specific characteristics (country dum-

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<sup>28</sup>Examples of studies with a limited set of observations are the ones using the CSES mass surveys (e.g., Dalton 2008; Dow 2011; Curini and Hino 2012) and also studies by Budge and McDonald (2006); Ezrow (2008) despite the use of the CMP data because they aggregate their data across countries or they have a limited time frame.

mies) and also for year specific effects, their inclusion is not the reason why our results differ from previous findings. This can be seen in Table A.4 where we compare the fixed effects model estimates with those of a model that omits country and year dummies (as for example in Curini and Hino 2012) and a random effects one.<sup>29</sup> While different in magnitude, the estimates regarding  $H1$  obtained under all three specifications are qualitatively identical, equally statistically significant, and on the same direction. Therefore, we attribute this significant difference with previous findings to our better data.

## 6 Concluding Remarks

Our work has implications on the design of electoral institutions as it surfaces an interesting trade-off between the need for more democratic pluralism and wider political representation (served by more proportional rules) and political stability and moderation (served more by majoritarian rules). Hence, the choice of one class of rules over the other is not as straightforward as one might think and a lot seems to depend on individual party-system characteristics and the attributes of each polity. Moreover, assumptions regarding voters' behavior, other than the ones explored in this paper, should be carefully explored in future studies. Possibility of abstention and of strategic voting are just two out of the many examples of such alternative behavioral elements that could be incorporated in the analysis.

As far as abstention is concerned, a simple extension of the model suggests that our results are robust to allowing voters to abstain. If for instance one considers that the society is normally distributed around the median and that alienated voters whose ideal

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<sup>29</sup>Since we have argued that we have limited within country variation on our binary electoral rule variable (in some countries there is at most one change while in others none) one might question whether the inclusion of country fixed effects in our model is appropriate. The reason is that under such specification the fixed effects estimated coefficient on the electoral rule disproportionality is in fact driven only by those few countries that had frequent “radical” changes in the electoral rule and therefore a random effects estimator may seem more appropriate. Since the results we obtain using a random effects estimator are qualitatively identical, albeit smaller in magnitude, equally statistically significant and in the same direction with both the fixed effects and the simple model estimates, we believe that there is no further reason to worry. In fact the use of both fixed and random effects allows us to frame the real impact of electoral rule disproportionality by estimating upper (FE) and lower (RE) bounds for the effect.

policies are “sufficiently” away from the parties’ platforms do not vote for the party that proposes a platform closer to their ideal policies but rather abstain then an increase in the electoral disproportionality would still reduce the level of polarization.

Finally, in our analysis voters are expressive and support the party that proposes the platform closest to their ideal point. But what if some voters behaves strategically? Intuition suggests that the presence of strategic voters provides further centrifugal incentives to political parties (Llavador, 2006; De Sinopoli and Iannantuoni, 2007). As before, a simple extension of the model shows that our main result would persist. That is, the degree of polarization would be higher when compared to the case of a completely non-strategic electorate but the direction of the effect of the degree of electoral rule disproportionality on the level of platform polarization would not be affected. But what if the electoral system itself endogenously determines the share of voters that behave strategically? To gain a complete picture of how disproportionality affects polarization such alternative channels, through which these two variables could possibly interact, should be further investigated.

## 7 Appendix

### 7.1 Proofs

*Proposition 1.* Follows proposition 2. □

*Proposition 2.* We prove our general result in four steps.

**Step 1 (Player  $j$  has a unique best response for every  $t_{-j}$ )** We notice that for any  $t_R \in [0, 1]$  we have that  $\hat{t}(t_L, t_R, n) = t_R$  whenever  $t_L = t_R$  and that  $\hat{t}(t_L, t_R, n) > t_R$  whenever  $t_L > t_R$ . Since, the utility of  $L$ ,  $u(\hat{t}, 0)$ , is strictly decreasing in  $\hat{t}$  it follows that the best response of party  $L$  to  $R$  playing  $t_R$ ,  $b_L(t_R)$ , can never be such that  $b_L(t_R) > t_R$ . In specific, if  $t_R = 0$  this implies that  $b_L(t_R) = t_R = 0$ . When  $t_R > 0$  we observe that  $\hat{t}(t_L, t_R, n) = t_R$  whenever  $t_L = t_R$  and that  $\hat{t}(t_L, t_R, n) < t_R$  whenever  $t_L = 0$ . Since, the utility of  $L$ ,  $u(\hat{t}, 0)$ , is strictly decreasing in  $\hat{t}$  it follows that the best response of party  $L$  to  $R$  playing  $t_R$ ,  $b_L(t_R)$ , can never be such that  $b_L(t_R) \geq t_R$  whenever  $t_R > 0$ .

Moreover, we know that for  $t_L < t_R$ :

$$\begin{aligned} \hat{t}(t_L, t_R, n) &= \frac{V_L(t_L, t_R)^n}{V_L(t_L, t_R)^n + V_R(t_L, t_R)^n} \times t_L + \frac{V_R(t_L, t_R)^n}{V_L(t_L, t_R)^n + V_R(t_L, t_R)^n} \times t_R = \\ &= \frac{F(\frac{t_L + t_R}{2})^n}{F(\frac{t_L + t_R}{2})^n + [1 - F(\frac{t_L + t_R}{2})]^n} \times t_L + \frac{[1 - F(\frac{t_L + t_R}{2})]^n}{F(\frac{t_L + t_R}{2})^n + [1 - F(\frac{t_L + t_R}{2})]^n} \times t_R. \end{aligned}$$

Observe that:

$$\frac{\partial u(\hat{t}, 0)}{\partial t_L} = \frac{\partial u(\hat{t}, 0)}{\partial \hat{t}} \times \frac{\partial \hat{t}(t_L, t_R, n)}{\partial t_L}.$$

Since we always have that  $\frac{\partial u(\hat{t}, 0)}{\partial \hat{t}} < 0$  it must be the case that  $\text{sign}[\frac{\partial u(\hat{t}, 0)}{\partial t_L}] = -\text{sign}[\frac{\partial \hat{t}(t_L, t_R, n)}{\partial t_L}]$ .

Now, notice that:

$$\frac{F(\frac{t_L + t_R}{2})^n}{F(\frac{t_L + t_R}{2})^n + [1 - F(\frac{t_L + t_R}{2})]^n} \times t_L$$

is strictly increasing in  $t_L \in [0, t_R]$  and that:

$$\frac{[1 - F(\frac{t_L + t_R}{2})]^n}{F(\frac{t_L + t_R}{2})^n + [1 - F(\frac{t_L + t_R}{2})]^n} \times t_R$$

is strictly decreasing in  $t_L \in [0, t_R]$ .

That is,  $\hat{t}(t_L, t_R, n)$  is strictly quasi-convex in  $t_L \in [0, t_R]$  and therefore  $u(\hat{t}, 0)$  is strictly quasi-concave in  $t_L \in [0, t_R]$ .

All the above suggest that for any  $t_R \in [0, 1]$  there exist a unique  $b_L(t_R)$ . Due to the symmetry of our game it is straightforward that  $b_R(t_L)$  is also single-valued.

**Step 2 (Existence of an equilibrium)** Following the arguments of Ortuño-Ortín (1997) and given the single-valuedness of  $b_L(t_R)$  and  $b_R(t_L)$  we can define  $B : [0, 1]^2 \rightarrow [0, 1]^2$  as  $B(t_L, t_R) = (b_R(t_L), b_L(t_R))$ . Notice that due to the continuity of  $F$  and  $u$  and Berge's maximum theorem it directly follows that  $B$  is continuous. That is, Brouwer's fixed point theorem applies in our case and guarantees existence of an equilibrium point.

**Step 3 (Uniqueness of the equilibrium)** Notice that  $\text{sign}[\frac{\partial u(\hat{t}, 0)}{\partial \hat{t}}] = -\text{sign}[\frac{\partial u(\hat{t}, 1)}{\partial \hat{t}}]$ . That is, our game is strictly competitive (Aumann, 1961; Friedman, 1983) and has the same properties as a zero-sum game. Assume that the game admits two equilibria  $(t_L^*, t_R^*)$  and  $(\hat{t}_L^*, \hat{t}_R^*)$  such that either  $t_L^* \neq \hat{t}_L^*$  or  $t_R^* \neq \hat{t}_R^*$  or both. Assume without loss of generality that at least  $t_L^* \neq \hat{t}_L^*$  holds. Since our game is strictly competitive it directly follows that  $(t_L^*, \hat{t}_R^*)$  and  $(\hat{t}_L^*, t_R^*)$  are also equilibria of the game. This implies  $b_L(t_R^*) = t_L^*$  and that  $b_L(\hat{t}_R^*) = \hat{t}_L^*$ . Since  $t_L^* \neq \hat{t}_L^*$  this contradicts the fact that  $b_L(t_R^*)$  is always single-valued. Therefore, the game must admit a unique equilibrium point.

**Step 4 (Characterization of the equilibrium)** Notice that  $t_L^* < t_R^*$  because in any other case at least one of the two parties is strictly better off by switching to her ideal policy. If  $0 < t_L^* < t_R^* < 1$  (interior equilibrium) it must be the case that  $\frac{\partial u(\hat{t}, 0)}{\partial \hat{t}_L^*} = \frac{\partial u(\hat{t}, 0)}{\partial \hat{t}_R^*} = 0$ . Algebraic manipulations of these two equilibrium conditions lead to a)  $F(\frac{t_L^* + t_R^*}{2}) = \frac{1}{2}$  and therefore to  $\frac{t_L^* + t_R^*}{2} = m$  and to b)  $\frac{1}{nf(m)} = t_R^* - t_L^*$ ; the degree of platform polarization is decreasing in  $n$ . Notice that when  $n \rightarrow \infty$  it holds that  $t_R^* \rightarrow t_L^*$  and when  $m = 0.5$  then  $t_R^* + t_L^* = 1$  and hence  $t_L^* = 1 - t_R^*$ .  $\square$

*Proposition 1.* Given that for the uniform distribution it holds that  $f(m) = 1$  and from Step 4 of Proposition 1 that  $\frac{1}{nf(m)} = t_R^{**} - t_L^{**}$  we can rewrite the latter as  $\frac{1}{n} = t_R^{**} - t_L^{**}$ . Moreover since for the uniform distribution  $m = 0.5$  and hence from step 4 we know that  $t_L^{**} = 1 - t_R^{**}$  we obtain that  $\frac{1}{n} = 2t_R^{**} - 1$  which implies that  $t_R^{**} = \frac{n+1}{2n}$  and therefore that  $t_L^{**} = \frac{n-1}{2n}$ .  $\square$

*Proposition 3.* We first check that the presented strategy profile is indeed a Nash equilibrium of the game. If we define  $t_R^\alpha = 1 - t_L^\alpha = \max\{\frac{1}{2}, \frac{n+1-\alpha(2n+1)}{2n(1-\alpha)}\}$  one can algebraically

verify that  $\pi_L(t_L^\alpha, t_R^\alpha) > \pi_L(t_L, t_R^\alpha)$  for every  $t_L \neq t_L^\alpha$ ;  $(t_L^\alpha, t_R^\alpha)$  is a Nash equilibrium. As far as uniqueness is concerned, we observe that  $\pi_L(t_L, t_R) + \pi_R(t_L, t_R) = 1$  for any  $(t_L, t_R) \in [0, 1]^2$ . Hence, our game is a constant-sum game. This implies that if another equilibrium  $(\hat{t}_L^\alpha, \hat{t}_R^\alpha)$  exists it should be the case that  $(\hat{t}_L^\alpha, \hat{t}_R^\alpha)$  is a Nash equilibrium too. But, as we argued,  $\pi_L(t_L^\alpha, t_R^\alpha) > \pi_L(t_L, t_R^\alpha)$  for every  $t_L \neq t_L^\alpha$ . Hence  $\pi_L(t_L^\alpha, t_R^\alpha) > \pi_L(\hat{t}_L^\alpha, \hat{t}_R^\alpha)$  and, thus,  $(\hat{t}_L^\alpha, \hat{t}_R^\alpha)$  cannot be an equilibrium.  $\square$

*Proposition 4.* Let us first describe some properties of an equilibrium and then proceed to the existence proof. Since we are interested in strategy profiles which induce a symmetric distribution of policy proposals about the center of the policy space, in equilibrium it must be the case that  $t_J^{***} < t_Q^{***} = 0.5 < t_Z^{***} = 1 - t_J^{***}$  for  $J, Q, Z \in \{L, C, R\}$  and  $\hat{t}(t_L^{***}, t_C^{***}, t_R^{***}, n) = \frac{1}{2}$ .<sup>30</sup> Notice that if  $Q = L$  ( $Q = R$ ) then party  $L$  ( $R$ ) has incentives to deviate away from 0.5 and bring the implemented policy closer to its ideal policy. Hence in equilibrium it must be that  $Q = C$ , that is, that  $t_C^{***} = 0.5$ . Moreover notice that if  $J = R$  and  $Z = L$  then party  $L$  ( $R$ ) has incentives to deviate (to offer the same platform as party  $R$  ( $L$ ) for example) and bring the implemented policy closer to its ideal policy. Hence, in equilibrium it should be the case that  $J = L$ ,  $Q = C$  and  $Z = R$ . That is, it should hold that  $t_L^{***} < t_C^{***} = 0.5 < t_R^{***} = 1 - t_L^{***}$ .

But could such a strategy profile be a Nash equilibrium of the game? Since in every such strategy profile we have that  $\hat{t}(t_L, t_C, t_R, n) = \frac{1}{2}$  then party  $C$  has no incentives to deviate. Moreover, observe that the utility of party  $L$  ( $R$ ) is strictly decreasing (increasing) in  $\hat{t}(t_L, t_C, t_R, n)$ . One can algebraically verify that a)  $\hat{t}(\frac{n-1}{2(n+1)}, 0.5, t_R, n) < \frac{1}{2}$  for every  $t_R \neq \frac{n+3}{2(n+1)}$ , b) and  $\hat{t}(t_L, 0.5, \frac{n+3}{2(n+1)}, n) > \frac{1}{2}$  for every  $t_L \neq \frac{n-1}{2(n+1)}$  and that c)  $\hat{t}(\frac{n-1}{2(n+1)}, 0.5, \frac{n+3}{2(n+1)}) = \frac{1}{2}$ . Hence, there exists a unique equilibrium and it is such that  $(t_L^{***}, t_C^{***}, t_R^{***}) = (\frac{n-1}{2(n+1)}, 0.5, \frac{n+3}{2(n+1)})$ .  $\square$

*Corollary 1.* From proposition 1 we have that  $(t_L^{**}, t_R^{**}) = (\frac{n-1}{2n}, \frac{n+1}{2n})$ . From proposition 4 we have that  $(t_L^{***}, t_R^{***}) = (\frac{n-1}{2(n+1)}, \frac{n+3}{2(n+1)})$ . It is clear that  $t_L^{***} < t_L^{**}$ ,  $t_R^{***} > t_R^{**}$  and

<sup>30</sup>The case  $t_L^{***} = t_C^{***} = t_R^{***} = 0.5$  is trivially ruled out as party  $L$  ( $R$ ) has clear incentives to deviate and bring the implemented policy nearer to its ideal policy.



hence  $t_R^{***} - t_L^{***} > t_R^{**} - t_L^{**}$  for any  $n > 1$  while for  $n = 1$  it holds that  $t_L^{***} = t_L^{**} = 0$ ,  $t_R^{***} = t_R^{**} = 1$ , and hence  $t_R^{***} - t_L^{***} = t_R^{**} - t_L^{**}$ .  $\square$

*Proposition 5.* Given the available actions for each party in the first stage there are eight policy-selection subgames. To fully characterize an equilibrium for the whole game we first need to see what happens in each of these subgames.

**Case 1 (No party enters)** In this case we have assumed that a status quo policy  $q \in [0, 1]$  is implemented (known to everybody).

**Case 2 (A single party enters)** If only party  $j$  entered then it selects  $t_j = \tau_j$  as it is the unique choice that maximizes  $\pi_j(\hat{t})$  (given that  $\hat{t} = t_j$  in this case). So if only  $L$  entered the race we have  $\hat{t} = t_L = \tau_L = 0$ , if only  $C$  entered the race we have  $\hat{t} = t_C = \tau_C = \frac{1}{2}$  and if only  $R$  entered the race we have  $\hat{t} = t_R = \tau_R = 1$ .

**Case 3 (Two parties enter)** If only party  $j$  and party  $k$  such that  $\tau_j < \tau_k$  entered then they select  $(t_j, t_k) = (\frac{n-1}{2n}, \frac{n+1}{2n})$  and  $\hat{t} = \frac{1}{2}$ . This is trivial for the case  $(j, k) = (L, R)$  (from Proposition 1) but it directly extends to the cases  $(j, k) = (L, C)$  and  $(j, k) = (C, R)$  as well. To see why it does, consider for example that only parties  $L$  and  $C$  entered the race. From Proposition 1 and 2 we know that if parties  $L$  and  $R$  compete there exists a unique equilibrium and in this unique equilibrium  $\hat{t} = \frac{1}{2}$ . Since in that case each party wishes to drag the implemented policy as near as possible to a different extreme policy it follows that, for party  $L$ , the strategy  $\frac{n-1}{2n}$  guarantees that  $\hat{t} \leq \frac{1}{2}$  independently of what  $R$  chooses and, for party  $R$ , the strategy  $\frac{n+1}{2n}$  guarantees that  $\hat{t} \geq \frac{1}{2}$  independently of what  $L$  chooses. Moreover, uniqueness implies that when  $t_L = \frac{n-1}{2n}$  ( $t_R = \frac{n+1}{2n}$ ) we have that  $\hat{t} < \frac{1}{2}$  ( $\hat{t} > \frac{1}{2}$ ) for any  $t_R \neq \frac{n+1}{2n}$  ( $t_L \neq \frac{n-1}{2n}$ ). Returning in our extended form game and in the case in which only parties  $L$  and  $C$  entered the race we notice with the help of the above that a)  $(t_L, t_C) = (\frac{n-1}{2n}, \frac{n+1}{2n})$  with  $\hat{t} = \frac{1}{2}$  must be an equilibrium because if  $L$  deviates to another policy we will get  $\hat{t} > \frac{1}{2}$  (party  $L$  will be strictly worse off) and if party  $C$  deviates to any other policy we will get  $\hat{t} < \frac{1}{2}$  (party  $C$  will be strictly worse off), and b) this equilibrium is unique. This is so because for any  $t_L$  the best response of  $C$  is  $t_C = 1 - t_L$  (so as to induce  $\hat{t} = \frac{1}{2}$ ) while for any  $t_C \neq \frac{n+1}{2n}$  party  $L$ 's best response

induces  $\hat{t} < \frac{1}{2}$ . All these obviously hold (in the reverse way) for the  $(j, k) = (C, R)$  case as well.

**Case 4 (All three parties enter)** If all three parties enter then we are in the case of Proposition 4 and  $(t_L, t_C, t_R) = (\frac{n-1}{2(n+1)}, 0.5, \frac{n+3}{2(n+1)})$  and  $\hat{t} = \frac{1}{2}$ .

So each policy selection subgame has essentially a unique equilibrium. This makes identification of a subgame perfect equilibrium (SPE) tractable.

First, we argue that in a SPE at least two parties should enter. If no party is expected to enter then the implemented policy will be  $q \in [0, 1]$ . In that case a party  $j$  with  $\tau_j \neq q$  is strictly better off by entering and, thus, implementing her ideal policy. If only one extreme party is expected to enter and implement its ideal policy then the other extreme party, for example, is better off by entering too and, thus, moving the implemented policy to  $\frac{1}{2}$ . If only the centrist party is expected to enter and implement its ideal policy ( $\frac{1}{2}$ ) then an extreme party has incentives to enter. By entering the extreme party will not affect the implemented policy (we argued in case 3 that if the centrist runs against an extremist then the implemented policy will be  $\frac{1}{2}$ ) but will enjoy an increase in the second element of its lexicographic preferences,  $\psi_j$ . This is so because if the extremist  $j$  does not enter and only the centrist party runs, then  $\psi_j = 0$  (because  $S_j = 0$  and  $c = 0$ ) but if it runs against the centrist it will enjoy  $\psi_j = 0$  (because  $S_j = \frac{1}{2}$  and  $c < \frac{1}{2}$ ).

Second, we will argue that if a SPE in which all three parties enter exists then generically no SPE exists in which only two parties enter. This is straightforward for the following reason. If a SPE in which all three parties enter exists then this implies that if a party expects that only the two other parties will run it also strictly prefers to run (apart from cases of measure zero in which for some party/ies  $S_j = \hat{c}$  in the equilibrium of the three party subgame); entering does not affect implemented policy but increases the value of  $\psi_j$ .

Third, we notice that in the equilibrium of the three party subgame  $\frac{\partial S_C}{\partial n} < 0$ . This implies that in the equilibrium of the three party subgame  $\frac{\partial S_L}{\partial n} = \frac{\partial S_R}{\partial n} > 0$ .

By a careful composition of the above we arrive to result. When  $n = 1$  we know

that in the equilibrium of the three party subgame  $S_L = S_R = \frac{1}{4}$  and  $S_C = \frac{1}{2}$ . Since  $\hat{c} < \frac{1}{4}$  it follows that for  $n = 1$  all three parties entering and selecting  $(t_L^{***}, t_C^{***}, t_R^{***}) = (\frac{n-1}{2(n+1)}, 0.5, \frac{n+3}{2(n+1)})$  is the a SPE of the game. This is so because if a party  $j$  deviated to not entering it would not affect the implemented policy but it would get a strictly lower  $\psi_j$ . Moreover, by our first and second argument it becomes evident that this SPE is unique. By increasing  $n$  our third observation dictates that the SPE in which all three parties enter is still the unique one as long as  $S_C > \hat{c}$ . Since  $\frac{\partial S_C}{\partial n} < 0$  and for  $n \rightarrow \infty$  we have that  $S_C \rightarrow 0$  there should exist some  $\hat{n} > 0$  such that for every  $n < \hat{n}$  we have  $S_C > \hat{c}$  and for every  $n > \hat{n}$  we have  $S_C < \hat{c}$ . For  $n < \hat{n}$  as stated above we have a unique SPE and in this equilibrium all three parties enter. For  $n > \hat{n}$  the unique SPE is such that only parties  $L$  and  $R$  enter and their platform choices are  $(t_L^{**}, t_R^{**}) = (\frac{n-1}{2n}, \frac{n+1}{2n})$ . Existence is established by the following argument. If only parties  $L$  and  $R$  are expected to run then  $C$  is strictly better off by not running. Its decision to enter does not affect the implemented policy but if it does not run it gets  $\psi_C = 0$  while if it runs it gets  $\psi_C < 0$ . Party  $L$  ( $R$ ) is strictly better off by running because if it does not the other extremist party will run alone and will thus implement its ideal policy, while if it runs the implemented policy will be  $\frac{1}{2}$ . Uniqueness is guaranteed by the facts that a) an equilibrium with three parties entering is not feasible (as party  $C$  is strictly better off by staying out when the other two are expected to run) and b) if another SPE exists it should be such that only parties  $L$  and  $C$  ( $R$  and  $C$ ) run. If party  $R$  expects that parties  $L$  and  $C$  will run then it is strictly better off by running. Its entry decision will not affect the implemented policy but it will increase its  $\psi_R$  from zero (in the case of no entry) to something strictly positive (in the case of entry). This is so because in the equilibrium of the three party subgame a)  $\psi_R > 0$  for  $n = 1$  and b)  $\frac{\partial \psi_R}{\partial n} = \frac{\partial S_R}{\partial n} > 0$ .  $\square$

## 7.2 Summary Statistics and Result Tables

Table A.1: Summary of empirical findings on Party-system Polarization (adapted from Curini & Hino, 2012)

Paper	Type of data	Sample size	Time-period	Measure of Polarization	Hypotheses Tested	Conclusion
Dow, 2001	National mass surveys	4 countries	1988-1994	Two-dimensional space using parties' and voters' positions	<i>Electoral System</i>	Not confirmed
Budge & McDonald, 2006	Comparative Manifesto Project (CMP)	17 countries	1945-1998	Unidimensional space using the distance between the two most extreme parties	<i>Electoral System</i>	Not confirmed
Dalton, 2008	Comparative Studies of Electoral Systems (CSES) mass surveys	35 countries (most with 1 observation)	1996-2006	Unidimensional space using <i>all parties'</i> positions weighted by vote-shares	<i>Electoral System &amp; Party-system size</i>	Both not confirmed
Ezrow, 2008	Expert-surveys and CMP for parties' positions/Eurobarometer for voters' positions	18 countries	1980-1990	Unidimensional space using <i>all parties'</i> positions weighted by vote-shares and voter distribution	<i>Electoral System &amp; Party-system size</i>	Both not confirmed
Andrews & Money, 2009	Comparative Manifesto Project (CMP)	20 countries	1945-1998	2-dimensional space using the distance between the two most extreme parties	<i>Electoral System &amp; Party-system size</i>	First not confirmed Second confirmed
Dow, 2011	CSES surveys	30 countries (most with 1 observation)	1996-2006	Same as in Ezrow 2008	<i>Electoral System &amp; Party-system size</i>	First confirmed Second not confirmed
Calvo & Hellwig, 2011	CSES-2 surveys	13 countries	2001-2006	Conditional Logit estimates based on Adams, Merrill & Grofman (2005) predictions	<i>Electoral System</i>	Hypothesis Conditionally confirmed
Curini and Hino, 2012	CSES-3surveys	36 countries (some with more than 1 observation)	1996-2006	Same as in Dalton 2008	<i>Electoral System Party-system size Coalition Habits &amp; Voters' Distribution</i>	First and second not confirmed. Third and fourth confirmed
<b>This paper</b>	<b>CMP, Carey-Hix Data Archive</b>	<b>Balanced panel of 23 OECD states</b>	<b>1960-2007</b>	<b>Same as in Dalton 2008; repeat estimates using Budge &amp; McDonald 2006</b>	<i>Electoral System Party-system size</i>	<b>First confirmed. Second conditionally confirmed</b>

Table A2: Summary Statistics of Key Explanatory Variables

Variable	Obs.	Mean	Std. Dev.	Min	Max
Electoral Rule Dummy (PR =1)	307	0.80	0.40	0	1
Type of Political Regime (Presidential = 1)	264	0.29	0.45	0	1
Coalition Habbits Dummy (Coal. Govt.= 1)	302	0.55	0.50	0	1
Polarization (Dalton Index 0-10)	307	1.70	0.79	0.23	5.14
Polarization (Distance between extremes)	307	2.43	1.10	0.34	6.55
Electoral District Magnitude (Average)	268	16.9	34.4	1	150
(Log of) Avg. Electoral District Magnitude	268	1.71	1.44	0	5.01
Years of consolidated Democracy	267	49.9	20.7	1	87
Effective Number of Electoral Parties	307	4.1	1.5	2.0	10.3
Actual Number of Parties	307	5.6	2.2	1	12
No of Parties participating in Gov't/Cabinet	264	2.0	1.3	1	6
Degree of Institutional Constraints (0-6)	307	2.2	1.5	0	5
GDP growth rate (in %)	307	3.5	2.6	-7.3	14.6
Government spending (% of GDP)	155	20.2	5.3	10.3	34.9
Unemployment rate (in %)	307	5.3	3.9	0.0	22.8
Gini coefficient of inequality	264	33.4	6.2	20.5	61.0

Note: Data reported at electoral not calendar year. Source: Comparative Political Studies Data Set 1, Carey-Hix Data Archive and the Manifesto Project Database. Period of observation: 1960 – 2007 for 23 OECD States.

Table A.3: The Impact of Electoral Rule Disproportionality on Platform Polarization in OECD Democracies (from 1960-2007)

Dependent Variable		Platform Distance between the Two Most Extreme Parties							
		Model 4.a			Model 4.b			Model 5.a	Model 5.b
Explanatory Variables		PR rule		Electoral District Magnitude			PR rule	District Magnitude	
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
H1	Electoral Rule Dummy (PR= 1)	2.291 (0.243)***	2.182 (0.252)***	2.238 (0.243)***	--	--	--	2.124 (0.339)***	--
H1	Log Avg. Electoral District Magnitude	--	--	--	0.401 (0.093)***	0.371 (0.094)***	0.401 (0.091)***	--	0.344 (0.112)***
H2	Effective Number of Parties (ENP)	0.112 (0.113)	--	--	0.169 (0.129)	--	--	0.118 (0.152)	0.134 (0.154)
H2	Log ENP	--	0.790 (0.464)*	--	--	1.104 (0.526)**	--	--	--
H2	Actual Number of Parties	--	--	0.062 (0.052)	--	--	0.066 (0.058)	--	--
	Country Dummies?	✓	✓	✓	✓	✓	✓	✓	✓
	Year Dummies?	✓	✓	✓	✓	✓	✓	✓	✓
	Other Institutional Controls?	No	No	No	No	No	No	Yes	Yes
	Economic Controls?	No	No	No	No	No	No	No	No
	R <sup>2</sup>	0.55	0.56	0.55	0.61	0.61	0.60	0.64	0.63
	N	307	307	307	255	255	255	237	237

\*  $p < 0.10$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$

Robust standard errors clustered at the country level reported in parentheses. Country and year dummies (fixed effects) are included in all specifications. In Model 5, other institutional controls include: a (dummy) variable indicating strong coalition habits and its interaction with ENP, the number of parties participating in government/cabinet, the type of political regime (presidentialism/parliamentarism), the degree of institutional constraints (a categorical variable taking values from 0 - 6), years of consolidated democracy, a (dummy) variable indicating government change and the ideological distance between incumbent and past government. Model 4.b has some missing values for the average electoral district size. In Model 4, the dependent variable (polarization) is measured as the distance between the two most extreme parties (as in Budge and McDonald 2006 and Andrews and Money 2009). In Models 4.a and 4.b we employ alternative measures for the size of the party-system in order to control for H2. In Models 5.a and 5.b we only use the *Effective Number of Parties* (ENP) for consistency with Table 1.

Table A.4: The Impact of Electoral Rule Disproportionality on Platform Polarization in OECD Countries (1960-2007): Random vs. Fixed Effects

Dependent Variable		Dalton Index of Party-System (Platform) Polarization							
		Curini & Hino (2012)		Models 1.a & 2.a			Model 6.a		
Explanatory Variables		Baseline Model		Fixed Effects			Random Effects		
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
H1	Electoral Rule Dummy (PR = 1)	0.377 (0.148)**	0.375 (0.183)**	1.423 (0.066)***	1.659 (0.181)***	1.326 (0.268)***	0.590 (0.230)***	0.471 (0.168)***	0.561 (0.196)***
H2	Effective Number of Parties (ENP)	0.020 (0.069)	0.084 (0.049)*	-0.038 (0.056)	0.009 (0.064)	-0.012 (0.151)	-0.010 (0.052)	0.028 (0.073)	0.142 (0.076)*
	Country Dummies?	X	X	✓	✓	✓	X	X	X
	Year Dummies?	X	X	X	✓	✓	X	✓	✓
	Random Effects?	X	X	X	X	X	✓	✓	✓
	Other Institutional Controls?	No	Yes	No	No	Yes	No	No	Yes
	R <sup>2</sup>	0.04	0.15	0.28	0.39	0.42	0.08	0.27	0.68
	N	307	237	307	307	237	307	307	237

\*  $p < 0.10$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$

Robust standard errors clustered at the country level reported in parentheses. The first two columns replicate the estimates using the Curini and Hino (2012) econometric specification on our data set. In columns 3 to 5 (Models 1.a and 2.a) country dummies (country fixed effects) are included in all specifications. In addition, in columns 4 and 5 we also include year dummies (year fixed effects). In columns 6 to 8 (Model 6.a) we estimate a random effects model (in columns 7 and 8 we also include year dummies). Other institutional controls include: a (dummy) variable indicating strong coalition habits and its interaction with ENP, the number of parties participating in government/cabinet, the type of political regime (presidentialism/parliamentarism), the degree of institutional constraints (a categorical variable taking values from 0 - 6), years of consolidated democracy, a (dummy) variable indicating government change and the ideological distance between incumbent and past government. Columns 2, 5 and 8 have less observations due to missing data for some institutional variables for the period from 1960 to 1980. In all models the dependent variable (platform polarization) is constructed as in Dalton (2008).

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