Spare parts inventory management: new evidence from distribution fitting

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IIF Workshop on Supply Chain Forecasting for Operations
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FORECASTING SPARE PARTS: WHICH DISTRIBUTION?

Intermittent demand

Inventory performance metrics:
- Achieved service level
- Average inventory on hand

- Which is the right distribution?
- Which is the right test to measure goodness-of-fit?
DISTRIBUTIONS IN THE LITERATURE

Compound distributions:
- Discrete time
  - $p_t \sim \text{Ge}(p)$
  - $z_t \sim (\ , \ ), \ \leftarrow$
- Continuous time
  - $p_t \sim (\ )$
  - $z_t \sim \text{Ge}(p), \text{Log}(p), \ \leftarrow$

Classic distributions:
- Normal, Gamma, Poisson, …

Figure: Syntetos et al. (2012)
KOLMOGOROV-SMIRNOV TEST

\[ F(x) \] – unknown real cumulative distribution of the data
\[ F_0(x) \] – supposed distribution
\[ F_n(x) \] – empirical distribution

\[ H_0 : F = F_0 \]

K-S statistic:
\[ D = \sup_x \left| F_n(x) - F_0(x) \right| \]

- Standard critical values are distribution-free for fully specified, continuous distribution
- Conservative test for discrete data

Massey (1951): The Kolmogorov-Smirnov Test for Goodness of Fit Journal of the American Statistical Association 46(253) 68-78
MODIFIED TESTS WITH FOCUS ON TAILS:  
2 – A MODIFIED ANDERSON-DARLING TEST

Classic A-D statistic:

\[ A = n \left[ F_n(x) - F(x) \right]^2 (F(x)) \, dx \]

with \( (u) = \frac{1}{u(1-u)} \)

The modified A-D test uses \( (u) = \frac{1}{1-u} \) weight only on the right tail!

- \( f \) – discrete with support \([1,k]\)
- \( p_j \) – probability of \( f(x) = j \)
- \( S_j \) – count of observations assuming value \( j \)
- \( T_j \) – expected number of observations assuming value \( j \)
- \( Z_j = S_j - T_j \)
- \( H_j = T_j / n \)

Modified A-D statistic:

\[ AU^2 = \frac{1}{n} \sum_{j=1}^{k} \frac{Z_j^2 p_j}{1 - H_j} \]

Sinclair et al. (1990): *Modified Anderson Darling test* Communications in Statistics-Theory and Methods 19(10) 3677-3686
MODIFIED TESTS WITH FOCUS ON TAILS:
2 – A MODIFIED K-S TEST

Classic K-S test

\[ x \rightarrow D(x) = \left| F_n(x) - F_0(x) \right| \]

Modified K-S test

\[ x \rightarrow A_i \rightarrow D(x) = \left| F_n(x) - F_0(x) \right| \]

\[ D = \sup_x |D(x)| \]
DATASETS

Dataset 1
- 4483 SKUs
- 3 years weekly data
- Mixed intermittency

Dataset 2 - RAF
- 5000 SKUs
- 7 years monthly data
- Very intermittent!

<table>
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<tr>
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<th>Min</th>
<th>Median</th>
<th>Max</th>
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### FITTING DEMAND PER PERIOD – DATASET 1

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<td>Str.</td>
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<td><strong>K-S modif.</strong></td>
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<td>99%</td>
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<td>20%</td>
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CLASSIFICATION SCHEME FIT

Figure: Syntetos et al. (2012)

Fit:
D1: 85%
D2: 68%

K-S modif.
## FITTING LEAD TIME DEMAND – DATASET 2

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<td>11%</td>
<td>13%</td>
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<tr>
<td>No</td>
<td>70%</td>
<td>92%</td>
<td>70%</td>
<td>52%</td>
<td>42%</td>
</tr>
<tr>
<td><strong>A-D modif.</strong></td>
<td></td>
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<td>37%</td>
<td>60%</td>
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<tr>
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<td>6%</td>
<td>10%</td>
<td>22%</td>
<td>14%</td>
<td>19%</td>
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<td>78%</td>
<td>84%</td>
<td>41%</td>
<td>26%</td>
<td>19%</td>
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</table>
• Inventory-up-to policy with backordering
• SBA method
• Initialization time: 4 years (48 obs.)

• 9 implemented policies:
  • Poisson
  • Normal
  • Gamma
  • NBD
  • StuttP
  • BestKSClassic
  • BestKSModified
  • BestADModified
  • ClassificationRule

• Classic version + Teunter and Duncan’ s correction:
  \[ \hat{L} = \hat{x} \times L \]
  \[ \hat{L} = \hat{z} + \hat{x}(L - 1) \]
  \[ \hat{L} = \hat{x} \times \sqrt{L} \]
Efficacy of Inventory Policies

Achieved Service Level

Target Service Level

Classic version

Teunter and Duncan’s correction

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Average inventory on hand vs. Achieved Service Level

- Poisson
- Normal
- Gamma
- NBD
- StuTP
- BestKSClassic
- BestKSMODified
- BestADModified
- ClassificationRule
Achieved Service Level

Average inventory on hand

- Poisson
- Normal
- Gamma
- NBD
- StuttP
- BestKSClassic
- BestKSModified
- BestADModified
- ClassificationRule
Achieved Service Level

Average inventory on hand

- Poisson
- Normal
- Gamma
- NBD
- StuttP
- BestKSClassic
- BestKSModified
- BestADMModified
- ClassificationRule
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Achieved Service Level

Average inventory on hand

Achieved Service Level
Achieved Service Level

Average inventory on hand

- Poisson
- Normal
- Gamma
- NBD
- StuttP
- BestKSClassic
- BestKSModified
- BestADMModified
- ClassificationRule
Achieved Service Level

Average inventory on hand

- Poisson
- Normal
- Gamma
- NBD
- StuttP
- BestKSClassic
- BestKSModified
- BestADMModified
- ClassificationRule
CONCLUSIONS AND FURTHER RESEARCH

• Distribution of demand per period mostly fits compound Poisson distributions.

• Lead time demand also mostly fits compound distributions, but results are worse using the modified tests – Right to use for inventory?

• BestKSModified policy is the most efficient EXCEPT for very high service levels (over 94%), where BestADModified is better.

• Need for classification rules for inventory applications.
THANK YOU FOR YOUR ATTENTION!