On the performance of temporal demand aggregation when optimal forecasting is used

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Motivation & business context

- High demand variability
  - Forecast accuracy
  - Simple vs. complex forecasting methods

- Presence of zero observations
  - Challenges in forecasting and stock control
  - Classical approaches are not appropriate
Some solutions

- Information sharing
  - Reduces variability (bullwhip effect)
- Improve forecasting methods
- Demand aggregation
  - Cross-sectional (hierarchical) demand aggregation
  - Temporal demand aggregation
    - Overlapping aggregation
    - Non-overlapping aggregation
Non-overlapping temporal aggregation

Weekly

Daily
Overlapping temporal aggregation

Weekly

Daily

Period
Research background

Temporal aggregation

- Empirical investigations: Nikolopoulos et al. (JORS, 2011), Babai et al. (OMEGA, 2012)
  - Lack of theoretical investigation and focus solely on intermittent demand series

- Theoretical analysis: Amemiya and Wu (1972), Tiao (1972)
  - Focus only on time series characterization and lack of studies on forecast accuracy or inventory implications

- Theoretical analysis: Rostami-Tabar et al. (NRL, 2013, 2014)
  - Under which conditions the demand aggregation (non-overlapping) approach outperforms the non-aggregation one?
  - Analytical results under ARMA-type demand processes and the Single Exponential Smoothing (SES) forecasting method.
  - No results under optimal forecasting methods!!
Findings from Rostami-Tabar et al. (NRL, 2013): Comparison at the disaggregate level

- No accuracy improvement by aggregating the smooth (high positive autocorrelation) series
- Accuracy improvement by increasing aggregation level \( m \)
- Accuracy improvement by setting a low smoothing \( \beta \) values
Findings from Rostami-Tabar et al. (NRL, 2014): Comparison at the aggregate level

\[ \alpha = 0.1 \quad m=2 \quad m=12 \]

- Aggregation approach is always preferred with longer forecast horizons
- Accuracy improvement by increasing aggregation level \( m \)
- Accuracy improvement by setting a low smoothing \( \beta \) values
What to forecast?

- Forecast horizon, Aggregate forecast

Non-Aggregation approach

Aggregation approach
System and assumptions

- Non-overlapping temporal aggregation
- The disaggregate demand follows a stationary process:
  - AutoRegressive of order one, AR(1)
- The forecasting method is the optimal Minimum Mean Square Error (MMSE) forecasting method
- The forecast accuracy is measured through the Mean Square Error (MSE)

\[
MSE = \text{Var}(\text{Forecast Error})
\]
Performance evaluation: MSE derivation (at the aggregate level)

**MSE before aggregation:**

\[
MSE_{BA} = \text{Var}(D_{T,1} - f_{t,m})
\]

\[
D_{T,1} = \sum_{l=1}^{m} d_{t+(l-1)}
\]

\[
f_{t,m} = E(D_{T,1} | d_{t-1}, d_{t-2}, d_{t-3}, \ldots)
\]

**MSE after aggregation:**

\[
MSE_{AA} = \text{Var}(D_{T,1} - F_{T,1})
\]

\[
F_{T,1} = E(D_{T,1} | D_{T-1}, D_{T-2}, \ldots)
\]

**Ratio**

\[
\text{Ratio} = \frac{MSE_{AA}}{MSE_{BA}}
\]
The MSE before aggregation is:

\[ MSE_{BA} = \frac{\sigma^2}{(1 - \phi)^2} \sum_{i=1}^{m} (1 - \phi^i)^2 \]

The MSE after aggregation is:

\[
MSE_{AA} = \begin{cases} 
\left(1 - \phi^2\right)^2 \left(1 - \phi^{2m}\right) \left(m + \sum_{k=1}^{m-1} 2(m-k)\phi^k\right) \sigma^2 \\
\left(1 - 2\phi^m \left(- (X + X\phi^{2m} - 2\phi^m) + \sqrt{\left(X + X\phi^{2m} - 2\phi^m\right)^2 - 4(1 - X\phi^m)^2} \right) \right) + \\
\left(1 - \phi^{2m}\right) \left(m + \sum_{k=1}^{m-1} 2(m-k)\phi^k\right) \sigma^2 \\
\left(1 - 2\phi^m \left(- (X + X\phi^{2m} - 2\phi^m) - \sqrt{\left(X + X\phi^{2m} - 2\phi^m\right)^2 - 4(1 - X\phi^m)^2} \right) \right) + \\
\left(1 - \phi^{2m}\right) \left(m + \sum_{k=1}^{m-1} 2(m-k)\phi^k\right) \sigma^2 \\
\end{cases}
\]

if \( \phi > 0 \)

\[
X = \frac{m + \sum_{k=1}^{m-1} 2(m-k)\phi^k}{\sum_{k=1}^{m} k\phi^k + \sum_{k=1}^{m-1} k\phi^{2m-k}}
\]
Numerical results and managerial implications

- For negative autocorrelation close to zero and low positive autocorrelation, almost the same performance of the two approaches.
- For high positive autocorrelation, the differences between two approaches becomes more significant and forecasting disaggregate data is more accurate.
- The aggregation level \( m \) does not play a significant role in terms of MSE forecast error.
Conclusion and future research

- Analytical derivation of the MSE expressions for the temporal aggregation approach under the optimal MMSE forecasting method;
- Numerical comparative results of the aggregation and non-aggregation approach;
- We show that by using the optimal forecasting method, regardless of the aggregation level and the process parameter, the aggregation approach is outperformed by the non-aggregation one;
- Analytical and numerical results for more general ARMA-type demand processes;
- Empirical performance of the aggregation approach under the optimal forecasting method
Thank you for your attention

Questions …