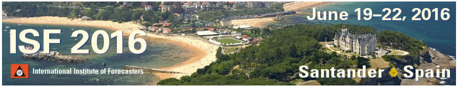


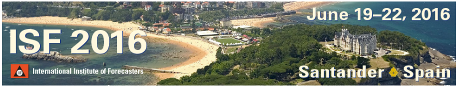
Optimal combination of volatility forecasts to enhance solar irradiation prediction intervals estimation

Juan R. Trapero [1] Alberto Martin [2]

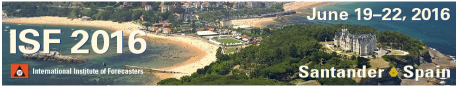
[1]Universidad de Castilla-La Mancha

[2]Agencia Estatal de Meteorología, AEMET

June 2016

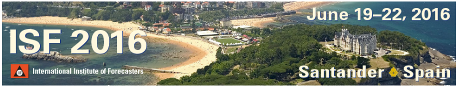

ISF 2016
June 19–22, 2016
International Institute of Forecasters
Santander Spain

Outline



Why is it important?

- Short-term forecasts are required to optimize operational planning of solar-power plants



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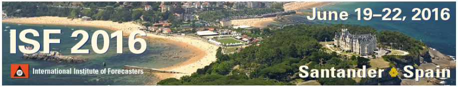
Reduction of 47.6 % of penalty costs by improving the forecasting technique in a Concentrating solar power plant (Kraas et al., 2013)



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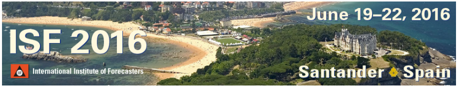

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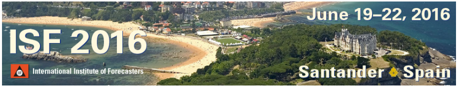
- Although stakeholders also require a measure of uncertainty (prediction interval), most of the works are focused on point forecasting
- In addition, forecast error is assumed to be normally i.i.d.

Research objectives

- Extend solar energy forecasting research beyond point forecasting.



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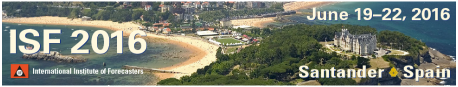

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- Explore non-parametric approaches as Kernel density estimators if forecast errors are not normal.
- Explore time-varying parametric volatility estimators (SES and GARCH) if forecast errors are not independent.
- Explore combination methods if any of the assumptions is hold.

Idea

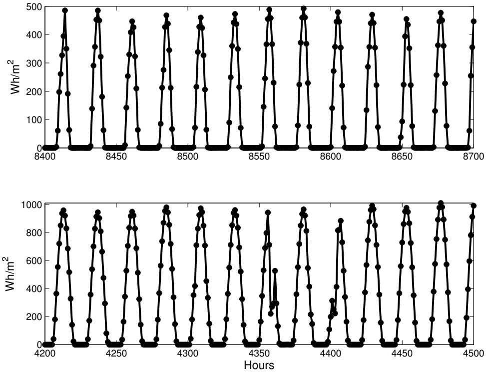
"Optimal" combination based on maximizing conditional coverage Christoffersen test p-value.

Case study

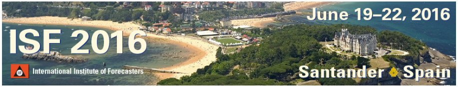
- We are going to focus on one-step-ahead uncertainty forecasts obtained from Global Horizontal Irradiation (GHI)
- This study can be extended to analyse the Direct Normal Irradiation (DNI)
- Spanish Institute of Concentrated-Photovoltaic Systems (ISFOC),

- 1,1 MW of Concentrated-Photovoltaic Energy (CPV)
- Hourly series: (01/2011)-(12/2011)

Case study

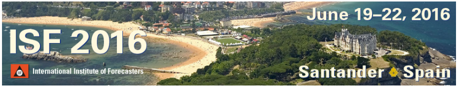


Example of hourly solar irradiation for GHI in winter (upper plot) and in summer (lower plot).



Literature review

- Prediction intervals.
  - Theoretical: $[L_{t+h}, U_{t+h}] = \left[F_{t+h} \pm z_{(1-p)/2} \cdot \sigma_{t+h}\right]$
    - L_{t+h}: lower interval
    - U_{t+h}: upper interval
    - F_{t+h}: point forecast
    - h: forecasting horizon
    - $z_{(1-p)/2}$: standard normal distribution table for a certain confidence level $100p\,\%$
    - σ_{t+h}:is the theoretical standard deviation determined by the h and the model forecasting parameters



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Forecasting hazard!

If the model is misspecified, the theoretical prediction intervals tend to be too narrow

Literature review

- Prediction intervals: If there are doubts about the validity of the "true" model, go **empirical**
  - Parametric (Normal): $[L_{t+h}, U_{t+h}] = \left[F_{t+h} \pm z_{(1-p)/2} \cdot \sigma_{t+h}\right]$
    - $\sigma^2_{t+h} = \frac{\sum_{i=t-n}^{t-1} (\epsilon_{i+h})^2}{n}$
    - ϵ_{i+h}: forecasting error at $i+h$
  - Non-parametric: $[L_{t+h}, U_{t+h}] = [F_{t+h} + Q_h((1-p)/2), F_{t+h} + Q_h((1+p)/2)]$
    - $Q_h(p)$: $100p\,\%$ forecast error quantile

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Bonus track :)

Deterministic (Numerical Weather Prediction) models are widely used. As they do not rely on a statistical model, the are well-suited for empirical approaches.

Benchmarks

- Point Forecast (F_{t+1}): $ARIMA(1,0,0) \times (1,1,0)_{24}$ (Reikard, 2009)

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  - **Parametric-Normal**: $Q_1(p) = z_{(1-p)/2} \cdot \sigma_{t+1}$
    - Single Exponential Smoothing: $\sigma^2_{t+1} = a\epsilon^2_t + (1-a)\sigma^2_t$
    - GARCH(1,1): $\sigma^2_{t+1} = \omega + a_1\epsilon^2_t + \beta_1\sigma^2_t$

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    - GARCH(1,1): $\sigma^2_{t+1} = \omega + a_1\epsilon^2_t + \beta_1\sigma^2_t$
  - **Non-parametric**: Kernel Density estimator

    $f(x) = \frac{1}{Nh}\sum_{j=1}^{N} K\left(\frac{x-X_j}{h}\right)$
    - $f(x)$ is probability density function of the forecast errors
    - N is the sample size
    - $K(\cdot)$ is the kernel smoothing function
    - h is the bandwidth.

Experimental setup

- Data (8,760 observations) have been split down in two parts.
- First part (4,392 observations)
  - to estimate ARIMA-GARCH model (in-sample data)
    - With nights (seasonality=24 hours)
    - Maximum likelihood based on a *t*-distribution with the Econometrics toolbox from MATLAB
  - to estimate SES parameter
    - Using ARIMA forecasts
    - Without nights
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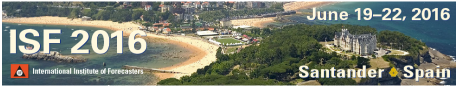

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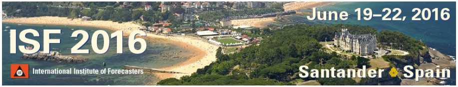
- Second part (4,368 observations) as out-of-sample data. Rolling origin experiment.
- SES slightly outperforms GARCH (RMSE of the variance)



Prediction intervals performance

- Comparison prediction intervals performance
  - **prediction interval coverage**: proportion of times that a forecast is included within the prediction interval (hit rate)
  - **average interval width**: interval range divided by its midpoint
  - **Conditional coverage test**: (Christoffersen, 1998) as a combination of tests for unconditional coverage and independence

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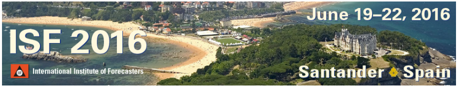
In summary

An ideal method should provide a *close prediction interval coverage* with regards to the desired confidence level, a *low average interval width* and *pass the conditional coverage test*.



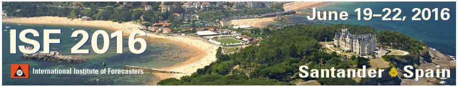
Results

Method	$p(\%)$	Hit Rate (%)	LR_u	LR_i	LR_c	Av. width
KERNEL	**80**	84.13	0.06	0.00	0.00	1.33
	85	89.21	0.03	0.00	0.00	1.46
	90	94.60	0.00	0.00	0.00	1.60
	95	97.14	0.06	0.24	0.09	1.74
SES	**80**	73.02	0.00	0.16	0.00	0.86
	85	78.10	0.00	0.55	0.00	0.93
	90	83.81	0.00	0.77	0.00	1.00
	95	87.94	0.00	0.75	0.00	1.09

- Likelihood ratio tests p-values for unconditional coverage (LR_u), independence (LR_i) and conditional coverage (LR_c)
- P-values lower than 0.05 mean that the null hypothesis of unconditional coverage, independence and conditional coverage, respectively, are rejected at the 5 % significance level.

Results

- KERNEL provides a higher hit rate, although it does not pass the independence test.
- SES passes the independence test, although it does not pass the unconditional coverage test.
- In conclusion, none of them pass the conditional coverage test.



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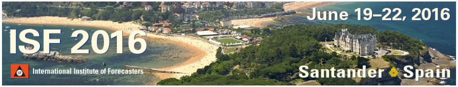
- Why don't we combine them?
- Combined prediction interval $[\hat{L}_c, \hat{U}_c]$:

$$\hat{L}_c = w \cdot \hat{L}_{Kernel} + (1 - w) \cdot \hat{L}_{SES}$$
$$\hat{U}_c = w \cdot \hat{U}_{Kernel} + (1 - w) \cdot \hat{U}_{SES}$$

where $0 < w < 1$ is a constant that maximizes the LR_c



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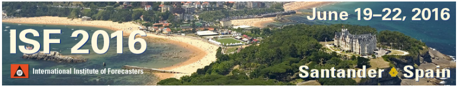
COMBINED	**80**	78.41	0.47	0.09	0.18	0.92
	85	83.49	0.44	0.09	0.17	0.99
	90	89.84	0.91	0.66	0.90	1.15
	95	95.24	0.86	0.74	0.93	1.32

Conclusions

- Kernel (non-parametric) provide a high prediction interval coverage but it also yields a high average interval width and lack of independence.
- SES (parametric) pass the independence test but they offer a prediction interval coverage excessively low.
- This work proposes a novel approach that combines a non-parametric and a parametric approach.
- The combination weight is obtained by maximizing the Christoffersen conditional coverage test p-value.
- The results show a good compromise between coverage and average interval width.



Thank you for your attention!

e-mail: juanramon.trapero@uclm.es
blog: http://blog.uclm.es/juanramontrapero/

This work has been supported by the Spanish Ministerio de Economía y Competitividad, under Research Grant no. DPI2015-64133-R (MINECO/FEDER/UE)